The Underemployment Trap*

Jie Duan†
National University of Singapore

Paul Jackson‡
National University of Singapore

April 4, 2023

Abstract

Many college graduates are underemployed, i.e., work in occupations that do not require a college degree. We document that underemployed workers are less likely to transition to a college occupation the longer they are underemployed and that longer underemployment histories are associated with lower wages in college occupations. To explain these findings, we develop a directed search model with unobserved heterogeneity, occupation-specific human capital, heterogeneous firms, and on the job search. Workers are uncertain about their suitability for college jobs and learn through search. Underemployment is generated by search and informational frictions as workers with a low expected job-finding probability in college occupations self-select into underemployment. Once underemployed, workers' college occupation-specific human capital decays. A quantitative decomposition shows that unobserved heterogeneity explains nearly 97% of the duration dependence in underemployment, and that information frictions play a significant role in both the existence of underemployment and the resulting duration dependence.

JEL Classification: E24; J24; J62; J64

Keywords: Underemployment; Duration Dependence; Unobserved Heterogeneity; Human Capital

---

*We thank Titan Alon, Ricardo Caballero, Andrea Chiavari, Ana Figueiredo, Naijia Guo, Andreas Mueller, Michelle Rendall, Serena Rhee, David Wiczer, seminar participants at Curtin University, Monash University, National University of Singapore, University of Aberdeen, UC Irvine, University of Melbourne, and participants at the 2022 Spring Midwestern Macroeconomics Meeting, 2022 Asian and Australasia Econometric Society Meetings, and the Inaugural Search and Matching Pacific in Asia-Pacific for their helpful comments and suggestions.

†Department of Economics, National University of Singapore. Blk AS1, #01-02, 1 Arts Link, Singapore 117570. *Email*: duanjie@u.nus.edu.

‡Department of Economics, National University of Singapore. Blk AS2, #04-22, 1 Arts Link, Singapore 117570. *Email*: ecspgj@nus.edu.sg.

1 Introduction

A significant fraction of recent college graduates in the US are underemployed, i.e., work in jobs that do not typically require a college degree. While underemployment is not a new phenomenon, it has gained considerable attention in the media since the Great Recession with a growing consensus that underemployed graduates are caught in an "underemployment trap", unable to escape their low-wage jobs.[1] Despite its traction in the media, research which studies underemployment is still in its infancy. We know that recent graduates are nearly ten times more likely to be underemployed than unemployed and that the underemployment rate is countercyclical (Barnichon and Zylberberg, 2019).[2] While this recent strand of research has made significant progress, very little is known about the quintessential underemployment duration, whether underemployed graduates are indeed stuck and, if so, what the sources of the underemployment trap are.

This paper studies the features and determinants of underemployment durations by first documenting several new stylized facts. Most prominently, we document negative duration dependence in underemployment. That is, the longer a worker has been underemployed, the less likely they are to transition into an occupation that requires a college degree. We then develop a directed search model which generates both underemployment and negative duration dependence in equilibrium. Finally, we quantitatively decompose duration dependence in underemployment into two classic channels: dynamic selection based on unobserved heterogeneity and structural duration dependence generated through the accumulation and decay of occupation-specific human capital.

We use the National Longitudinal Survey of Youth 1997 (NLSY97) to document three facts. First, the average underemployment duration is nearly eighteen months. Second, the probability an underemployed graduate transitions to a college occupation significantly decreases in the length of their underemployment spell. For example, a worker who has been underemployed for a year is nearly 40% less likely to exit underemployment than a newly underemployed worker. Finally, college graduates with a longer underemployment history (accumulated experience in non-college occupations) earn lower wages in college occupations than their observationally equivalent peers. In particular, one month of additional underemployment history is associated with 0.13% lower wages in college occupations. Accounting for the average length of an underemployment spell, these effects are comparable to the effect of the average unemployment spell on wages.

[1] For a few examples, see "First jobs matter: Avoiding the underemployment trap" by Michelle Weise and "College Grads May Be Stuck in Low-Skill Jobs" by Ben Casselman.

[2] The underemployment rate is typically placed at nearly 40%. See Abel et al. (2014), Barnichon and Zylberberg (2019), BGT and SI (2018), and Jackson (2023).

To explain these facts, we develop a model of underemployment grounded in the environments of Gonzalez and Shi (2010) and Menzio and Shi (2011). Workers enter the labor market and direct their search towards non-college or college jobs. The first key ingredient is that workers can be of either limited- or broad-suitability, where a worker's type determines the probability they will produce output at any given college job. As in Gonzalez and Shi (2010), there is symmetric incomplete information regarding a worker's type and learning occurs through search. A worker who is unsuccessful in searching for a college job becomes more pessimistic about their ability to produce in any given college job. Those with a low expected suitability self-select into underemployment and continue to search on the job. It is at this stage where we introduce the model's second key ingredient: working in non-college jobs leads to both the accumulation of non-college and decay of college occupation-specific human capital. Our formalization of human capital follows the view that human capital is occupation-specific (Kambourov and Manovskii, 2009a,b) and is in line with recent frameworks which model the decay of occupation-specific skills when they are under-utilized (e.g., Lise and Postel-Vinay (2020)).

The model produces a simple optimality condition relating the marginal cost and benefit of an underemployed worker transitioning to a college job and encompasses three channels through which the model generates negative duration dependence. First, workers with a longer underemployment duration are more likely to be limited suitability types and are less likely to match with any given college job. This is the unobserved heterogeneity channel. The second channel is the accumulation of non-college occupation-specific human capital where additional underemployment history makes workers more productive in non-college jobs, thereby reducing the marginal benefit of exiting underemployment. The final channel is the decay of college occupation-specific human capital. Workers with a longer underemployment history are less productive in college jobs, which also reduces the marginal benefit of transitioning from a non-college to college job. Taken together, negative duration dependence is generated through a combination of dynamic selection (the unobserved heterogeneity channel) and structural duration dependence (the accumulation and decay of occupation-specific human capital).

The model is calibrated to match the stylized facts and to decompose the model generated duration dependence into the effect of unobserved heterogeneity and changes to occupation-specific human capital. The model can match well, among other moments, the path of the job-to-job transition probability between non-college and college jobs as a function of underemployment duration and the effect of additional underemployment history on wages in college jobs. In our main quantitative exercise, we shut down the human capital dynamics so that unobserved heterogeneity is the only source of duration dependence. The model with only unobserved heterogeneity can explain nearly 97.2% of the duration dependence observed in the data. Moreover, when we shut down the unobserved heterogeneity channel, the model fails to generate underemployment in equilibrium. This underscores the role of information frictions in explaining both the existence of underemployment and the ensuing duration dependence. To further demonstrate the importance of information frictions, we show that the version of the model where a worker's type is public information cannot generate nearly enough duration dependence relative to what is observed in the data. This is because, after removing information frictions, only limited suitability workers select into underemployment.

We also use the calibrated model to study the contribution of the key channels to aggregate outcomes. Removing changes in occupation-specific human capital causes the unemployment (underemployment) rate to dramatically decrease (increase), as workers are more willing to select into underemployment if they do not face the loss of college-specific skills. Eliminating information frictions causes the underemployment rate among broad-suitable workers to fall to 0% while outcomes among limited suitability workers are largely unchanged. As the majority of workers in our parameterization are of limited suitability, the removal of information frictions has little effect on aggregate outcomes. Therefore, the quantitative findings suggest that information frictions play a large role in generating duration dependence but have very little effect on aggregate outcomes while occupation-specific human capital dynamics can have a large influence on aggregate outcomes and a minimal effect on duration dependence in underemployment.

As a final exercise, we simulate the model to assess the role of bad luck versus sorting in generating long underemployment durations. We find that even broad-suitable workers who take longer to find their first job, which can occur out of bad luck due to search frictions, do not experience significantly longer underemployment spells than their lucky peers. This again points to the role of sorting in generating duration dependence in underemployment. In other words, while many underemployed graduates appear to be stuck in their current jobs, our model and quantitative findings indicate that this phenomenon is not primarily caused by the job itself. Rather, it is more a result of the worker's individual characteristics, as well as the presence of search and information frictions.

We conclude our analysis by providing two forms of suggestive evidence supporting the presence of unobserved heterogeneity among underemployed workers. First, we find that there is less residual wage inequality among workers who experience long underemployment spells. Second, we find that workers who exit underemployment after one year or longer experience slower wage growth in college occupations than their otherwise observationally equivalent peers who encounter shorter underemployment spells.

Our paper relates to the recent and growing literature which studies underemployment. We are to the best of our knowledge unaware of any study which has documented the average length of an underemployment spell, negative duration dependence in underemployment, or the effect of underemployment history on wages in college jobs. As for the theory, the vast majority of existing models generate equilibrium underemployment in random search environments (e.g., Shephard and Sidibé (2022) and Jackson (2023)).[3] An exception is Barnichon and Zylberberg (2019), where workers direct their search to islands indexed by the productivity of vacancies posted there. A key ingredient in their framework is wage competition between workers, which generates a ranking mechanism whereby firms prefer to hire high-skill applicants. Relative to these studies, we develop an environment with directed search and competitive search equilibrium as in Moen (1997) and propose information frictions as a source of underemployment. Moreover, our model generates duration dependence in underemployment, which is absent from the aforementioned models. Finally, our quantitative analysis offers new insights into the sources of both duration dependence and the existence of underemployment by underscoring the role of both heterogeneity and information frictions.

Underemployment is closely related to the literature on skill mismatch. A set of recent papers include Guvenen et al. (2020), Lise and Postel-Vinay (2020), and Baley et al. (2022). In these papers, workers' abilities are measured across three dimensions: (math, verbal, social) or (cognitive, manual, interpersonal) using Armed Services Vocational Aptitude Battery (ASVAB) scores in the National Longitudinal Survey of Youth 1979 (NLSY79). While this is a very informative and innovative approach to studying the micro and macro implications of skill mismatch over both the life- and business-cycle, we complement this literature by focusing on educational mismatch (i.e., a discrepancy between the worker's educational attainment and degree requirements of the job). The first reason for this is that ASVAB scores are collected before most individuals in the NLSY79 are 18 years old and therefore do not account for skills acquired in college. Second, we view educational attainment and occupational college degree requirements as summary measures of the worker's skills and occupation's skill requirements which may include, but are not limited to, the categories listed above.[4] Finally, underemployment has garnered significant attention following the sullied outcomes of recent college graduates during and after the Great Recession and as many countries continue to explore policies to increase the supply

[3]See Albrecht and Vroman (2002), Gautier (2002), Dolado et al. (2009), and Coskun (2020).

[4]We measure verbal, math, and social skill requirements across occupations as in Guvenen et al. (2020) and show that skill requirements in each category are positively correlated with education requirements. We also use these measures of skill requirements to help correct for measurement error in measuring occupational mobility between non-college and college occupations.

of college graduates. A more thorough understanding of the determinants and properties of underemployment has the potential to speak to both of these areas of discourse.

We draw on the insights from the literature which has studied duration dependence in unemployment.[5] Our finding that unobserved heterogeneity explains almost all the duration dependence in underemployment is consistent with an emerging body of evidence showing that dynamic selection can account for a vast majority of duration dependence in unemployment (Mueller et al., 2021; Alvarez et al., 2019; Jarosch and Pilossoph, 2019).

Finally, our modelling of human capital decay during underemployment is inspired by the literature on skill loss during unemployment (Pissarides, 1992; Ljungqvist and Sargent, 1998; Ortego-Marti, 2016; Laureys, 2021). While our quantitative model includes skill loss during unemployment, we complement this literature by documenting the effect of underemployment history on wages in college occupations and by modelling the loss of college occupation-specific human capital during underemployment.

The rest of this paper is organized as follows. Section 2 presents the empirical evidence. Section 3 introduces the environment, while Section 4 defines a stationary equilibrium and characterizes the sources of duration dependence. Section 5 presents our quantitative analysis. Section 6 provides suggestive evidence on the presence of unobserved heterogeneity. Finally, Section 7 concludes.

2 Empirical Evidence

In this section, we document several stylized facts. First, underemployment is both more prevalent and persistent than unemployment. Second, underemployment exhibits negative duration dependence. Third, workers who have accumulated more experience in non-college occupations earn lower wages in college occupations.

2.1 Data

Our data sources are the National Longitudinal Survey of Youth 1997 (NLSY97) and the Occupational Informational Network (O*NET). As the NLSY97 and O*NET are well-known surveys, we delegate detailed descriptions of them and our constructed samples to Appendix C.

We use the NLSY97 to construct a weekly employment history of college graduates

[5] Recent references include, but are not limited to, Baydur and Xu (2020), Jarosch and Pilossoph (2019), Barnichon and Figura (2015), Doppelt (2016), Fernández-Blanco and Preugschat (2018) and Kospentaris (2021).

starting from when they enter the labor market until 2011.[6] An individual's labor market history begins when they graduate with a bachelors degree or above and, in subsequent years, are not enrolled in college. Starting from 8,984 respondents, we arrive at a sample of $N = 996$ individuals who obtained a bachelors degree before 2011 and have a complete set of employment records and time-varying individual characteristics.

To identify whether a respondent is underemployed (working in a non-college occupation) or properly employed (working in a college occupation), we follow Abel et al. (2014) and Jackson (2023) by defining an individual with at least a bachelors degree to be underemployed if they work in a non-college occupation, i.e., an occupation that requires less than a bachelors degree. Non-college (college) occupations are those where an average of less (more) than 50% of respondents in O*NET releases 5.0 through 16.0 state that a bachelors degree or above is necessary to perform that occupation.[7] We find that 108/298 (36%) of occupations in our sample are college occupations.[8]

Measuring occupational mobility is prone to measurement error due to the vast number of occupations (Moscarini and Thomsson, 2007). While this concern is mitigated in our analysis because we are only interested in transitions between two broad groups of occupations (non-college and college), we correct for measurement error in occupation switches by identifying "genuine" switches. We treat all switches between a non-college and college occupation that are also accompanied by a change in employer as genuine. For within-firm occupation changes, we use a correction based on occupation skill requirements. The first step of our approach is to measure verbal, math, and social skill requirements for each occupation following Guvenen et al. (2020), which produces a skill requirement vector, $\mathbf{r}_i$, for each occupation i.[9] We then compute the angular distance between skill requirement vectors of occupation i and j, denoted by $\phi(\mathbf{r}_i, \mathbf{r}_j)$, and choose a threshold, $\bar{\phi}$, so that within-firm transitions are genuine if $\phi(\mathbf{r}_i, \mathbf{r}_j) \geq \bar{\phi}$.[10]

[6] Our analysis requires observing consecutive employment records. Therefore, we do not use any post-2011 employment records, as this is when the NLSY97 switched from recording employment records on an annual to biennial basis.

[7] Occupations around the 50% threshold are listed in Online Appendix Table C5. While the 50% cutoff is stark, it produces an aggregate underemployment rate that is very similar to alternative approaches to measure occupational educational requirements. For example, BGT and SI (2018) use educational requirements included in online job ads while Barnichon and Zylberberg (2019) use the BLS 2012 Occupational Outlook Handbook. All three approaches find an underemployment rate of approximately 40%.

[8] The ten college and non-college occupations with the highest number of observations are enumerated in Online Appendix Table C6.

[9] Online Appendix C.1 provides more particulars on measuring skill requirements and also shows that education and skill are positively correlated with each other.

[10] The threshold, $\bar{\phi}$, is chosen so that the average correlation in skill requirements between the new college occupation and previous non-college occupation is close to zero. This approach follows Baley et al. (2022), who use this method to identify career transitions. See Appendix A.3 for further details.

Table 1: Frequency and Duration Across Labor Market Statuses

Labor force status	Unemployed	Underemployed	Properly employed
Ratio	0.031	0.392	0.522
Duration (months)	2.39	18.22	22.62

Notes: First row does not sum to one as 5.6% of observations are outside the labor force.

2.2 The Prevalence and Persistence of Underemployment

Our first objective is to highlight the prevalence of underemployment. To do so, we calculate the fraction of a respondent's employment record they spend outside the labor force, unemployed, underemployed, and properly employed. As seen in the first row of Table 1, on average, respondents spent 39.2% (3.1%) of their post-graduate career underemployed (unemployed). We also find that 625 respondents (62.8%) are never unemployed and that 348 (34.9%) never experience underemployment. Figure 1 shows the fraction of each year of potential experience a worker spends underemployed and properly employed. While the prevalence of underemployment decreases with potential experience, underemployment is still widespread 5+ years into one's career.

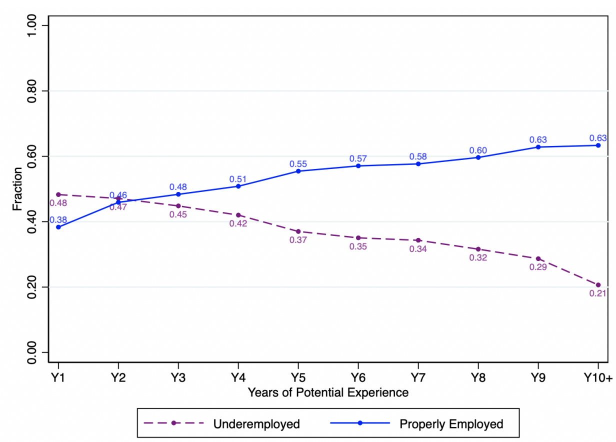


Figure 1: Underemployment and Proper Employment over Years of Potential Experience

As for the persistence of underemployment, the second row of Table 1 shows that the average underemployment duration is around 18 months, which is nearly eight times longer than the average unemployment duration.

2.3 Duration Dependence

Extensive research has documented negative duration dependence in unemployment (Kroft et al., 2016; Jarosch and Pilossoph, 2019; Kospentaris, 2021). In this section, we apply these techniques to examine whether there is duration dependence in underemployment. We define exiting underemployment to be when a worker transitions from a non-college to a college occupation between week t and $t+1$.[11]

Our objective is to estimate the negative exponential relationship between the transition probability from underemployment to proper employment and the worker's underemployment duration (in months).[12] Specifically, we estimate the following functional form via weighted nonlinear least squares:

$$D(\tau) = b_1 + (1 - b_1)\exp(-b_2 \times \tau), \tag{1}$$

where τ is a worker's underemployment duration and $D(\tau)$ is the average exit probability at duration τ relative to the average exit probability of workers who have been underemployed for less than one month. To estimate (1), we need estimates of the average exit probabilities from underemployment to proper employment at each duration τ. To obtain these, we begin by estimating

$$y_{imt} = \beta\tau_{imt} + \delta_m + \delta_t + \epsilon_{imt}, \tag{2}$$

where y_{imt} is a dummy equal to one if individual i transitioned from underemployment to proper employment in month m, τ is underemployment duration, δ_m is a month fixed effect, and δ_t is a year fixed effect. We then compute the predicted transition probabilities at each duration $\tau \in \{0, 1, \ldots, 25\}$, normalize the transition probability at $\tau = 0$ to one, and compute the relative exit probabilities for $\tau = \{1, 2, \ldots, 25\}$.

