

The Janus cosmological model: an answer to the deep crisis of today's cosmology.

J.P.Petit¹ , G.d'Agostini² Nathalie Debergh³

Manaty Research Group

Keywords : Negative energy, negative mass, dark matter, dark energy, Janus cosmological model.

Abstract : After having evoked the deep crisis that cosmology and physics know, we examine the different attempts to introduce negative masses in the cosmological model. Only the Janus cosmological model presents both a mathematical and physical coherence, as well as numerous observational confirmations. Presenting itself as a geometrical extension of general relativity, it represents a possible solution to the crisis of cosmology.

1 - Introduction :

Physics and cosmology have been in perfect crisis since 1970. In 2006 the physicist Lee Smolin published a book entitled "The trouble with physics" [1] the same year Peter Woit published "Not even wrong! " [2]. Between 1900 and 1970 theoretical physics, experimental physics and observational techniques progressed hand in hand. Sometimes the experiment or the observation raised a question to which theorists then answered, possibly by introducing a profound paradigm shift in our way of conceiving space, time, matter, the universe. Sometimes the theoretical models were ahead of the observation, as it was the case with the prediction of the existence of antimatter by Dirac. Through an anecdote, we can evoke the fact that theoretical advances were sometimes received with irony. Thus Niels Bohr had said, after this announcement made by Dirac, that "this theory was good for capturing elephants. Dirac's theory is hung on a tree in Africa. The elephants read it and are so flabbergasted that they can be easily captured".

In the sixties, cosmology, where a large consensus had been reached, which translated into the idea that the cosmological constant should be either zero or negligible, was practically reduced to the measurement of the density of the universe, by comparing it to a critical density of 10^{-29} grams per cubic centimeter. Depending on the case, one chose one of the three models derived from Friedmann's solutions.

The theory of quarks had allowed the emergence of a standard model whose elegance and coherence were admired. Particle physics added new members to the bestiary with supersymmetry but, all of a sudden, Nature refused to confirm these predictions. The new superparticles did not appear. The crisis spread to cosmology and astrophysics. The dynamics of galaxies and the observed strong gravitational lensing effects could no

1 Jean-pierre.petit@manaty.net

2 gilles.dagostini@manaty.net

3 Nathalie.Debergh@manaty.net

longer work with visible matter. We had to conjecture the existence of another one, of unknown nature, to which we gave the word dark matter. But for the moment it is only a name, hiding our ignorance. To explain the extraordinary homogeneity of the primitive universe, it was considered that the universe could have undergone an extremely brutal expansion phase, under the effect of the field, immediately associated with a new particle: the inflaton. But today there are as many models of inflatons as there are researchers working on this subject.

As if this problem was not enough, the evidence of the acceleration of the cosmic expansion required that we consider a new form of energy, also black.

The years pass. All attempts to identify these components, as important by their effects as mysterious by their nature, were failures.

If it takes years before we can give an identity to all these unknown components, can we really speak of a standard cosmological model?

For forty years, string theory seemed to represent a new paradigm shift. Historically, it will undoubtedly be an interlude one day. After a short period of skepticism, the majority of theorists threw themselves into the adventure, which gave the opportunity to publish thousands of articles and to present hundreds of theses. But when one builds a model it is still desirable that it proposes explanations of phenomena, or predicts them. Half a century later, one conclusion is obvious. String theory is no more than a mathematical gesticulation⁴.

In their books Smolin and Woit called for the emergence of new ideas⁵. In what follows we propose to study the benefit and cost of introducing particles with negative masses and energies into the model.

2 – Attempts to introduce negative masses in cosmology.

The main article appeared in 1957 [5]. The author introduces then two types of masses, inertial and gravitational, which are identified in General Relativity⁶. Any attempt to introduce these negative masses in the General Relativity model is then confronted with the runaway phenomenon, which can be summarized in these two sentences:

- Positive masses attract all types of masses
- Negative masses repel all types of masses.

⁴ But the four representatives of theoretical physics at the Advanced Studies Institute of Princeton, to which Einstein and Kurt Gödel belonged, are string specialists. Because there is nobody else.

⁵ Smolin only saw a justification for launching a "quantum gravitation" where space and time themselves would be quantized. But for the moment, it doesn't work. Probably because the approach considered (Carlo Rovelli) is not the right one. Apart from that, Smolin refused any other approach, refusing to answer to our mailings, transmitted by the mathematician Alain Connes, Field medalist and preface of his work.

⁶ Their identity reflects the "principle of equivalence".

Under these conditions, if two masses of opposite signs are brought together, the negative mass runs away, pursued by the positive mass. In this phenomenon of mutual acceleration, energy is conserved since the kinetic energy of the negative mass is negative. A physicist can hardly imagine that such a behavior could be part of a new physics. This article provokes some comments ([6]-[12]).

