What is the Best Gameplay for Zombie Dice?

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1 Introduction

Zombie Dice is a game which was released by Steve Jackson Games in 2010. The main pursuit within the game is to attain as many brains as possible without getting shot. Given that the success of a player is partially driven by the luck of the role of their dice, the probability of different outcomes will be considered. Nevertheless, luck is not our only component for success. This is a game also led by each players' decisions, where being risky or conservative can affect every game.

Our report is intended to divulge into the outcomes of different players' decisions. We will attempt to mirror the real-life results of various choices within the game using the programming language, *R*.

For those unfamiliar with Zombie Dice, we will now explain the rules of the game that our simulation follows.

2 Gameplay

This report will be considering the original release of the game where we have thirteen six-sided dice. The possible outcomes of rolling a die: are a brain, which we want; a footstep, which is where our decisions come to play; or a shotgun, which we definitely don't want. Of the thirteen, we have:

- Six green dice which consist of three brains, two footsteps and a shotgun
- Four yellow dice which consist of two brains, two footsteps and two shotguns
- Three red dice which consist of one brain, two footsteps and three shotguns

In order to win the game, we need to attain thirteen or more brains...without getting "killed" first that is.

Each turn begins with a player rolling three dice. If the player rolls a brain or a shotgun, it is set to the side. If we roll a footstep, we keep it and decide what to do next. We may either continue our turn or end it - unless we have already rolled three shotguns and we die. If we choose to continue, we pick up additional dice to the number of footsteps so that we again have three dice to roll. We then repeat the above procedure.

As aforementioned, if we accumulate three shotguns in a single turn, our turn ends as we have, unfortunately, "died". If this happens, any brains that we collected during our turn are abandoned and we gain nothing from our turn. If we end our turn before our demise, we add our brains to our score from the previous rounds.

If you're lucky enough to reach thirteen brains, you can still continue your turn to try to gain even more. This is useful as every player must have an equal number of turns before the game is fully finished. At the end of the round, the player with the highest score wins - *not* necessarily the player who had thirteen brains first.

For additional insight, a detailed overview of the rules for the original release is provided on the distributor's website. [1]

3 Our Strategies

When approaching Zombie Dice, various different strategies are plausible. This investigation began with designing six distinct players. The following will be referred to in this document as our **Standard Strategies**:

- Random Player:- a random player where there is no logic to their strategy
- Strategy One:- a player who rolls until they have one shotgun
- Strategy Two:- a player who stops after two shotguns
- Strategy Three:- a player who after collecting two shotguns rolls one more time and then stops
- Strategy Four:- a player who combines the above strategies, depending on their current score (i.e. when their score is equal to zero, it adopts Strategy Two; when their score is the maximum, it adopts Strategy One; when their score is the minimum, it adopts Strategy Three; otherwise, it adopts Strategy Two)
- Strategy Five:- a player who stops after obtaining at least four brains, or after getting two shotguns

These players differ by their stopping points. When playing a board game where the player dictates their fate, the outcome is dependent upon stopping points. Our task is to now identify what the most *successful* stopping points are. We could continue by immediately jumping into the analysis of these six strategies. However, we wish to further test if we can make strategies that are not only logical in general gameplay, but also as smart as possible in order to win the highest number of games.

To do this, we also consider the following strategies which are our **Enhanced Strategies**:

- "Optimal" Strategy:- a player who follows all of the steps dictated within an article [2] which similarly explores the best player strategy for Zombie Dice
- Alun's Strategy:- a player that calculates the expected shotguns depending on what dice have already been thrown
- James' Strategy:- a player that bases its strategy off of the probability of throwing different combinations of dice; the corresponding stopping points depend on how likely it is the player will roll a good combination of dice.
- Niamh's Strategy:- a player that makes decisions depending on a generalized probability calculator based off of what dice are left, also accounting for several extreme cases
- *Note: Each of the last three strategies are further described within Section 6.1*

The first enhanced strategy aforementioned, the "optimal" strategy, scrutinizes the probability and expectation surrounding Zombie Dice. The stopping points are then given based off of their findings. This approach to the game is one which attempts to combine the effect of the probability of each roll and the ability to make the appropriate decisions alongside this. This strategy follows a similar mindset as the ones each of us have created as our own "best" strategies.

4 Results and Analysis

The analysis of our strategies will be split into two separate but very similar parts. The first part of analysis consists of the Standard Strategies and the second consists of the Enhanced Strategies. As such, we have collected our data separately.

After designing the various strategies, we proceeded by running the simulator of the game for 100,000 trials for each test group. We also randomised the order of the players for each trial to make the collection of data fair. This is due to the fact that some strategies perform better in certain positions rather than others. We created subsets of our trials by removing any ties that occurred; the reasoning for this is that we wanted to analyse the scenarios where the different strategies were fully successful. Therefore, the metrics that we have investigated are features of the winners of each game.

To obtain data on the different metrics, our simulator was reworked to output the desired data for every run of the game. The outputs of our simulation were the winning player, winning score, the total number of rounds in the game, the average number of brains per round survived for the winning player and the average number of shotguns per round for the winning player. Obviously 100,000 trials provides us with a lot of insight, to give an overview of this data we have included Table 1.

