## Inequality Decomposition by Population Subgroups Author(s): Anthony F. Shorrocks

Source: *Econometrica,* Vol. 52, No. 6 (Nov., 1984), pp. 1369-1385 Published by: [Econometric Society](http://www.jstor.org/publisher/econosoc)

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***Econometrica,*** Vol. 52, No. 6 (November, 1984)

INEQUALITY DECOMPOSITION BY POPULATION SUBGROUPS

BY ANTHONY F. SHORROCKS 1

This paper examines the implications of imposing a weak aggregation condition on inequality indices, so that the overall inequality value can be computed from information concerning the size, mean, and inequality value of each population subgroup. It is shown that such decomposable inequality measures must be monotonic transformations of addi­ tively decomposable indices. The general functional form of decomposable indices is derived without assuming that the measures are differentiable. The analysis is suitable for extension to the many other kinds of indices for which a similar relationship between the overall index value and subaggregates is desirable.

I. INTRODUCTION

MosT INDEX NUMBERS in common usage exhibit some kind of decomposition property that enables the overall index value to be computed from subaggregates. These subaggregates are typically based on grouping together observations which share a common characteristic. Thus, for instance, aggregate price and quantity indices are normally derived from subaggregates corresponding to commodity categories. Since the level of disaggregation and the grouping criteria are largely a matter of choice, it is clearly important that the overall index value should not depend on how the subcategories are selected. This consideration results in constraints being placed on the index numbers, and consequent restrictions on the types of functional forms that are admissible. It is not the intention of this paper to examine in full generality the question of consistent aggregation and the implications for the functional forms of index numbers. Instead, the relevant issues will be explored in detail in one specific context-that of inequality indices. At the same time, however, the framework of analysis appears suitable for extension to a wide variety of other index numbers, for which similar results may be expected to obtain.

A number of recent articles have been directed at the relationship between overall inequality values and the inequality levels corresponding to population subgroups. (See, for example, Blackorby et al. [3], Bourguignon [4], Cowell [5], Cowell and Kuga [6], Das and Parikh [7], Shorrocks [12], and Toyoda [13].) The central issue concerns the circumstances under which aggregate inequality can be expressed as a function of the subgroup inequality levels. If this can be done satisfactorily, the way is open to decompose the overall inequality level into the inequality contributions associated with each of the subgroups, or, alternatively, to aggregate upwards from the subgroup values to derive the composite figure. When the decomposition (or aggregation) is additive, the inequality measure ***I***

1 I am particularly indebted to Professor J. Aczel, Chuck Blac;korby, Frank Cowell, Terence Gorman, and Dr. C. T. Ng for comments on, and assistance with, the results in this paper. The research was carried out in conjunction with a project financed by the SSRC.

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satisfies the constraint

(1) ***I(x)*** = ***I(x1 , • • • ,*** x*G*) =I. w*g*l(x*g*) + ***B***

*g*

where x*i*, ..., x*G* represents any partition of the distribution x into G subgroups; and where the coefficients w*g* and the "between-group" term, ***B,*** depend only on subgroup means and population sizes. Shorrocks [12] shows that any such ***additively decomposable*** inequality measure, which is also differentiable, must take the form

(2) ***I(x)*** = a(*µ,*, n) I. {cf>(x;)-<f,(µ,)}

where *µ,* and n respectively denote the mean and population size of If/(·) is also scale invariant (homogeneous of degree zero in x) and replication invariant (unchanged when the population and distribution are replicated), the only admiss­ ible indices belong to the single parameter "Generalized Entropy" family**2**

***x.***

}{(X·)

1 1 c

(3) I*c*(x) =- --;I. *\_!.* -1 , *c¥0,* 1,

n c(c-1) *µ,*

with corresponding (limiting) expressions for *c* = 0 and *c* = 1.