Years later, this possibility is again considered [13], without any more success. Let us quote the author's conclusion :

- Indeed, what I am writing may be called science fantasy, and the busy reader is fully entitled to turn the page. My reason for continuing is to see whether the properties of the hypothetical universe suggest why the real universe contains only positive mass. My intention is summarized by Einstein's metaphorical phrase : « "What interests me is whether God had choice in the creation of the world" ».

For the sake of completeness, let us quote two other attempts, one in string theory [29] and the other in a more classical approach [30], but where the essential paradigmatic leap, the passage to a system of two coupled field equations, is not made, so that the authors run up against the runaway paradox, inevitable for any attempt to introduce negative masses into the Einstein equation.

Still trying to introduce these negative masses into General Relativity, i.e., into Einstein's field equation, we find the suggestion that the negative energy of these masses could mimic the cosmological constant [14]. But, under these conditions the density of the latter must remain constant during the cosmic evolution. The author then suggests a mechanism of continuous creation of these negative masses, not described, which does not clarify the problem.

There are then heuristic attempts⁷ [15] in which the following interaction laws are given, free of theoretical justifications.

- Masses of the same sign attract each other according to Newton's law
- Masses of opposite signs repel each other according to "anti-Newton".

Such a heuristic model is equivalent to restoring the principles of action-reaction and equivalence in this set of masses. By taking densities and temperatures equal in absolute value for the two entities, one obtains then a phenomenon of percolation (see Fig. 1).

⁷ Heuristic approach: we formulate a hypothesis on the interaction laws, without theoretical justifications, and we look at what results from it, through numerical simulations.

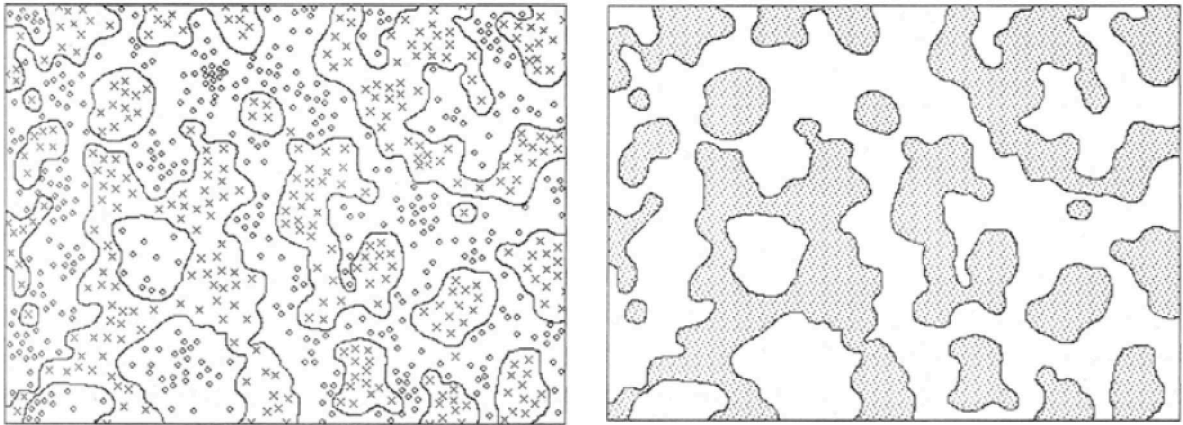


Fig.1 : Percolation, from figures 7 and 8 of [15].

It is seen that the two types of masses tend to separate, giving a pattern that does not seem to be able to be connected to any astronomical observation. This 1990 result⁸ was based on 250 positive and 250 negative mass points. Seeing this result we had the idea to give the negative points a mass of an order of magnitude greater than that of the positive points. The effect of this heuristic approach was immediate. The negative masses immediately formed a regular network of clusters, confining the positive mass in the residual space. (see Fig. 2 and Fig. 3).

