The Underemployment Trap

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Online Appendix

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A Supplementary Empirical Appendix

A.1 Further Details on Occupation Skill Requirements

To measure the distance in skill requirements between occupations, we start by measuring the occupation's requirement along three different skill dimensions. Specifically, each occupation is represented by a three-dimensional vector $(r_{verbal}, r_{math}, r_{social})$ where r_{verbal} measures the occupation's verbal skill requirement, r_{math} measures the math/quantitative skill requirement, and r_{social} captures the social skill requirement.

To measure verbal and mathematical skill requirements, we strictly follow the methodology used by Guvenen et al. (2020). The first step is to construct four scores for each occupation. The scores are: (i) word knowledge, (ii) paragraph comprehension, (iii) arithmetic reasoning, and (iv) mathematics knowledge. To construct these scores, we first select 26 O*NET descriptors that are chosen by the Defense Manpower Data Center (DMDC) and are listed in the top part of Table A1. In the raw data, these descriptors range in value from 0 to 5. We re-scale their values in each year to fall between 0 and 1 and then take the average value for each descriptor between 2003 and 2011. Finally, we construct a weighted average in each of the four skill categories using the weights matrix provided by the DMDC. For example, to construct the word knowledge score in occupation o, $S_{o,wk}$, we compute

$$S_{o,wk} = \sum_{i=1}^{26} s_{o,i} * \omega_{wk,i}, \tag{A.1}$$

where $s_{o,i}$ is descriptor i's average value between 2003 and 2011 for occupation o and $\omega_{wk,i}$ is the weight given to descriptor i in the category of word knowledge.

Second, we normalize the standard deviation of each score to one and reduce these four scores into two composite indicators, r_{verbal} and r_{math}, by applying principal component analysis (PCA). To be specific, the verbal skill is the first principal component of word knowledge and paragraph comprehension, and the math skill is the first principal component of arithmetic reasoning and mathematics knowledge. The verbal and math skills are then converted into percentile ranks among all occupations as the scale of each component is somewhat arbitrary.

The social skill requirement can be identified similarly. By applying PCA to six scaled O*NET descriptors, we construct a single index to reflect the social skill requirement and then apply the percentile transform as above. The six descriptors used to construct the social skill requirement are listed at the bottom of Table A1. Based on the skill requirement along each dimension $(r_{verbal}, r_{math}, r_{social})$, we proceed to calculate the average skill requirement for each occupation by taking the unweighted average across the three di-

Table A1: List of Descriptors

Panel A: Verbal and Math Skills	
Oral Comprehension	Written Comprehension
Deductive Reasoning	Inductive Reasoning
Information Ordering	Mathematical Reasoning
Number Facility	Reading Comprehension
Mathematics Skill	Science
Technology Design	Equipment Selection
Installation	Operation and Control
Equipment Maintenance	Troubleshooting
Repairing	Computers and Electronics
Engineering and Technology	Building and Construction
Mechanical	Mathematics Knowledge
Physics	Chemistry
Biology	English Language
Panel B: Social Skills	
Social Perceptiveness	Coordination
Persuasion	Negotiation
Instructing	Service Orientation

mensions.

Next, we examine the relationship between skill and education requirements. Table A2 lists the skill and education requirements of the five most common college and non-college jobs in our sample. College jobs are associated with higher skill requirements along each skill dimension, as well as the average skill requirement. Figure A1 further demonstrates by plotting the average skill requirement among non-college and college jobs for verbal, math, social and average skill requirements. The same pattern emerges. In particular, college jobs have significantly higher skill requirements.

Finally, Figure A2 presents a heat map demonstrating the correlation between skill and education requirements. Darker shades of red indicate a stronger positive correlation. The first column represents the percentage of respondents in the O*NET surveys who state that a bachelors degree or higher is needed to perform a certain occupation. The second column is a binary variable that indicates whether more than 50% of respondents indicate that a bachelors degree or higher is necessary to perform the occupation. The results show a positive correlation between education and skill requirements.

Table A2: Skill Requirements of Five Most Common College/Non-college Jobs

Occupation title	Verbal	Math	Social	Avg.
Panel A: College jobs				
Elementary and middle school teachers	0.82	0.77	0.85	0.81
Registered nurses	0.76	0.67	0.80	0.74
Accountants and auditors	0.64	0.86	0.33	0.61
Secondary school teachers	0.84	0.81	0.92	0.85
Social workers	0.24	0.14	0.96	0.44
Panel B: Non-college jobs				
First-line supervisors/managers of retail sales workers	0.33	0.44	0.56	0.44
Retail salespersons	0.10	0.21	0.20	0.17
Sales representatives, wholesale and manufacturing	0.28	0.37	0.67	0.44
Secretaries and administrative assistants	0.40	0.23	0.18	0.27
Customer service representatives	0.34	0.31	0.30	0.32