This table alone conveys the success of each individual strategy, however, there are some interesting observations. For example, Strategy Three wins 8.122% of the trials which is approximately 20% less than Strategy Two but has a higher average winning score. Hence, our focus now turns to a more in depth analysis of the results we have collected. This may help us to understand the reasons why certain strategies performed better than others.

Player Strategy	% of Wins	Av. Winning Score	Av. # of Rounds to Win	Av. # of B	Av. # of S
Random Player	4.627	14.222	5.048	3.063	1.266
Strategy One	9.854	14.423	4.891	3.235	1.289
Strategy Two	28.232	15.007	4.418	3.807	2.125
Strategy Three	8.122	15.247	4.113	4.336	2.338
Strategy Four	24.218	14.640	4.528	3.612	1.862
Strategy Five	24.948	14.269	4.969	3.023	1.601
“Optimal” Strategy	20.796	14.549	4.600	3.416	1.708
Alun’s Strategy	27.545	14.871	4.221	3.852	2.011
James’ Strategy	25.250	14.632	4.336	3.637	1.878
Niamh’s Strategy	26.408	14.846	4.320	3.724	1.996

Table 1: Averages Collected from our Samples

4.1 What is the Best Strategy?

4.1.1 Proportion of Wins

When deciding on the best strategy for a game, everyone’s first inclination is to see how many games it wins. To get a better visualisation in this context we have used pie charts. Due to the large sample size we used and the randomness of our trials, we are confident that our data reflects the true proportion of wins.

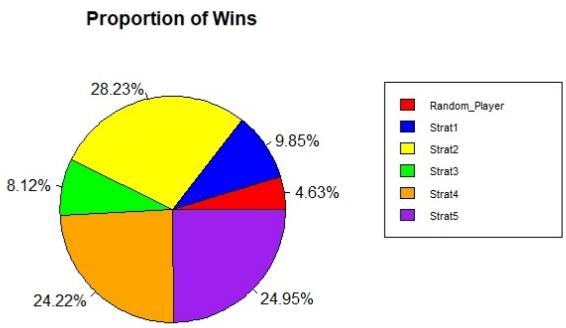

Figure 1: Pie Chart of the Proportion of Wins for the Standard Strategies

In Figure 1, we can see the main contenders for the highest number of wins are: Strategy Two, Four and Five. The main correlation between these strategies is the common stopping point when they have two shotguns. This is logical if we consider the probability of our dice factoring in to our success. If we have obtained two shotguns, our likelihood of dying and losing the brains we have collected over our turn is high. This is reflected in the performance of Strategy Three, which after collecting two shotguns rolls one more time. With a percentage of wins at 8.122%, this shows that the risk of rolling one more time at this stage is not rewarded.

Nonetheless, simply stopping before collecting two shotguns may also be a detrimental decision. This is because, prior to obtaining two shotguns, you are more likely to collect more brains over your turn - giving you a better chance of getting a higher score. This is reflected in the percentage of wins Strategy One gets at 9.854% which is considerably lower than Strategy Two.

From solely looking at the Standard Strategies it is apparent that Strategy Two, arguably one of the simplest strategies, performs the best. Despite Strategy Four and Five trying to develop Strategy Two and performing well they still don’t perform to the standard of the original. This may be due to the development not being a beneficial addition to the strategy. Consequently, there is the high possibility of there existing better strategies than Strategy Two that simply optimize what Strategy Four and Five have tried to introduce.

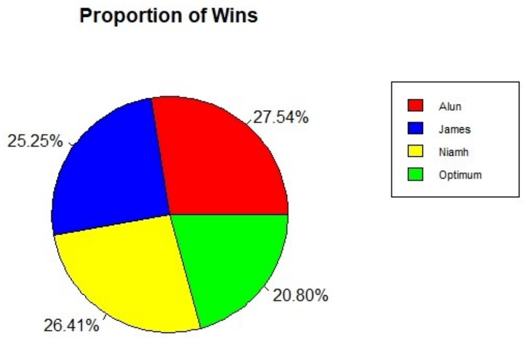

Figure 2: Pie Chart of the Proportion of Wins for the Enhanced Strategies

The proportions of the results for Figure 2 are evenly spread, suggesting that the Enhanced Strategies perform similarly. It is clear that the "optimal" strategy is not actually optimal as it has been beaten by all of our own strategies. This might be due to the fact it took a simplistic approach to the optimal player. Within the report [2], it is stated that their final strategy is "slightly less optimal than using the enormous data set, but it is vastly easier". Here, they are referring to the large number of outcomes that can occur and that they have made a generalisation to simplify this.

Another feasible reason for the "Optimal" Player being beaten is that the other strategies all include statements dependent on if any of the other players have surpassed twelve brains. If all previous players have taken their turn before you, there is no need to play safe. This shows awareness of the players which brings another dimension to the strategy.

Due to the closeness in the percentage of wins for the other strategies, it is difficult to fully conclude which method is optimal without further analysis. This may even extend to looking into what approach is taken towards the probability driving the game. It may even be plausible to find which method results in finding the best overall player.

4.1.2 Average Number of Brains and Shotguns

To be able to further understand why the strategies performed as they did, we align more attention towards the detailed aspects of their performances. To do this, we observed the average number of brains and shotguns per round over the 100,000 trials as displayed in Table 1. This should give an inference into how well the winning strategies perform in each round, rather than a simple overview of the entire game.