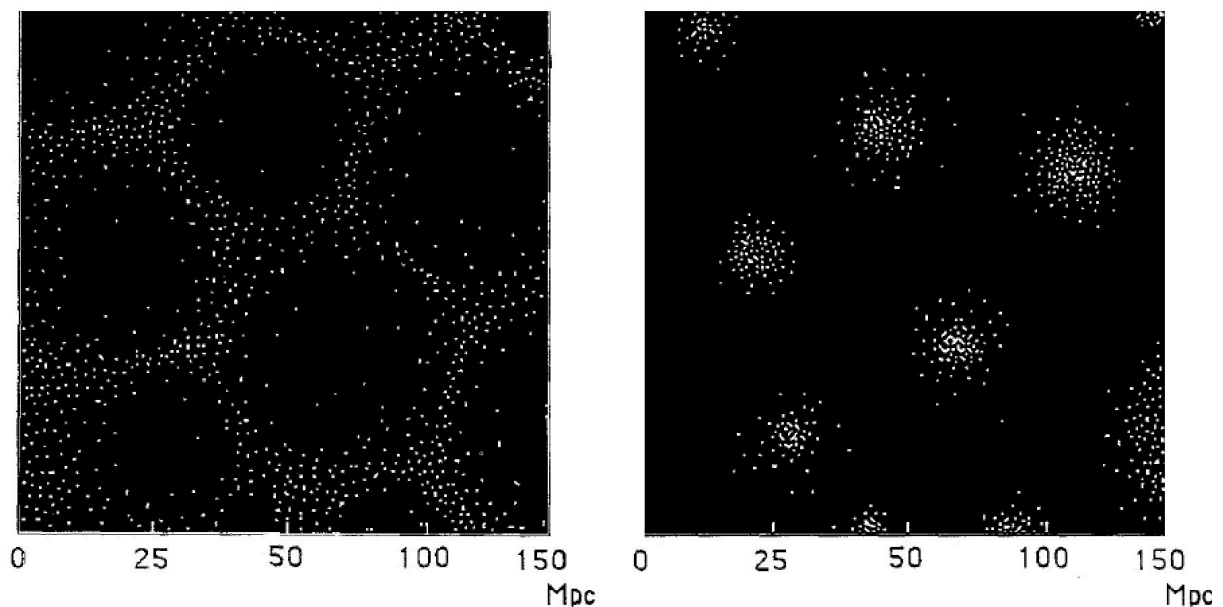


Fig.2 : Two-dimensional (2D) simulation [17]. On the right the regular distribution of negative mass clusters. On the left the positive mass lacunar distribution, confined in the residual space.

Ratio of negative mass density to positive mass density: 50.

⁸ Using the "Rolls" of the computers of the time, a Cray-one! Today any PC can do ten times better (but cannot handle 3D)

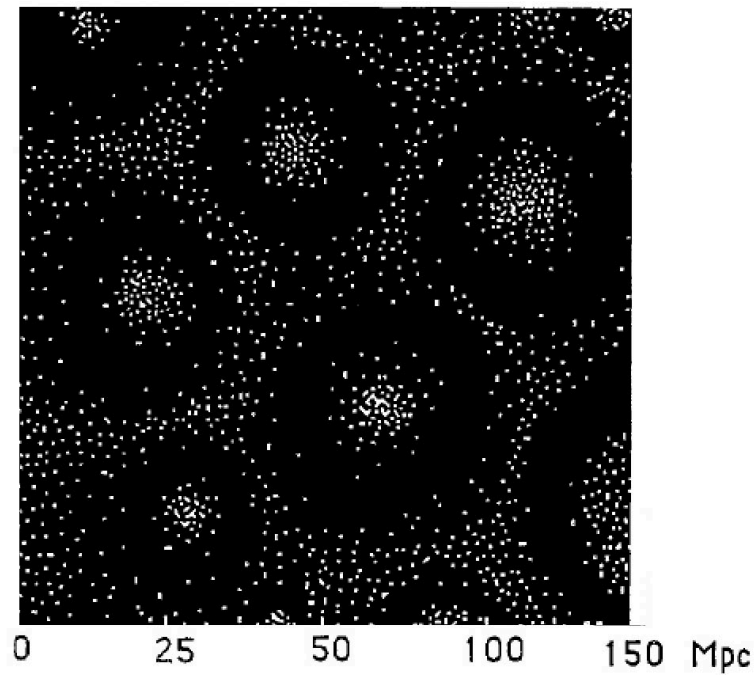


Fig.3 : The two images, superimposed

This suggested that the key to the large-scale structure of the universe could arise from a very dissymmetric situation. Moreover, the fact that positive masses were the first to give rise to clusters could be explained by the fact that the Jeans time associated with them was then lower :

$$(1) \quad t_J^{(-)} = \frac{1}{\sqrt{4\pi G |\rho^{(-)}|}} \ll t_J^{(+)} = \frac{1}{\sqrt{4\pi G \rho^{(+)}}}$$

In a three-dimensional (3D) extension (which our computational means at the time did not allow) this suggests a large-scale structure of the universe (VLS) in the form of joined soap bubbles (see Fig. 4).

