PROBLEMAS SOBRE EL BIONOMIO DE NEWTON CON VALORES NATURALES DEL EXPONENTE

1. Desarrollar $(x^2+2x)^4$:
   Tomemos como modelo el desarrollo de $(a+b)^4$, y sustituyamos a por x^2 y b por $2x$:

$$(a+b)^4 = \binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(x^2+2x)^4 = (x^2)^4 + 4(x^2)^3 2x + 6(x^2)^2(2x)^2 + 4x^2(2x)^3 + (2x)^4 = x^8 + 8x^7 + 24x^6 + 32x^5 + 16x^4$$

2. ¿Cuál es el desarrollo de $(a-b)^5$?
   Basta observar que $a-b$ puede escribirse de la forma $a+(-b)$; por lo tanto,

$$(a-b)^5 = (a+(-b))^5 = a^5 + 5a^4(-b) + 10a^3(-b)^2 + 10a^2(-b)^3 + 5a(-b)^4 + (-b)^5 =$$

$$= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$$

   Todos los términos en los que el exponente de $-b$ es impar son negativos, y son positivos los términos en los que dicho exponente es par.

3. Del desarrollo de $(x^2-3x)^6$ sólo nos interesa el término quinto. ¿Cuál es?

$$T_5 = \binom{6}{4}(x^2)^{6-4}(-3x)^4 = 15x^4 \cdot 81x^4 = 1215x^8$$

4. Escribe el término de grado 8 en el desarrollo de $\left(3x^2+\dfrac{1}{x}\right)^7$.
   Supongamos que el término buscado es T_k, es decir, que ocupa el lugar k:

$$T_k = \binom{7}{k-1}(3x^2)^{7-(k-1)}\left(\frac{1}{x}\right)^{k-1} = \binom{7}{k-1}(3x^2)^{8-k}\frac{1}{x^{k-1}} = \frac{\binom{7}{k-1}3^{8-k}x^{2(8-k)}}{x^{k-1}}$$

   El grado del término es el exponente definitivo de x, que sería la diferencia entre los dos exponentes $2(8-k)$ y $k-1$, puesto que para dividir dos potencias de x basta restar los exponentes del numerador y del denominador. Por consiguiente:

$$2(8-k)-(k-1) = 8 \rightarrow 16-2k-k+1 = 8 \rightarrow -3k = -9 \Rightarrow k = 3$$

   Es decir, el término de grado 8 es el *tercero*:

$$T_3 = \binom{7}{2}(3x^2)^5\left(\frac{1}{x}\right)^2 = 5103x^8$$

PROBLEMAS SOBRE EL BIONOMIO DE NEWTON EXTENDIDO A VALORES NO NATURALES DEL EXPONENTE

1. Expand $(1+x)^{-2}$ up to the term in x^4.

   a Use your expansion to estimate $\frac{1}{1.5^2}$

   b Comment on the level of accuracy of your answer.

2. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^5.

   a Use your expansion to estimate $\frac{1}{\sqrt{1.01}}$

   b Comment on the level of accuracy of your answer.

3. Find the term in x^6 in the expansions of:

   a $(1-x)^{1.1}$

   b $\sqrt{1-x}$

   c $(1+2x)^{-1}$

   d $\sqrt{1-2x}$

4. Expand, using the Binomial Theorem, up to the term in x^5, the following:

   a $\frac{1}{\sqrt{1+3x}}$

   b $(1-3x)^{0.7}$

   c $\frac{1}{\sqrt[3]{1+2x}}$

   d $2\sqrt{1+x}$

5. Consider the expression $\frac{3}{(1-x)^2}$

   a Use the Binomial theorem to develop a series expansion.

   b Substitute $x = 0.2$ into the first seven terms of your expansion

   c Use your expansion to approximate $\frac{3}{0.9^2}$

6. Find the term in x^5 in the binomial expansion of $4\sqrt{1-2x}$.

7. Consider the expression $4+x$.

   a Write the expression in the form $A(1+Bx)$ where A & B are constants.

   b Use your expression to find a series expansion for $\sqrt{4+x}$.

   c Hence find the square root of 4.1 correct to 5 significant figures.

8. Consider the expression $\frac{1}{(1-5x)^3}$

   a Find the first three terms in the Binomial Expansion of this expression.

   b Find the coefficient of the term in x^6.

   Your answer to part b suggests that the size of the terms might be growing and the series diverging even if $|x| < 1$. Us a value of x = 0.5 to answer the rest of this question.

c Find the ratio of term 2 to term 1. Are the terms growing in size or decreasing?

d Find the ratio of term 4 to term 3. Are the terms growing in size or decreasing?

e Find the ratio of term 7 to term 6. Are the terms growing in size or decreasing?

f Is this series a viable method of making numerical approximations.

9. Find the first seven terms in the expansion of $\frac{1}{\sqrt[4]{1+x}}$.

a Find the value of $\frac{1}{\sqrt[4]{1.1}}$ to the maximum accuracy permitted by your series.

b Find the absolute error of your estimate from part a.

c Find the percentage error of your estimate from part a.

10. Find the coefficient of the term in x^4 in the binomial expansion of $(1-2x)^{-0.1}$.

11. Use a series method to find the value of √2 correct to 4 significant figures.

Soluciones

1. $1-2x+3x^2-4x^3+5x^4$ a 0.5265 b Not even correct to 1 s.f.

2. $1-0.5x+0.375x^2-0.3125x^3+0.273438x^4-0.246094x^5$ a 0.9951

b High accuracy > 6 s.f.

3. a $0.00295474x^6$ b $-0.0205078x^6$

c $64x^6$ d $-1.3125x^6$

4. a $1.-1.5x+3.375x^2-8.4375x^3+22.1484x^4-59.8008x^5+164.452x^6$

b $1.-2.1x-0.945x^2-1.2285x^3-2.11916x^4-4.19594x^5-9.02127x^6$

c $1+\frac{2}{3}x-\frac{4}{9}x^2+\frac{40}{81}x^3-\frac{160}{243}x^4+\frac{704}{729}x^5-\frac{9856}{6561}x^6$

d $2+x-\frac{1}{4}x^2+\frac{1}{8}x^3-\frac{5}{64}x^4+\frac{7}{128}x^5-\frac{21}{512}x^6$

5. a $3+6x+9x^2+12x^3+15x^4+18x^5+21x^6$

b 4.6871

c 3.7037

6. $-3.5x^5$

7. a $4+x=4\left(1+\frac{1}{4}x\right)$ b $\sqrt{4+x}=\sqrt{4\left(1+\frac{1}{4}x\right)}=2\sqrt{1+\frac{1}{4}x}$

c $2.+0.25x-0.015625x^2+0.00195313x^3-\ldots$ d 2.0249

8. a $1+15x+150x^2$ b 437 500

c 7.5 growing. d 4.167 growing, but not so rapidly.

e 3.33 growth slowing. f No. It converges too slowly.

9. $1.-0.25x+0.15625x^2-0.117188x^3+0.0952148x^4-0.0809326x^5+0.070816x^6$

a 0.976454 b < 0.0000001 c $<0.00001\%$

10. 0.4774

11. 1.414