(i) Verification of Data:- As we have taken a sample average, we want to verify these results to ensure they are statistically accurate and haven't been heavily affected by outliers. This was done by conducting a number of hypothesis tests. We first investigate the Random Player. As we ran the simulation 100,000 times, we have one sample with 100,000 observations. Thus, to do a hypothesis test for the mean brains per round we must affirm that our data meets certain conditions in order to decide which test to do. The first test was for symmetry. We used the MGG test [3] to conduct this.

- Hypotheses - H_0: the distribution of our data is symmetric; H_1: the distribution of our data is asymmetric
- Test Statistic - T = 25.673
- P-Value - p < 2.2e-16
- Decision - At a 5% significance level we reject the null hypothesis
- Conclusion - There is statistical evidence to reject the null hypothesis that the distribution of our data is symmetric

Furthermore, as normally distributed data is symmetric, there is also statistical evidence to reject any hypothesis of our data being normally distributed. As there is enough statistical evidence to reject the assumptions of normality and symmetry, we will use Fisher's one sample permutation test [4] to test for the mean brains per round:

- Hypotheses - H_0: $\mu = 3.063$; H_1: $\mu \neq 3.063$
- Test Statistic - $T = 2.122143$
- P-Value - $p = 0.98$
- At a 5% significance level, we do not to reject the null hypothesis
- There is not statistical evidence to reject the hypothesis that are sample mean of the average brains per round is the true mean for the Random Player.

The process detailed above was followed for every single strategy, all of which received similar results and are contained within Section 6.2. All the data on the strategies for average shotguns and brains failed the assumptions of normality and symmetry, resulting in the use of Fisher's one sample permutation test. Therefore, there was not enough statistical evidence to reject the null hypothesis' that our sample means were the true population means. Due to our verification, we can now trust the results in Table 1 for the average number of brains and shotguns per round.

(ii) Analysis:- We begin by looking at the results for the Standard Strategies. From first inspection, the Random Player and Strategy One have the lowest shotguns on average per turn. They both had the lowest percentage of wins notwithstanding this. The inference which can be gained from this is that the strategies both stop their turns prematurely. Consequently, in comparison to some more successful players, they do not gain a large amount of shotguns but they also don't gain a large amount of brains on the turns they survive.

When comparing Strategy Two and Three, we see that Strategy Three gains a larger amount of brains *and* shotguns. We can infer from this, due to Strategy Three not performing as well as Strategy Two, that despite it taking more risks and gaining more brains, it also gains more shotguns and so is more likely to be killed before it can improve its score.

Thus far, we can observe that we need to find a balance between being risky and not ending our turn prematurely and being conservative and stopping when the odds are against us. This is elaborated through the results for Strategy Four. This strategy was more conservative than Strategy Two and yet its overall percentage of wins is approximately 4% less. So we need to turn our focus towards finding more optimal stopping points.

Strategy Five is a peculiar case. It does not fit into the trend of our results so far where between 3.6 and 3.9 brains and 2 shotguns on average is a rewarding combination. Every single other strategy uses the common structure of the number of shotguns determining the strategy to follow.

Strategy Five steps away from this and is a unique case because of its success despite not following the general trend of the other successful strategies. Rather than following shotguns then brains, it uses brains then shotguns. As the rest of our strategies all follow the shotguns to brains method, we are unable to test if the alternative approach could be developed to become an even better strategy; further investigation would be required.

Observing the Enhanced Strategies, the previously mentioned trend is continued. The closer we get to 3.9 brains and 2 shotguns on average, the better the percentage of wins. Due to this reaffirming what has been seen above, we are safe to assume that combining the correct balance of riskiness and playing safe will result in a successful strategy. It is not solely determined by who gets the largest amount of brains or shotguns on average.

4.2 How many Rounds does a Game Last?

4.2.1 When is our Game Likely to Finish?

For some insight into how long a game of Zombie Dice will last, we will investigate the number of rounds that it takes the Enhanced Strategies to win. To gain an accurate representation of the likelihood that our game finishes after a certain number of rounds, we will look at the plot of the empirical cumulative distribution function (ECDF). As we have used a very large sample size of 100,000 trials, we are confident that this converges to the true cumulative distribution function.

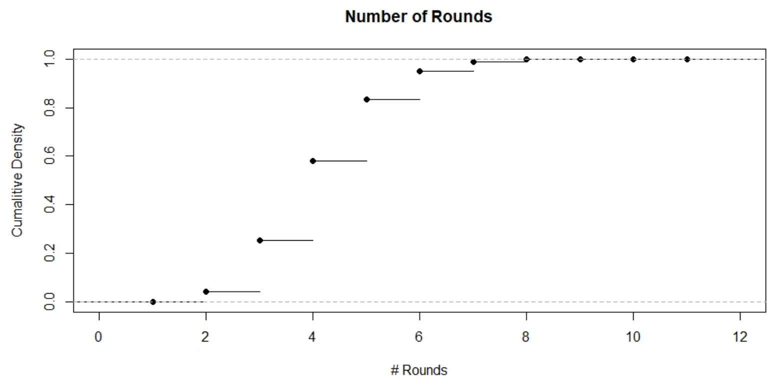

Figure 3: Empirical Cumulative Distribution Plot of the Number of Rounds for the Enhanced Strategies

As expected, the cumulative density increases as the number of rounds increase. In Figure 3, it is noticeable that, after around six rounds, the game is very likely to end soon. In particular, approximately 95% of games finish between two and six rounds.