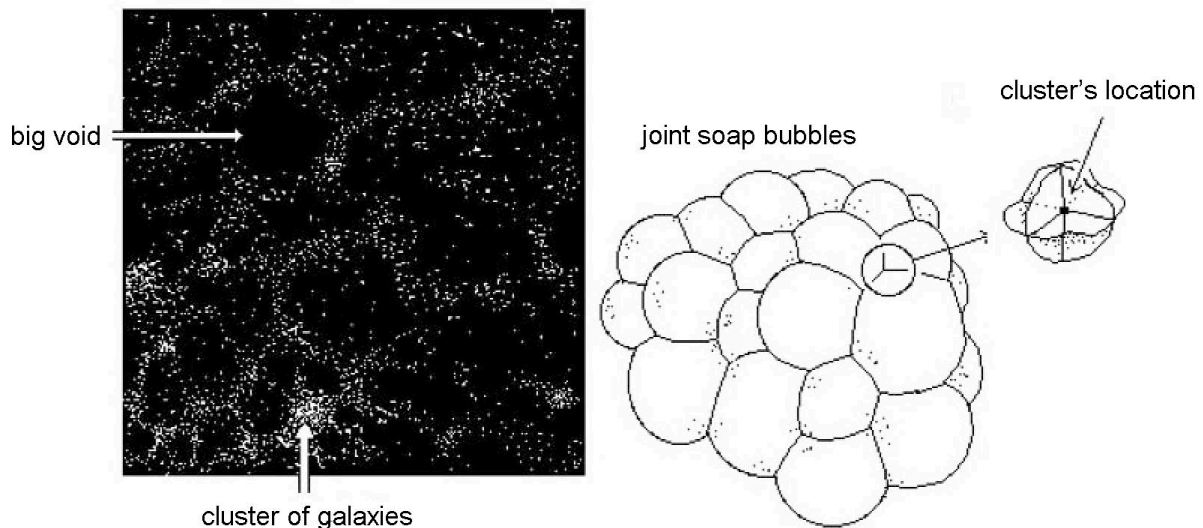


Fig.4 : The joint soap bubbles model

At the center of each cell, there is a cluster of negative mass. Three cells meet along a segment, likely, still by gravitational instability, to give rise to a filamentary structure. Four cells meet along a point in the vicinity of which we will then have a cluster of galaxies. At the moment of the constitution of this structure on a very large scale, the matter is brutally compressed according to plates, which allow a rapid radiative cooling, destabilizing, generating the birth of galaxies.

This idea is reinforced by the discovery, in 2017, of the Dipole Repeller phenomenon [16]. A cluster of negative mass, occupying the center of this great void, is then responsible for the effect of repulsion produced, on the neighboring galaxies.

Another observational aspect lends credence to this model of large-scale structure of the universe. We know that galaxies with high redshift⁹ ($z > 7$) have very low luminosities. They are then considered as dwarfs. This model provides a different interpretation: objects of negative mass, producing on photons of positive energy a negative gravitational lensing [17], [18]), it would be galaxies of a normal size whose luminosity would be attenuated by this phenomenon. Combining these two aspects, finer measurements of attenuation of this luminosity of the galaxies in the background would then provide an assessment of the size of the object.

3 – Towards a new model based on a new geometric paradigm.

It is clear that if we stay in the geometrical framework proposed by Einstein, where the trajectories of masses and photons are interpreted as metric solutions¹⁰ $g_{\mu\nu}$ of a single field equation, the runaway phenomenon is inevitable. The only way to escape this

⁹ With great speed of escape, therefore very distant and therefore very young.

¹⁰ Trajectoires calculées à l'aide de ces fonctions $g_{\mu\nu}$, solutions de l'équation de champ.

paradox, hardly acceptable to a physicist, is to consider that the trajectories of positive masses and photons of positive energy on the one hand, and of negative masses and photons of negative energy on the other hand, are deduced from two different metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$. This leads to a new geometrical model where the single M^4 manifold¹¹ is equipped with not one, but two metric fields¹². The first attempt in this direction presents this situation as resulting from the interaction between two "branes". In this model the elements of the two populations are supposed to interact through gravitons of non-zero mass ([19],[20]). This non-zero mass of the gravitons¹³ means that this model also belongs to the *massive gravity*, and is thus presented as a *massive bigravity*. An assumption which only complicates the model, by giving birth to the classical ghost instability¹⁴. Hereafter the action¹⁵ corresponding to this model:

(2)

$$S = \int d^4x \sqrt{-g_R} (M_R^2 R(g_R) - \Lambda_R) + \int d^4x \sqrt{-g_L} (M_L^2 R(g_L) - \Lambda_L) + \int d^4x \sqrt{-g_R} L(\Phi_R, g_R) \\ + \int d^4x \sqrt{-g_L} L(\Phi_L, g_L) - \mu^4 \int d^4x (g_R g_L)^{1/4} V(g_L, g_R)$$

Any point of the first branch, the "right" branch, marked by the letter R , is interacting with a conjugate point of the second branch, the "left" branch, marked by the letter L .

