Theorem. *10. The boundary of a set is closed.*

Def: A point p, is a boundary point of the set D , if $\forall$ *nbhd* N of p , N contains points of both D and D^c.

Proof: Let D be a point set. Suppose that $bd(D)$ is not closed. This means that there is some limit point l of $bd(D)$ *s.t.* $l \notin bd(D)$.

Since l is a limit point of $bd(D)$, $\forall$ *nhbd* N of l , $\exists$ a point $q \in bd(D) \cap N \setminus \{l\}$. N is also a *nhbd* of $q \in bd(D)$.

We have shown $\forall$ *nbhd* N of p , N has point in both D and D^c , so it follows by the definition of boundary that $l \in bd(D)$.

This means $bd(D)$ contains all its limits points.

$\therefore$ The boundary of a set closed. $\square$