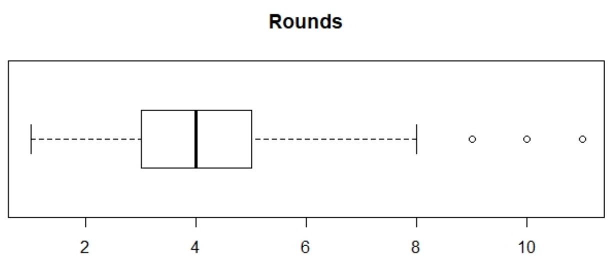

Figure 4: Boxplot of Enhanced Strategies for Rounds

We can see from Figure 4 that the upper tail of the boxplot is at eight rounds with everything after this being outliers. This is verified in our ECDF. As we can see with almost certainty that we will finish within eight rounds because there is a very small chance that the game will take as long as this to finish.

4.2.2 Who Wins Efficiently?

Being a smart player is not exclusively limited to getting the largest percentage of wins - a good player will also win a game efficiently. Our observations in Table 2 try to scrutinize this. This table is a collection of data for each strategy (also including the strategies' groups as a whole) of the five point summaries of their boxplots for the number of rounds to win. Unsurprisingly, the Standard Strategies take a larger number of rounds to win than the Enhanced. This reaffirms the theory that a smarter player finishes a game efficiently. We will now study the Enhanced Strategy in more depth, though a boxplot of the results for the Standard Strategies are included in Section 6.3.

Player Strategy	Lower Tail	Lower Quartile	Median	Upper Quartile	Upper Tail
Standard Strategies	1	4	5	6	9
Strategy One	1	4	5	6	9
Strategy Two	1	3	4	5	8
Strategy Three	1	3	4	5	8
Strategy Four	3	4	4	5	6
Strategy Five	3	4	5	6	9
Random Player	1	4	5	6	9
Enhanced Strategies	1	3	4	5	8
"Optimal" Player	3	4	5	5	6
Alun's Strategy	1	3	4	5	8
James' Strategy	3	4	4	5	6
Niamh's Strategy	1	3	4	5	8

Table 2: Five Point Summaries for the Number of Rounds to Win for the Different Strategies

The "Optimal" Player has the largest median within the group, as expected. The three other strategies are fairly alike, given that they share the same median. If we observe the tails of the boxplots, this is where our strategies differ. Alun and Niamh's strategy share identical results within Table 2. They have a smaller value for their lower tail compared to James' strategy. Nonetheless, James' strategy has a smaller IQR. From all of this, it is possible to infer that all three of our own strategies win fairly efficiently. It can be argued that James' strategy wins in a smaller number of rounds more reliably; albeit Alun and Niamh's strategy also are able to win in an even smaller number of rounds at times. Consequently, it is down to the reader's personal opinion as to which triumph is prioritised.

4.3 Is there a Relationship between the Winning Score and the Number of Rounds a Game Lasts?

This is an interesting question as one may reasonably assume that the more rounds you play, the more brains you can collect and the higher the winning score will be. Thus, we have decided to investigate on whether there is a linear relationship between the winning score and the number of rounds a game takes for the Standard Strategies and the Enhanced Strategies. The winning score, denoted S, will be collected from all 100,000 trials as will the number of rounds a game takes, denoted R. We will start with the Standard Strategies.

To see if there is a linear relationship between S and R, we will be using a simple regression model as we have one dependent variable, S, and one predictor, R. Thus, for our first model the regression equation will be:

- $S = \beta_0 + \beta_1 \times R + \epsilon$, where β_0 and β_1 are unknown coefficients and ϵ is the error term

By running the model we observe the following information:

- The estimated coefficients of the model are $\hat{\beta_0} = 13.666428$ and $\hat{\beta_1} = 0.245883$
- The rounds, R, and the intercept are highly significant
- Within the hypothesis tests for the coefficients $H_0 : \beta_0 = 0$ vs $H_1 : \beta_0 \neq 0$ and $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$, the null hypotheses were rejected at a 5% significance level
- The coefficient of determination $R^2 = 0.03318$

The value for R^2 tell us that the relationship between the winning score and the number of rounds in a game is not linear. This is illustrated within Figure 5.

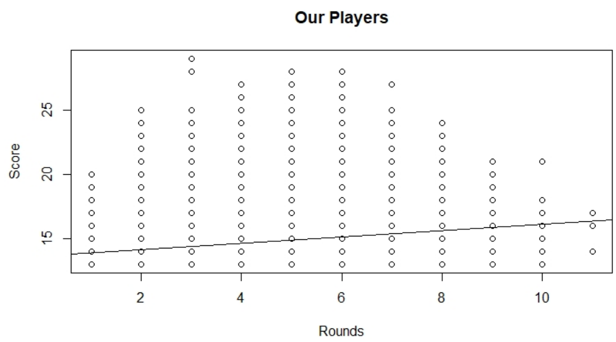

Figure 5: Scatterplot of Winning Scores and Rounds for Enhanced Strategies

Furthermore, many of the assumptions needed for a linear regression model were not satisfied. For example, we can see that normality is violated within Figure 6. As the plot of the residual values do not fit on a straight diagonal line, we can safely assume that it does not fit the Normal Distribution. Therefore, a linear regression model is not suitable to describe the relationship between the winning score and the number of rounds a game lasted.

