Not a Real World Problem - Possible Editorial

PROBLEM LINK: Div-1 Contest https://www.codechef.com/xxxxxxx
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DIFFICULTY: MEDIUM-HARD
PREREQUISITES: Max-flow min-cut theorem
PROBLEM:
Given N particles located at integers positions (x_i, y_i) of a grid and being E the set of pairs of particles located at adjacent positions, the problem ask us to choose a sign to each of the particles' value p_i in order to maximize the total grid value given by

$$V = \sum_{(i,j)\in E} p_i p_j \; + \sum_{i=1\ldots N} p_i \, h(x_i, y_i) \tag{1}$$

where $h(x_i, y_i)$ is the value of the cell containing particle i.
QUICK EXPLANATION: The problem can be transformed to a minimum cut problem where each of the connected components defined by the cut corresponds to the set of particles with the same sign assigned.
EXPLANATION:
Let's see that we can transform the problem into a minimum cut one.
Consider an undirected graph where each node corresponds to a particle and the edges connect particles located at adjacent cells. The value of the edge connecting particles i and j is given by $|p_i \, p_j|$. The graph we build this way includes so far the "interactions" between particles (first therm of expression 1).
We need to take account also the contribution of the second therm. To do that, let's consider two additional nodes, s and t, the former connected to every node located at position with non-negative h, the latter connected to every node located at position with non-positive h. The value given to each of these new edges is $|p_i \, h(x_i, y_i)|$.
As you can see, the values of the edges corresponds to the absolute value of the summands of expression 1.
If we define $A = E \cup \{s, t\}$, that is, the union of the pairs of adjacent cells and the nodes s and t, we can transform V in the following way

$$V = \sum_{(i,j)\in A} J_i J_j \qquad \text{where } J_i = p_i \text{ if } i \text{ is a particle; or } J_i = h_i \text{ if } i \in \{s, t\}$$

Let's classify our nodes in two sets:

$$S = \{\text{nodes that contribute to a positive value } J_i\}$$

$$T = \{\text{nodes that contribute to a negative value } J_i\}$$

S contains node s and every node to which we assign a positive sign. T contains node t and every node to which we assign a negative sign.
Now V can be expressed as

$$V = \sum_{(i,j)\in A} |J_i J_j| \; - \; 2 \sum_{i\in S; j\in \bar{S}} |J_i J_j| \tag{2}$$

Where we have used the fact that only the therms that combines one element of S with one element of T will contributes with negative product. As we have summed these therms at first, we should subtract them twice.
In order to choose signs of J_i's that maximize V, the first therm of last expression is irrelevant because it sums up over all nodes and takes absolute values. So, our goal is to minimize the second therm, that depends on which particles belongs to S and which ones to T.

As we said, the only contributions to the second therm comes from interactions between one of the particles of S with one of the particles of T, and those interactions are represented in our graph by the values of the edges. So, the problem of minimizing the second therm of eq. 2 is equivalent to the problem of finding a minimum cut that splits the graph in two components.
The Max Flow - Min Cut Theorem states that the minimum cut is equal to the max flow of the network. So we can arrive to the answer of our problem solving by using Edmond-Karp or Dinic's algorithms to get the max flow (min cut) and replace it in equation 2.
Finally, in order to find the sign of each particle, once he have calculated the max flow, we can employ DFS over the residual graph to find the nodes that belong to the S component (or T). More specifically, all the nodes that we should associate positive values of p are reachable from s considering only edges with positive residual capacity.

Almost a real world problem:

With some minor modifications, this problem is used to model ferromagnetic (and antiferromagnetic) behaviours in materials. The V function corresponds to $-Energy$ of the system, the p values to magnetic moments of particles and h to the position-dependent value of an external magnetic field. Maximizing V implies minimizing the energy of the system. You can further read on the Ising model here (link).

$$J_i == iJi == j$$

SOLUTIONS:
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[details="Tester's Solution"] indent whole code by 4 spaces [/details]
[details=.Editorialist's Solution"] indent whole code by 4 spaces [/details]