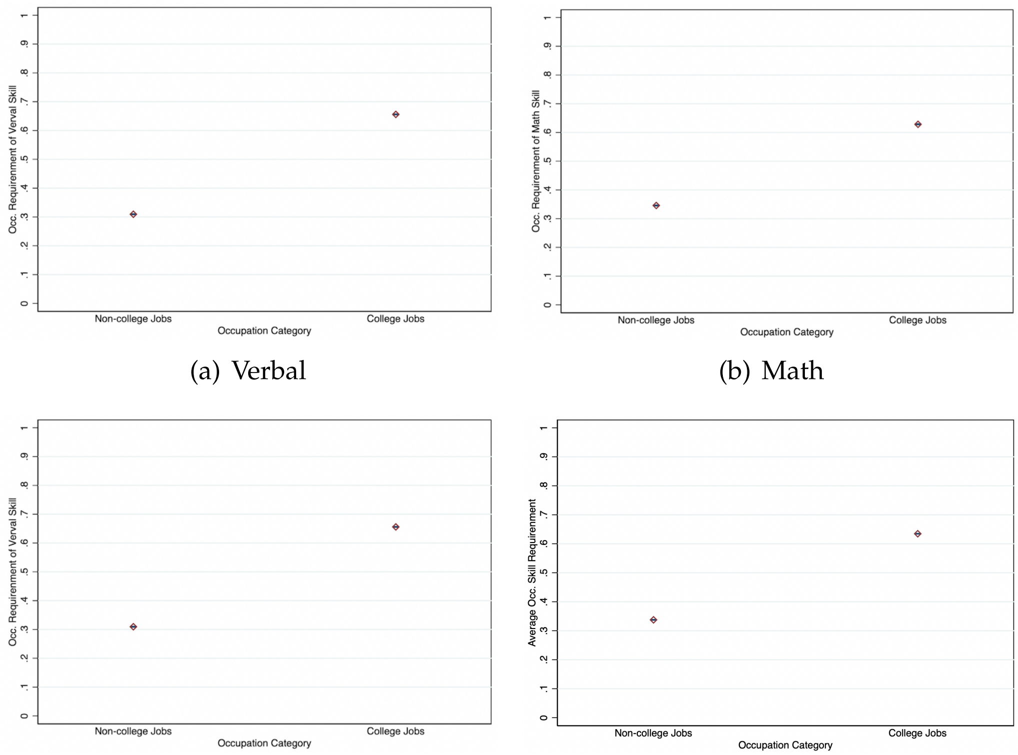
A.2 Skill Distance and the Scarring Effects of Underemployment

To further support the notion of the accumulation and decay of occupation-specific human capital, we study how the scarring effects of underemployment vary with the distance in skill requirements between a worker's current college occupation and previous non-college occupation. The idea here is that if the distance in required skills between the two occupations is larger, then the skills required by the new college occupation would have been used less intensively in the previous non-college occupation and thus experienced a greater rate of decay, ultimately leading to larger scarring effects. To assess this hypothesis, we estimate the following regression:

$$w_{imt} = \alpha \text{Underhis}_{imt} + \gamma \phi_{imt} + \zeta \text{Underhis}_{imt} \times \phi_{imt} + \Gamma \cdot X_{imt} + \delta_i + \varepsilon_{imt}, \tag{A.2}$$

where ϕ_{imt} is the distance in skill requirements between individual i's current college occupation and their most recent non-college occupation and X contains the same controls as in equation (4) in the main text. We use two measures of distance. The first is the Euclidean distance while the second is the angular distance as in Baley et al. (2022). The estimation of (A.2) only includes observations among individuals currently employed in a college occupation and who have been previously underemployed. Moreover, we restrict to those individuals where the average skill level in their current college occupation is higher than their previous non-college occupation.

Table A3 contains the results. Column (1) reveals that a larger Euclidean distance is associated with significantly higher scarring effects. In particular, the average Euclidean



(a) Verbal

(b) Math

(c) Social

(d) Average

Figure A1: Comparison of Skill Requirements among Non-college Jobs Versus College Jobs

Notes: Graph shows 95% confidence intervals. We test the null hypothesis that the verbal/math/social/average skill requirement of non-college jobs is the same as that of college jobs against the alternative that the skill requirement of non-college jobs is below that of college jobs, and the test results indicate a p-value less than 0.01, supporting the alternative hypothesis.

distance of 0.75 in transitions between non-college and college jobs leads to a 3.4% rise in wage scars due to additional underemployment in previous non-college jobs. Column (2) echoes this result by examining the angular distance between skill requirement vectors. An average angular distance of 28.04 in transitions between non-college and college jobs results in an 4.8% increase in wage scars caused by additional underemployment in previous non-college jobs.

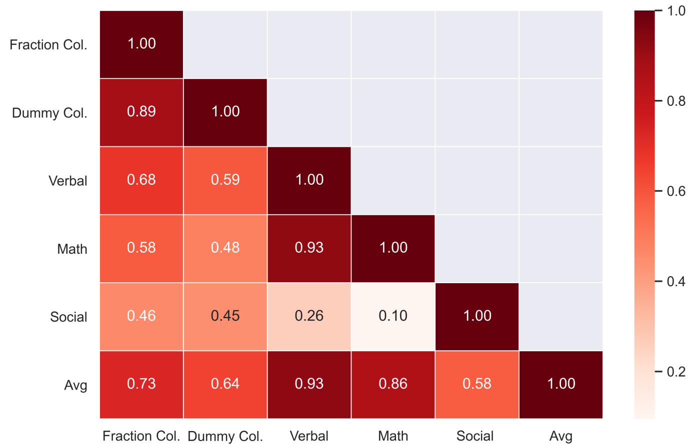


Figure A2: Correlation Between Education and Skill Requirements

Table A3: Skill Distance and the Scarring Effects of Underemployment

	(1)	(2)
Underhis	0.0173	0.0374***
	(0.0106)	(0.0098)
Underhis*Euclidean distance	-0.0454***	
	(0.0103)	
Underhis*Angular distance		-0.0017***
		(0.0004)
N	16,594	16,594
R^2	0.924	0.923