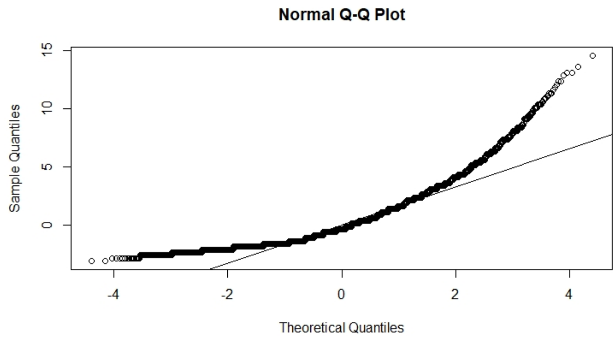

Figure 6: Q-Q Plot of the Residual Values of the First Model

We tried to improve our model by transforming both variables, R and S, in such a way to try to meet the assumptions needed for a regression model. Various transformations were trialed. Ultimately it was decided that the closest we could get to meeting the assumption of normality for residual values was by using the following model:

- $1/S^2 = \beta_0 + log(R) + \epsilon$

This model attempts to see if there was a linear relationship between the reciprocal of the winning score squared and the logarithm of the number of rounds a game lasts. Nevertheless, this model not only failed the assumption of normality of the residual values, but also gave a coefficient of determination value of $R^2 = 0.03901$. This is visible within Figure 7.

Thus, we have not been able to find any sort of linear relationship between the winning score and the number of rounds a game takes for the enhanced strategies. From this, we are able to deduce that the increase of rounds within a game will not fundamentally increase the scores within the game. This is easily contextualized through thinking about the scenarios where you can gain zero points in a round or you can gain a very large amount of brains in a single round. So when the rounds increase, there is not always a similar increase in score.

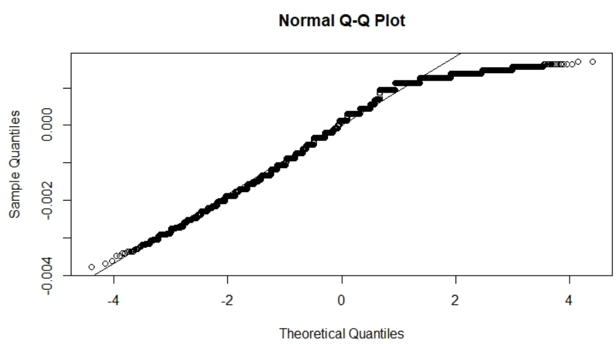

Figure 7: Q-Q Plot of the Residual Values of the Second Model

A very similar outcome is found from the Standard Strategies and the details behind this result are contained within Section 6.4.

5 Conclusion: What *is* the Best Strategy?

5.1 Standard Strategies

Within Table 1, we immediately see which is the best player strategy within this group. Strategy Two wins approximately 3% more than its closest rival, Strategy Five. From this alone we can deduce that stopping at two shotguns is a logical decision. In simple terms, due to Strategy Two solely being conservative when a high risk is involved, it results in a better performance than those who either stop prematurely or are too risky.

5.2 Enhanced Strategies

Moving on from Strategy Two, the Enhanced Strategies are where a more in depth method is tested. We have already established that a strategy should not stop prematurely and should not take risks where it is unlikely to be rewarded. The best strategy is the one that takes this and strengthens itself further through the logic of the situation. Through considering the factor of the probability, a player will undoubtedly perform even better - if it is done correctly.

The "Optimal" Strategy attempts to consider the expectation of brains and shotguns. By using a simpler version rather than accounting for the thousands of different scenarios that can occur, it results in its percentage of wins being the worst in the group by almost 6%. So we can deduce that using simplified methods can negatively impact your performance.

The three other strategies base their stopping points off of various methods of probability theory. The difference in overall performance between the strategies is marginal as seen by the difference between the proportion of wins between them. However, it is clear from our results that basing your strategy upon the expected number of shotguns your player can receive gives you a slight edge compared to looking at the probability of rolling different combinations of dice or getting a shotgun or a brain.

Although these strategies are smart and perform well, it is unlikely they will be performed in practice. Due to the complicated arithmetic that would be involved to calculate the various probabilities, a much simpler strategy would have to be used. Regardless, based off of our numerous observations, Alun's strategy is the one which is likely to be the best performer. Within this document we have also come across scenarios where further investigation would be required to fully rule out its possibility of being improved and becoming a better strategy. Nevertheless, exclusively looking at the Enhanced Strategies, we can conclude that Alun's strategy is the best one to implement within Zombie Dice.

References

[1] Steven Jackson Games. Zombie Dice Rules. `http://www.sjgames.com/dice/zombiedice/img/ZDRules_English.pdf`. Rules of Original Release.

[2] Heather L. Cook and David G. Taylor. Zombie dice: An optimal play strategy. `https://arxiv.org/pdf/1406.0351.pdf`. Published: 02/06/2014.