Although an additive form of decomposition has particular attractions, it seems an unnecessarily severe restriction to impose as a general requirement.A weaker aggregation structure may well preserve many of the advantages of additively decomposable measures while allowing a greater variety of admissible indices, with the different perceptions of inequality that these would imply. This paper assumes a very weak aggregation condition, requiring only that the overall inequality level is some general function of the subgroup means, population sizes, and inequality values. As the additive nature of the aggregation requirement is completely relaxed, the corresponding inequality measures are described as being just ***decomposable*** or ***aggregative.***

The following section sets out the notation and formal definitions employed in the paper. Section 3 explores the initial implications of decomposability and shows that a decomposable inequality measure must be an increasing transforma­ tion of an additively decomposable index. This result is achieved without presum­ ing that either the inequality index or the aggregator function is differentiable. The analysis is extended in Section 4 by first demonstrating that the transformed version of the index can be chosen to satisfy equation (2), and then establishing the general form of a decomposable inequality measure that is also scale invariant and/ or replication invariant. This again relies on just continuity, rather than differentiability, of the relevant functions. Section 5 summarizes the main results and indicates potential directions for further research.

For the purposes of exposition, the analysis is developed in terms of the

inequality of income, but is clearly applicable to many other types of distributions.

**2** The Generalized Entropy family is also discussed in Cowell and Kuga [6].

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1. **DEFINITIONS AND PRELIMINARY RESULTS**

*Let* X; *E* ***9ll++*** *represent the (positive) income of person i, and x=(x*1 , • • • , *xn )E*

***9/l*** *:+ be an income distribution vector for a population containing n individuals.*

*The set of all stric:tly positive income vectors of dimension at least m is denoted*

*by Xm =U:-m* ***9/l*** *+. It will be convenient to write the dimension of any vector*

***x*** *(i.e., the population size) as* ***n(x)*** *and the mean of* ***x*** *as* ***µ,(x),*** *so* ***8(x)****=* ***(µ,(x), n(x))*** *can be regarded as a "parameter vector" for the distribution* ***x.*** *Income distributions drawn from X1 with a common parameter vector 8 comprise*

*the set S(8)={xEXd8(x)=8}. The notation xis employed as shorthand for the "equalized version" of* ***x,*** *defined by* ***n(x)****=* ***n(x)*** *and X;=****µ,(x)*** *for all i.*

*An inequality index is defined as a function l: X1 ➔* ***9/l*** *with the properties:*

*(4a)*

*(4b)*

*(4c)*

***l(x)*** *is a continuous and symmetric function of* ***x,***

***l(x)* =** *0 (a normalization condition),*

***l(Bx)*** *<* ***l(x)*** *for all bistochastic matrices* ***B*** *that are not permutation matrices (strict Schur-concavity).*

*N1ote***2** *th3at, strictly speaking,* ***l*** *refers to a countable collection of functions 1 , 1 1 , where I": is an inequality index constructed for n-person*

**,** ... , ***9/l*** *:+ ➔* ***9/l***

*distributions (and satisfying (4)). However no ambiguity arises, and the notation is simplified, by writing* ***l(x)****=* ***l"("'>(x).*** *Condition (4c) is equivalent to the strict version of the Pigou-Dalton principle of transfers: any positive mean preserving transfer from one individual to a richer person increases inequality. Conditions (4b) and (4c) together ensure that* ***l(x)*** *> 0 whenever* ***x*** *,c x.*

*The other principal assumption to be imposed on the inequality index is tha3t of decomposability. An index* ***l*** *will be said to be* ***decomposable*** *or* ***aggregative***

*iff there exists an "aggregator" function* ***A*** *such that*

*(5)* ***l(x,y)****=****A(I(x),µ,(x), n(x),l(y),µ,(y), n(y))***

***=A(I(x), 8(x),l(y), 8(y))*** *for all x,yEX1 ,*

*where* ***A*** *is continuous and strictly increasing in the index values* ***l(x),l(y).****4 In the decomposition sense,equation (5) implies that when any population number­*

*ing* ***n(x,y);;,*** *2 is divided into disjoint and exhaustive su****)****bgroups numbering*