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

A.3 Supplementary Tables and Figures

Table A4: Descriptive Statistics

	N	Mean	Std. Dev	Min	Max
Gender	996	0.54	0.50	0	1
– Male	439	0	0	0	0
– Female	557	1	0	1	1
Birth year	996	1982.06	1.40	1980	1984
– 1980	176	-	-	-	-
– 1981	207	-	-	-	-
– 1982	202	-	-	-	-
– 1983	204	-	-	-	-
– 1984	207	-	-	-	-
Race	996	3.24	1.19	1	4
– Black	157	1	0	1	1
– Hispanic	135	2	0	2	2
– Mixed race (Non-Hispanic)	14	3	0	3	3
– Non-Black / Non-Hispanic	690	4	0	4	4
ASVAB percentile	882	69.25	23.11	3	100
Region	230,791	2.64	1.01	1	4
– Northeast	39,460	1	0	1	1
– North Central	55,447	2	0	2	2
– South	85,363	3	0	3	3
– West	50,521	4	0	4	4
Highest degree	1,112	4.42	0.73	4	7
– BA	740	4	0	4	4
– MA	214	5	0	5	5
– PhD	9	6	0	6	6
– Professional degree	33	7	0	7	7
Major	994	0.34	0.475	0	1
– Arts and Social Sciences	723	0	0	0	0
– STEM	271	1	0	1	1
Employed hours	206,177	44.28	11.30	10	98
Real wage	206,872	17.61	21.59	1.01	519.65
Potential experience (in months)	232,953	42.92	26.39	1	127
Student loan currently owed	232,953	0.26	2.60	0	120
Family income per-capita	212,420	37.71	38.62	0	421.37
College GPA	232,661	3.23	0.41	1.88	5

Table A5: Occupations within 5 Percentage Points of the 50% Threshold

Occupation title	Freq. of college requirement
Geological and petroleum technicians	46.19
Other life, physical, and social science technicians	48.72
Sales representatives, wholesale and manufacturing	49.76
Designers	50.13
Directors, religious activities and education	50.97
Religious workers, all other	50.97
Cost estimators	51.03
Producers and directors	51.57
Construction managers	52.10
Judges, magistrates, and other judicial workers	53.40
Writers and authors	53.82
Other business operations specialists	54.15
Network systems and data communications analysts	54.49

Table A6: Top 10 College and Non-college Occupations

College occupations	*N*	Non-college occupations	*N*
Elementary/middle school teachers	11,771	Managers of retail sales workers	6,788
Registered nurses	5,990	Retail salespersons	3,806
Accountants and auditors	5,762	Sales representatives	3,297
Secondary school teachers	5,761	Secretaries and admin. assistants	3,136
Social workers	4,703	Customer service representatives	2,978
Managers, all other	3,989	Police and sheriff's patrol officers	2,973
Financial managers	3,559	Managers of office and admin. support workers	2,692
Other teachers and instructors	3,517	Waiters and waitresses	2,481
Computer software engineers	3,396	Cashiers	1,879
Marketing and sales managers	3,225	Loan counselors and officers	1,866

Table A7: Underemployment and College Major

Major	N	Respondents	Underemp. ratio
A: Arts and Social Sciences			
Liberal arts and science	104	2	0.337
International relations and affairs	156	1	0.122
Social work	187	1	0.989
Archaeology	291	1	0.808
Hotel/Hospitality management	500	3	0.790
Pre-law	531	2	0.452
Other – Recoded to human services	578	3	0.351
Home economics	595	4	0.606
Area studies	709	2	0.234
Anthropology	709	6	0.068
Theology/religious studies	1,148	5	0.462
Philosophy	1,361	5	0.505
Foreign languages	2,244	8	0.311
English	4786	24	0.335
Political science and government	5,422	26	0.375
Economics	5,636	16	0.265
History	5,832	32	0.482
Nursing	6,378	28	0.035
Sociology	6,905	31	0.283
Criminology	7,022	31	0.573
Fine and applied arts	11,928	45	0.635
Psychology	16,015	69	0.398
Communications	17,910	68	0.442
Education	20,574	99	0.242
Business management	56,538	211	0.484
All Arts and Social Sciences	174,059	723	**0.415**
B: STEM			
Pre-vet	156	1	0.865
Nutrition/dietetics	365	2	0.399
Pre-med	448	4	0.213
Agriculture/natural resources	2,163	8	0.626
Mathematics	2,621	13	0.491
Interdisciplinary studies	2,622	12	0.387
Physical sciences	3,131	16	0.262
Architecture/environmental design	3,132	15	0.213
Other health professions	7,982	38	0.393
Biological sciences	10,430	49	0.251
Computer/information science	12,634	52	0.446
Engineering	13,096	61	0.312
All STEM	58,780	271	**0.357**