[3] Yulia R. Gel Weiwen Miao, Gel and Joseph L. Gastwirth. *Random Walk, Sequential Analysis and Related Topics: A New Test of Symmetry about an Unknown Median.* WSPC. Published: 06/12/2006.

[4] Sir Ronald A. Fisher. *The Design of Experiments (4th ed.).* Oliver and Boyd. Published: 1947.

6 Appendix

6.1 Additional Descriptions of Strategies

6.1.1 Alun's Strategy

The expected number of shotguns are the backbone of this player, the strategy has been developed such that the current amount of shotguns, dice left and dice selected (which were footsteps) are all factors that are considered to give the expected shotguns for each throw of the dice. The player checks the expected shotguns if all dice left (DL) are thrown, this is denoted by *ExpAll*. The equation for *ExpAll* is:

$$ExpAll = \sum DL_G \cdot \frac{1}{6} + \sum DL_Y \cdot \frac{1}{3} + \sum DL_R \cdot \frac{1}{2}$$

The subscripts denote the colour of the dice. Due to linearity of expectation, the expected value of shotguns for all remaining dice can be scaled down to 1, 2 or 3 dice dependent on the amount of footsteps(F) achieved in the previous throw.

$$E[aX] = aE[X]$$

Therefore for each scenario, *ExpAll* needs to be divided by the amount of dice left in the cup and multiplied by how many dice are needed. If footsteps are thrown in a previous throw, the expected shotguns when throwing these again are calculated as *ExpF* in the function. The formula for expected shotguns from footsteps is:

$$ExpF = \sum F_G \cdot \frac{1}{6} + \sum F_Y \cdot \frac{1}{3} + \sum F_R \cdot \frac{1}{2}$$

There are special cases when there are insufficient dice left to pick from. During these, the expected shotguns need to be calculated when there are 13 dice back in the cup. The expected shotguns with 13 dice is $\frac{23}{6}$ and is denoted by *ExpDZero*, this of course needs to be scaled appropriately to the situation as before for *ExpAll*. As there were only a few of these special cases, they could be structured separately. To start, the situations are split up by the amount of Footsteps gathered, then by the number of dice left.

As a collection, *ExpAll*, *ExpF* and *ExpDZero* can give a value for the expected shotguns for the next throw. Alongside the shotguns acquired already, the total expected shotguns after the next throw can be calculated. Setting the total expected shotguns to be less than a certain value (*Risklevel*) allows for some variety in the risk allowed while playing. It was found that a *Risklevel* of 2.4 was optimal through trial and error during preliminary trials. Further decisions were also added for the level of *Risklevel*. These decisions were based on the brains collected, dice left and other players score.

The decisions for the *Risklevel* are all considered by the player.

6.1.2 James' Strategy

This player function starts off by adopting three different small strategies depending on the score.

For the first strategy the player adopts a simple strategy for when their score is equal to zero. The player looks at how many shotguns they have collected, if it is zero they carry on playing. Otherwise they stop playing after collecting a certain number of brains depending on whether they have collected one or two shotguns.

The second strategy is for when a different player reaches 13 or more brains prior to this player's turn. In this scenario, the player will keep on playing until they have collected enough brains on their turn to equal or beat the maximum score. If this is the case and if they are the last player to have their turn, they stop playing. If they are not the last player, they will observe the players going after them (and their scores). If they have a high score, the player stops playing after collecting enough brains to beat a score of 15.

The third strategy is used if successive players have a really high score and the player has a score of 9 or more. In this case, the player stops playing after collecting enough brains to beat a score of 14.

After this the player calculates the probabilities of getting different combinations of dice from the dice that are remaining. This may be the probability of picking up one dice, two dice or three dice. This is achieved through conditional probability. The player has a set of predetermined combinations of three dice that they think is a good combination to roll; these are three green dice, two green and a yellow, two green and a red, two yellows and a green and three yellows (order does not matter). The rest of the combinations are considered to be bad.

The players main strategy initially looks at how many shotguns they have collected. If they have zero shotguns, they carry on playing. If they have one shotgun the player will then look at the number of footsteps they have collected. If they have none, they will see if the probability of rolling a good combination of dice is greater than rolling a bad combination. This will determine the number of brains the player stops at. This is the process for when the player has one footstep and two footstep. They will look at the colour of the footsteps that they will be re-rolling and determine whether or not the probability of them picking out the required dice to make a good combination is greater than the bad combination. The number of brains the player stops at will then be determined.

There are also specific scenarios for when the player has one or two dice left. In the case where the player knows exactly which dice they will be throwing, they know whether or not they are rolling a good combination and what brains they would want to stop at. For the scenario when the player has one dice left and one footstep, they will look at the footsteps they have and then look at the dice they have left. Following this, based on the likelihood of getting a good combination of dice, they will determine the number of brains to stop at. This is the same for all cases when the player will be selecting out of the refilled cup of dice. The strategy for when the player has collected two shotguns is exactly the same as for one shotgun , varying after collecting less brains where it will be more more conservative.

6.1.3 Niamh's Strategy

The player keeps track of what dice have or haven't been thrown. This helps the player calculate the probabilities of brains and shotguns for different combinations of dice. To calculate this, the twenty-seven combinations of rolls are accounted for.

