

1. Suppose you have a secret number (make it an integer) between 1 and 8. We will call it your initial number at step 1. Now for each time step, your secret number will change according to the following rules:

Flip a coin.

If the coin turns up heads, then increase your secret number by one (8 increases to 1).

If the coin turns up tails, then decrease your secret number by one (1 decreases to 5).

Repeat n times and record the evolving history of your secret number.

2. What is the probability of getting a sum of 9 when two dice are thrown?
3. Peter and Paul play a simple game involving repeated tosses of a fair coin. In a given toss, if heads is observed, Peter wins 1 coin from Paul, otherwise if tails is tossed, Peter gives 1 coin to Paul. If Peter starts with zero coins, we are interested in his fortune as the game is played for 50 tosses.
4. Consider a European-style roulette which includes numbers between 0 and 36. In roulette there are many different betting alternatives but we consider here the simplest case where we bet on a single number. If we pick the right number then we win 35 times what we bet.

Consider the following scenario. We start playing roulette with a fixed budget. Every spin of the roulette costs one unit of budget. We play until we run out of budget and we always bet on the same number.

It takes two inputs: the *budget* and the *number we decided to play all the time*. It outputs our budget throughout the whole game until it ends.

Let's play one game with a budget of 15 and betting on the number 8.

Questions:

1. What is the probability that the game last exactly 15 spins if I start with a budget of 15 and bet on the number 8?
 2. What is the average length of the game starting with a budget of 15 and betting on the number 8?
5. Estimate pi-value using Monte Carlo Simulation.

6. What is the expected number of rolls needed to see all 6 sides of a fair die?
7. Draw $n = 100$ numbers from the discrete probability with given distribution: $X = \{-40, 14, -33, 2, 0, 15\}$ and $P = \{0.2, 0.1, 0.05, 0.3, 0.25, 0.1\}$. Calculate the expected value and variance for both experimental and theoretical data.
8. A coin is tossed repeatedly until head comes up for the 8th time. What is the probability this happens on the 20th toss?
9. Draw numbers from given density function $f(x)$ using inverse function method. Plot the obtained distribution. $f(x) = 2 \cdot (x + 1)/9, (-1, 2)$.
10. Using acceptance-rejection method generate numbers corresponding to given density function distribution. Plot it. $f(x) = \sin^2(x - 0.5) \cdot \exp(-x^2) + 0.2 \cdot \exp(-x^2), [-3, 3]$.
11. Suppose a weighted coin comes up heads with probability $= 1/5$. How many flips do you think it will take for the first tail appear? Use code to estimate this average.