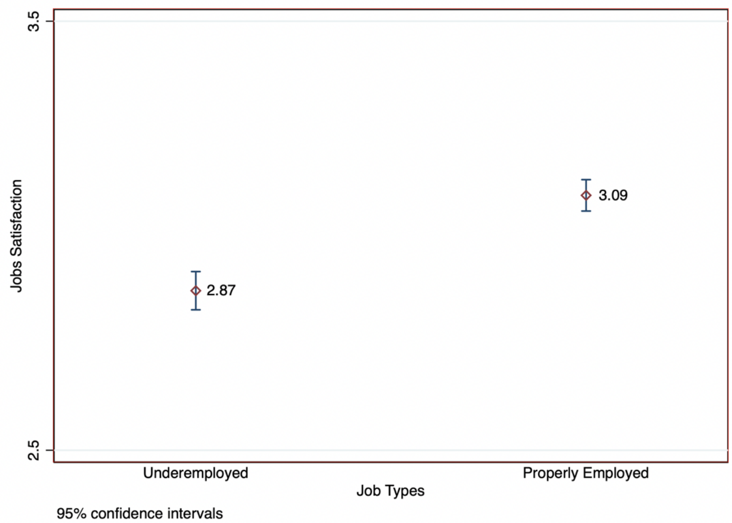
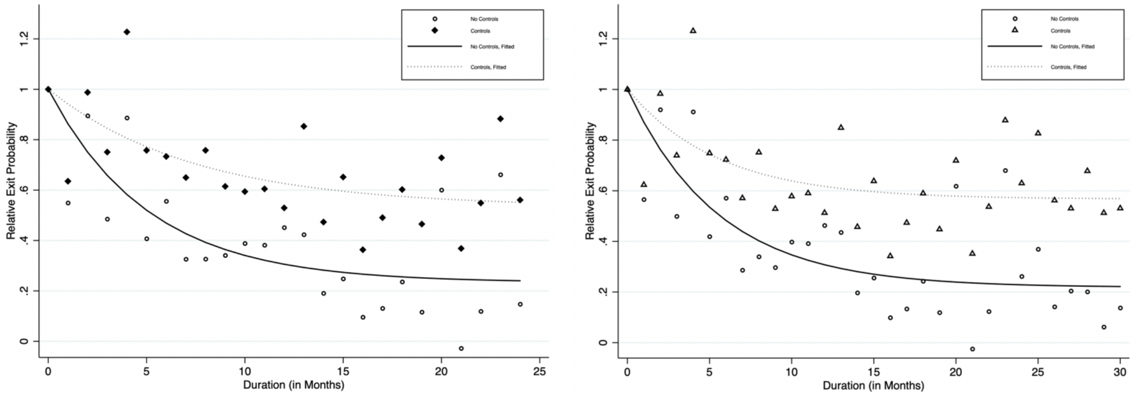


Figure A3: Job Satisfaction among Underemployed versus Properly Employed

Notes: We test the null hypothesis that the job satisfaction is equal among the underemployed versus the properly employed against the alternative that the job satisfaction among the properly employed is different from the job satisfaction among the underemployed. The p-value is less than 0.01, indicating significant differences in job satisfaction between the two groups.



(a) Duration Dependence (Extra transition time) (b) Duration Dependence Capped at 30 Months

Figure A4: Duration Dependence

Table A8: Scarring Effects (1-digit Industry and Occupation Codes)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0166***		-0.0164***	-0.0148***		-0.0145***
	(0.0010)		(0.0010)	(0.0011)		(0.0010)
Underhis		0.0006***	0.0006***		0.0010***	0.0009***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × *Unhis*				-0.0011		-0.0002
				(0.0013)		(0.0013)
College × Underhis					-0.0023***	-0.0022***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.778	0.777	0.778	0.774	0.774	0.774

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 1-digit level), and region.

Table A9: Scarring Effects (1-digit Occupation FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0154***		-0.0153***	-0.0153***		-0.0150***
	(0.0010)		(0.0010)	(0.0011)		(0.0011)
Underhis		0.0004***	0.0004***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				0.0002		0.0008
				(0.0014)		(0.0014)
College × Underhis					-0.0022***	-0.0021***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172149	172149	172149	172149	172149	172149
R^2	0.784	0.784	0.784	0.784	0.784	0.785

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 1-digit level), and region.

Table A10: Scarring Effects (2-digit Occupation FE)

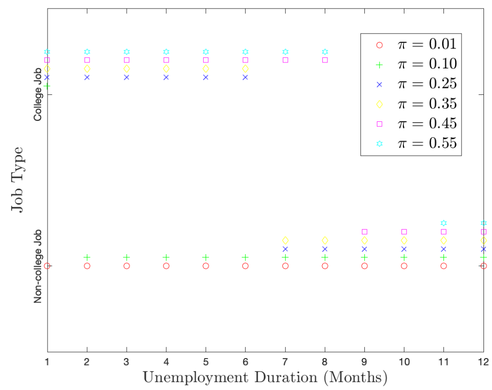
	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0144***		-0.0141***
	(0.0009)		(0.0009)	(0.0010)		(0.0010)
Underhis		0.0003***	0.0003***		0.0005***	0.0005***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				-0.0004		-0.0000
				(0.0013)		(0.0013)
College × Underhis					-0.0017***	-0.0016***
					(0.0001)	(0.0001)
2-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172,149	172,149	172,149	172,149	172149	172,149
R^2	0.791	0.790	0.791	0.791	0.791	0.791

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and region.