The player then divulges into its main strategies, these include: the race to the finish strategy, the catch up strategy and the regular playing strategy. If the player has the maximum score which is greater than or equal to twelve, then it checks if it is the last player or not. If it is, it only tries to obtain one more brain and ends it turn to automatically win the game. If it isn't, it checks if any of the other players have a score greater than nine (i.e. ones that could catch up) and tries to gain a larger amount of brains to increase its chances of securing the win. Otherwise, it simply stops after a number of brains.

The catch up strategy is followed for when the player has the minimum score and the maximum score is larger by more than or equal to three points (substantially behind, essentially). Depending on how many shotguns have already been rolled, the player uses the probability that has been calculated to judge when to stop. If the probability of a brain is twice that of a shotgun and it has collected zero shotguns so far, it doesn't stop until it has gained seven brains that turn. A similar approach is followed for the probabilities where they are equally likely or a shotgun is more likely. When two shotguns have been rolled, the stopping points for brains are reduced by one. When two have been rolled, it stops at one brain regardless of probability.

The regular playing strategy is similarly split into scenarios for the number of shotguns that have been rolled. For each scenario, extreme cases are also observed. For example, if there are no red or yellow dice left, the stopping point is a very large amount of brains because they are highly likely. The other extreme cases are for when there are no red or green left or no yellow and green left (stop at a much lower number of brains). The general cases are then looked into as seen previously, if the probability of brains is a certain scaling of that of shotguns, then follow a certain stopping point. If the probability of a brain is much smaller than that of a shotgun, then stop at a fewer number of brains. For when one or more shotgun has been rolled, the stopping points are reduced accordingly.

6.2 Hypothesis Tests for Average Brains and Shotguns

Random Player

Average Brains
- Symmetry Test: Test statistic: 25.673, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.063$, H_1: $\mu \neq 3.063$, test statistic: 2.122143, p-value: 0.98 and so we accept the null hypothesis

Average Shotguns
- Symmetry Test: Test statistic: 3.321, p-value: 0.022 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.266$, H_1: $\mu \neq 1.266$, test statistic: 1.450159, p-value: 0.9578 and so we do not reject the null hypothesis

Strategy One

Average Brains
- Symmetry Test: Test statistic: 29.448, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.235$, H_1: $\mu \neq 3.235$, test statistic: 0.1986075, p-value: 0.9992 and so we do not reject the null hypothesis

Average Shotguns
- Symmetry Test: Test statistic: 21.92, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.289$, H_1: $\mu \neq 1.289$, test statistic: 1.772639, p-value: 0.937 and so we do not reject the null hypothesis

Strategy Two

Average Brains
- Symmetry Test: Test statistic: 66.286, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.807$, H_1: $\mu \neq 3.807$, test statistic: 5.904134, p-value: 0.9806 and so we do not reject the null hypothesis

Average Shotguns
- Symmetry Test: Test statistic: 172.45, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 2.125$, H_1: $\mu \neq 2.125$, test statistic: 0.9427128, p-value: 0.9668 and so we do not reject the null hypothesis

Strategy Three

Average Brains
- Symmetry Test: Test statistic: 38.788, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 4.336$, H_1: $\mu \neq 4.336$, test statistic: 0.966619, p-value: 0.9942 and so we do not reject the null hypothesis

Average Shotguns
- Symmetry Test: Test statistic: 2.521, p-value: 0.01 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 2.338$, H_1: $\mu \neq 2.338$, test statistic: 0.1067619, p-value: 0.9952 and so we do not reject the null hypothesis

Strategy Four

Average Brains

- Symmetry Test: Test statistic: 59.508, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.612$, H_1: $\mu \neq 3.612$, test statistic: 10.86655, p-value: 0.9612 and so we do not reject the null hypothesis

Average Shotguns

- Symmetry Test: Test statistic: -87.03, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.862$, H_1: $\mu \neq 1.862$, test statistic: 5.039056, p-value: 0.9232 and so we do not reject the null hypothesis

Strategy Five

Average Brains

- Symmetry Test: Test statistic: 6.4855, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.023$, H_1: $\mu \neq 3.023$, test statistic: 11.60436, p-value: 0.9226 and so we do not reject the null hypothesis

Average Shotguns

- Symmetry Test: Test statistic: -38.487, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.601$, H_1: $\mu \neq 1.601$, test statistic: 6.832356, p-value: 0.9006 and so we do not reject the null hypothesis

"Optimal" Strategy

Average Brains

- Symmetry Test: Test statistic: 31.45, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.416$, H_1: $\mu \neq 3.416$, test statistic: 4.483743, p-value: 0.9796 and so we do not reject the null hypothesis

Average Shotguns

- Symmetry Test: Test statistic: -26.657, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.708$, H_1: $\mu \neq 1.708$, test statistic: 4.524146, p-value: 0.8954 and so we do not reject the null hypothesis

Alun's Player Function

Average Brains

- Symmetry Test: Test statistic: 63.165, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.852$, H_1: $\mu \neq 3.852$, test statistic: 5.906619, p-value: 0.9748 and so we do not reject the null hypothesis

Average Shotguns

- Symmetry Test: Test statistic: 9.002, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 2.011$, H_1: $\mu \neq 2.011$, test statistic: 4.39554, p-value: 0.9318 and so we do not reject the null hypothesis