While we can estimate (1) using the results from (2), this specification does not control for any observable individual characteristics that are correlated with the worker's likelihood of exiting underemployment. To address this, we estimate

$$y_{imt} = \beta\tau_{imt} + \Gamma \cdot X_{imt} + \delta_m + \delta_t + \epsilon_{imt}, \tag{3}$$

where the only difference relative to (2) is the inclusion of X_{imt}, a vector containing gen-

[11] We also consider a more flexible criterion for exiting underemployment where we allow for a transition time of three weeks. For example, if a worker spends three weeks unemployed between underemployment and proper employment, we code it as a direct transition between underemployment and proper employment. The duration dependence demonstrated under this criterion is very similar to our baseline specification. See Online Appendix Figure C4.

[12] We group all workers with an underemployment duration of 25 months or longer together. This helps make our estimates more precise as the number of transitions out of underemployment at durations greater than or equal to 25 months is minimal. Moreover, as seen in Figure 2, the relative exit probability begins to flatten after durations of 15 months or longer. We also increased the maximum underemployment duration from 24 months to 30 months. The results are largely unchanged, as seen from Online Appendix Figure C4.

der, race, age, gender interacted with race, ASVAB quartile, family income per-capita, outstanding student loan debt, the highest degree ever obtained, gender interacted with the highest education obtained, undergraduate GPA, undergraduate major category (STEM or Arts and Social Sciences), and current level of job satisfaction.[13]

Figure 2 displays the transition probability from underemployment to proper employment at each duration τ relative to a newly underemployed worker. The circles and triangles represent the average predicted transition probabilities generated by equations (2) and (3), respectively, while the smooth curves are the result of estimating the nonlinear model, (1), on each set of relative transition probabilities. We indeed observe a sharp decline in the relative transition probability over the first year of underemployment and, while the relative transition probability continues to decline, it does level off at higher underemployment durations.[14] For example, after controlling for observable characteristics, a worker who is underemployed for one year is approximately 40% less likely to transition to a college job than a newly underemployed worker while those who have been underemployed for two years or more are approximately 45% less likely to exit underemployment. The shape of the decline in the relative exit probability is indicative of dynamic selection. If some workers, based on unobservable characteristics, have a high probability to exit underemployment, then these workers exit at short underemployment durations and this leaves workers with a low probability of exiting underemployment in the pool of workers who experience long spells of underemployment. In addition to dynamic selection, however, there could also be structural forces which result in longer underemployment durations causing each individual worker's probability of exiting underemployment to decline. To examine if there is evidence supporting this, we proceed to estimate the scarring effects of underemployment.

2.4 Scarring Effects of Underemployment

It is well documented that longer unemployment durations are associated with lower reemployment wages.[15] These scarring effects are consistent with two prominent explanations for structural duration dependence: human capital depreciation and statistical

[13]Job satisfaction ranges from 1 (dislike very much) to 5 (like very much). Online Appendix Figure C3 shows that the average job satisfaction level is significantly higher in college occupations. Online Appendix Table C7 lists the majors within the STEM and Arts and Social Sciences categories and shows that STEM (Arts and Social Sciences) majors spent an average of 35.7% (41.5%) of their labor market history underemployed.

[14]The duration dependence we observe in underemployment is similar to the duration dependence in unemployment reported in Jarosch and Pilossoph (2019). Both patterns exhibit a sharp initial decline and then level off around a relative exit probability of nearly 50% after controlling for observable characteristics.

[15]See Addison and Portugal (1989), Neal (1995), Ortego-Marti (2016, 2017a,b), and Schmieder et al. (2016).

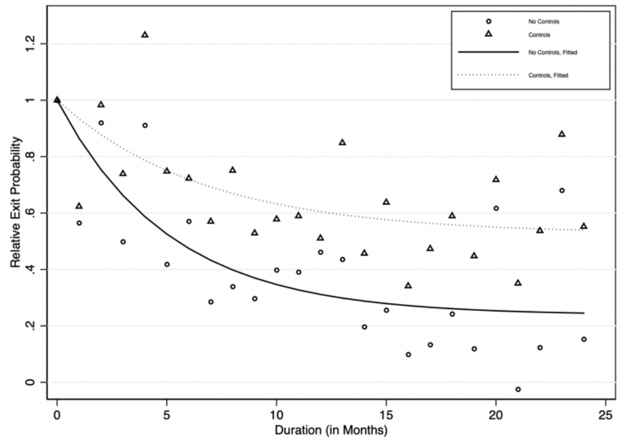


Figure 2: Duration Dependence

discrimination. In the former, a worker's skills depreciate over the course of their unemployment spell, which lowers both their productivity and chance to find a job. In the latter, a longer unemployment duration signals that the worker is less productive.

In this section, we assess whether a worker with a longer underemployment history earns lower wages in college occupations than their observationally equivalent peers. The regression model is given by

$$w_{imt} = \alpha \text{Underhis}_{imt} + \beta \text{College}_{imt} + \mu \text{Underhis}_{imt} * \text{College}_{imt} + \Gamma \cdot X_{imt} + \delta_i + \varepsilon_{imt}. \quad (4)$$

The dependent variable is individual i's log hourly real wage in month m and year t, Underhis is accumulated experience in non-college occupations, and College is a dummy for whether individual i is employed in a college occupation. The vector X includes a cubic polynomial in potential experience, annual regional unemployment rate, annual aggregate unemployment rate, a quadratic polynomial in age, family income per-capita, outstanding student loan debt, current level of job satisfaction, region, two-digit industry, and individual fixed effects. From (4), $\alpha + \mu$ captures the effect of an additional month of underemployment history on wages in college occupations.

Table 2 presents the results. Column (1) shows that an additional month of unemployment history is associated with a 1.45% decrease in wages, which is consistent with prior literature (Neal, 1995; Ortego-Marti, 2016). Columns (2) and (3) show that an additional month of underemployment history is associated with an 0.03% increase in wages. From column (4), an additional month of unemployment history has the same effect on both college and non-college jobs. Columns (5) and (6) show that an additional month of underemployment is associated with $0.06\% - 0.07\%$ higher wages in non-college jobs and

Table 2: Scarring Effects of Unemployment and Underemployment

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0139***		-0.0136***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0006***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
Unhis*College				-0.0009		-0.0004
				(0.0013)		(0.0013)
Underhis*College					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
Occupation (2-digit) FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.791	0.790	0.791	0.782	0.782	0.783

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). All regressions include the full set of control variables introduced after equation (4), individual, industry (2-digit), and region fixed effects.

0.13% lower wages in college occupations.[16]

Table 2 shows that workers with a longer underemployment history earn lower wages in college occupations than their observationally equivalent peers.[17] We reconcile these findings and allow for structural duration dependence in our model through the growth and decay of occupation-specific human capital. Underemployed workers accumulate non-college occupation-specific human capital. At the same time, working in non-college jobs implies that workers are not utilizing their college occupation-specific human capital. This approach to modelling human capital dynamics is in line with evidence supporting the view that human capital is occupation-specific (Kambourov and Manovskii, 2009a,b).

To further support the notion of the accumulation and decay of occupation-specific human capital, Section C.2 in the Online Appendix studies how the scarring effects of underemployment vary with the distance in skill requirements between a worker's cur-

[16]We conduct several robustness exercises. First, we use one-digit industry and occupation fixed effects (Online Appendix Table C8). Second, we control for one-digit or two-digit occupation fixed effects in all specifications to address the concern of occupation heterogeneity (Altonji and Shakotko, 1987) (Online Appendix Tables C9 and C10). Finally, we include month and year fixed effects (Online Appendix Table C11). The estimated scarring effects are very similar across all specifications.

[17]While the effect of an additional month of underemployment history on wages in college jobs is smaller than the effect of an additional month of unemployment history, one needs to bear in mind that, as shown in Table 1, the average underemployment duration is nearly eight times longer than the average unemployment duration and that recent college graduates are nearly thirteen times more likely to be underemployed than unemployed.

rent college occupation and previous non-college occupation. The idea here is that if the distance in required skills between the two occupations is larger, then the skills required by the new college occupation would have been used less intensively in the previous non-college occupation and thus experienced a greater rate of decay, ultimately leading to larger scarring effects. Online Appendix Table C3 shows that the scarring effects of underemployment are significantly larger and increasing in the gap in skill requirements between the college and non-college occupation.

In summary, this section has established the prevalence and persistence of underemployment among recent graduates, documented negative duration dependence in underemployment, and shown that longer underemployment histories are associated with lower wages in college occupations. Our primary goals for the rest of the paper are twofold. First is to write a model with two classic channels, dynamic selection based on unobserved heterogeneity and human capital dynamics, which can generate duration dependence in underemployment.[18] Second is to quantify the contributions of unobserved heterogeneity and human capital to duration dependence in underemployment.

3 Environment

Time, Agents, and Preferences Time is discrete and goes on forever. There is a measure 1 of workers and a large measure of firms. Workers are endowed with an indivisible unit of labor. All agents are risk neutral and share the discount factor $\beta \in (0,1)$. Each firm corresponds to one job which can either be vacant or filled. Firms are indexed by $\chi \in X = \{n, c\}$ where n (c) denotes a non-college (college) job.

Workers are ex-ante heterogeneous in their suitability for college jobs. Workers are of either broad (H) or limited (L) suitability. The mass of type H workers is given by $\pi \in (0,1)$. A type i worker is suitable for a given college job with probability a^i where $a^H > a^L$. If a worker is not suitable for a college job, the match produces zero output. All workers produce output in non-college jobs with probability one.

Workers can also differ in their labor market history. We denote $v \in \Upsilon = \{0, 1, \ldots, \bar{v}\}$ as the number of periods a worker has been unemployed while $\tau \in T = \{0, 1, \ldots, \bar{\tau}\}$ denotes the worker's underemployment history (i.e., experience in non-college jobs). All workers with unemployment (underemployment) history $v \geq \bar{v}$ ($\tau \geq \bar{\tau}$) form a homogeneous group regarding their unemployment (underemployment) history.

[18] We abstract from statistical discrimination based on a worker's labor market history for tractability and also based on the results of Jarosch and Pilossoph (2019) who find that statistical discrimination in callback rates of unemployed workers is largely a result of dynamic selection based on unobserved heterogeneity, which we allow for in our model.

Information There is symmetric incomplete information regarding a worker's suitability, i.e., neither workers nor firms know whether a worker is of broad or limited suitability. A worker's unemployment and underemployment history is public information.

Technology Firms operate a technology that maps one unit of suitable labor into $y_\chi(\tau)$ units of output where $y_\chi \colon T \to \mathbb{R}_+$, $y_n'(\tau) \geq 0$, $y_c'(\tau) \leq 0$, and $y_c(\tau) > y_n(\tau)$ for all $\tau \in T$. The relationship between y_χ and τ captures the accumulation (decay) of non-college (college) occupation-specific human capital.[19]

Firms who operate college jobs have access to the following hiring technology. Upon meeting a worker, the firm observes a private signal which perfectly identifies unsuitable workers. Neither workers nor other firms observe the signal. Firms can then hire suitable workers and reject unsuitable ones.

Suitability and Skills Our notion of suitability follows Gonzalez and Shi (2010) and Kospentaris (2021). Suppose that a worker's skills can be represented by a multidimensional skill vector as in Guvenen et al. (2020), Lise and Postel-Vinay (2020), and Baley et al. (2022). We interpret suitability as a measure of how well the worker's skills align with the requirements of college jobs (where skill requirements can potentially differ across college jobs). We then view a worker's occupation-specific human capital as the quantity of skills, within those categories, that the worker has on the intensive margin, which can either accumulate or decay as the worker's labor market history evolves.

The Labor Market The labor market is organized in a continuum of submarkets indexed by $\omega = (\chi, v, \tau, x) \in X \times \mathrm{Y} \times T \times \mathbb{R}$. In submarket ω, type χ firms search for workers with labor market history (v, τ) and offer suitable workers an employment contract worth $x \in \mathbb{R}$ in lifetime utility.[20] Submarkets need not be indexed by the worker's expected suitability because, as we explain below, a worker's labor market history (v, τ) is a sufficient statistic for their expected suitability in college jobs.

Timing Each period is divided into four stages: search, matching, entry/production, and exit.

[19]In the quantitative analysis, we allow for the production technology to be a function of the worker's unemployment history to capture skill loss during unemployment. See Section 5.1.

[20]We could alternatively assume that firms post a menu of contracts with one contract for each type of worker. However, Menzio and Shi (2010b) show that firms find it optimal to offer a menu of contracts such that the menu specifies a value of zero to all but one type of worker.

Stage 1 Firms decide whether to create a vacancy and, if so, which submarket to post it in. Workers choose which submarket to search in. Firms incur a cost $k_\chi > 0$ to open and maintain a type χ vacancy for one period. Workers who begin the period unemployed (employed) are endowed with 1 ($\lambda \in [0,1]$) unit(s) of search intensity.

Stage 2 Suitable workers and vacancies who search in the same submarket are brought together by a constant returns to scale matching technology. Let $v(\omega)$ denote the number of vacancies created in submarket ω. Further, let $u^i(\omega)$ and $e^i(\omega)$ denote the measure of unemployed and employed workers, respectively, of suitability type i searching in submarket ω. The effective measure of suitable units searching in submarket ω is given by

$$\psi(\omega) = \sum_i a^i(\omega)[u^i(\omega) + \lambda e^i(\omega)], \tag{5}$$

where

$$a^i(\omega) = \begin{cases} 1, & \text{if } \chi = n, \\ a^i, & \text{if } \chi = c. \end{cases} \tag{6}$$

Equation (6) specifies that all workers are suitable if they are searching in a submarket with non-college jobs. The number of matches between suitable workers and vacancies is determined by the matching function $F(\psi(\omega), v(\omega))$. We define $\theta(\omega) \equiv v(\omega)/\psi(\omega)$ as effective tightness in submarket ω. The probability that a suitable unemployed worker matches with a vacancy is given by $p(\theta(\omega)) = F/\psi$ where p: $\mathbb{R}_+ \to [0,1]$ is twice continuously differentiable, strictly increasing, strictly concave, $p(0) = 0$, and $p(\infty) = 1$. Suitable workers who are currently searching on the job find a match with probability $\lambda p(\theta(\omega))$. The probability that a vacancy matches with a suitable worker is given by $q(\theta(\omega)) = F/v$ where $q : \mathbb{R}_+ \to [0,1]$ is twice continuously differentiable, strictly decreasing, strictly convex, $q(0) = 1$, and $q(\infty) = 0$.

Stage 3 A measure δ of workers enter the economy at the beginning of the stage, have their suitability type determined by nature, and begin their career unemployed. Unemployed workers produce b units of output and matches between a worker with underemployment history τ and type χ firm produce $y_\chi(\tau)$ units of output. After production occurs, workers employed in college jobs with underemployment history $\tau > 0$ regain their college-specific skills with probability $\phi \in [0,1]$ and have their underemployment history reset to $\tau = 0$.

Stage 4 A fraction δ of workers exit the labor market. There are no exogenous transitions of workers from employment to unemployment.

Learning and Beliefs Workers learn about their suitability type as they search for college jobs. We denote μ as the worker's expectation that they will produce output in a given college job. Workers share the common initial belief upon entering the labor market: $\mu_0 = \pi a^H + (1-\pi)a^L$. It follows that, after applying Bayes rule, unemployed workers who search for and do not find a college job update their beliefs to

$$\hat{\mu} \equiv H(p, \mu) = a^H - \frac{(a^H - \mu)(1 - pa^L)}{1 - p\mu}, \tag{7}$$

where $p = p(\theta(\omega))$ is the job finding probability of a suitable worker in submarket ω Now consider an underemployed worker who enters the period with beliefs μ. If they search for a college job in submarket ω and are suitable, then they will find a job with probability $\lambda p(\theta(\omega))$. It follows that underemployed workers who do not find a college job update their beliefs to $H(\lambda p, \mu)$.

Equation (7) also captures firms' beliefs about a worker's suitability. This follows from our assumption of symmetric incomplete information and, hence, leads to workers and firms sharing the same expectation regarding a worker's suitability. It follows that workers with the same labor market history (v, τ) will have spent the same number of periods not matched with a college job and are observationally equivalent. Therefore, the worker's labor market history is a sufficient statistic for their expected suitability.[21]

Contractual Environment Employment contracts offered by firms are bilaterally efficient. That is, the contract offered by the firm maximizes the sum of the worker's lifetime utility and the firm's lifetime profits. As shown by Menzio and Shi (2009, 2011), a contract which maximizes the firm's profits will also maximize the joint value of the match in several contractual environments, including when the contract space is complete and when employment contracts can specify a hiring fee and a wage as a function of the aggregate state of the economy, the worker's observable characteristics, and the productivity of the match. As there are multiple environments which lead to bilaterally efficient employment contracts, we only assume that the underlying contractual environment is such that

[21] Our assumption of no exogenous separations of workers from employment to unemployment is important for generating this feature of the model. As we show in Section 4, employment in college jobs is an absorbing state. It follows that a firm who meets a worker with labor market history (v, τ) knows that the worker was unemployed for v periods before becoming underemployed for τ periods. Moreover, in equilibrium, firms know how many times the worker's expected suitability was updated according to (7).

workers and firms can exploit all of their gains from trade.

4 Equilibrium

We focus on stationary equilibria. Section 4.1 describes the value functions and maximization problems. Section 4.2 introduces free entry of firms. Section 4.3 formally defines an equilibrium and characterizes the solution to the worker's search problems.

4.1 Value Functions

Let $V_{u,\chi}(v,\mu)$ denote the lifetime utility of an unemployed worker who has been unemployed for v periods, has expected suitability μ, and searches in a submarket with type χ jobs. Further, we denote $V_u(v,\mu)$ as the value of unemployment where

$$V_u(v,\mu) = \max_{\chi\in\{n,c\}} V_{u,\chi}(v,\mu). \tag{8}$$

Consider an unemployed worker at the beginning of the production stage. They produce b units of output and remain in the labor market with probability $1-\delta$. Suppose that the worker searches for a non-college job in submarket $\omega = (n,\hat{v},0,x)$ in the next period's search stage where $\hat{v} = \min\{v+1,\bar{v}\}$.[22] The worker finds a match with probability $p(\theta(\omega))$. If the worker matches with a firm, their continuation value is x. Workers who do not match with a firm remain unemployed. However, workers who search for a non-college job do not update their expected suitability as searching for non-college jobs provides the worker no information regarding their suitability for college jobs. If the worker remains unemployed, their continuation value is $V_u(\hat{v},\mu)$. It follows that $V_{u,n}(v,\mu)$ satisfies

$$V_{u,n}(v,\mu) = b + \beta(1-\delta)\{V_u(\hat{v},\mu) + R_n(x,V_u(\hat{v},\mu))\}, \tag{9}$$

where

$$R_\chi(x,V_u(\hat{v},\mu)) = \max_x p(\theta(\chi,\hat{v},0,x))(x - V_u(\hat{v},\mu)). \tag{10}$$

Now suppose that the unemployed worker searches for a college job. There are two differences relative to searching for a non-college job. First, workers with an expected suitability μ expect to match with a college job in the next period's matching stage with probability $\mu p(\theta(\omega))$. Second, workers who search for a college job and do not find one

[22] As there are no separations from employment to unemployment, all unemployed workers have underemployment history $\tau = 0$.

update their expected suitability to $\hat{\mu} = H(p(\theta(\omega)), \mu)$. It follows that $V_{u,c}(v, \mu)$ satisfies

$$V_{u,c}(v, \mu) = b + \beta(1 - \delta)\{V_u(\hat{v}, \hat{\mu}) + \mu R_c(x, V_u(\hat{v}, \hat{\mu}))\}. \tag{11}$$

We denote $\omega_u^*(v, \mu)$ as the policy function associated with an unemployed worker with unemployment history v and expected suitability μ.