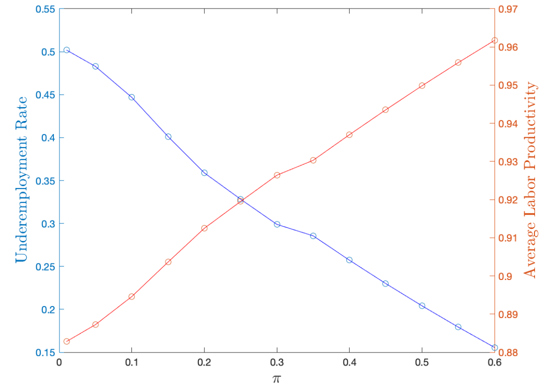
Table A11: Scarring Effects (Year and Month FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0144***		-0.0143***	-0.0137***		-0.0134***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				-0.0012		-0.0007
				(0.0013)		(0.0013)
College × Underhis					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
2-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.792	0.791	0.792	0.783	0.783	0.784

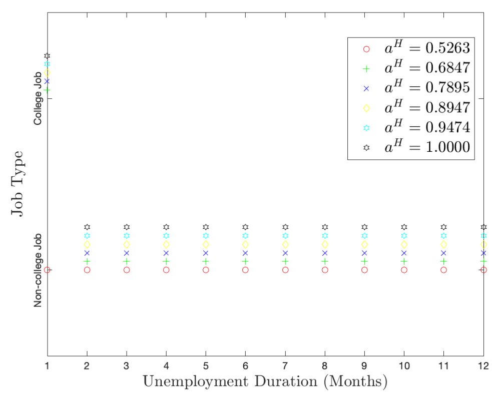
Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for calendar year, calendar month, individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and regions.



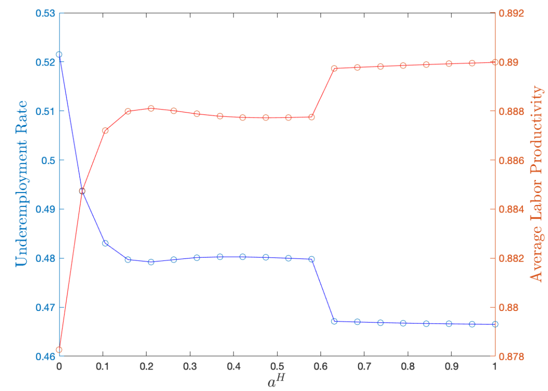
(a) Policy Function of Unemployed Workers (π)



(b) Underemployment and Productivity (π)



(c) Policy Function of Unemployed Workers (a^H)



(d) Underemployment and Productivity (a^H)

Figure A5: Comparative statics. Composition of Workers (π) and Suitability of Type H workers (a^H)

Table A12: Residual Wage Dispersion

	90/50 ratio	50/10 ratio
Full sample	1.609	1.739
A: After the first underemployment spell		
$<$ 1 year to exit underemployment	1.411	1.505
$\geq$ 1 year to exit underemployment	1.244	1.285
B: Proper employment spell following underemployment		
$<$ 1 year to exit underemployment	1.342	1.414
$\geq$ 1 year to exit underemployment	1.281	1.334

Notes: Regression specification controls for unemployment history, a cubic in potential experience, individual, regional, and 2-digit occupation fixed effects.

Table A13: Wage Growth with All College Jobs Observations

	(1)	(2)	(3)	(4)
Long	-0.0146*	-0.0143*	-0.0135	-0.0145*
	(0.0083)	(0.0082)	(0.0086)	(0.0088)
N	2045	2045	1998	1998
R^2	0.023	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. Results are robust to clustering at the birth year, or contemporary year levels. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table A14: Wage Growth with Consideration of Previous Underemployment Spell

	(1)	(2)	(3)	(4)
Long	-0.0184**	-0.0178**	-0.0163*	-0.0172*
	(0.0084)	(0.0084)	(0.0088)	(0.0091)
N	2045	2045	1998	1998
R^2	0.024	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. Results are robust to clustering at the highest education, or contemporary year levels. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

A.4 Empirical Moments for Calibration

A.4.1 Wage Premium

Table A15 contains results from estimating equation (25) in the main text. Each respective column represents a different combination of control variables and fixed effects as indicated in the table. Column (4) represents our preferred specification that is used to calibrate the model in Section 5.2.