They introduce Lagrangian densities in the action: the Ricci terms $\sqrt{-g_R} R(g_R)$ and $\sqrt{-g_L} R(g_L)$, the terms corresponding to positive matter $\sqrt{-g_R} L(\phi_R, g_R)$ and negative matter ¹⁶ $\sqrt{-g_L} L(\phi_L, g_L)$. All those are based on the corresponding four-dimensional

¹¹ A "manifold" is simply a continuous space, consisting of points (on the basis of which one can construct first and second derivatives).

¹² The mathematical tool to calculate distances. In this model, between two points there are two different distances.

¹³ Free of any foundation.

¹⁴ "Ghost instability" which stems from the fact that gravitons would contribute to the gravity field, which makes the system self unstable (...).

¹⁵ This idea of "action" derives from the principle of least action of the Frenchman Pierre de Maupertuis (1698-1759). The trajectories are "optimal" paths along which the integral defined by the scalar S is minimal. This constraint leads to "a calculation of variation" (it is the metric, which then becomes the unknown).

¹⁶ The way of approaching the problem, which was defined by David Hilbert to support the empiricist approach of Albert Einstein, is to compose the action (which became the "Hilbert-Einstein action"). It has an undeniable "kitchen" side. In the integral we put "the Ricci scalar" R , which is a scalar related to the curvature of space. It is zero if the space is flat. It is the presence of this term that gives the first member of the Einstein equation, at the end of a "variation calculation" (of the metric and of the Ricci tensor which is deduced from it). As there are two Ricci scalars, each attached to each of these two "geometries", the calculation of the variations will give the first two members of two field equations. And we add "Lagrangian densities". In General Relativity there is only

hypervolumes $\sqrt{-g^R} dx^0 dx^1 dx^2 dx^3$ and $\sqrt{-g^L} dx^0 dx^1 dx^2 dx^3$. They introduce an interaction term : $\mu^4 (g_R g_L)^{1/4} V(g_R, g_L)$ based on an "average volume factor"¹⁷ $(g_R g_L)^{1/4}$. The interest of this paper is that the Lagrangian derivation shows for the first time the form of the system of two coupled field equations, linked to such a bimetric model¹⁸:

(3)

$$\begin{aligned} 2M_L^2 \left(R_{\mu\nu}(g^L) - \frac{1}{2} g_{\mu\nu}^L R(g^L) \right) + \Lambda_L g_{\mu\nu}^L &= t_{\mu\nu}^L + T_{\mu\nu}^L \\ 2M_R^2 \left(R_{\mu\nu}(g^R) - \frac{1}{2} g_{\mu\nu}^R R(g^R) \right) + \Lambda_R g_{\mu\nu}^R &= t_{\mu\nu}^R + T_{\mu\nu}^R \end{aligned}$$

In this equation the tensors¹⁹ $T_{\mu\nu}^L$ and $T_{\mu\nu}^R$ represent the matter fields of the two types of masses, while the terms $t_{\mu\nu}^L$ and $t_{\mu\nu}^R$ are then responsible for what can be called an "induced geometry effect", i.e. the way the geometry of one of the two populations is influenced by the matter field of the second. But their test does not lead to any model, because they cannot specify the nature of the interaction terms²⁰.

A more sophisticated attempt [18] is based on the action²¹:

(4)

$$\begin{aligned} S = \int d^4 x \sqrt{-g} ((g)_R / 8 \pi G + \mathcal{L}(\psi)) + \sqrt{-h} P_h (\underline{\mathcal{L}}(\underline{\phi})) \\ + \int d^4 x \sqrt{-h} ((h)_R / 8 \pi G + \underline{\mathcal{L}}(\underline{\phi})) + \sqrt{-g} P_g (\mathcal{L}(\psi)) \end{aligned}$$

The metrics are then marked with the letters g and h . We find the two Lagrangian densities. The author manages the interaction between the two populations with the help of applications to which she gives the name "pull over". Her coupled field equations system is then Qui donne cette fois encore les deux équations de champ couplées avec les termes d'interaction.²² :

(5)

one term L, which gives the tensor of the second member. Here there are two. The coefficients and are the volume elements in these Riemanian spaces.

¹⁷ A heuristic hypothesis by Damour and Kogan, without geometric justification.

¹⁸ Thus Damour and Kogan produced the first "bimetric" model leading to two coupled field equations. For them the two entities are supposed to "inhabit" different "branes".

¹⁹ The "tensor" formalism means that the equations and solutions obtained do not depend on the chosen coordinate system. The only invariant objects are the geodesics and the lengths measured along them (think of the S2 sphere).

²⁰ Damour and Kogan will fail to define them, in a 45-pages paper from 2004.

