Theorem: A segment is Connected.

Proof: Suppose (a, b) is not connected. Then $(a, b) = H \cup K$ where H and K are non-empty disjoint mutually separated sets .

Consider (a, z) , where z is a variable. Consider $U = \{z$ in $(a, b) : (a, z)$ subset $H\}$. Let $W =$ l.u.b of U. We know that $W \leq b$.

$\hookrightarrow$ Case 1: $W = b$. If this happens K will be empty. This contradicts that K is non-empty.

$\hookrightarrow$ Case 2: $W < b$.

1 Claim: W is a limit point of H. Assume it's not. Then $\exists\ \epsilon > 0$ s.t. $(W - \epsilon, W + \epsilon) \cap H = \emptyset$. Then $\exists$ smaller numbers than W that are also upper bounds of U.

2 Claim: W is a limit point of K. Assume it's not. Then $\exists\ \epsilon > 0$ s.t. $(W - \epsilon, W + \epsilon) \cap K = \emptyset$. Since everything between $(W, W + \epsilon)$ is in H(Part of U) , W itself would not be a l.u.b. or even upper bound. This is a contradiction.

WLOG , assume W is in H,

Since K is mutually separated from H and W is a limit point of K and H , this contradicts our first assumption that $(a, b) = H \cup K$.

$\therefore$ A segment is Connected $\square$