Table A15: The Wage Premium of College Jobs

	(1)	(2)	(3)	(4)
College Job	0.3849***	0.3857***	0.2600***	0.2597***
	(0.0192)	(0.0196)	(0.0218)	(0.0223)
Exp	-0.0087***	-0.0026	0.0013	-0.0062***
	(0.0023)	(0.0020)	(0.0019)	(0.0021)
Exp^2	0.0002***	0.0000	-0.0000	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	-0.0000	0.0000	0.0000**	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annual Urate				-0.0179**
				(0.0070)
Annual Urate				-0.0083
				(0.0064)
Age				0.4603***
				(0.0920)
Age^2				-0.0084***
				(0.0018)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,085	10,988	10,988	10,988
R^2	0.843	0.853	0.894	0.894

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of *College*. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

A.4.2 Unemployment and Underemployment Rates

The calculation of the unemployment and underemployment rates involves a three-step procedure. First, the number of individuals in the categories of unemployment, underemployment, and proper employment are counted for each week. Second, the proportion of each labor status category is calculated by dividing the headcount of each category by the total headcount observed during that week. Finally, both an unweighted average and a weighted average are computed across all weeks, with the number of observations in that week serving as the weighting factor. Table A16 contains the results.

Table A16: Composition of Labor Force Statuses

	Unemployed	Underemployed	Properly employed	NILF
Unweighted	0.032	0.408	0.497	0.063
Weighted	0.027	0.416	0.503	0.054

Table B.1: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.050	$\partial \log(w_n)/\partial v$	-0.014	-0.014
Underemployment rate	0.416	0.416	$\partial \log(w_c)/\partial v$	-0.014	-0.014
Average EE probability	0.014	0.011	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.256	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.710	-	-	-

B Model with Permanent Skill Loss

In an effort to gain further insight into the contribution of skill loss to duration dependence, we alter the full quantitative model to include permanent skill loss by setting the probability of regaining college-specific skills, ϕ, to zero. We re-calibrate the model with $\phi = 0$ and proceed to perform the same decomposition exercises as in the main text.

B.1 Calibration

The calibration procedure is identical to that of the full model outlined in the main text, except for setting $\phi = 0$. The comparison between the model and target moments can be found in Table B.1, and the model-generated path is shown in Figure B.1(a). Except for the unemployment rate, the calibrated model closely matches the empirical targets. The updated parameters are reported in Table B.2.

Table B.2: Parameter Values with Permanent Skill Loss

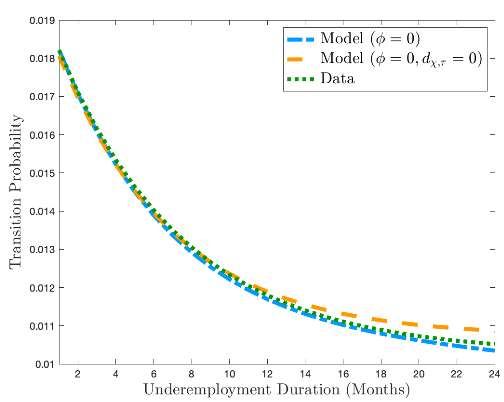
	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.010	a^H	Suitability pr.: type H	0.357
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.048
g_n	Non-college productivity	0.751	ϕ	Pr. of regaining college skills	0.000
b	Utility while unemployed	0.609	$d_{c,v}$	College skill loss: unemp.	-0.014
k_n	Non-college vacancy cost	2.332	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	1.343	$d_{n,v}$	Non-college skill loss: unemp.	-0.014
λ	Employed search intensity	0.761	$d_{n,\tau}$	Growth of non-college skills	0.001

B.2 Decomposition

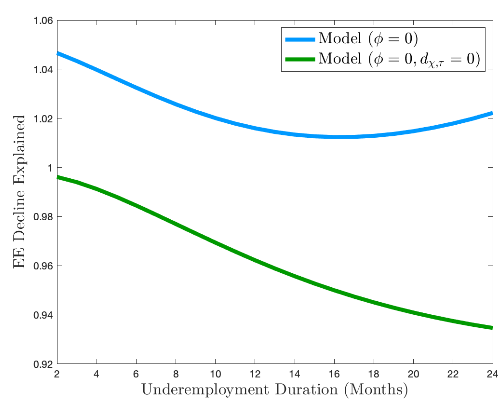
In this section, we evaluate the relative contributions of unobserved heterogeneity and changes to occupation-specific human capital in generating duration dependence when the skill loss is permanent. Beginning with Figure B.1(a), we present the path of the transition probability from non-college to college occupations from the data, a full version of the model with permanent skill change, and a version of the model where we shut off the accumulation and decay of occupation-specific human capital during underemployment. In practice, this requires setting $d_{\chi,\tau} = 0$ for $\chi \in X$ while all other parameters remain unchanged.

Next, we examine what proportion of the decrease in the transition probability at each underemployment duration relative to the transition probability at $\tau = 1$ observed in the data can be explained by the full model and the version with unobserved heterogeneity only. As depicted in Figure B.1(b), the duration dependence produced by the full model with permanent skill evolution exceeds that in the data, whereas the model with only unobserved heterogeneity explains at least 94% of the decline.

To obtain an overall decomposition, we calculate the weighted average of the fraction explained by unobserved heterogeneity for all τ. The weights are the proportion of underemployed workers who are employed at each duration τ in the steady state. As a result, we find that the full model can explain 102.3% of the observed decrease in the transition probability from non-college to college jobs. If we eliminate the dynamics of human capital during underemployment, the model with unobserved heterogeneity still explains 94.6% of the duration dependence.