²¹ Second attempt, from 2008, also in Physical Review D, by Sabine Hossenfelder. We find the two metrics, the two Ricci scalars and the "Lagrangian densities"

²² Which again gives the two field equations coupled with the interaction terms.

$$\begin{aligned}
({}^g)R_{\kappa\nu} - \frac{1}{2}g_{\kappa\nu}({}^g)R &= T_{\kappa\nu} - V\sqrt{\frac{h}{g}}a_{\nu}^{\kappa}a_{\kappa}^{\nu}T_{\nu\kappa} \\
({}^h)R_{\nu\kappa} - \frac{1}{2}h_{\nu\kappa}({}^h)R &= T_{\nu\kappa} - W\sqrt{\frac{g}{h}}a_{\kappa}^{\nu}a_{\nu}^{\kappa}T_{\kappa\nu}
\end{aligned}$$

The mapping a corresponds to the hypothesis made concerning the coupling mode between the two metrics²³

$$(6) \quad \delta h_{\kappa\lambda} = -[a^{-1}]_{\kappa}^{\mu}[a^{-1}]_{\lambda}^{\nu}\delta g_{\mu\nu}$$

The conservativity of the two equations, resulting from the form of their first two members is then ensured. If the mathematical method is rigorous, the benefit that results from the approach, in terms of comparison with observations, derives from the choice, obviously free, of the sign of terms present in the action (4). At the time she composed her article, in 2008, the scientific community had not yet agreed on the fact that the expansion of the universe is not slowed down, but accelerated ([21],[3],[4]). In particular, her hypothesis leads to a violation of the equivalence principle in the negative mass population, and she concludes that this is a generic property of bimetric models²⁴. In our unpublished work²⁵[22] her method is taken up again, with different choices of signs, restoring the principle of equivalence within this population of negative mass. This work represents then the justification of the various works, evoked above, until then founded on a heuristic hypothesis. This model, named Janus Cosmological Model (JCM), is based on the fundamental assumption of a deep dissymmetry between the mass and negative energy populations and the mass and negative energy populations. Here is the system of equations that results from this approach, here in mixed notation²⁶ :

(7)

²³ This is the key to the construction of its solution: an application linking the differentials of the two metrics. Between brackets of tensors, to be defined.

²⁴ In order to fit with the observational data of the time, she violates the equivalence principle of GR (equality between inertial mass-the way a particle behaves in a field- and gravitational mass-the way it contributes to this field). In doing so, it loses all the benefit that the rigor of its mathematical approach gave it and does not manage to fit with observations

²⁵ Refusal of four journals to simply submit this paper for review by referees (or admission of failure of the journals to find a referee willing to do this analysis work).

²⁶ T. Damour, in 2019, believed he had found a crippling flaw in this model, which was quickly eliminated by tinkering (the "hats"), with the second tensors of the second members that reflect "induced curvature effects". This is not a tinkering with the model. In fact it is the satisfaction of the "Bianchi mathematical conditions" that determine the shape of these tensors. But he never read our answer.

$$R^{(+)\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}R^{(+)} = \chi^{(+)} \left[T^{(+)\nu}_{\mu} + \sqrt{\frac{g^{(-)}}{g^{(+)}}} \hat{T}^{(-)\nu}_{\mu} \right]$$

$$R^{(-)\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}R^{(-)} = -\chi^{(-)} \left[\sqrt{\frac{g^{(+)}}{g^{(-)}}} \hat{T}^{(+)\nu}_{\mu} + T^{(-)\nu}_{\mu} \right]$$

The interaction laws are derived from the Newtonian approximation, and are those that were chosen heuristically. The form of the tensors responsible for the effects of induced geometry is determined by the conservativity and equilibrium relations in the two equations, derived from the Bianchi identities. When we study, for example, the induced geometry of a species, under the action of the field created by the second species, present in a sphere of constant density, the relation simply translates the fact that in this mass, positive or negative, the pressure force balances the force of gravity [23]. This is ensured on the tensor in the form:

(8)

$$\hat{T}^{(+)\nu}_{\mu} = \begin{pmatrix} \rho^{(+)}c^{(+2)} & 0 & 0 & 0 \\ 0 & p^{(+)} & 0 & 0 \\ 0 & 0 & p^{(+)} & 0 \\ 0 & 0 & 0 & p^{(+)} \end{pmatrix} \quad \hat{T}^{(-)\nu}_{\mu} = \begin{pmatrix} \rho^{(-)}c^{(-2)} & 0 & 0 & 0 \\ 0 & p^{(-)} & 0 & 0 \\ 0 & 0 & p^{(-)} & 0 \\ 0 & 0 & 0 & p^{(-)} \end{pmatrix}$$

This model is supported by the numerous results having flowed from the heuristic approach:

- Explanation of the confinement of galaxies and clusters as well as of the flatness of the rotation curves of galaxies in the periphery.
- Explanation of the strong gravitational lensing effects in the vicinity of galaxies and galaxy clusters.
- Spiral structure as a result of the interaction between galaxies and their negative mass environment.
- Since the two types of masses are mutually exclusive, the density of negative mass, very low in galaxies, is negligible in the neighborhood of the Sun. The first equation of the system is then identified with the Einstein equation. The local relativistic verifications, Mercury's perihelion advance, deviation of light rays by the Sun, is thus assured.