Consider a worker with labor market history (v, τ) and expected suitability μ who is currently matched with a non-college job at the beginning of the production stage. We denote $V_{e,n}(v, \tau, \mu)$ as the sum of the worker's lifetime utility and firm's profits. In the current period, the sum of the worker's utility and firm's profits are equal to the output of the match, $y_n(\tau)$. The worker exits the market with probability δ. In this case, the worker's continuation utility and firm's continuation profit are both equal to 0. With probability $1 - \delta$, the worker does not exit the market. However, the worker and firm separate with probability $\lambda \mu p(\theta(\omega))$ where ω is the submarket where the worker searches for a new match in the subsequent search stage. The worker's continuation utility is x and the firm's continuation profits are 0. Finally, the worker and firm remain matched with probability $1 - \lambda \mu p(\theta(\omega))$. In this case, the sum of the worker's continuation utility and the firm's continuation profit is $V_{e,n}(v, \hat{\tau}, \hat{\mu})$ where $\hat{\tau} = \min\{\tau + 1, \bar{\tau}\}$ and $\hat{\mu} = H(\lambda p(\theta(\omega)), \mu)$. It follows that $V_{e,n}(v, \tau, \mu)$ satisfies

$$V_{e,n}(v, \tau, \mu) = y_n(\tau) + \beta(1 - \delta)\{V_{e,n}(v, \hat{\tau}, \hat{\mu}) + \lambda \mu S(v, \hat{\tau}, \hat{\mu})\}, \tag{12}$$

where

$$S(v, \hat{\tau}, \hat{\mu}) = \max_x p(\theta(c, v, \hat{\tau}, x))(x - V_{e,n}(v, \hat{\tau}, \hat{\mu})). \tag{13}$$

We denote $\omega_{e,n}^*(v, \tau, \mu)$ as the policy function associated with equation (12). Note that equations (12)-(13) assume that underemployed workers will only search in submarkets with college jobs and, hence, will always update their beliefs in the event they do not find a match. We will prove this in Section 4.3, but the intuition is straightforward. Due to employment contracts being bilaterally efficient, workers currently employed in type χ jobs cannot realize any more gains from trade by transitioning to another type χ job.

Finally, consider a worker with labor market history (v, τ) and expected suitability μ who is matched with a college job. The worker and firm produce $y_c(\tau)$ units of output in the current period. Then, with probability ϕ, the worker regains their college occupation-specific human capital and produces $y_c(0)$ units of output for the remainder of the match. Formally, this is equivalent to the worker's underemployment history being reset to $\tau =$

0. The sum of the worker's lifetime utility and the firm's profits, $V_{e,c}(v,\tau,\mu)$, satisfies

$$V_{e,c}(v,\tau,\mu) = y_c(\tau) + \beta(1-\delta)\{\phi V_{e,c}(v,0,\mu) + (1-\phi)V_{e,c}(v,\tau,\mu)\}. \tag{14}$$

Equation (14) is simpler than (12) because workers employed at a college job will remain in their current match until they exit the market. This is another result we will prove in Section 4.3. The intuition is that if, given the worker's labor market history, they produce more output in a match with a college job than a non-college job, they would have no incentive to search for a non-college job.

4.2 Free Entry

Working back to the search stage, firms decide whether to create a vacancy or not and, if yes, which submarket to post it in. The firm's cost to create a type χ vacancy is k_χ. The firm's benefit from creating a vacancy in submarket $\omega = (\chi, v, \tau, x)$ is given by $q(\theta(\chi,v,\tau,x))\{V_{e,\chi}(v,\tau) - x\}$ where $q(\theta(\chi,v,\tau,x))$ is the probability of matching with a worker, $V_{e,\chi}(v,\tau) = V_{e,\chi}(v,\tau,\mu)$ is the joint value of a match, and x is the portion of the joint surplus that the firm delivers to the worker.[23]

In any submarket visited by a positive number of workers, tightness is consistent with the firm's incentives to create vacancies if and only if

$$k_\chi \geq q(\theta(\chi,v,\tau,x))\{V_{e,\chi}(v,\tau) - x\}, \tag{15}$$

and $\theta(\chi,v,\tau,x) \geq 0$ with complementary slackness. We restrict attention to equilibria in which $\theta(\chi,v,\tau,x)$ satisfies the complementary slackness condition in every submarket, even those that are not visited by workers. That is

$$\theta(\chi,v,\tau,x) = \begin{cases} q^{-1}\left(\frac{k_\chi}{V_{e,\chi}(v,\tau)-x}\right), & \text{if } k_\chi = q(\theta(\chi,v,\tau,x))\{V_{e,\chi}(v,\tau) - x\}, \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Equation (16) demonstrates the well-known trade-off present in models of competitive search. Workers find jobs with a higher probability in submarkets with higher tightness but are also offered employment contracts with a lower value to ensure that the free entry condition is satisfied.

[23] The equivalence between $V_{e,\chi}(v,\tau)$ and $V_{e,\chi}(v,\tau,\mu)$ follows from the fact that the worker's labor market history is a sufficient statistic for their expected suitability.

4.3 Equilibrium Definition and Characterization

Definition 1. Let $A = [a^L, a^H]$. A stationary recursive equilibrium (RE) consists of a belief function $\hat{\mu}(p,\mu): [0,1] \times A \to A$, a market tightness function $\theta(\omega): X \times \Upsilon \times T \times \mathbb{R} \to \mathbb{R}_+$, a value function for unemployed workers, $V_u(v,\mu): \Upsilon \times A \to \mathbb{R}$, a policy function for unemployed workers, $\omega_u^*(v,\mu): \Upsilon \times A \to X \times \mathbb{R}$, a joint value function for the worker-firm match, $V_{e,\chi}(v,\tau,\mu): X \times \Upsilon \times T \times A \to \mathbb{R}$, a policy function for the worker-firm match, $\omega_{e,\chi}^*(v,\tau,\mu): X \times \Upsilon \times T \times A \to X \times \mathbb{R}$, and a distribution of workers across the states of employment. The functions satisfy the following conditions. First, a worker updates their beliefs according to (7) when they search in a submarket with type $\chi = c$ jobs and do not find one. Second, $\theta(\omega)$ satisfies (16) for all $\omega \in X \times \Upsilon \times T \times \mathbb{R}$. Third, $V_u(\tau,\mu)$ satisfies (8) for all $(v,\mu) \in \Upsilon \times A$ and $\omega_u^*(v,\mu)$ is the associated policy function. Fourth, $V_{e,n}(v,\tau,\mu)$ and $V_{e,c}(v,\tau,\mu)$ satisfy equations (12) and (14) for all $(v,\tau,\mu) \in \Upsilon \times T \times A$ and $\omega_{e,\chi}^*(v,\tau,\mu)$ for $\chi \in X$ are the associated policy functions. Finally, the distribution of workers satisfies the laws of motion specified in Appendix B.1.

As established by Menzio and Shi (2010a, 2011) for directed search models with free entry and bilateral efficiency and subsequently Schaal (2017) for similar environments with two-sided heterogeneity, a RE exists, is unique, and is block-recursive. We now formally define a block-recursive equilibrium.

Definition 2. A block-recursive equilibrium (BRE) is a RE where the value and policy functions are independent of the distribution of workers across the states of employment.

The existence of a BRE follows from our assumption of directed search. Workers self-select into different submarkets based on their current employment status and labor market history. As a result, firms know they will only meet one type of worker, based on observable characteristics, in the submarket they post vacancies in. Second, the hiring protocol and matching function specification of Gonzalez and Shi (2010) implies that firms do not need to keep track of the composition of suitable workers in each submarket. Hence, tightness in each submarket is independent of the distribution of workers across employment statuses and the composition of worker suitability. It then follows that a worker's value and policy functions are also independent of the distribution of workers.

We conclude our description of the equilibrium by characterizing the solution to the workers' search problems. To begin, we note that in any submarket with $\theta(\omega) > 0$, we can use equation (16) to express the value of the employment contract, x, as a function of the job type, a worker's labor market history, expected suitability, and market tightness:

$$x(\chi, v, \tau, \mu, \theta) = V_{e,\chi}(v, \tau, \mu) - \frac{k_\chi}{q(\theta)}. \tag{17}$$

Combining equation (17) with (10) allows us to rewrite the optimization problem of an unemployed worker who began the previous production stage with unemployment history v, now has unemployment history $\hat{v}$, and searches for a college job as a choice of market tightness:

$$R_c(x, V_u(\hat{v}, \hat{\mu})) = \max_{\theta \geq 0} -k_c\theta + p(\theta)(V_{e,c}(\hat{v}, 0, \mu) - V_u(\hat{v}, \hat{\mu})). \tag{18}$$

The worker's choice of θ satisfies

$$k_c \geq p'(\theta)(V_{e,c}(\hat{v}, 0, \mu) - V_u(\hat{v}, \hat{\mu})), \tag{19}$$

and $\theta \geq 0$ with complementary slackness. Consider a submarket with $\theta > 0$. Equation (19) states that the marginal cost of searching in a market with higher tightness, k_c, is equal to the marginal benefit of searching in a submarket with higher tightness, where the marginal benefit is the product of the increased probability of finding a job, $p'(\theta)$, and the net surplus of forming a college match. Similarly, an unemployed worker's optimal choice of tightness in a submarket for non-college jobs satisfies:

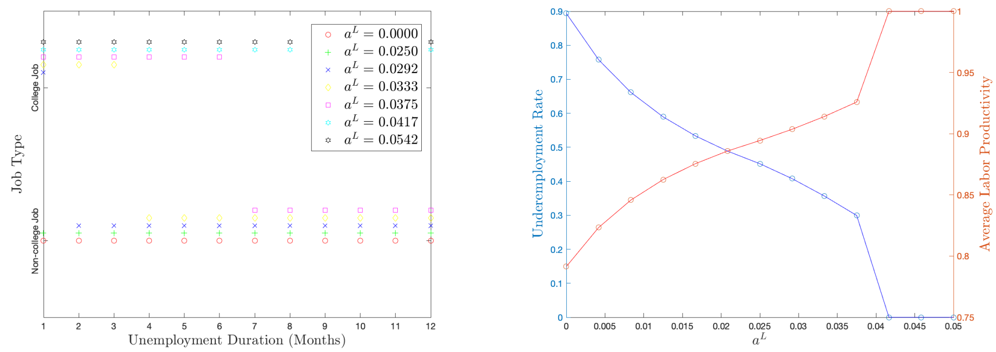
$$k_n \geq p'(\theta)(V_{e,n}(\hat{v}, 0, \mu) - V_u(\hat{v}, \mu)). \tag{20}$$

We denote $\theta^*_{c,\hat{v},\mu}$ as the solution to (19) and $\theta^*_{n,\hat{v},\mu}$ as the solution to (20).

Now consider an unemployed worker's choice of which type of job to search for:

$$\max \Bigg\{ \overbrace{V_u(\hat{v}, \mu) - k_n\theta^*_{n,\hat{v},\mu} + p(\theta^*_{n,\hat{v},\mu})(V_{e,n}(\hat{v}, 0, \mu) - V_u(\hat{v}, \mu))}^{\text{Search for a non-college job}}, \\ \underbrace{V_u(\hat{v}, \hat{\mu}) + \mu[-k_c\theta^*_{c,\hat{v},\mu} + p(\theta^*_{c,\hat{v},\mu})(V_{e,c}(\hat{v}, 0, \mu) - V_u(\hat{v}, \hat{\mu}))]}_{\text{Search for a college job}} \Bigg\}. \tag{21}$$

Let $\varrho_{n,\hat{v}}$ denote the fraction of unemployed workers with unemployment history $\hat{v}$ who search for a non-college job. It follows that $\varrho_{n,\hat{v}} = 1$ if the net benefit of searching for a non-college job is higher than searching for a college job, $\varrho_{n,\hat{v}} \in [0, 1]$ if the net benefit of searching for a non-college job is equal to that of a college job, and $\varrho_{n,\hat{v}} = 0$ if searching for a college job delivers a higher net benefit than searching for a non-college job.



(a) Policy Function of Unemployed Workers (b) Underemployment and Productivity

Figure 3: Suitability of Type L Workers (a^L)

Equation (21) delivers some insight into the trade-offs faced by an unemployed worker. Transitioning to a college job may generate a larger net surplus than forming a match with a non-college job. However, the worker discounts this surplus by their expected suitability. Thus, if a worker's expected suitability is low enough, they may find it more worthwhile to search for a non-college job. Figure 3 demonstrates this by displaying comparative statics with respect to a^L, the probability that a type L worker is suitable for any given college job.[24] Panel (a) shows the policy function of unemployed workers, where a marker next to "Non-college Job" ("College") on the vertical axis indicates that an unemployed worker with unemployment duration v searches for a non-college ("College") job. For example, when $a^L = 0$, all the red circles are next to "Non-college Job", indicating that workers only search for non-college jobs. As a^L increases, workers begin to search for college jobs. In fact, when a^L increases from 0.037 to 0.042, workers' initial beliefs are high enough that they forgo searching for non-college jobs altogether. Panel (b) shows the corresponding implications for the aggregate underemployment rate and average labor productivity. As initial beliefs increase and workers search less for non-college jobs, the underemployment rate decreases and average labor productivity increases. When workers stop searching for non-college jobs in panel (a), this corresponds with the underemployment rate in panel (b) decreasing from nearly 30% to 0%.

Figure 3(a) not only emphasizes the role of initial beliefs in generating underemployment, it displays the sequential search pattern that unemployed workers follow. Proposition 1 summarizes.

[24]The parameter values used in this exercise are the calibrated parameters presented in Table 4 with one exception. The quantitative model allows for skill loss during unemployment. To produce Figure 3, we shut down this channel to make the model the same as in the main text. Online Appendix Figure C5 shows similar patterns by displaying comparative statics with respect to π and a^H.

Proposition 1. *Suppose that $V_{u,n}(\tilde{v}, \mu) > V_{u,c}(\tilde{v}, \mu)$ for some $\tilde{v} \in \Upsilon \backslash \{\bar{v}\}$. Then $V_{u,n}(v, \mu) > V_{u,c}(v, \mu)$ for all $v \in \{\tilde{v} + 1, \ldots, \bar{v}\}$.*

All proofs are delegated to Appendix B. Proposition 1 states that if an unemployed worker of duration $\tilde{v}$ finds it optimal to search for a non-college job, then they will continue to search for a non-college job for the remainder of their unemployment duration. To see this, consider a worker who begins their career searching for college jobs. If the worker does not find a match, they will downgrade their expected suitability. If their expected suitability is low enough that it becomes more worthwhile to search for non-college jobs, the unemployed worker will not search for a college job from unemployment again. If the worker were to consider searching for a college job after searching for a non-college job, their expected suitability would be unchanged from when the worker previously decided that it was optimal to search for a non-college job. Hence, the worker would still find it optimal to search for a non-college job.

Next, we show that workers employed in college jobs do not search on the job.

Proposition 2. *Consider a worker with history (v, τ) and expected suitability μ who is currently employed in a college job. Then $\theta^*_{\chi,v,\tau,\mu} = 0$ for all $(\chi, v, \tau, \mu) \in X \times \Upsilon \times T \times A$.*

Suppose that a worker currently employed at a college job were to search for a different college job. As we have assumed that employment contracts are bilaterally efficient, there is no additional surplus that can be generated by switching to a different college job as the joint surplus of their current employment relationship has already been maximized. So, the worker will not search for a different college job. Now suppose that the worker searches for a non-college job. Following our assumptions of the production technology, the worker is already in the employment relationship in which they are the most productive. It follows that the worker would not generate any additional surplus by transitioning from a college to a non-college job. Hence, once a worker becomes employed in a college job, they remain in that match until exiting the labor market.

Now consider an underemployed worker with history (v, τ) and expected suitability μ. By a similar intuition as in Proposition 2, underemployed workers have no incentive to search for a non-college job. Therefore, we restrict our attention to an underemployed worker's choice of tightness in submarkets with complex jobs, which satisfies:

$$k_c \geq p'(\theta)(V_{e,c}(v, \tau, \mu) - V_{e,n}(v, \tau, \hat{\mu})), \tag{22}$$

and $\theta \geq 0$ with complementary slackness. Assuming an interior solution, the underemployed worker's optimal choice of tightness, $\theta^*_{e,c}(\upsilon, \tau, \mu)$, solves

$$k_c = p'(\theta^*_{c,\upsilon,\tau,\mu})\Delta(\upsilon, \tau, \mu), \tag{23}$$

where $\Delta(\upsilon, \tau, \mu) = V_{e,c}(\upsilon, \tau, \mu) - V_{e,n}(\upsilon, \tau, \hat{\mu})$. Given that underemployed workers transition to a college job with probability $\mu\lambda p(\theta^*_{c,\upsilon,\tau,\mu})$, we can see three channels at work which create duration dependence in underemployment. First, workers with a higher underemployment history have a lower expected suitability and thus transition from non-college to college jobs at a lower rate. In other words, the pool of long-term underemployed workers will be skewed towards those who are of limited suitability. The second and third channels follow from the changes in occupation-specific human capital. Additional underemployment history reduces (increases) the worker's productivity in college (non-college) jobs. Both of these effects put downward pressure on the surplus generated by the worker escaping underemployment, $\Delta(\upsilon, \tau, \mu)$, and therefore the probability of suitable workers matching with college jobs, $\lambda p(\theta^*_{c,\upsilon,\tau,\mu})$. To further emphasize the second and third channels, Proposition 3 studies a version of the model where human capital dynamics are the only sources of duration dependence.

Proposition 3. *Assume that $a^H = a^L = 1$. Further, define $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$ and θ_τ as tightness which satisfies:*

$$k_c \geq p'(\theta_\tau)\Delta(\tau), \tag{24}$$

where $\theta_\tau \geq 0$ with complementary slackness. Finally, define $\tilde{\Delta}(\tau)$ as a function of τ and exogenous parameters as shown in Appendix B.4. We have the following results:

(i) $\Delta(\tau)$ is strictly decreasing in τ.

(ii) $\lambda p(\theta_\tau)$ is weakly decreasing in τ.

(iii) If $k_c > p'(0)\tilde{\Delta}(\tilde{\tau})$ for some $\tilde{\tau} \leq \bar{\tau}$, then $\lambda p(\theta_\tau) = 0$ for all $\tau \in \{\tilde{\tau}, \ldots, \bar{\tau}\}$.

Part (i) states that the surplus a worker generates by exiting underemployment is decreasing in their underemployment duration. Part (ii) states that the probability a worker transitions from a non-college to college job, $\lambda p(\theta_\tau)$ is weakly decreasing in τ. This is intuitive and follows from part (i). As additional underemployment experience reduces the surplus of transitioning out of underemployment, less college vacancies are created in submarkets for workers with a longer underemployment history. Finally, part (iii) shows that if the surplus generated by the worker transitioning out of underemployment is small enough, then $\theta_\tau = 0$ solves (24) and workers who reach an underemployment duration of $\tau = \tilde{\tau}$ remain underemployed until exiting the labor market.