(a) Transition Probability Path



(b) Fraction Explained at each τ

Figure B.1: Duration Dependence Decomposition

C Model with Full Information

This appendix provides further details on the model with full information referenced in Section 5.3 in the main text. For brevity, we only present the details in the environment and equilibrium that are new relative to the baseline model presented in Section 3.

C.1 Environment

Workers learn their suitability type upon entering the labor market. A worker's suitability type is public information. The labor market continues to be organized in a continuum of submarkets. In the full information case, however, submarkets are also indexed by the suitability type of workers searching in that particular submarket. Denoting $A = \{L, H\}$ and an individual's worker suitability type by i, the labor market is now organized in a continuum of submarkets indexed by $\omega = (\chi, i, v, \tau, x) \in X \times A \times \Upsilon \times T \times \mathbb{R}$. That is, in submarket ω, type-χ firms search for a type-i worker with labor market history (v, τ) and offer suitable workers an employment contract worth x in lifetime utility.

C.2 Value Functions

The value functions in the full information version of the model are very similar to those in the baseline model. The main exception is that a worker's suitability type, i, and probability of being suitable for a college job, a^i, take the place of, μ, their expected suitability. Here is the value of an unemployed worker of suitability type i who searches for a non-college job:

$$V_{u,n}(v,i) = b + \beta(1-\delta)\{V_u(\hat{v},i) + R_n(x, V_u(\hat{v},i))\}, \tag{C.1}$$

where

$$V_u(v,i) = \max\{V_{u,n}(v,i), V_{u,c}(v,i)\}, \tag{C.2}$$

is the value of unemployment for a type-i worker with unemployment history v and

$$R_\chi(x, V_u(\hat{v},i)) = \max_x p(\theta(\chi, i, \hat{v}, 0, x))(x - V_u(\hat{v},i)). \tag{C.3}$$

The value of searching for a college job satisfies:

$$V_{u,c}(v,i) = b + \beta(1-\delta)\{V_u(\hat{v},i) + a^i R_c(x, V_u(\hat{v},i))\}. \tag{C.4}$$

The sum of the worker's lifetime utility and firm's profits in a match between a non-college job and type-i worker with history (v, τ) is given by:

$$V_{e,n}(v,\tau,i) = y_n(v,\tau) + \beta(1-\delta)\{V_{e,n}(v,\hat{\tau},i) + \lambda a^i S(v,\hat{\tau},i)\}, \tag{C.5}$$

where

$$S(v,\hat{\tau},i) = \max_x p(\theta(c,i,v,\hat{\tau},x))(x - V_{e,n}(v,\hat{\tau},i)). \tag{C.6}$$

Finally, sum of the worker's lifetime utility and the firm's profits in a match between a college job and a type-i worker with history (v, τ), $V_{e,c}(v,\tau,i)$, satisfies

$$V_{e,c}(v,\tau,i) = y_c(v,\tau) + \beta(1-\delta)\{\phi V_{e,c}(1,0,i) + (1-\phi)V_{e,c}(v,\tau,i)\}. \tag{C.7}$$

C.3 Free Entry

In any submarket visited by a positive number of workers, tightness is consistent with the firm's incentives to create vacancies if and only if

$$k_\chi \geq q(\theta(\chi,i,v,\tau,x))\{V_{e,\chi}(v,\tau,i) - x\}, \tag{C.8}$$

and $\theta(\chi,i,v,\tau,x) \geq 0$ with complementary slackness. We restrict attention to equilibria in which $\theta(\chi,i,v,\tau,x)$ satisfies the complementary slackness condition in every submarket, even those that are not visited by workers. That is

$$\theta(\chi,i,v,\tau,x) = \begin{cases} q^{-1}\left(\frac{k_\chi}{V_{e,\chi}(v,\tau,i)-x}\right), & \text{if } k_\chi = q(\theta(\chi,i,v,\tau,x))\{V_{e,\chi}(v,\tau,i) - x\}, \\ 0, & \text{otherwise.} \end{cases} \tag{C.9}$$

C.4 Laws of Motion

Let $u_i(v)$ denote the measure of workers of suitability type i who begin the period unemployed with unemployment history v. The law of motion is given by

$$\hat{u}_i(v) = \begin{cases} (1-\delta)\delta\pi^i, & \text{for } v = 1, \\ (1-\delta)u_i(v_-)[\varrho_{i,n,v_-}(1-p(\theta^*_{i,n,v_-})) + \varrho_{i,c,v_-}(1-a^i p(\theta^*_{i,c,v_-}))] & \text{for } v \in \{2,\dots,\bar{v}-1\}, \\ (1-\delta)\sum\limits_{v=\bar{v}}^{\bar{v}+1} u_i(v_-)[\varrho_{i,n,v_-}(1-p(\theta^*_{i,n,v_-})) + \varrho_{i,c,v_-}(1-a^i p(\theta^*_{i,c,v_-}))] & \text{for } v = \bar{v}, \end{cases} \tag{C.10}$$

where $\pi^H = \pi$, $\pi^L = 1-\pi$, $\hat{u}_i(v)$ is the measure of unemployed workers with unemployment history v and suitability type i at the beginning of the next period, $v_- \equiv v - 1$,

$\varrho_{i,\chi,v} \in [0,1]$ is the fraction of unemployed workers with suitability type i and unemployment history v who search for type χ jobs, and $\theta^*_{i,\chi,v}$ denotes tightness associated with the policy function of unemployed workers with suitability type i and unemployment history v who search for type χ jobs.