Beyond the construction of an exact solution, exploiting the dissymmetry between the two populations has led to an excellent agreement with the data from the observations of type Ia supernovae [24] (see Fig 5).

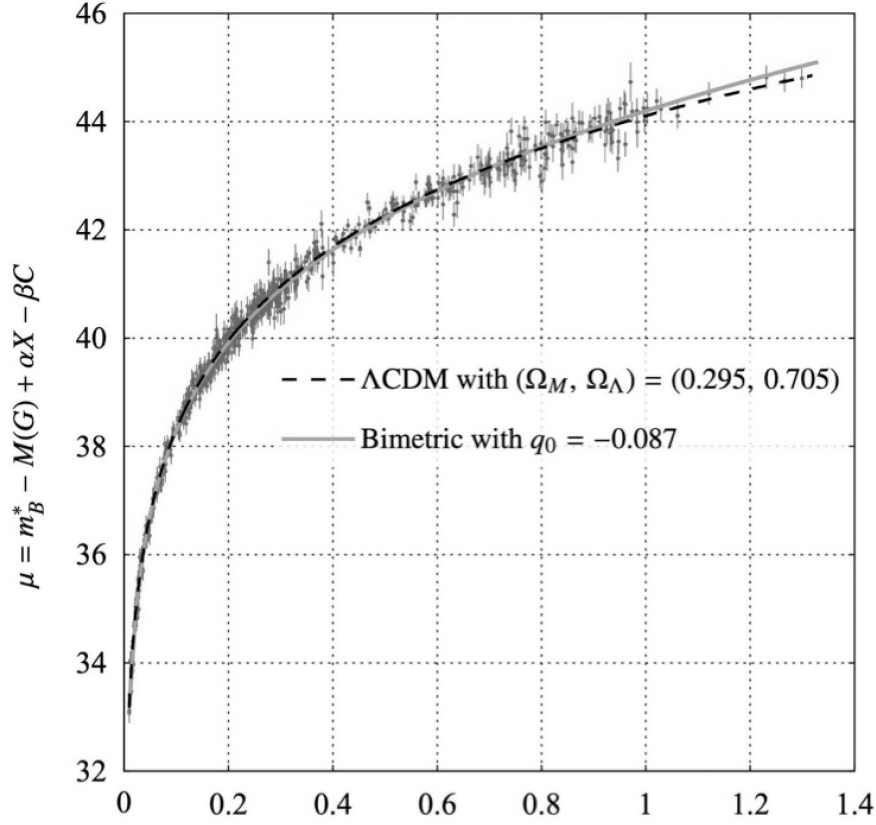


Fig. 5 : Hubble diagram for two models (linear redshift scale) [24]

4 – Conclusion.

The question of the introduction of negative masses has been strongly opposed by the emergence of the runaway phenomenon ([5],[13]), which is difficult to integrate in physics and cosmology. This concept of negative mass has also been invoked in a reinterpretation of some particle physics phenomena [25]. It is mentioned in the string theory [26], or in a reinterpretation of the geometry associated to the Schwarzschild solution ([27], [28]). The integration in a mathematically and physically coherent Janus cosmological model, leading to a large number of observational confirmations, makes this approach, which implies a deep paradigmatic leap concerning the geometry of the universe, a solution to the current crisis in cosmology and astrophysics.

References :

- [1] L.Smolin « The Trouble With Physics. The Rise of String Theory, the Fall of Science, and What Comes Next ». 2006 Houghton Mifflin Ed.
- [2] P. Woit « Not Even Wrong ». 2006 Jonathan Cape ed. London
- [3] Perlmutter, S., et al. 1999, ApJ, 517, 565