In summary, the model generates duration dependence in underemployment due to dynamic selection of workers based on unobserved heterogeneity and structural duration dependence due to (i) deterioration of college occupation-specific human capital and (ii) accumulation of non-college occupation-specific human capital. We now proceed to quantitative analysis where our primary objective is to decompose the model generated duration dependence into the three aforementioned channels.

5 Quantitative Analysis

This section calibrates the model and performs quantitative exercises. Section 5.1 presents the quantitative model. Section 5.2 details the calibration strategy. Sections 5.3 and 5.4 study the contribution of unobserved heterogeneity and occupation-specific human capital to duration dependence and aggregate outcomes. Finally, Section 5.5 disentangles the role of sorting versus bad luck in generating long underemployment durations.

5.1 Quantitative Model

We introduce two extensions to the baseline environment. First, we allow for skill loss during unemployment. A match between a suitable worker and type χ job produces $y_\chi(v,\tau)$ units of output where $y_\chi : \Upsilon \times T \to \mathbb{R}_+$, $\partial y_\chi(v,\tau)/\partial v < 0$, $\partial y_n(v,\tau)/\partial \tau > 0$, $\partial y_c(v,\tau)/\partial \tau < 0$ and $y_c(v,\tau) > y_n(v,\tau)$ for all $(v,\tau) \in \Upsilon \times T$. After producing in stage 3, a worker employed in a college job with $\max\{v-1,\tau\} > 0$ regains their college-specific skills with probability ϕ and produces $y_c(1,0)$ units of output for the remainder of the match.[25] Second, as wages are indeterminate under bilaterally efficient contracts, we assume that employment contracts can specify a one-time hiring fee paid by the worker to the firm upon formation of the match and wages as a function of the match productivity, worker's observable characteristics, and the aggregate state of the economy. As discussed in Menzio and Shi (2009), this contract ensures that workers fully internalize the effect of their decision to search on the job on the joint surplus of the match by paying a wage that is equal to marginal product while the hiring fee splits the match surplus.[26]

[25] All workers have an unemployment history of at least $v = 1$ as workers enter the economy in stage 3 and thus spend one period unemployed before searching for a job.

[26] This approach follows Schaal (2017) and Baley et al. (2022). See Stevens (2004) and Menzio and Shi (2009) for more details.

5.2 Calibration

A unit of time is one month, $\bar{v} = 12$, and $\bar{\tau} = 24$. The discount factor is set to $0.95^{1/12}$, which targets an annual interest rate of 5%. The matching function is $F(\psi, v) = \frac{\psi v}{\psi + v}$ and the production technology is $y_\chi(v, \tau) = g_\chi e^{(d_{\chi,v}(v-1)+d_{\chi,\tau}\tau)}$. We normalize the economy by setting $g_c = 1$. The flow value of unemployment, b, is chosen so that the ratio of b to average labor productivity is 0.71 following Hall and Milgrom (2008). We then calibrate the production parameters $\{g_n, d_{c,v}, d_{c,\tau}, d_{n,v}, d_{n,\tau}\}$ to match several moments. First is the wage premium of college jobs, i.e., how much higher a worker's wage is in a college occupation than an observationally equivalent worker's wage in a non-college occupation. To obtain this target, we estimate the following regression:

$$w_{imt} = \beta \text{College}_{imt} + \Gamma \cdot X_{imt} + \delta_i + \varepsilon_{imt}, \tag{25}$$

where w_{imt} is the log real hourly wage of an individual i in month m and year t, College is an indicator for whether individual i works in a college occupation in month m and year t, and X contains a cubic in potential experience, regional annual unemployment rate, aggregate annual unemployment rate, a quadratic in age, industry (2-digit), regional, and individual fixed effects. We follow Barnichon and Zylberberg (2019) in estimating (25) only on "marginally" underemployed workers, i.e., those workers who transitioned from proper employment to underemployment and back to proper employment in an effort to control for selection based on unobservable characteristics into underemployment. Estimating (25) yields a college job wage premium of 0.2597.[27] We also target the effects of accumulated unemployment and underemployment histories on log wages in both non-college and college jobs presented in column (6) of Table 2. To target these wage moments, we solve the model and subsequently simulate employment histories with a length of 43 months for an initial group of 10,000 workers who are replaced with a new worker upon exiting the labor market.[28] We then estimate the same regressions as in column (6) of Table 2 and column (4) of Online Appendix Table C15 using the simulated data. We repeat this simulation 100 times and take the average regression coefficients across all simulations.

This leaves seven parameters, $\{\delta, k_n, k_c, \lambda, a^L, a^H, \pi\}$, to calibrate. We calibrate these parameters via simulated method of moments by first targeting an unemployment rate of 8.1% and underemployment rate of 41.6%.[29] We also target the path of transition prob-

[27] Barnichon and Zylberberg (2019) find a wage premium of nearly 28% using the CPS. Online Appendix Table C15 contains further details and alternate specifications.

[28] We chose 43 months as this is the average amount of potential experience in our sample.

[29] The unemployment rate we target is the nonemployment rate. We target the nonemployment rate because, in the data, there are, a significant number of transitions from not in the labor force to employment.

Table 3: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.081	$\partial \log(w_n)/\partial v$	-0.014	-0.014
Underemployment rate	0.416	0.416	$\partial \log(w_c)/\partial v$	-0.014	-0.014
Average EE probability	0.014	0.012	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.262	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.710	-	-	-

abilities from non-college to college occupations that is produced by estimating the conditional duration dependence via equation (3). With this path in hand, we compute the average transition probability between non-college and college jobs:

$$\text{Average EE probability} = \sum_{\tau=0}^{24} \frac{e_n(\tau)}{e_n} \text{EE}_\tau, \tag{26}$$

where $e_n(\tau)$ is the number of workers with an underemployment duration τ, $e_n = \sum_{\tau=0}^{24} e_n(\tau)$ is the total number of workers employed in non-college jobs, and EE_τ is the probability of transitioning from a non-college to college job at duration τ. We find and target an average monthly transition probability of 0.0142.

While each of these parameters impacts more than one moment, one can interpret $\{k_n, k_c\}$ as targeting the unemployment and underemployment rates, $\{\delta, \lambda\}$ as targeting the average transition probability from non-college to college jobs, and $\{a^L, a^H, \pi\}$ as targeting the path of transition probabilities between non-college and college occupations. Table 3 shows that the model matches the empirical targets well. Further, Figure 4 shows that the model generated path of transition probabilities from non-college to college jobs almost exactly matches its empirical counterpart.

Table 4 displays the calibrated parameter values. Here we highlight the values of $\{a^L, a^H, \pi\}$. We first note that the calibrated value of π is 0.055, indicating that a large fraction of workers has limited suitability. Second, while both values of a^L and a^H are significantly less than one, we find that $a^H/a^L = 15.3$. So, broad suitable workers are nearly fifteen times more likely to be suitable for any given college job.

For context, the average weekly weighted transition probability from outside the labor force (unemployment) to employment is 0.075 (0.045). See Online Appendix Table C16.

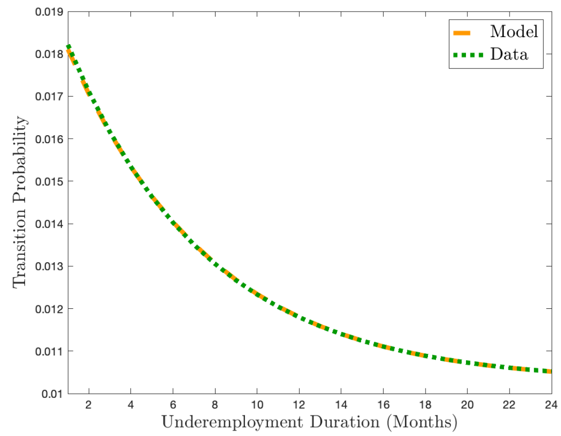


Figure 4: Duration Dependence in the Model and Data

5.3 Decomposing Duration Dependence

This section evaluates the relative contributions of unobserved heterogeneity and changes to occupation-specific human capital in generating duration dependence. Beginning with Figure 5(a), we present the transition probability from non-college to college occupations from the data, full version of the model, and version of the model where we shut off the accumulation and decay of occupation-specific human capital. The model with only unobserved heterogeneity generates a substantial amount of duration dependence.

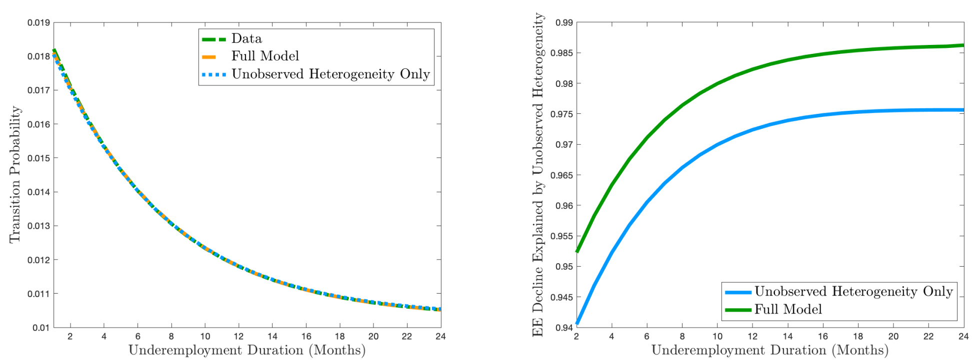
We next ask what percentage of the decline in the transition probability at each underemployment duration relative to the transition probability at $\tau = 1$ observed in the data can be explained by the full model and the version with unobserved heterogeneity only. Figure 5(b) illustrates that the full model can generate, at a minimum, 95% of the decline in the transition probability where the model with only unobserved heterogeneity explains at least 94% of the decline. At durations of more than 18 months, the model with only unobserved heterogeneity can explain close to 97% of the decline.

To arrive at an aggregate decomposition, we compute the weighted average of the fraction explained by unobserved heterogeneity over all τ. The weights are the fraction of underemployed workers who, in the steady-state, are employed at each duration τ. From this exercise, we find that the full model can explain 98.3% of the observed decline in the transition probability from non-college to college occupations. After removing human capital dynamics, the model with unobserved heterogeneity can still explain 97.2% of the duration dependence. Therefore, we conclude that unobserved heterogeneity is by far the biggest driver of duration dependence in our model and that incorporating changes in occupation-specific human capital has a very marginal effect on the model's ability

Table 4: Parameter Values

	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.022
δ	Entry/exit probability	0.011	a^H	Suitability pr.: type H	0.321
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.056
g_n	Non-college productivity	0.768	ϕ	Pr. of regaining college skills	0.547
b	Utility while unemployed	0.634	$d_{c,v}$	College skill loss: unemp.	-0.014
k_n	Non-college vacancy cost	1.382	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	1.617	$d_{n,v}$	Non-college skill loss: unemp.	-0.014
λ	Employed search intensity	0.826	$d_{n,\tau}$	Growth of non-college skills	0.001

to match the duration dependence observed in the data. In fact, when one gives skill loss what is arguably its best chance to contribute to duration dependence by making skill loss permanent, unobserved heterogeneity can still explain 94.61% of the duration dependence observed in the data.[30]

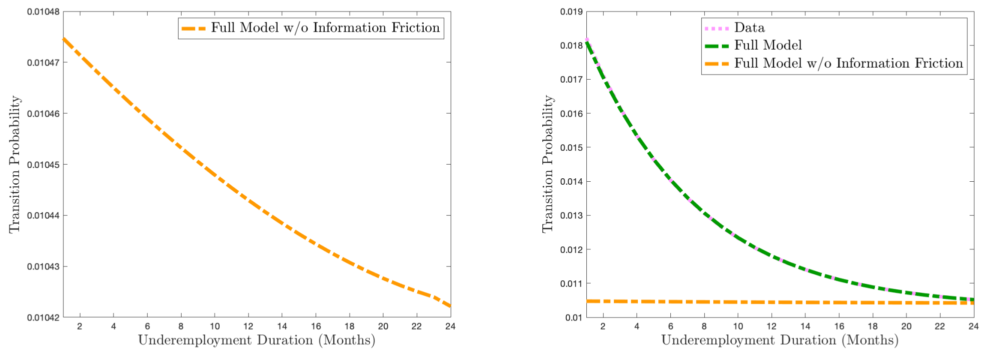


(a) Transition Probability Path (b) Fraction Explained at each τ

Figure 5: Duration Dependence Decomposition

To further demonstrate the role of unobserved heterogeneity, we turn off this channel by setting $a^L = a^H = 1$ while all other parameters take the values in Table 4. The main result from this exercise is that workers never select into underemployment. Thus, both the existence of underemployment and the resulting duration dependence are closely tied to the presence of unobserved heterogeneity.

[30]This simply requires setting $\phi = 0$ and re-calibrating the model. See Online Appendix D for further details on this calibration and decomposition exercise.



(a) No Information Frictions

(b) Model and Data Comparisons

Figure 6: Duration Dependence with and without Information Frictions

To conclude this section, we ask to what extent information frictions matter in generating underemployment and duration dependence. To answer this, we solve the full information version of our model which continues to feature heterogeneity in suitability and shocks to occupation-specific human capital. The only difference between this model and the baseline framework is that a worker's suitability type is public information.[31]

Two findings emerge from removing information frictions. First, under the calibrated values in Table 4, broad suitable workers never search for a non-college job. Therefore, the pool of underemployed workers contains only limited-suitability workers. Second, as seen in Figure 6, the full information model generates a negligible amount of duration dependence, as the transition probability decreases from 0.01048 at $\tau = 1$ to 0.01042 at $\tau = 24$. It follows that information frictions are a key ingredient to generating a pattern of duration dependence commensurate with what is observed in the data.

5.4 Aggregate Effects

With an understanding of the contribution of the key channels and frictions to duration dependence in hand, this section evaluates the effect of the same forces on aggregate outcomes to gain a more in-depth understanding of their macroeconomic implications.

Table 5 presents the effect of removing the key ingredients in the model on the unemployment rate, underemployment rate, and average labor productivity. Column (1) shows the outcomes produced by the baseline model. Column (2) shows that removing the accumulation and decay of human capital causes the unemployment rate to decrease from 8.1% to 4.3% as, without the threat of skill loss during underemployment, unemployed workers search for non-college jobs which have a higher job finding probability.

[31] Online Appendix E provides more details on the full information version of the model.

Table 5: Frictions and Aggregate Outcomes

	(1)	(2)	(3)	(4)
Unemployment rate	0.081	0.043	0.035	0.083
Underemployment rate	0.416	0.476	0.000	0.417
Labor productivity	1.000	0.990	1.119	0.999
Heterogeneous suitability	✓	✓		✓
Human capital dynamics	✓		✓	✓
Information frictions	✓	✓		

As a result of the change in search behavior among the unemployed, the flow of workers into underemployment increases, causing the underemployment rate to increase. As more workers are underemployed, labor productivity decreases by 1%.

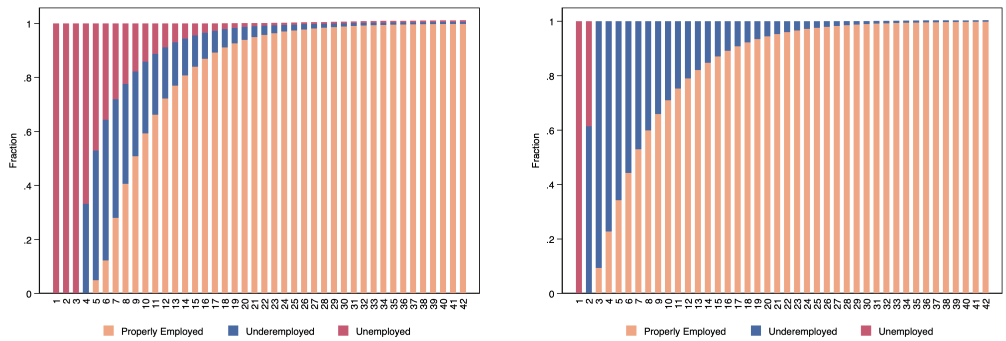
Column (3) illustrates the effect of removing heterogeneity in suitability by assuming all workers are suitable for any given college job with probability one. The first row shows a decline in unemployment as all workers are suitable for college jobs with probability one and thus face a higher job finding probability in submarkets for college jobs. The second row suggests that human capital dynamics alone do not result in any instances of underemployment, as all workers search for college jobs. Finally, labor productivity increases by 11.9% as all workers are employed in college jobs, which are more productive.

The first row of column (4) shows that removing information frictions increases the unemployment rate from 8.1% to 8.3%, as broad-suitability workers always direct their search to college jobs upon entering the labor market, which have lower job-finding probabilities than submarkets with non-college jobs.[32] The second row in column (4) shows that the aggregate underemployment rate is largely unchanged. However, this masks substantial heterogeneity: the underemployment rate among broad (limited) suitable workers is 0% (44.2%). Finally, removing information frictions causes average labor productivity to decrease by 0.1%. This effect is small because the composition of employment is largely unchanged relative to the baseline outcome.

5.5 Sorting and Bad Luck

To this point, our main quantitative finding is that a vast majority of the duration dependence observed in the data can be accounted for by unobserved heterogeneity among

[32] Recall from Section 5.3 that, in the full information version of the model, broad-suitable workers never search for non-college jobs.



(a) Unlucky Workers (b) Lucky Workers

Figure 7: Percentage of Time Spent in Various Labor Market Statuses

college graduates. A related and open question is what role "bad luck" plays in generating long underemployment durations. What we mean by bad luck is whether, because of the search frictions, some broad suitable workers experience long unemployment durations, select into and get stuck in underemployment.[33]

To evaluate the role of bad luck, we simulate the model and compare two groups of broad-suitable workers. The first group, "lucky", are those who find their first job within three months of entering the labor market. The second group, "unlucky", are those who take more than three months to find their first job. Figure 7 compares the fraction of each month spent in each labor market status across the two groups. From panel (a), unlucky workers gradually transition into underemployment after spending their first three months unemployed. Despite a slow start, the fraction of time spent underemployed is not noticeably higher than the lucky group. In fact, the average underemployment duration among the unlucky (lucky) group is 5.86 (5.98) months.

While there are no large differences in the incidence of longer underemployment spells across lucky and unlucky broad-suitable workers, there are significant differences in labor productivity across the two groups. Table 6 shows that the longer unemployment spell to begin the career, and hence higher amount of human capital decay, follows unlucky workers throughout their career as they, on average, have lower labor productivity in both non-college and college jobs. While the productivity gap exists in both types of jobs, it is much larger in non-college jobs as many workers eventually regain their college-specific skills after exiting underemployment.

[33]Our motive for assessing the role of bad luck is normative. The extent that a social planner would like to reduce the incidence of long underemployment spells is, in part, determined by how likely it is that a long-term underemployed worker would match with a more productive firm.

