**Theorem**: A bounded closed set is compact.

*Proof*: Let $M$ be a bounded closed set. Let's assume that $M$ is an infinite set.

Now, let $\{A_i\}$ be an infinite subcollection of $M$. Since $M$ is bounded, there exists a closed square $S$ that contains $M$. According to theorem 29, the square union its interior is compact. Therefore, the set $S$ is also compact.

Since $\{A_i\}$ is an infinite subcollection of $M$, and $M \subseteq S$, $\{A_i\}$ is also an infinite subcollection of $S$. Since $S$ is compact, there exists a point $p \in S$ which is a limit point of this subcollection $\{A_i\}$.

We need to show that $p \in M$. Since $M$ is closed, it contains all its limit points. Since $p$ is a limit point of the subcollection of $M$, $p$ must belong to $M$.

Thus, we have shown that for every infinite subcollection of $M$, there exists a point $p \in M$ which is a limit point of this subcollection.

$\therefore$ $M$ is compact. $\square$