Now let $e_{i,\chi}(v,\tau)$ denote the measure of workers with suitability type i and history (v,τ) who are employed at type χ jobs at the beginning of the period. The law of motion for $e_{i,n}(v,\tau)$ is given by

$$\hat{e}_{i,n}(v,\tau) = \begin{cases} (1-\delta)\varrho_{i,n,v} u_i(v) p(\theta^*_{i,n,v}), & \text{for } \tau = 1, \\ (1-\delta) e_{i,n}(v,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,v,\tau_-})), & \text{for } \tau \in \{2,\dots,\bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_{i,n}(v,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,v,\tau_-})), & \text{for } \tau = \bar{\tau}, \end{cases} \tag{C.11}$$

where $\tau_- \equiv \tau - 1$, $\theta^*_{i,\chi,v,\tau}$ is tightness associated with the policy function of an employed worker with suitability type i and history (v,τ) in a submarket with type χ jobs.

The law of motion for $e_{i,c}(v,\tau)$ is given by

$$\hat{e}_{i,c}(v,\tau) = \begin{cases} (1-\delta)[e_c(v,\tau) + u_i(v)\varrho_{i,c,v} a^i p(\theta^*_{i,c,v}) + \phi(e_{i,c} - e_{i,c}(1,0) + e^*_{i,n} + u^*_{i,c})], & \text{for } v = 1 \text{ and } \tau = 0, \\ (1-\delta)(1-\phi)[u_i(v)\varrho_{i,c,v} a^i p(\theta^*_{i,c,v}) + e_{i,c}(v,\tau)], & \text{for } v \geq 2 \text{ and } \tau = 0, \\ (1-\delta)(1-\phi)[e_{i,n}(v,\tau)\lambda a^i p(\theta^*_{i,c,v,\tau}) + e_{i,c}(v,\tau)], & \text{for } v \geq 2 \text{ and } \tau \geq 1, \end{cases} \tag{C.12}$$

where $e_{i,c} = \sum_{v\in \Upsilon}\sum_{\tau \in T} e_{i,c}(v,\tau)$ is the total measure of type-i workers employed in college jobs to begin the period, $e^*_{i,n} = \lambda \sum_{v\in\Upsilon}\sum_{\tau\in T} e_{i,n}(v,\tau) a^i p(\theta^*_{i,c,v,\tau})$ is the total measure of type-i workers who transitioned from a non-college to college job within the period, and $u^*_{i,c} = a^i \sum_{v=2}^{\bar{v}} u_i(v)\varrho_{i,c,v} p(\theta^*_{i,c,v})$ is the total measure of unemployed workers with unemployment history $v \in \{2,\dots,\bar{v}\}$ who found a college job in the previous period.

C.5 Equilibrium Definition

Definition 1. A stationary recursive equilibrium (RE) consists of a market tightness function $\theta(\omega)\colon X \times A \times \Upsilon \times T \times \mathbb{R} \to \mathbb{R}_+$, a value function for unemployed workers, $V_u(v,i)\colon \Upsilon \times A \to \mathbb{R}$, a policy function for unemployed workers, $\omega^*_u(v,i)\colon \Upsilon \times A \to X \times \mathbb{R}$, a joint value function for the worker-firm match, $V_{e,\chi}(v,\tau,i) : X \times \Upsilon \times T \times A \to \mathbb{R}$, a policy function for the worker-firm match, $\omega^*_{e,\chi}(v,\tau,i)\colon X \times \Upsilon \times T \times A \to X \times \mathbb{R}$, and a distribution of workers across the states of employment. The functions satisfy the following conditions. First, $\theta(\omega)$ satisfies (C.9) for all $\omega \in X \times A \times \Upsilon \times T \times \mathbb{R}$. Third, $V_u(\tau,i)$ satisfies (C.2) for all $(v,i) \in \Upsilon \times A$ and $\omega^*_u(v,i)$ is the associated policy function. Fourth,

$V_{e,n}(v,\tau,i)$ and $V_{e,c}(v,\tau,i)$ satisfy equations (C.5) and (C.7) for all $(v,\tau,i) \in \Upsilon \times T \times A$ and $\omega^*_{e,\chi}(v,\tau,i)$ is the associated policy function. Finally, the distribution of workers satisfies the laws of motion specified in Section C.4.

As in the baseline model, the labor market segments into submarkets. Here, the submarkets are indexed, in part, by the worker's suitability type. Hence, firm entry into each submarket is independent of the distribution of workers across employment statuses and suitability. We conclude the description of the full information model by defining a block-recursive equilibrium.

Definition 2. A block-recursive equilibrium (BRE) is a RE where the value and policy functions are independent of the distribution of workers across the states of employment.

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