Table 6: Labor Productivity Among Unlucky and Lucky Workers

	College matches	Non-college matches
Unlucky	0.991	0.743
Lucky	0.998	0.766

6 Suggestive Evidence

The results in Section 5 suggest that unobserved heterogeneity in workers' suitability for college jobs plays a large role in generating duration dependence in underemployment. In this section, we provide two sources of suggestive evidence from hourly wage data in the NLSY to support this finding. While wages are not a function of a suitability in the theory, our model has several natural implications for comparing wages in college occupations between workers who experience short- and long-underemployment spells.

First, suppose that wages were a function of a worker's suitability type. This would occur, for example, if limited suitability workers produced less output than broad suitable workers. If this were the case, then the amount of residual wage dispersion among workers who transition from non-college to college jobs at relatively short underemployment durations would be larger than the amount of dispersion among the group who take longer to transition out of underemployment. In other words, there should be more wage inequality among observationally equivalent workers in the former group than the latter because the latter is primarily comprised of limited suitability workers. To investigate this, we estimate residual wage inequality in jobs held after a worker exits underemployment. The approach, which follows Acemoglu (2002), starts by estimating

$$w_{it} = \Gamma \cdot X_{it} + \delta_i + \varepsilon_{it}, \tag{27}$$

where w_{it} is an individual i's log hourly real wage at time t and X is a set of controls that includes years of potential experience (cubic), the annual unemployment rate, the annual regional unemployment rate, age (quadratic), family income per-capita, outstanding student loan debt, current level of job satisfaction, region, and 2-digit occupation and industry fixed effects. Finally, δ_i is an individual fixed effect. After estimating (27), we compute the ratio of the 90th to 50th and 50th to 10th percentile of the residuals.

The first row of Table 7 shows the 90/50 and 50/10 ratios for our entire sample. We see that the 90/50 ratio is 1.53 and the 50/10 ratio is 1.57, which is slightly below the

Table 7: Residual Wage Dispersion

	90/50 ratio	50/10 ratio
Full sample	1.525	1.569
A: After the first underemployment spell		
<1 year to exit underemployment	1.325	1.306
≥ 1 year to exit underemployment	1.201	1.196
B: Proper employment spell following underemployment		
<1 year to exit underemployment	1.319	1.315
≥ 1 year to exit underemployment	1.238	1.239

typical range of 1.7-1.9 (Hornstein et al., 2011).[34] Panel A restricts the sample to wages earned in college jobs following a worker's first underemployment spell and shows that the 90/50 (50/10) ratio is 10% (9%) larger in the group that exits underemployment in less than one year. Panel B shows that this pattern also emerges when we focus on the proper employment spell which immediately follows a spell of underemployment.

Our second form of evidence compares wage growth in college jobs between workers who experience short and long underemployment spells. The intuition is the following. If broad suitable workers have a higher ability to learn new skills, then wage growth in college jobs should be higher among the group of workers who experience short underemployment durations, as this group contains most of the broad-suitable workers who experience underemployment. To investigate this, we estimate the following regression:

$$\Delta w_{it} = \beta \text{Long}_{it} + \Gamma \cdot X_{it} + \varepsilon_{it}, \tag{28}$$

where Δw_{it} is the difference in the log of average hourly real wage between quarter $t-1$ and quarter t in workers' first proper employment and Long is equal to 1 if the worker's first underemployment spell lasted a year or longer and 0 otherwise. The vector X includes a cubic in potential experience, highest level of education, race, gender, quadratic in age, and 2-digit occupation and industry fixed effects. Table 8 contains the results and shows, across all specifications, workers who experience longer underemployment spells

[34] If we only include a standard set of controls used in the residual wage dispersion literature (see, e.g., Ortego-Marti (2016)), the 90/50 and 50/10 ratios are larger and fall within the range of 1.7-1.9. Moreover, the gap in the 90/50 and 50/10 ratios between the two groups of workers to exit underemployment is widened relative to those shown in Table 7. See Online Appendix Table C12.

Table 8: Wage Growth After Exiting Underemployment

	(1)	(2)	(3)	(4)
Long	-0.0233**	-0.0230**	-0.0224**	-0.0233**
	(0.0091)	(0.0091)	(0.0096)	(0.0099)
N	1815	1815	1779	1779
R^2	0.025	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. Significance levels are indicated as follows: *(p<0.10), **(p<0.05), ***(p<0.01).

exhibit lower wage growth in college jobs than their observationally equivalent peers.[35]

7 Conclusion

This paper has studied underemployment durations among recent college graduates in the US. Using the NLSY97, we have shown that (i) the average underemployment duration is eighteen months long, (ii) the probability a worker exits underemployment decreases in their underemployment duration, and (iii) longer underemployment histories are associated with lower wages in college occupations.

To explain these facts, we developed a directed search model with unobserved heterogeneity, occupation-specific human capital, heterogeneous firms, and on the job search. Workers, who are uncertain of their suitability for college jobs, learn about their job-finding probability in college jobs through search. Underemployment is generated when workers with a low expected suitability self-select into non-college jobs. Underemployed workers face both the accumulation of non-college and decay of college skills, creating duration dependence in underemployment. A quantitative analysis shows that unobserved heterogeneity is a large contributor to both the existence of underemployment and the negative duration dependence observed in the data.

[35]Column (1) contains the set of control variables described in the main text. Column (2) additionally controls for one year lag of the unemployment rate. Column (3) further controls for marital status. In column (4), we include the interaction between marital status and gender, as well as marital status and age. All results remain significant if we cluster standard errors at the birth year, highest education, or the contemporary year levels. We also expanded the sample to include all college job observations. The takeaway remains the same and the results can be found in Online Appendix Table C13. Further, we introduced a new binary variable to capture the length of time it takes for a worker to transition out of their most recent underemployment spell (and not just the first one). Again, the results are unchanged and can be found in Online Appendix Table C14.

References

ABEL, J. R., R. DEITZ, AND Y. SU (2014): "Are Recent College Graduates Finding Good Jobs?" *Current Issues in Economics and Finance*, 20.

ACEMOGLU, D. (2002): "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, 40, 7–72.

ADDISON, J. T. AND P. PORTUGAL (1989): "Job Displacement, Relative Wage Changes, and Duration of Unemployment," *Journal of Labor economics*, 7, 281–302.

ALBRECHT, J. AND S. VROMAN (2002): "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43, 283–305.

ALTONJI, J. G. AND R. A. SHAKOTKO (1987): "Do Wages Rise with Job Seniority?" *Review of Economic Studies*, 54, 437–459.

ALVAREZ, F. E., K. BOROVIČKOVÁ, AND R. SHIMER (2019): "Decomposing Duration Dependence in a Stopping Time Model," Working paper.

BALEY, I., A. FIGUEIREDO, AND R. ULBRICHT (2022): "Mismatch Cycles," *Journal of Political Economy*, 130, 2943–2984.

BARNICHON, R. AND A. FIGURA (2015): "Labor Market Heterogeneity and the Aggregate Matching Function," *American Economic Journal: Macroeconomics*, 7, 222–249.

BARNICHON, R. AND Y. ZYLBERBERG (2019): "Underemployment and the Trickle-Down of Unemployment," *American Economic Journal: Macroeconomics*, 11, 40–78.

BAYDUR, I. AND J. XU (2020): "Decomposing duration dependence: Skill depreciation vs. statistical discrimination," Working paper.

BGT AND SI (2018): "The Permanent Detour: Underemployment's Long-Term Effects on the Careers of College Grads," Tech. rep., Burning Glass Technologies and Strada Institute for the Future of Work, technical report.

COSKUN, S. (2020): "Young, Educated, Unemployed," Working paper.

DOLADO, J. J., M. JANSEN, AND J. F. JIMENO (2009): "On-the-job Search in a Matching Model with Heterogeneous Jobs and Workers," *The Economic Journal*, 119, 200–228.

DOPPELT, R. (2016): "The Hazards of Unemployment: A Macroeconomic of Job Search and Résumé Dynamics," Working paper.

FERNÁNDEZ-BLANCO, J. AND E. PREUGSCHAT (2018): "On the effects of ranking by unemployment duration," *European Economic Review*, 104, 92–110.

GAUTIER, P. A. (2002): "Unemployment and Search Externalities in a Model with Heterogeneous Jobs and Workers," *Economica*, 69, 21–40.

GONZALEZ, F. M. AND S. SHI (2010): "An Equilibrium Theory of Learning, Search, and Wages," *Econometrica*, 78, 509–537.

GUVENEN, F., B. KURUSCU, S. TANAKA, AND D. WICZER (2020): "Multidimensional Skill Mismatch," *American Economic Journal: Macroeconomics*, 12, 210–244.

HALL, R. E. AND P. R. MILGROM (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98, 1653–1674.

HORNSTEIN, A., P. KRUSSELL, AND G. L. VIOLANTE (2011): "Frictional Wage Dispersion in Search Models: A Quantitative Assessment," *American Economic Review*, 101, 2873–2898.

JACKSON, P. (2023): "Equilibrium underemployment," *Labour Economics*, 81, 102334.

JAROSCH, G. AND L. PILOSSOPH (2019): "Statistical Discrimination and Duration Dependence in the Job Finding Rate," *Review of Economic Studies*, 86, 1631–1665.

KAMBOUROV, G. AND I. MANOVSKII (2009a): "Occupational Mobility and Wage Inequality," *Review of Economic Studies*, 76, 731–759.

——— (2009b): "Occupational Specificity of Human Capital," *International Economic Review*, 50, 63–115.

KOSPENTARIS, I. (2021): "Unobserved heterogeneity and skill loss in a structural model of duration dependence," *Review of Economic Dynamics*, 39, 280–303.

KROFT, K., F. LANGE, M. J. NOTOWIDIGDO, AND L. F. KATZ (2016): "Long-term Unemployment and the Great Recession: The Role of Composition, Duration Dependence, and Nonparticipation," *Journal of Labor Economics*, 34, S7–S54.

LAUREYS, L. (2021): "The Cost of Human Capital Depreciation during Unemployment," *Economic Journal*, 131, 827–850.

LISE, J. AND F. POSTEL-VINAY (2020): "Multidimensional Skills, Sorting, and Human Capital Accumulation," *American Economic Review*, 110, 2328–2376.

LJUNGQVIST, L. AND T. J. SARGENT (1998): "The European Unemployment Dilemma," *Journal of Political Economy*, 106, 514–550.

MENZIO, G. AND S. SHI (2009): "Efficient Search on the Job and the Business Cycle," Working paper.

——— (2010a): "Block recursive equilibria for stochastic models of search on the job," *Journal of Economic Theory*, 145, 1453–1494.

——— (2010b): "Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations," *American Economic Review*, 100, 327–332.

——— (2011): "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy*, 119, 468–510.

MOEN, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

MOSCARINI, G. AND K. THOMSSON (2007): "Occupational and Job Mobility in the US," *Scandinavian Journal of Economics*, 109, 807–836.

MUELLER, A. I., J. SPINNEWIJN, AND G. TOPA (2021): "Job Seekers' Perceptions and Employment Prospects: Heterogeneity, Duration Dependence, and Bias," *American Economic Review*, 111, 324–363.

NEAL, D. (1995): "Industry-Specific Human Capital: Evidence from Displaced Workers," *Journal of Labor Economics*, 13, 653–677.

ORTEGO-MARTI, V. (2016): "Unemployment history and frictional wage dispersion," *Journal of Monetary Economics*, 78, 5–22.

——— (2017a): "Differences in skill loss during unemployment across industries and occupations," *Economics Letters*, 161, 31–33.

——— (2017b): "Loss of skill during unemployment and TFP differences across countries," *European Economic Review*, 100, 215–235.

PISSARIDES, C. A. (1992): "Loss of Skill During Unemployment and the Persistence of Employment Shocks," *Quarterly Journal of Economics*, 107, 1371–1391.

SCHAAL, E. (2017): "Uncertainty and Unemployment," *Econometrica*, 85, 1675–1721.

SCHMIEDER, J. F., T. VON WACHTER, AND S. BENDER (2016): "The Effect of Unemployment Benefits and Nonemployment Durations on Wages," *American Economic Review*, 106, 739–777.

SHEPHARD, A. AND M. SIDIBÉ (2022): "Schooling Investment, Mismatch, and Wage Inequality," Working paper.

STEVENS, M. (2004): "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *Review of Economic Studies*, 71, 535–551.

Appendix

A Empirical Appendix

A.1 National Longitudinal Survey of Youth 1997 (NLSY97)

The NLSY97 is a nationally representative survey that tracks the lives of 8,984 American youth born between 1980 and 1984, with the initial mandate of studying long-term employment processes. As such, it covers employment activities that can affect the ability to obtain and perform a job (such as education, training, etc.), as well as other sections on marriage, fertility, household composition, health, and so on. The ongoing survey has been conducted in 19 waves, on an annual basis from 1997 through 2011 and biennially thereafter. All respondents were ages 12-17 at their first interview.

Our sample construction begins with selecting individuals who obtained a bachelors degree or above and have at least one year of labor market experience before 2011. As the analysis of duration dependence requires consecutive employment records, 171 out of 1,974 college graduates with at least one year of an employment record are dropped because of missing employment records in some weeks and 651 are dropped because they have a missing occupation code. Next, we drop 6 respondents who are always enrolled at school after college graduation. Finally, all employment records start from completely entering the labor market, leaving 996 respondents with at least one year of employment records after their last enrollment in school. Online Appendix Table C4 provides summary statistics.

The complete employment history includes weekly working hours, employment status, 2002 Census Industry and Occupation Codes, and real hourly wage.[36] The individual characteristics we focus on include gender, age, race, Armed Services Vocational Aptitude Battery (ASVAB) percentile, education history (including the highest education and the graduation date), college major, college GPA, student loan debt, and family income per-capita. As for the college major, we focus on primary majors self-reported term by term, and consider the major reported the most times during the undergraduate period as the college major of a certain respondent. In terms of college GPA, we take the average GPA across all available reported terms. Last, the financial information has also been extracted to pin down the amount of student loan currently owed and the family income per-capita, where the latter is calculated by dividing the total family income by the household size.

[36] We deflate the hourly rate of pay with the Consumer Price Index. We then code the deflated hourly pay rate to missing if that is less than \$1 or more than \$1,000. We code weekly working hours to missing if less than 10 or more than 98 hours. Note that the average working hours per week among underemployed (properly employed) workers is 42.97 (45.25).

Both debt and family income are in thousands. We also consider the possibility of dual jobs and find that less than 13% of observations have more than one job. For these observations, we code the main job to be the one with the highest real wage.

A.2 Occupational Information Network (O*NET)

The O*NET measures occupational requirements and worker attributes. It is composed of four survey questionnaires covering skills, knowledge, generalized work activities, and work context, and all of them are answered by job incumbents at a variety of business work sites. Notably, it reports the required level of education to perform a job under the domain of worker requirements, which enables us to determine whether an occupation is one that typically requires a bachelors degree and above.

A.3 Within-firm Transitions

As discussed in the main text, there may be measurement error in within-firm occupational transitions. We attempt to correct this error by identifying "genuine" within-firm occupation switches from non-college to college occupations. To do so, we measure the angular distance between the skill requirement of the new college occupation and the previous non-college occupation. Specifically, let $\phi\colon \mathbb{R}^3 \times \mathbb{R}^3 \to [0, \pi/2]$, and define the angular distance between two skill vectors $\mathbf{r}_i$ and $\mathbf{r}_j$ as

$$\phi(\mathbf{r}_i, \mathbf{r}_j) = \cos^{-1}\left(\frac{\mathbf{r}_i \cdot \mathbf{r}_j'}{\|\mathbf{r}_i\|\|\mathbf{r}_j\|}\right). \tag{A.29}$$

A within-firm transition from an occupation i to another different occupation j is treated as a "genuine" transition if and only if $\phi(\mathbf{r}_i, \mathbf{r}_j) \geq \bar{\phi}$ where $\bar{\phi}$ is chosen so that the average correlation in skill requirements in "genuine" switches is close to zero. We set $\bar{\phi} = 18.094$, which results in a correlation in skill requirements among within-firm occupation switches of 0.0048. In our sample, 46/96, or 48% of within-firm transitions from non-college to college jobs have been corrected using the angular-distance strategy, which is close to the propensity of switching careers, 42.1%, obtained by Baley et al. (2022).

An alternative measure of distance between skill requirements is the Euclidean distance, which not only captures the difference of skill mix, but also the level of skill requirement in each aptitude. Formally, the Euclidean distance between occupation i and j

is given by

$$\psi(\mathbf{r}_i, \mathbf{r}_j) = \sqrt{\sum_{k=1}^{3} (r_{i,k} - r_{j,k})^2}, \tag{A.30}$$

where $r_{i,k}$ is occupation i's requirement in aptitude k (verbal, math, and social).

B Theoretical Appendix

B.1 Laws of Motion

Let $u(v)$ denote the measure of workers who begin the period unemployed with unemployment history v. The law of motion for unemployed workers is given by

$$\hat{u}(v) = \begin{cases} (1-\delta)\delta, & \text{for } v = 1, \\ (1-\delta)u(v_-)[\varrho_{n,v_-}(1 - p(\theta^*_{n,v_-})) + \varrho_{c,v_-}(1 - \mu_{v_-} p(\theta^*_{c,v_-}))], & \text{for } v \in \{2, \dots, \bar{v}-1\}, \\ (1-\delta)\sum_{v=\bar{v}}^{\bar{v}+1} u(v_-)[\varrho_{n,v_-}(1 - p(\theta^*_{n,v_-})) + \varrho_{c,v_-}(1 - \mu_{v_-} p(\theta^*_{c,v_-}))], & \text{for } v = \bar{v}, \end{cases} \tag{B.1}$$

where $\hat{u}(v)$ is the measure of unemployed workers with unemployment history v at the beginning of the next period, $v_- \equiv v - 1$, $\varrho_{\chi,v} \in [0,1]$ is the fraction of unemployed workers with unemployment history v who search for type χ jobs, μ_v is the expected suitability of an unemployed worker with unemployment history v, and $\theta^*_{\chi,v}$ is tightness associated with the policy function of unemployed workers with unemployment history v who search for type χ jobs. From (B.1), the measure of unemployed workers who begin the next period unemployed with history $v \in \{2, \dots, \bar{v}\}$ is given by those who began the previous period unemployed and did not find a job or exit the economy. For $v = 1$, the measure of unemployed workers is simply given by the new entrants to the labor market during stage 3 in the previous period who did not exit in stage 4.