- [4] Riess A. G. 2000, PASP, 112, 1284
 - [5] H. Bondi: Negative mass in General Relativity : Negative mass in General Relativity. Rev. of Mod. Phys., Vol 29, N°3, july1957
 - [6] P.Morisson: Am. J. Phys. **26**,358 (1958)
 - [7] Matz and Kaempffer Bull. American Phys. Soc. **3**,317(1958)
 - [8] L.I.Shiff Proc. Nat. Acad. Sci. **45**, 69 (1959)
 - [9] C. de Beauregard. Comptes Rendus Acad. Sc. 252, 1737 (1961)
 - [10] Winterberg. Nuovo Cimento **19**, 186 (1961)
 - [11] Ya. Terlertsty. Int. Conf. Theory of Gravit. King's College (1961)
 - [12] A. Inomata and D. Peak Nuovo Cimento **63**, 132 (1969)
 - [13] H.Bonnor Negative mass and general relativity. Gen. Relat. And Gravit. Nov 1989 vol **21** pp 1143-1157
 - [14] J.Farnes : A unifying theory of dark energy and dark matter : Negative mass and matter creation within a modified Λ CDM framework. Astronomy and Astrophysics (2018)
 - [15] J.P.Petit : The missing mass problem. Il Nuovo Cimento B Vol. **109** July 1994, pp. 697-710
 - [16] The Dipole Repeller : Y Hoffman, D.Pomarède, R.B.Tully, H.Courtois. Nature Astronomy 2017 1 , 0036
 - [17] J.P.PETIT, Twin Universe Cosmology, Astrophys. and Sp. Science, **226**, 273-307, 1995.
 - [18] S. Hossenfelder : A bimetric Theory with Exchange Symmetry. Phys. Rev. D78, 044015, 2008 and arXiv : 0807.2838v1 (gr-qc)17 july 2008
 - [19] Damour T. , Kogan I I. Effective Lagrangians and universality classes of nonlinear bigravity Phys. Rev. D **66** (2002) 104024. hep-th/0206042.
 - [20] Damour T. , Kogan I. I. , Papazoglou A. Non-linear bigravity and cosmic acceleration Phys. Rev. D **66** (2002) 104025. hep-th/0206044.
 - [21] Riess, A. G., et al. 1998, ApJ, 116, 1009
 - [22] J.P.Petit, G,d'Agostini, N.Debergh : The Janus Cosmological Model : (2021)
- <https://hal.archives-ouvertes.fr/hal-03285671/document>

- [23] J.P.Petit, G. D'Agostini, N.Debergh : Physical and mathematical consistency of the Janus Cosmological Model (JCM). Progress in Physics **15** issue 1 (2019)
- [24] G. DAgostini and J.P.Petit : Constraints on Janus Cosmological model from recent observations of supernovae type Ia, Astrophysics and Space Science, 363:139 (2018).
<https://doi.org/10.1007/s10509-018-3365-3>
- [25] Theory of negative mass cyclotron resonance. D.C.Mattis M.J.Stevenson Phys. Rev. Lett. **3** , 19 (1959)
- [26] Negative energy in string theory? D.Brill, G.T.Horowitz, Phys. Lett. B Vo. 262 issue 4 (1991)
- [27] J.P.Petit, G.D'Agostini : Cancellation of the singularity of the Schwarzschild solution with natural mass inversion process. Mod. Phys. Lett. A vol. **30** n°9 (2015)
- [28] Negative mass scenario and Schwarzschild spacetime in general relativity. S.Bondarenko, Mod. Phys. Lette A- vol. **34** n°11 (2019)
- [29] F.Winterberg : Planck Mass Rotors as Cold Dark Matter and Quintessence. Verlag der Zeitschrift für Naturforschung. Tübingen. 2003.
- [30] J.Belletête and M.J.Paranjape : On negative mass. Int. Jr. of Mod. Phys. Voo.22 n°12, 1341017 (2013). arXiv:1304.1566v1 [gr-qc] A apr. 2013.
Doi/10.1142/50218271813410174

Referees' reports :

Manuscript ID: 793564

Authors: Nathalie Debergh, J.-P Petit and G D'agostini

Journal: **Frontiers in Physics, section Cosmology**

Article type: Original Research

Submitted on: 12 Oct 2021

First referee :

Author elucidates the idea of negative mass in cosmological models. After the historical review of different attempts to explore this controversial idea, author states that the so called Janus Cosmological Model (JCM) is a good candidate for the modeling of negative mass. It is stressed that the basic argument in favor of the Janus Cosmological Model comes out both from the heuristic supposition and the definite observational confirmation. The major result of the paper is in detailed justification of various works supporting the idea of negative mass in cosmology. The results of this paper are new and physically interesting. This paper, in spite of some formal presentation, may be useful for a wide scientific community in view of the fundamental importance in understanding the physical details of cosmological expansion. The manuscript is well organized, clearly written and may be published as it is. I recommend to publish this paper in Frontiers in Physics.

Check List

- a. Is the quality of the figures and tables satisfactory? Yes
- b. Does the reference list cover the relevant literature adequately and in an unbiased manner? Yes
- c. Are the statistical methods valid and correctly applied? (e.g. sample size, choice of test) Yes
- d. Are the methods sufficiently documented to allow replication studies? Yes

QUALITY ASSESSMENT (x/5) :

Rigor 4

Quality of the writing 3

Overall quality of the content 3

Interest to a general audience 4

Second referee :

The introduction of negative masses in physics is generally considered as a problem. However, there are some minority but authoritative currents that hypothesize this possibility. The article is an introductory review, and should be commended for its intellectual honesty.