***n(x), n(y);;,*** *1, so that the overall income distribution* ***(x,y*** *is partitioned into*

*the subgroup vectors* ***x*** *and* ***y,*** *then the overall inequality level* ***l(x,y)*** *can be expressed in terms of the subgroup means, sizes, and inequality values. Alterna­ tively, in the aggregation sense, condition (5) states that if two groups with*

**3** This is the term used by Bourguignon [4, p. 903].

**4** Although (5) is defined in terms of 2-group aggregation only, in contrast to equation (I), the aggregation can be extended by recursion to any number of subgroups. The explicit assumption that A is strictly increasing in the index argument is technically unnecessary, since it is a consequence of assumption (4c). For any change in the value of ***I(x)*** can be generated by a sequence of transfers, all of which are either progressive or regressive, and the value of ***I(x,y)*** must therefore change in the same direction, respecting the movement in the subgroup index values.

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*distributions x and* ***y*** *are combined into a single population, then calculation of aggregate inequality only requires information on the size, mean income, and inequality value of the component distributions. For the purposes of this paper, either of these interpretations can be used, although the analysis tends to be framed in terms of the aggregation of groups into larger populations.*

*Two further assumptions frequently imposed on inequality measures concern the reaction of the index to a replication of the population and its distribution, and to a proportional change in all incomes. An inequality index* ***I*** *is* ***replication invariant,*** *or satisfies the principle of* ***population replication,*** *if*

1. ***I(Rr x)*** = ***I(x, x, . . . , x)* = *I(x)*** *for all* ***x****EX1 and all r*

*where* ***Rr*** *is a "replicator matrix" of dimension n(x) x rn(x), taking the form* ***Rr****=[* ***E, E,*** *. .. ,****E]****for some identity matrix* ***E****of suitable dimension. An inequality index* ***I*** *is* ***scale invariant*** *(or* ***income homogeneous,*** *or* ***mean independent)*** *if* ***I(x)*** *is homogeneous of degree zero in all incomes.*

*This section concludes with a simple result on functional equations used extensively in the subsequent analysis.*

***If G,f,,f****2,* ***and*** *2* ***are continuous and strictly monotonic func­ tions �*** *➔ �* ***and if***

*LEMMA 1: F, gi, g*

***p-'(f,(u)*** +***fi(v))* = *o-1(g,(u)+gi(v)), then there exist constants a, b, and c such that***

***f1(u)****=****cg,(u)+a,***

***fi(v)* = *cgi(v)+b,***

***F(w)=cG(w)+a+b.***

***Furthermore, iffk(O)* = *gk(O)* =** *0* ***fork=****l,2,* ***then a = b* =** *0.*

*PROOF: Setting g1 (* ***u)*** *=* ***x, gi(v)****=****y*** *we obtain*

*f1 o g11 (x)* +*f2 o g2 1 (y)=****F*** *o o-'(x* + *y)*

*where* **O** *denotes composition of functions. This is a Pexider equation whose solution [1, p. 142] is*

*f,* 0 *g11 (x)=****ex +a, f2*** o *g2 1 (y)=****cy+b,***

*F* **0** *a-'(z)= CZ* ***+a+b.***

*The result then follows immediately.*

1. *PROPERTIES OF DECOMPOSABLE INEQUALITY MEASURES*

*It is clear from the requirements of (4) and (5) that any transformation J* **=** *F(I)*

*of any decomposable inequality index* ***I(x)*** *will also be a decomposable inequality*

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*index, as long as* ***F*** *is continuous, strictly increasing, and preserves the origin. The decomposition and inequality index characteristics are also retained if F is allowed to vary continuously with the mean of* ***x,*** *and to depend in an arbitrary way on the dimension* ***n(x)*** *of* ***x.*** *Performing such transformations will alter the structure of the corresponding aggregator function defined over the subgroup index and parameter values. The results of this section demonstrate that a suitable transformation can always be found to convert any decomposable inequality index* ***I*** *into an* ***additively*** *decomposable inequality index J. More specifically* ***J*** *can be made to satisfy the equation*