Now let $e_\chi(v, \tau)$ denote the measure of workers with history (v, τ) and are employed at type χ jobs at the beginning of the period. The law of motion for $e_n(v, \tau)$ is given by

$$\hat{e}_n(v, \tau) = \begin{cases} (1-\delta)u(v)\varrho_{n,v} p(\theta^*_{n,v}), & \text{for } \tau = 1, \\ (1-\delta)e_n(v, \tau_-)(1 - \lambda\mu_{v,\tau_-} p(\theta^*_{c,v,\tau_-})), & \text{for } \tau \in \{2, \dots, \bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_n(v, \tau_-)(1 - \lambda\mu_{v,\tau_-} p(\theta^*_{c,v,\tau_-})) & \text{for } \tau = \bar{\tau}, \end{cases} \tag{B.2}$$

where $\mu_{v,\tau}$ is a worker with history (v, τ)'s expected suitability, $\tau_- \equiv \tau - 1$, and $\theta^*_{\chi,v,\tau}$ is tightness associated with the policy function of an employed worker with history (v, τ) in a submarket with type χ jobs. From (B.2), workers who begin the next period employed in non-college jobs and with $\tau = 1$ are comprised of unemployed workers who matched with a non-college job in the previous period. Workers who begin the next period with at least two periods of underemployment history are comprised of those who began the previous period underemployed and did not transition to a college job. All respective measures are multiplied by $(1-\delta)$ as this is the fraction of workers who remain in the labor market across periods.[37] The law of motion for $e_c(v,\tau)$ is given by

$$\hat{e}_c(v,\tau) = \begin{cases} (1-\delta)\big[u(v)\varrho_{c,v}\mu_v p(\theta^*_{c,v}) + e_c(v,\tau) + \phi(\sum_{\tau=1}^{\bar{\tau}}[e_c(v,\tau) + e_n(v,\tau)\lambda\mu_{v,\tau}p(\theta^*_{c,v,\tau})])\big], & \text{for } \tau = 0, \\ (1-\delta)(1-\phi)[e_c(v,\tau) + e_n(v,\tau)\lambda\mu_{v,\tau}p(\theta^*_{c,v,\tau})], & \text{for } \tau \in \{1,\ldots,\bar{\tau}\}. \end{cases} \tag{B.3}$$

The measure of workers who work in college jobs and have zero underemployment experience consists of unemployed workers who find a college job, those workers who are already employed in college jobs with $\tau = 0$, and finally a fraction ϕ of those employed in a college job with $\tau \geq 1$ or who transitioned from a non-college to college job and regained their college-specific skills. The measure of workers employed in college jobs with $\tau > 0$ is given by a fraction $1-\phi$ of workers who began the previous period either already employed in a college job or transitioned from a non-college job to a college job and did not regain their college-specific skills.

B.2 Proof of Proposition 1

To establish the result, it is sufficient to show that if a worker with unemployment history v and beliefs μ searches for a non-college job, then they would make the same choice with unemployment history $\hat{v}$ and beliefs μ in the following period, i.e. $V_u(v,\mu) = V_u(\hat{v},\mu)$. We show this through backwards induction. Consider $v = \bar{v}-1$. We have

$$\begin{aligned} V_u(\bar{v}-1,\mu) = \max\big\{& b + \beta(1-\delta)\{V_u(\bar{v},\mu) - k_n\theta_{n,\bar{v}} + p(\theta_{n,\bar{v}})[V_{e,n}(\bar{v},0,\mu) - V_u(\bar{v},\mu)]\}, \\ & b + \beta(1-\delta)\{V_u(\bar{v},\hat{\mu}) + \mu\big[-k_c\theta_{c,\bar{v}} + p(\theta_{c,\bar{v}})[V_{e,c}(\bar{v},0,\mu) - V_u(\bar{v},\hat{\mu})]\big]\}\big\}, \end{aligned} \tag{B.4}$$

$$\begin{aligned} V_u(\bar{v},\mu) = \max\big\{& b + \beta(1-\delta)\{V_u(\bar{v},\mu) - k_n\theta_{n,\bar{v}} + p(\theta_{n,\bar{v}})[V_{e,n}(\bar{v},0,\mu) - V_u(\bar{v},\mu)]\}, \\ & b + \beta(1-\delta)\{V_u(\bar{v},\mu) + \mu\big[-k_c\theta_{c,\bar{v}} + p(\theta_{c,\bar{v}})[V_{e,c}(\bar{v},0,\mu) - V_u(\bar{v},\mu)]\big]\}\big\}. \end{aligned} \tag{B.5}$$

[37] We have simplified equations (B.2)-(B.3) by accounting for the fact that workers employed in college jobs will not transition to a non-college job.

Suppose that the worker with an unemployment history $\bar{v}-1$ searches for a non-college job. From equations (B.4) and (B.5), $V_u(\bar{v},\mu) = V_u(\bar{v}-1,\mu)$ as $\hat{\mu} = \mu$, i.e., the worker's expected suitability does not change after searching for a non-college job, and both $V_{e,n}$ and $V_{e,c}$ are not direct functions of a worker's unemployment history. It follows that a worker who searches for a non-college job with an unemployment history $\bar{v}-1$ will also search for a non-college job with unemployment history $\bar{v}$. We can then have

$$V_u(\bar{v}-2,\mu) = \max\big\{b+\beta(1-\delta)\{V_u(\bar{v},\mu) - k_n\theta_{n,\bar{v}-1} + p(\theta_{n,\bar{v}-1})[V_{e,n}(\bar{v}-1,0,\mu) - V_u(\bar{v},\mu)]\},$$

$$b+\beta(1-\delta)\{V_u(\bar{v},\hat{\mu}) + \mu\big[-k_c\theta_{c,\bar{v}-1} + p(\theta_{c,\bar{v}-1})[V_{e,c}(\bar{v}-1,0,\mu) - V_u(\bar{v},\hat{\mu})]\big]\}\big\}. \quad \text{(B.6)}$$

Note that the only (potential) difference between equations (B.4) and (B.6) are the values of $\theta_{\chi,\bar{v}-1}$ and $\theta_{\chi,\bar{v}}$. However, we know that

$$\theta_{n,\bar{v}-1} = \arg\max\{-k_n\theta + p(\theta)[V_{e,n}(\bar{v}-1,0,\mu) - V_u(\bar{v},\mu)]\}, \quad \text{(B.7)}$$

$$\theta_{n,\bar{v}} = \arg\max\{-k_n\theta + p(\theta)[V_{e,n}(\bar{v},0,\mu) - V_u(\bar{v},\mu)]\}. \quad \text{(B.8)}$$

As $V_{e,n}$ is independent of a worker's unemployment history, it follows that $\theta_{n,\bar{v}-1} = \theta_{n,\bar{v}}$. The same argument can be used to show $\theta_{c,\bar{v}-1} = \theta_{c,\bar{v}}$. Hence, $V_u(\bar{v}-2,\mu) = V_u(\bar{v}-1,\mu)$. Iterating backwards establishes $V_u(v,\mu) = V_u(\hat{v},\mu)$ for all $v \in \Upsilon$ if a worker searches for non-college jobs with unemployment history v and expected suitability μ. ∎

B.3 Proof of Proposition 2

Suppose that a worker who is currently employed in a college job searches in a submarket for college jobs. Their submarket choice is given by

$$\theta = \arg\max\{-k_c\theta + p(\theta)(V_{e,c}(v,\tau,\mu) - V_{e,c}(v,\tau,\hat{\mu}))\}. \quad \text{(B.9)}$$

Clearly the solution to (B.9) is $\theta = 0$. If the worker chose $\theta > 0$ and found another college job, then the value of their employment relationship is unchanged from the value of their current employment relationship, $V_{e,c}(v,\tau,\mu)$.

Now suppose that the worker searches for a non-college job. Their submarket choice is given by

$$\theta = \arg\max\{-k_n\theta + p(\theta)(V_{e,n}(v,\tau,\mu) - V_{e,c}(v,\tau,\mu))\}. \quad \text{(B.10)}$$

A worker would not transition to a non-college job to only transition back to a college job in the future as underemployment leads to depreciation of college occupation-specific human capital. Therefore, if the worker transitions to a non-college job, they will remain in a non-college job until they exit the labor force. It follows that the sum of the worker's lifetime utility and firm's profits in a non-college job is bounded by

$$V_{e,n}(v,\tau,\mu) = \frac{y_n(\bar{\tau})}{1-\beta(1-\delta)}. \tag{B.11}$$

If, however, the worker were to remain employed in the college job until exiting the labor force, the value of their current employment relationship would be given by

$$V_{e,c}(v,\tau,\mu) = \frac{y_c(\tau)}{1-\beta(1-\delta)(1-\phi)} + \frac{\phi\beta(1-\delta)y_c(0)}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]}. \tag{B.12}$$

Clearly $V_{e,c}(v,\tau,\mu) > V_{e,n}(v,\tau,\mu)$ as we have assumed $y_c(\tau) > y_n(\tau)$ for all $\tau \in T$. Therefore, the solution to (B.10) is $\theta = 0$. ■

B.4 Proof of Proposition 3

Consider the model without heterogeneity in a worker's suitability type by simply assuming all workers produce output at college jobs with probability 1. We denote $V_{e,\chi}(\tau)$ as the sum of the worker's utility and firm's profits in a match between a type χ job and worker with underemployment history τ. It is straightforward to show:

$$V_{e,n}(\tau) = y_n(\tau) + \beta(1-\delta)\{V_{e,n}(\hat{\tau}) + \lambda p(\hat{\theta})\Delta(\hat{\tau})\}, \tag{B.13}$$

$$V_{e,c}(\tau) = \frac{y_c(\tau)[1-\beta(1-\delta)] + y_c(0)\beta(1-\delta)\phi}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]}, \tag{B.14}$$

where $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$ and $\hat{\theta}$ solves

$$k_c \geq p'(\hat{\theta})\Delta(\hat{\tau}). \tag{B.15}$$

Part (i): We proceed via proof by contradiction. Suppose that $\Delta'(\tau) > 0$. Consider $V_{e,n}(\bar{\tau})$ and $V_{e,n}(\bar{\tau}-1)$. It is easy to show that

$$V_{e,n}(\bar{\tau}) - V_{e,n}(\bar{\tau}-1) = y_n(\bar{\tau}) - y_n(\bar{\tau}-1) > 0. \tag{B.16}$$

Now consider

$$V_{e,n}(\bar{\tau}-1) - V_{e,n}(\bar{\tau}-2) = y_n(\bar{\tau}-1) - y_n(\bar{\tau}-2) + \beta(1-\delta)\{V_{e,n}(\bar{\tau}) - V_{e,n}(\bar{\tau}-1) + \lambda p(\theta^*)\Delta(\bar{\tau}) - \lambda p(\theta^{**})\Delta(\bar{\tau}-1)\}. \quad \text{(B.17)}$$

From (B.17), $V_{e,n}(\bar{\tau}-1) - V_{e,n}(\bar{\tau}-2) > 0$ as $y_n(\bar{\tau}-1) > y_n(\bar{\tau}-2)$, $V_{e,n}(\bar{\tau}) > V_{e,n}(\bar{\tau}-1)$ from equation (B.16), and (assuming interior solutions), $\lambda p(\theta^*)\Delta(\bar{\tau}) > \lambda p(\theta^{**})\Delta(\bar{\tau}-1)$ as $\Delta(\bar{\tau}) > \Delta(\bar{\tau}-1)$ (by assumption) and $\theta^* > \theta^{**}$ following (B.15). We can extend this logic to show that $V_{e,n}(\tau) < V_{e,n}(\hat{\tau})$ for all $\tau \in \{1, 2, \ldots, \bar{\tau}-1\}$ and $\hat{\tau} = \min\{\tau+1, \bar{\tau}\}$. In other words, under the assumption that $\Delta(\tau)$ is increasing in τ, $V_{e,n}(\tau)$ is also increasing in τ. However, we can see from (B.14) that $V_{e,c}(\tau)$ is decreasing in τ as $y_c'(\tau) < 0$. Hence, $\Delta(\tau) = V_{e,c}(\tau) - V_{e,n}(\tau)$ is decreasing in τ, which is a contradiction.

Part (ii): We now proceed to show that θ is (weakly) decreasing in τ. In the main text, we showed that the optimal choice of θ satisfies

$$k_c \geq p'(\theta)\Delta(\tau). \quad \text{(B.18)}$$

For this part of the proof, we assume an interior solution to (B.18). Following part (i), where we have shown $\Delta'(\tau) < 0$, it follows that the optimal θ which satisfies equation (B.18) is decreasing in τ as $p(\theta)$ is strictly concave and, hence, $p'(\theta)$ is strictly decreasing in θ. As $p(\theta)$ is strictly increasing in θ, it follows that $\lambda p(\theta)$ is strictly decreasing in τ for all τ such that $\theta > 0$ satisfies (B.18). If $\theta = 0$ solves (B.18) for some $\tau^* \in T$, it follows that $p(\theta) = 0$ for all $\tau \in \{\tau^*, \ldots, \bar{\tau}\}$. Hence, $\lambda p(\theta)$ is weakly decreasing in τ.

Part (iii): Consider $\tau = \tilde{\tau} \in T$ where $k_c > p'(0)\Delta(\tilde{\tau})$. It follows that $\theta = 0$ solves (B.18) as $p'(\theta)$ is strictly decreasing in θ. Moreover, as $\Delta'(\tau) < 0$, $\lambda p(\theta_\tau) = 0$ for all $\tau \in \{\tilde{\tau}, \ldots, \bar{\tau}\}$.

Our final objective is to arrive at a condition that determines value of $\tilde{\tau}$. To do so, we define $\tilde{\Delta}(\tau)$ as the surplus generated by a worker transitioning from a non-college to a college job when $\theta_\tau = 0$ for all $\tau \in \{\hat{\tau}, \ldots, \bar{\tau}\}$ where $\hat{\tau} = \min\{\tau+1, \bar{\tau}\}$. The surplus in a match with a college job is given by (B.14). It is straightforward to show that the surplus in a match with a non-college job, $V_{e,n}(\tau)$, when $\theta_\tau = 0$ for all $\tau \in \{\hat{\tau}, \ldots, \bar{\tau}\}$ is given by

$$V_{e,n}(\tau) = \mathbb{I}_{\{\tau<\bar{\tau}\}} \sum_{x=\tau}^{\bar{\tau}-1} \beta^{x-\tau}(1-\delta)^{x-\tau} y_n(x) + \frac{\beta^{\bar{\tau}-\tau}(1-\delta)^{\bar{\tau}-\tau} y_n(\bar{\tau})}{1-\beta(1-\delta)}, \quad \text{(B.19)}$$

where $\mathbb{I}_{\{\tau<\bar{\tau}\}}$ is an indicator equal to 1 if $\tau < \bar{\tau}$ and 0 otherwise. Hence, $\tilde{\Delta}(\tau)$ is given by

$$\tilde{\Delta}(\tau) = \frac{y_c(\tau)[1-\beta(1-\delta)] + y_c(0)\beta(1-\delta)\phi}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\phi)]} - \mathbb{I}_{\{\tau<\bar{\tau}\}} \sum_{x=\tau}^{\bar{\tau}-1} \beta^{x-\tau}(1-\delta)^{x-\tau} y_n(x) + \frac{\beta^{\bar{\tau}-\tau}(1-\delta)^{\bar{\tau}-\tau} y_n(\bar{\tau})}{1-\beta(1-\delta)}. \tag{B.20}$$

To summarize, in the case where $\tilde{\tau} \leq \bar{\tau}$, we have that $\lambda p(\theta_\tau)$ is strictly decreasing in τ for all $\tau \in \{1, \ldots, \tilde{\tau}\}$ and $\lambda p(\theta_\tau) = 0$ for all $\tau \in \{\tilde{\tau}, \ldots, \bar{\tau}\}$. If $\tilde{\tau} > \bar{\tau}$, then $\lambda p(\theta_\tau)$ is strictly decreasing in τ for all $\tau \in T$. ∎

Online Appendix (Not for Publication)

C Supplementary Empirical Appendix

C.1 Further Details on Occupation Skill Requirements

To measure the distance in skill requirements between occupations, we start by measuring the occupation's requirement along three different skill dimensions. Specifically, each occupation is represented by a three-dimensional vector $(r_{verbal}, r_{math}, r_{social})$ where r_{verbal} measures the occupation's verbal skill requirement, r_{math} measures the math/quantitative skill requirement, and r_{social} captures the social skill requirement.

To measure verbal and mathematical skill requirements, we strictly follow the methodology used by Guvenen et al. (2020). The first step is to construct four scores for each occupation. The scores are: (i) word knowledge, (ii) paragraph comprehension, (iii) arithmetic reasoning, and (iv) mathematics knowledge. To construct these scores, we first select 26 O*NET descriptors that are chosen by the Defense Manpower Data Center (DMDC) and are listed in the top part of Table C1. In the raw data, these descriptors range in value from 0 to 5. We re-scale their values in each year to fall between 0 and 1 and then take the average value for each descriptor between 2003 and 2011. Finally, we construct a weighted average in each of the four skill categories using the weights matrix provided by the DMDC. For example, to construct the word knowledge score in occupation o, $S_{o,wk}$, we compute

$$S_{o,wk} = \sum_{i=1}^{26} s_{o,i} * \omega_{wk,i}, \tag{C.1}$$

where $s_{o,i}$ is descriptor i's average value between 2003 and 2011 for occupation o and $\omega_{wk,i}$ is the weight given to descriptor i in the category of word knowledge.

Second, we normalize the standard deviation of each score to one and reduce these four scores into two composite indicators, r_{verbal} and r_{math}, by applying principal component analysis (PCA). To be specific, the verbal skill is the first principal component of word knowledge and paragraph comprehension, and the math skill is the first principal component of arithmetic reasoning and mathematics knowledge. The verbal and math skills are then converted into percentile ranks among all occupations as the scale of each component is somewhat arbitrary.

The social skill requirement can be identified similarly. By applying PCA to six scaled O*NET descriptors, we construct a single index to reflect the social skill requirement and then apply the percentile transform as above. The six descriptors used to construct the social skill requirement are listed at the bottom of Table C1. Based on the skill require-

Table C1: List of Descriptors

Panel A: Verbal and Math Skills	
Oral Comprehension	Written Comprehension
Deductive Reasoning	Inductive Reasoning
Information Ordering	Mathematical Reasoning
Number Facility	Reading Comprehension
Mathematics Skill	Science
Technology Design	Equipment Selection
Installation	Operation and Control
Equipment Maintenance	Troubleshooting
Repairing	Computers and Electronics
Engineering and Technology	Building and Construction
Mechanical	Mathematics Knowledge
Physics	Chemistry
Biology	English Language
Panel B: Social Skills	
Social Perceptiveness	Coordination
Persuasion	Negotiation
Instructing	Service Orientation

ment along each dimension $(r_{verbal}, r_{math}, r_{social})$, we proceed to calculate the average skill requirement for each occupation by taking the unweighted average across the three dimensions.

Next, we examine the relationship between skill and education requirements. Table C2 lists the skill and education requirements of the five most common college and non-college jobs in our sample. College jobs are associated with higher skill requirements along each skill dimension, as well as the average skill requirement. Figure C1 further demonstrates by plotting the average skill requirement among non-college and college jobs for verbal, math, social and average skill requirements. The same pattern emerges. In particular, college jobs have significantly higher skill requirements.

Finally, Figure C2 presents a heat map demonstrating the correlation between skill and education requirements. Darker shades of red indicate a stronger positive correlation. The first column represents the percentage of respondents in the O*NET surveys who state that a bachelors degree or higher is needed to perform a certain occupation. The second column is a binary variable that indicates whether more than 50% of respondents indicate that a bachelors degree or higher is necessary to perform the occupation. The results show a positive correlation between education and skill requirements.