James' Player Function

Average Brains

- Symmetry Test: Test statistic: 27.238, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.637$, H_1: $\mu \neq 3.637$, test statistic: 3.424569, p-value: 0.9826 and so we do not reject the null hypothesis

Average Shotguns

- Symmetry Test: Test statistic: -78.773, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.878$, H_1: $\mu \neq 1.878$, test statistic: 7.905403, p-value: 0.867 and so we do not reject the null hypothesis

Niamh's Player Function

Average Brains

- Symmetry Test: Test statistic: 42.76, p-value < 2.2e-16 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 3.724$, H_1: $\mu \neq 3.724$, test statistic: 2.25018, p-value: 0.9878 and so we do not reject the null hypothesis

Average Shotguns

- Symmetry Test: Test statistic: -3.1265, p-value: 0.01 and so we reject the null hypothesis
- Permutation Test: H_0: $\mu = 1.996$, H_1: $\mu \neq 1.996$, test statistic: 2.582431, p-value: 0.9524 and so we do not reject the null hypothesis

6.3 Rounds for the Standard Strategies

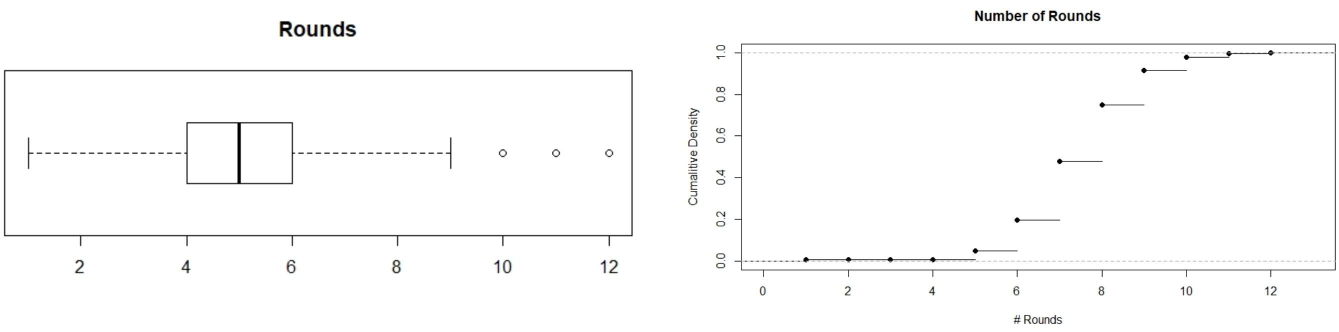

Figure 8: Boxplot and ECDF Plot of the Number of Rounds for the Standard Strategies

6.4 Regression Analysis for the Standard Strategies

This section will cover the regression analysis of the winning score, denoted S, and the number of rounds a game takes, denoted R. The data collected for this analysis will be from the 100,000 trials of the Standard Strategies. We will be testing to see if there is a linear relationship between S and R. For our first model the first regression equation will be:

- $S = \beta_0 + \beta_1 \times R + \epsilon$

By running the model we observe the following information:

- The estimated coefficients of the model are $\hat{\beta_0} = 13.958936$ and $\hat{\beta_1} = 0.151232$
- The rounds R and the intercept are highly significant in determining the score
- Within the hypothesis tests for the coefficients $H_0 : \beta_0 = 0$ vs $H_1 : \beta_0 \neq 0$ and $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$, the null hypotheses were rejected at a 5% significance level
- The coefficient of determination $R^2 = 0.013$

The value for R^2 tell us that the relationship between the winning score and the number of rounds it takes to finish a game is not linear. Additionally, the assumption of normality for residual values is not satisfied as displayed in:

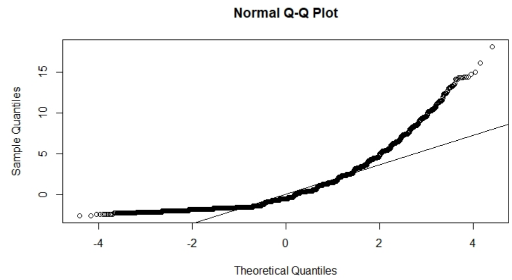

Figure 9: Q-Q Plot of the Residual Values of the First Model

As this assumption was not satisfied we tried changing our model by transforming both variables, R and S. After running various different models, the model we got closest to meeting the assumption of normality for residual values is displayed below:

- $1/log(S)^2 = \beta_0 + \beta_1 \times \sqrt{R}$

As displayed in Figure 10, this model still failed the assumption of normality for residual values. Additionally, $R^2 = 0.01498$, therefore, this model is also not suitable for linear regression. None of the models we look at were suitable for linear regression for S and R and we were not able to find one.

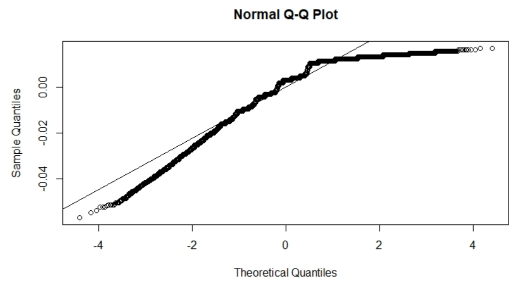

Figure 10: Q-Q Plot of the Residual Values of the Second Model