Table C2: Skill Requirements of Five Most Common College/Non-college Jobs

Occupation title	Verbal	Math	Social	Avg.
Panel A: College jobs				
Elementary and middle school teachers	0.82	0.77	0.85	0.81
Registered nurses	0.76	0.67	0.80	0.74
Accountants and auditors	0.64	0.86	0.33	0.61
Secondary school teachers	0.84	0.81	0.92	0.85
Social workers	0.24	0.14	0.96	0.44
Panel B: Non-college jobs				
First-line supervisors/managers of retail sales workers	0.33	0.44	0.56	0.44
Retail salespersons	0.10	0.21	0.20	0.17
Sales representatives, wholesale and manufacturing	0.28	0.37	0.67	0.44
Secretaries and administrative assistants	0.40	0.23	0.18	0.27
Customer service representatives	0.34	0.31	0.30	0.32

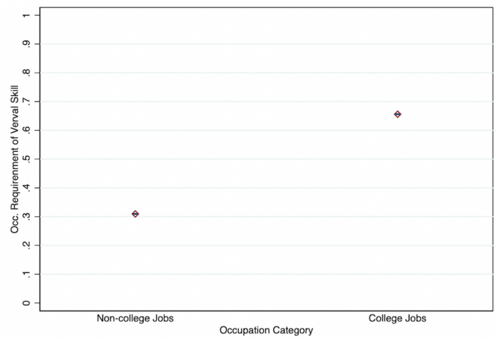
C.2 Skill Distance and the Scarring Effects of Underemployment

To further support the notion of the accumulation and decay of occupation-specific human capital, we study how the scarring effects of underemployment vary with the distance in skill requirements between a worker's current college occupation and previous non-college occupation. The idea here is that if the distance in required skills between the two occupations is larger, then the skills required by the new college occupation would have been used less intensively in the previous non-college occupation and thus experienced a greater rate of decay, ultimately leading to larger scarring effects. To assess this hypothesis, we estimate the following regression:

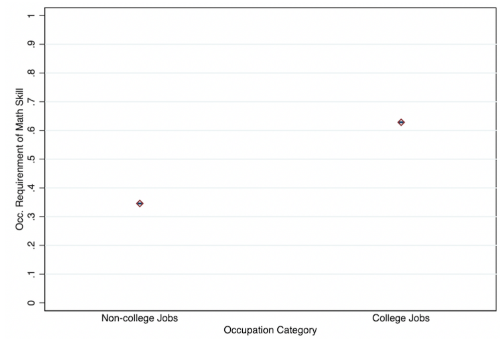
$$w_{imt} = \alpha \text{Underhis}_{imt} + \gamma \phi_{imt} + \zeta \text{Underhis}_{imt} \times \phi_{imt} + \Gamma \cdot X_{imt} + \delta_i + \varepsilon_{imt}, \tag{C.2}$$

where ϕ_{imt} is the distance in skill requirements between individual i's current college occupation and their most recent non-college occupation and X contains the same controls as in equation (4) in the main text. We use two measures of distance. The first is the Euclidean distance while the second is the angular distance as in Baley et al. (2022). The estimation of (C.2) only includes observations among individuals currently employed in a college occupation and who have been previously underemployed. Moreover, we restrict to those individuals where the average skill level in their current college occupation is higher than their previous non-college occupation.

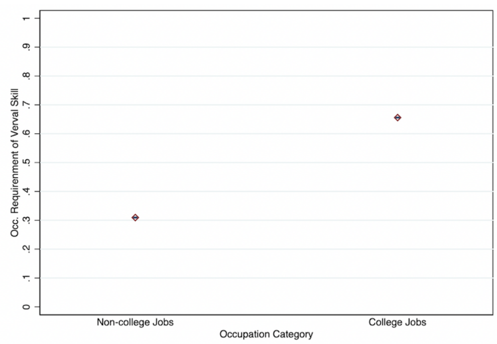
Table C3 contains the results. Column (1) reveals that a larger Euclidean distance is associated with significantly higher scarring effects. In particular, the average Euclidean



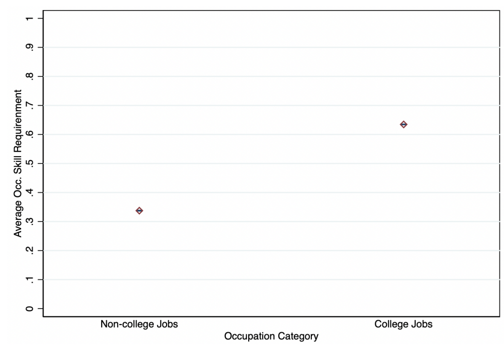
(a) Verbal



(b) Math



(c) Social



(d) Average

Figure C1: Comparison of Skill Requirements among Non-college Jobs Versus College Jobs

Notes: Graph shows 95% confidence intervals. We test the null hypothesis that the verbal/math/social/average skill requirement of non-college jobs is the same as that of college jobs against the alternative that the skill requirement of non-college jobs is below that of college jobs, and the test results indicate a p-value less than 0.01, supporting the alternative hypothesis.

distance of 0.75 in transitions between non-college and college jobs leads to a 3.4% rise in wage scars due to additional underemployment in previous non-college jobs. Column (2) echoes this result by examining the angular distance between skill requirement vectors. An average angular distance of 28.04 in transitions between non-college and college jobs results in an 4.8% increase in wage scars caused by additional underemployment in previous non-college jobs.

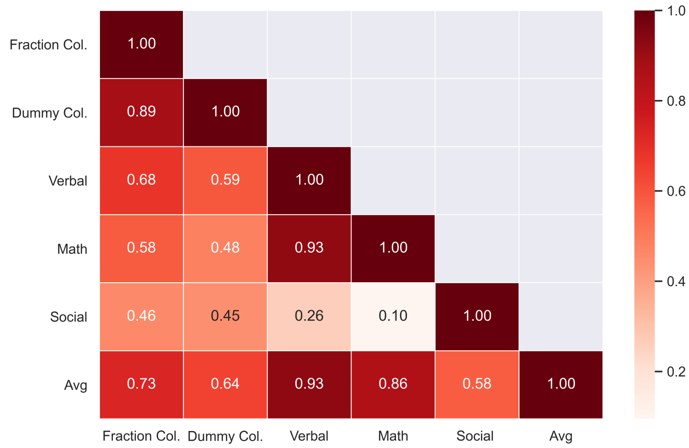


Figure C2: Correlation Between Education and Skill Requirements

Table C3: Skill Distance and the Scarring Effects of Underemployment

	(1)	(2)
Underhis	0.0173	0.0374***
	(0.0106)	(0.0098)
Underhis*Euclidean distance	-0.0454***	
	(0.0103)	
Underhis*Angular distance		-0.0017***
		(0.0004)
N	16,594	16,594
R^2	0.924	0.923

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

C.3 Supplementary Tables and Figures

Table C4: Descriptive Statistics

	N	Mean	Std. Dev	Min	Max
Gender	996	0.54	0.50	0	1
– Male	439	0	0	0	0
– Female	557	1	0	1	1
Birth year	996	1982.06	1.40	1980	1984
– 1980	176	-	-	-	-
– 1981	207	-	-	-	-
– 1982	202	-	-	-	-
– 1983	204	-	-	-	-
– 1984	207	-	-	-	-
Race	996	3.24	1.19	1	4
– Black	157	1	0	1	1
– Hispanic	135	2	0	2	2
– Mixed race (Non-Hispanic)	14	3	0	3	3
– Non-Black / Non-Hispanic	690	4	0	4	4
ASVAB percentile	882	69.25	23.11	3	100
Region	230,791	2.64	1.01	1	4
– Northeast	39,460	1	0	1	1
– North Central	55,447	2	0	2	2
– South	85,363	3	0	3	3
– West	50,521	4	0	4	4
Highest degree	1,112	4.42	0.73	4	7
– BA	740	4	0	4	4
– MA	214	5	0	5	5
– PhD	9	6	0	6	6
– Professional degree	33	7	0	7	7
Major	994	0.34	0.475	0	1
– Arts and Social Sciences	723	0	0	0	0
– STEM	271	1	0	1	1
Employed hours	206,177	44.28	11.30	10	98
Real wage	206,872	17.61	21.59	1.01	519.65
Potential experience (in months)	232,953	42.92	26.39	1	127
Student loan currently owed	232,953	0.26	2.60	0	120
Family income per-capita	212,420	37.71	38.62	0	421.37
College GPA	232,661	3.23	0.41	1.88	5

Table C5: Occupations within 5 Percentage Points of the 50% Threshold

Occupation title	Freq. of college requirement
Geological and petroleum technicians	46.19
Other life, physical, and social science technicians	48.72
Sales representatives, wholesale and manufacturing	49.76
Designers	50.13
Directors, religious activities and education	50.97
Religious workers, all other	50.97
Cost estimators	51.03
Producers and directors	51.57
Construction managers	52.10
Judges, magistrates, and other judicial workers	53.40
Writers and authors	53.82
Other business operations specialists	54.15
Network systems and data communications analysts	54.49

Table C6: Top 10 College and Non-college Occupations

College occupations	N	Non-college occupations	N
Elementary/middle school teachers	11,771	Managers of retail sales workers	6,788
Registered nurses	5,990	Retail salespersons	3,806
Accountants and auditors	5,762	Sales representatives	3,297
Secondary school teachers	5,761	Secretaries and admin. assistants	3,136
Social workers	4,703	Customer service representatives	2,978
Managers, all other	3,989	Police and sheriff's patrol officers	2,973
Financial managers	3,559	Managers of office and admin. support workers	2,692
Other teachers and instructors	3,517	Waiters and waitresses	2,481
Computer software engineers	3,396	Cashiers	1,879
Marketing and sales managers	3,225	Loan counselors and officers	1,866

Table C7: Underemployment and College Major

Major	N	Respondents	Underemp. ratio
A: Arts and Social Sciences			
Liberal arts and science	104	2	0.337
International relations and affairs	156	1	0.122
Social work	187	1	0.989
Archaeology	291	1	0.808
Hotel/Hospitality management	500	3	0.790
Pre-law	531	2	0.452
Other – Recoded to human services	578	3	0.351
Home economics	595	4	0.606
Area studies	709	2	0.234
Anthropology	709	6	0.068
Theology/religious studies	1,148	5	0.462
Philosophy	1,361	5	0.505
Foreign languages	2,244	8	0.311
English	4786	24	0.335
Political science and government	5,422	26	0.375
Economics	5,636	16	0.265
History	5,832	32	0.482
Nursing	6,378	28	0.035
Sociology	6,905	31	0.283
Criminology	7,022	31	0.573
Fine and applied arts	11,928	45	0.635
Psychology	16,015	69	0.398
Communications	17,910	68	0.442
Education	20,574	99	0.242
Business management	56,538	211	0.484
All Arts and Social Sciences	174,059	723	**0.415**
B: STEM			
Pre-vet	156	1	0.865
Nutrition/dietetics	365	2	0.399
Pre-med	448	4	0.213
Agriculture/natural resources	2,163	8	0.626
Mathematics	2,621	13	0.491
Interdisciplinary studies	2,622	12	0.387
Physical sciences	3,131	16	0.262
Architecture/environmental design	3,132	15	0.213
Other health professions	7,982	38	0.393
Biological sciences	10,430	49	0.251
Computer/information science	12,634	52	0.446
Engineering	13,096	61	0.312
All STEM	58,780	271	**0.357**

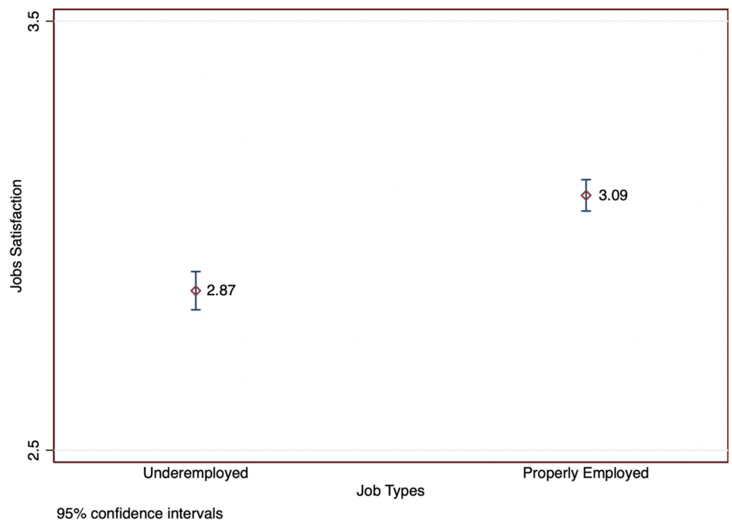
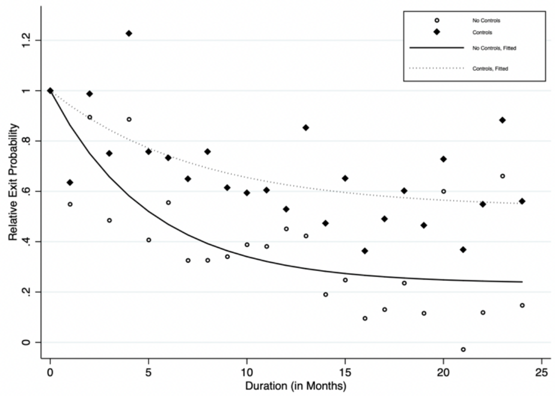
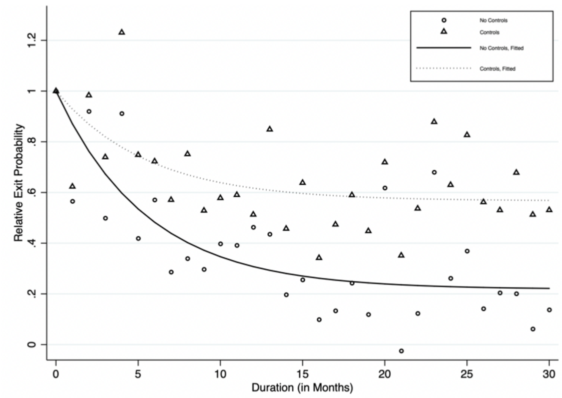


Figure C3: Job Satisfaction among Underemployed versus Properly Employed

Notes: We test the null hypothesis that the job satisfaction is equal among the underemployed versus the properly employed against the alternative that the job satisfaction among the properly employed is different from the job satisfaction among the underemployed. The p-value is less than 0.01, indicating significant differences in job satisfaction between the two groups.



(a) Duration Dependence (Extra transition time)



(b) Duration Dependence Capped at 30 Months

Figure C4: Duration Dependence

Table C8: Scarring Effects (1-digit Industry and Occupation Codes)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0166***		-0.0164***	-0.0148***		-0.0145***
	(0.0010)		(0.0010)	(0.0011)		(0.0010)
Underhis		0.0006***	0.0006***		0.0010***	0.0009***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × *Unhis*				-0.0011		-0.0002
				(0.0013)		(0.0013)
College × Underhis					-0.0023***	-0.0022***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.778	0.777	0.778	0.774	0.774	0.774

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 1-digit level), and region.

Table C9: Scarring Effects (1-digit Occupation FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0154***		-0.0153***	-0.0153***		-0.0150***
	(0.0010)		(0.0010)	(0.0011)		(0.0011)
Underhis		0.0004***	0.0004***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				0.0002		0.0008
				(0.0014)		(0.0014)
College × Underhis					-0.0022***	-0.0021***
					(0.0002)	(0.0002)
1-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172149	172149	172149	172149	172149	172149
R^2	0.784	0.784	0.784	0.784	0.784	0.785

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 1-digit level), and region.

Table C10: Scarring Effects (2-digit Occupation FE)

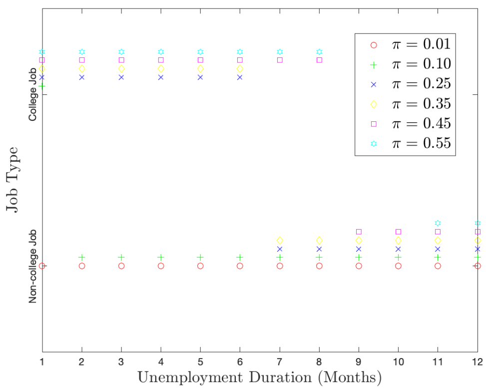
	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0145***		-0.0145***	-0.0144***		-0.0141***
	(0.0009)		(0.0009)	(0.0010)		(0.0010)
Underhis		0.0003***	0.0003***		0.0005***	0.0005***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College × Unhis				-0.0004		-0.0000
				(0.0013)		(0.0013)
College × Underhis					-0.0017***	-0.0016***
					(0.0001)	(0.0001)
2-digit Occupation FE	✓	✓	✓	✓	✓	✓
N	172,149	172,149	172,149	172,149	172149	172,149
R^2	0.791	0.790	0.791	0.791	0.791	0.791

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and region.

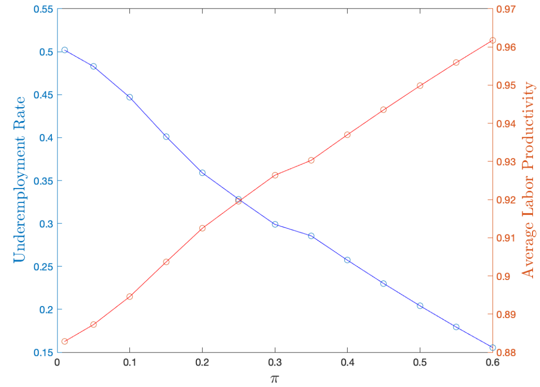
Table C11: Scarring Effects (Year and Month FE)

	(1)	(2)	(3)	(4)	(5)	(6)
Unhis	-0.0144***		-0.0143***	-0.0137***		-0.0134***
	(0.0009)		(0.0009)	(0.0011)		(0.0011)
Underhis		0.0003***	0.0003***		0.0007***	0.0007***
		(0.0001)	(0.0001)		(0.0001)	(0.0001)
College $\times$ Unhis				-0.0012		-0.0007
				(0.0013)		(0.0013)
College $\times$ Underhis					-0.0020***	-0.0019***
					(0.0002)	(0.0002)
2-digit Occupation FE	✓	✓	✓			
N	172,149	172,149	172,149	172,149	172,149	172,149
R^2	0.792	0.791	0.792	0.783	0.783	0.784

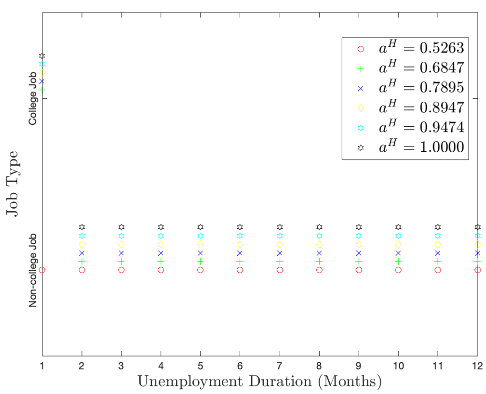
Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The regressions consider all control variables, such as potential experience (measured in months), regional and annual unemployment rates, age, age squared, per-capita family income, student loan debt (in thousands), and job satisfaction. Furthermore, the analysis includes fixed effects for calendar year, calendar month, individuals, industries (at the 2-digit level), occupations (at the 2-digit level), and regions.



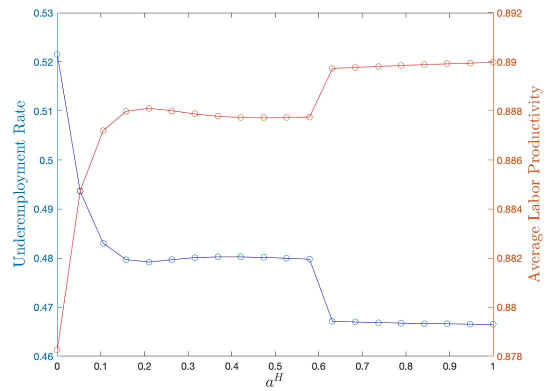
(a) Policy Function of Unemployed Workers (π)



(b) Underemployment and Productivity (π)



(c) Policy Function of Unemployed Workers (a^H)



(d) Underemployment and Productivity (a^H)

Figure C5: Comparative statics. Composition of Workers (π) and Suitability of Type H workers (a^H)

Table C12: Residual Wage Dispersion

	90/50 ratio	50/10 ratio
Full sample	1.609	1.739
A: After the first underemployment spell		
$<$ 1 year to exit underemployment	1.411	1.505
$\geq$ 1 year to exit underemployment	1.244	1.285
B: Proper employment spell following underemployment		
$<$ 1 year to exit underemployment	1.342	1.414
$\geq$ 1 year to exit underemployment	1.281	1.334

Notes: Regression specification controls for unemployment history, a cubic in potential experience, individual, regional, and 2-digit occupation fixed effects.

Table C13: Wage Growth with All College Jobs Observations

	(1)	(2)	(3)	(4)
Long	-0.0146*	-0.0143*	-0.0135	-0.0145*
	(0.0083)	(0.0082)	(0.0086)	(0.0088)
N	2045	2045	1998	1998
R^2	0.023	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. Results are robust to clustering at the birth year, or contemporary year levels. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

Table C14: Wage Growth with Consideration of Previous Underemployment Spell

	(1)	(2)	(3)	(4)
Long	-0.0184**	-0.0178**	-0.0163*	-0.0172*
	(0.0084)	(0.0084)	(0.0088)	(0.0091)
N	2045	2045	1998	1998
R^2	0.024	0.029	0.029	0.030

Notes: Standard errors are clustered at the individual level. Results are robust to clustering at the highest education, or contemporary year levels. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$).

C.4 Empirical Moments for Calibration

C.4.1 Wage Premium

Table C15 contains results from estimating equation (25) in the main text. Each respective column represents a different combination of control variables and fixed effects as indicated in the table. Column (4) represents our preferred specification that is used to calibrate the model in Section 5.2.

Table C15: The Wage Premium of College Jobs

	(1)	(2)	(3)	(4)
College Job	0.3849***	0.3857***	0.2600***	0.2597***
	(0.0192)	(0.0196)	(0.0218)	(0.0223)
Exp	-0.0087***	-0.0026	0.0013	-0.0062***
	(0.0023)	(0.0020)	(0.0019)	(0.0021)
Exp^2	0.0002***	0.0000	-0.0000	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Exp^3	-0.0000	0.0000	0.0000**	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regional Annual Urate				-0.0179**
				(0.0070)
Annual Urate				-0.0083
				(0.0064)
Age				0.4603***
				(0.0920)
Age^2				-0.0084***
				(0.0018)
Individual FE	✓	✓	✓	✓
Region FE		✓	✓	✓
2-digit Industry FE			✓	✓
N	11,085	10,988	10,988	10,988
R^2	0.843	0.853	0.894	0.894

Notes: Robust standard errors are in parentheses. *($p < 0.10$), **($p < 0.05$), ***($p < 0.01$). The wage premium of college jobs (i.e., being properly employed) is captured by the coefficient of *College*. Notice that we only include "marginal" underemployed workers (workers who used to be properly employed and just moved down the job ladder) in the sample, which is the same approach as in Barnichon and Zylberberg (2019).

C.4.2 Unemployment and Underemployment Rates

The calculation of the unemployment and underemployment rates involves a three-step procedure. First, the number of individuals in the categories of unemployment, underemployment, and proper employment are counted for each week. Second, the proportion of each labor status category is calculated by dividing the headcount of each category by the total headcount observed during that week. Finally, both an unweighted average and a weighted average are computed across all weeks, with the number of observations in that week serving as the weighting factor. Table C16 contains the results.

Table C16: Composition of Labor Force Statuses

	Unemployed	Underemployed	Properly employed	NILF
Unweighted	0.032	0.408	0.497	0.063
Weighted	0.027	0.416	0.503	0.054

Table D.1: Model and Data Comparison

Moment	Target	Model	Moment	Target	Model
Unemployment rate	0.081	0.050	$\partial \log(w_n)/\partial v$	-0.014	-0.014
Underemployment rate	0.416	0.416	$\partial \log(w_c)/\partial v$	-0.014	-0.014
Average EE probability	0.014	0.011	$\partial \log(w_n)/\partial \tau$	0.001	0.001
College job premium	0.260	0.256	$\partial \log(w_c)/\partial \tau$	-0.001	-0.001
b/[Average labor productivity]	0.710	0.710	-	-	-

D Model with Permanent Skill Loss

In an effort to gain further insight into the contribution of skill loss to duration dependence, we alter the full quantitative model to include permanent skill loss by setting the probability of regaining college-specific skills, ϕ, to zero. We re-calibrate the model with $\phi = 0$ and proceed to perform the same decomposition exercises as in the main text.

D.1 Calibration

The calibration procedure is identical to that of the full model outlined in the main text, except for setting $\phi = 0$. The comparison between the model and target moments can be found in Table D.1, and the model-generated path is shown in Figure D.1(a). Except for the unemployment rate, the calibrated model closely matches the empirical targets. The updated parameters are reported in Table D.2.

Table D.2: Parameter Values with Permanent Skill Loss

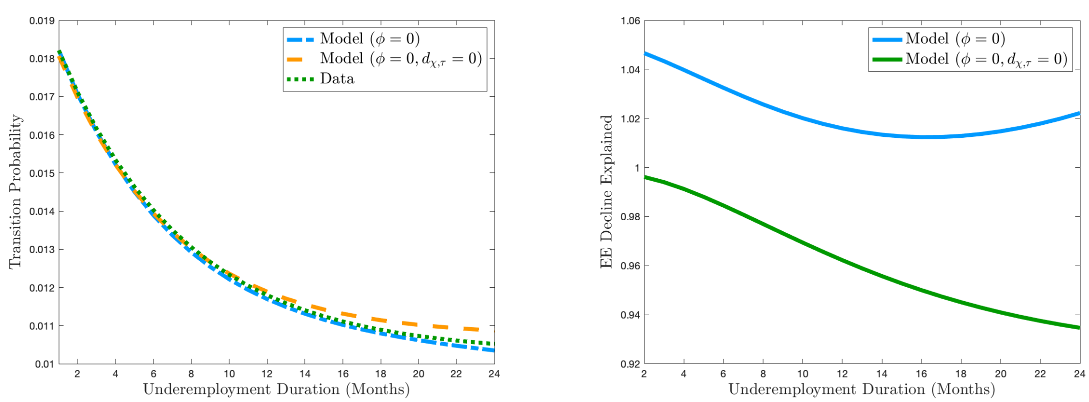
	Definition	Value		Definition	Value
β	Discount factor	0.996	a^L	Suitability pr.: type L	0.023
δ	Entry/exit probability	0.010	a^H	Suitability pr.: type H	0.357
g_c	College productivity	1.000	π	Pr. of being a type H worker	0.048
g_n	Non-college productivity	0.751	ϕ	Pr. of regaining college skills	0.000
b	Utility while unemployed	0.609	$d_{c,v}$	College skill loss: unemp.	-0.014
k_n	Non-college vacancy cost	2.332	$d_{c,\tau}$	College skill loss: underemp.	-0.001
k_c	College vacancy cost	1.343	$d_{n,v}$	Non-college skill loss: unemp.	-0.014
λ	Employed search intensity	0.761	$d_{n,\tau}$	Growth of non-college skills	0.001

D.2 Decomposition

In this section, we evaluate the relative contributions of unobserved heterogeneity and changes to occupation-specific human capital in generating duration dependence when the skill loss is permanent. Beginning with Figure D.1(a), we present the path of the transition probability from non-college to college occupations from the data, a full version of the model with permanent skill change, and a version of the model where we shut off the accumulation and decay of occupation-specific human capital during underemployment. In practice, this requires setting $d_{\chi,\tau} = 0$ for $\chi \in X$ while all other parameters remain unchanged.

Next, we examine what proportion of the decrease in the transition probability at each underemployment duration relative to the transition probability at $\tau = 1$ observed in the data can be explained by the full model and the version with unobserved heterogeneity only. As depicted in Figure D.1(b), the duration dependence produced by the full model with permanent skill evolution exceeds that in the data, whereas the model with only unobserved heterogeneity explains at least 94% of the decline.

To obtain an overall decomposition, we calculate the weighted average of the fraction explained by unobserved heterogeneity for all τ. The weights are the proportion of underemployed workers who are employed at each duration τ in the steady state. As a result, we find that the full model can explain 102.3% of the observed decrease in the transition probability from non-college to college jobs. If we eliminate the dynamics of human capital during underemployment, the model with unobserved heterogeneity still explains 94.6% of the duration dependence.



(a) Transition Probability Path

(b) Fraction Explained at each τ

Figure D.1: Duration Dependence Decomposition

E Model with Full Information

This appendix provides further details on the model with full information referenced in Section 5.3 in the main text. For brevity, we only present the details in the environment and equilibrium that are new relative to the baseline model presented in Section 3.

E.1 Environment

Workers learn their suitability type upon entering the labor market. A worker's suitability type is public information. The labor market continues to be organized in a continuum of submarkets. In the full information case, however, submarkets are also indexed by the suitability type of workers searching in that particular submarket. Denoting $A = \{L, H\}$ and an individual's worker suitability type by i, the labor market is now organized in a continuum of submarkets indexed by $\omega = (\chi, i, v, \tau, x) \in X \times A \times \Upsilon \times T \times \mathbb{R}$. That is, in submarket ω, type-χ firms search for a type-i worker with labor market history (v, τ) and offer suitable workers an employment contract worth x in lifetime utility.

E.2 Value Functions

The value functions in the full information version of the model are very similar to those in the baseline model. The main exception is that a worker's suitability type, i, and probability of being suitable for a college job, a^i, take the place of, μ, their expected suitability. Here is the value of an unemployed worker of suitability type i who searches for a non-college job:

$$V_{u,n}(v, i) = b + \beta(1-\delta)\{V_u(\hat{v}, i) + R_n(x, V_u(\hat{v}, i))\}, \tag{E.1}$$

where

$$V_u(v, i) = \max\{V_{u,n}(v, i), V_{u,c}(v, i)\}, \tag{E.2}$$

is the value of unemployment for a type-i worker with unemployment history v and

$$R_\chi(x, V_u(\hat{v}, i)) = \max_x p(\theta(\chi, i, \hat{v}, 0, x))(x - V_u(\hat{v}, i)). \tag{E.3}$$

The value of searching for a college job satisfies:

$$V_{u,c}(v, i) = b + \beta(1-\delta)\{V_u(\hat{v}, i) + a^i R_c(x, V_u(\hat{v}, i))\}. \tag{E.4}$$

The sum of the worker's lifetime utility and firm's profits in a match between a non-college job and type-i worker with history (v, τ) is given by:

$$V_{e,n}(v,\tau,i) = y_n(v,\tau) + \beta(1-\delta)\{V_{e,n}(v,\hat{\tau},i) + \lambda a^i S(v,\hat{\tau},i)\}, \tag{E.5}$$

where

$$S(v,\hat{\tau},i) = \max_x p(\theta(c,i,v,\hat{\tau},x))(x - V_{e,n}(v,\hat{\tau},i)). \tag{E.6}$$

Finally, sum of the worker's lifetime utility and the firm's profits in a match between a college job and a type-i worker with history (v, τ), $V_{e,c}(v,\tau,i)$, satisfies

$$V_{e,c}(v,\tau,i) = y_c(v,\tau) + \beta(1-\delta)\{\phi V_{e,c}(1,0,i) + (1-\phi)V_{e,c}(v,\tau,i)\}. \tag{E.7}$$

E.3 Free Entry

In any submarket visited by a positive number of workers, tightness is consistent with the firm's incentives to create vacancies if and only if

$$k_\chi \geq q(\theta(\chi,i,v,\tau,x))\{V_{e,\chi}(v,\tau,i) - x\}, \tag{E.8}$$

and $\theta(\chi,i,v,\tau,x) \geq 0$ with complementary slackness. We restrict attention to equilibria in which $\theta(\chi,i,v,\tau,x)$ satisfies the complementary slackness condition in every submarket, even those that are not visited by workers. That is

$$\theta(\chi,i,v,\tau,x) = \begin{cases} q^{-1}\left(\frac{k_\chi}{V_{e,\chi}(v,\tau,i)-x}\right), & \text{if } k_\chi = q(\theta(\chi,i,v,\tau,x))\{V_{e,\chi}(v,\tau,i) - x\}, \\ 0, & \text{otherwise.} \end{cases} \tag{E.9}$$

E.4 Laws of Motion

Let $u_i(v)$ denote the measure of workers of suitability type i who begin the period unemployed with unemployment history v. The law of motion is given by

$$\hat{u}_i(v) = \begin{cases} (1-\delta)\delta\pi^i, & \text{for } v = 1, \\ (1-\delta)u_i(v_-)[\varrho_{i,n,v_-}(1-p(\theta^*_{i,n,v_-})) + \varrho_{i,c,v_-}(1-a^i p(\theta^*_{i,c,v_-}))] & \text{for } v \in \{2,\ldots,\bar{v}-1\}, \\ (1-\delta)\sum\limits_{v=\bar{v}}^{\bar{v}+1} u_i(v_-)[\varrho_{i,n,v_-}(1-p(\theta^*_{i,n,v_-})) + \varrho_{i,c,v_-}(1-a^i p(\theta^*_{i,c,v_-}))] & \text{for } v = \bar{v}, \end{cases} \tag{E.10}$$

where $\pi^H = \pi$, $\pi^L = 1 - \pi$, $\hat{u}_i(v)$ is the measure of unemployed workers with unemployment history v and suitability type i at the beginning of the next period, $v_- \equiv v - 1$,

$\varrho_{i,\chi,v} \in [0,1]$ is the fraction of unemployed workers with suitability type i and unemployment history v who search for type χ jobs, and $\theta^*_{i,\chi,v}$ denotes tightness associated with the policy function of unemployed workers with suitability type i and unemployment history v who search for type χ jobs.

Now let $e_{i,\chi}(v,\tau)$ denote the measure of workers with suitability type i and history (v,τ) who are employed at type χ jobs at the beginning of the period. The law of motion for $e_{i,n}(v,\tau)$ is given by

$$\hat{e}_{i,n}(v,\tau) = \begin{cases} (1-\delta)\varrho_{i,n,v}u_i(v)p(\theta^*_{i,n,v}), & \text{for } \tau = 1, \\ (1-\delta)e_{i,n}(v,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,v,\tau_-})), & \text{for } \tau \in \{2,\ldots,\bar{\tau}-1\}, \\ (1-\delta)\sum_{\tau=\bar{\tau}}^{\bar{\tau}+1} e_{i,n}(v,\tau_-)(1-\lambda a^i p(\theta^*_{i,c,v,\tau_-})), & \text{for } \tau = \bar{\tau}, \end{cases} \tag{E.11}$$

where $\tau_- \equiv \tau - 1$, $\theta^*_{i,\chi,v,\tau}$ is tightness associated with the policy function of an employed worker with suitability type i and history (v,τ) in a submarket with type χ jobs.

The law of motion for $e_{i,c}(v,\tau)$ is given by

$$\hat{e}_{i,c}(v,\tau) = \begin{cases} (1-\delta)[e_c(v,\tau) + u_i(v)\varrho_{i,c,v}a^i p(\theta^*_{i,c,v}) + \phi(e_{i,c} - e_{i,c}(1,0) + e^*_{i,n} + u^*_{i,c})], & \text{for } v = 1 \text{ and } \tau = 0, \\ (1-\delta)(1-\phi)[u_i(v)\varrho_{i,c,v}a^i p(\theta^*_{i,c,v}) + e_{i,c}(v,\tau)], & \text{for } v \geq 2 \text{ and } \tau = 0, \\ (1-\delta)(1-\phi)[e_{i,n}(v,\tau)\lambda a^i p(\theta^*_{i,c,v,\tau}) + e_{i,c}(v,\tau)], & \text{for } v \geq 2 \text{ and } \tau \geq 1, \end{cases} \tag{E.12}$$

where $e_{i,c} = \sum_{v\in\Upsilon}\sum_{\tau\in T} e_{i,c}(v,\tau)$ is the total measure of type-i workers employed in college jobs to begin the period, $e^*_{i,n} = \lambda\sum_{v\in\Upsilon}\sum_{\tau\in T} e_{i,n}(v,\tau)a^i p(\theta^*_{i,c,v,\tau})$ is the total measure of type-i workers who transitioned from a non-college to college job within the period, and $u^*_{i,c} = a^i\sum_{v=2}^{\bar{v}} u_i(v)\varrho_{i,c,v}p(\theta^*_{i,c,v})$ is the total measure of unemployed workers with unemployment history $v \in \{2,\ldots,\bar{v}\}$ who found a college job in the previous period.

E.5 Equilibrium Definition

Definition 3. A stationary recursive equilibrium (RE) consists of a market tightness function $\theta(\omega)\colon X \times A \times \Upsilon \times T \times \mathbb{R} \to \mathbb{R}_+$, a value function for unemployed workers, $V_u(v,i)\colon \Upsilon \times A \to \mathbb{R}$, a policy function for unemployed workers, $\omega^*_u(v,i)\colon \Upsilon \times A \to X \times \mathbb{R}$, a joint value function for the worker-firm match, $V_{e,\chi}(v,\tau,i) : X \times \Upsilon \times T \times A \to \mathbb{R}$, a policy function for the worker-firm match, $\omega^*_{e,\chi}(v,\tau,i)\colon X \times \Upsilon \times T \times A \to X \times \mathbb{R}$, and a distribution of workers across the states of employment. The functions satisfy the following conditions. First, $\theta(\omega)$ satisfies (E.9) for all $\omega \in X \times A \times \Upsilon \times T \times \mathbb{R}$. Third, $V_u(\tau,i)$ satisfies (E.2) for all $(v,i) \in \Upsilon \times A$ and $\omega^*_u(v,i)$ is the associated policy function. Fourth,

$V_{e,n}(\upsilon, \tau, i)$ and $V_{e,c}(\upsilon, \tau, i)$ satisfy equations (E.5) and (E.7) for all $(\upsilon, \tau, i) \in \Upsilon \times T \times A$ and $\omega^*_{e,\chi}(\upsilon, \tau, i)$ is the associated policy function. Finally, the distribution of workers satisfies the laws of motion specified in Section E.4.

As in the baseline model, the labor market segments into submarkets. Here, the submarkets are indexed, in part, by the worker's suitability type. Hence, firm entry into each submarket is independent of the distribution of workers across employment statuses and suitability. We conclude the description of the full information model by defining a block-recursive equilibrium.

Definition 4. A block-recursive equilibrium (BRE) is a RE where the value and policy functions are independent of the distribution of workers across the states of employment.