***J(x,y)*** *=****J(x) +J(y) +J(x,****ji) for all* ***x,y****EX*1

*which will be referred to as the* ***normal form*** *equation for a decomposable inequality index. It follows that, for any decomposable inequality index "in normal form," the corresponding aggregator function is additive in the subgroup index values. Note that the original index* ***I*** *and the transformed measure J will be ordinally equivalent in the restricted sense of agreeing in their ranking of pairs of distributions with identical means and population sizes.*

**THEOREM** *1:* ***If I is a decomposable inequality index, there exists another index J(x)* = *F(I(x),8(x)) such that***

1. ***J(x,y)****=****J(x) +J(y) +J(x,****.fl* ***for allx,y****EX***2, *where***

*(Sa)* ***F(I,****8) is* ***continu ou s and strictly increasing in I,***

*(Sb)* ***F(O,8)=0 for all8.***

*PRooF: 5 Let I' be any decomposable inequality index and define @* **=**

*{****8(x)lx*** E *X*2}. *For all 8* E *@, S(8) is a connected, open subset of X2 containing more than one element. Hence, using the properties (4),*

***I'[ S(8)]* = *{ I'(x)lx****E S(****8)}* =** *[O, t(****8))***

*where t(8) may be finite or infinite. Choose any function* ***1/1(u,*** *8), continuous and strictly increasing in* ***u,*** *such that* ***I(x) ifJ(I'(x),8(x))*** *and J[ S(****8)]*** *[O,1)*

**=** =**=**

***U.*** *Since* ***I*** *inherits the symmetry and decomposition characteristics of* ***I',*** *there exists an aggregator function A such that*

***I(x,y)* = *A(I(x),8(x), I(y),8(y))*** *for all* ***x,****yEX*2

*where A is continuous and strictly increasing in the index arguments.*

*For* ***g****=1, 2, 3, choose any fixed* ***8i****E@ and any* ***Xg****E S(****8g).*** *Writing* ***xgh*** *=* ***(xg,xh),8ih 8(xgh)*** *and* ***Ik I(xk),*** *we obtain*

**==**

***Igh* = *I(xgh )* = *I(xg,xh)***

**= *A(I(xg),8;, I(xh),8t)* = *Ag,h(Ig, Ih )***

*5 The initial part of this proof is similar to Gorman's [9] overlapping theorem for separable functions.*

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and

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A,*2* ,iAdI,, 1*2* ) , /3) **=** A(/1*2* , 8f2, /3, 8t}

= I(x,*2* , x*3* ) = I(x,, X*23* )

**=** A(I,, 8f, 1*23* , 8f3)

**=** A,,*2* il,, A*2* ,*3* U*2* , /*3* ))

where A*1c, 1 (* u, v) = A(u, 8t, v, 8t) is continuous and strictly increasing in u and

v. Now select any continuous, strictly increasing functions </> *12* and </>*23* such that A*g*,h= </>*g* h **O** A*g*,h: *u* X *u ➔* **onto** u. Then

AA , ,iAA ,,i(Ii, 1 ) , /3) \_= A1 ,i<l>-,1 ° AA ,, (1,, 1 ) , /3)

*2 2 2 2 2 2*

**=** A,*2* ,iA,,i(I,, 1*2* ), /*3* )

**=** A1,*23* U*1* , </>23*1* **0** A*2* ,il2, /3))

= A,,*2* 3(/*1* , A*2*il*2* , /3))

where /**1 ,** *2 , 3 ,* A,**2** and A*2* **3** can take any value in the interval given a suitable choice of X*g* ES(8:) . By an.appropriate normalization of A,2,*3* and A*1* ,2*3* onto the interval the prerequisites of Aczel's theorem on associative functions [1, p. 312, Corollary 1], are satisfied. Hence there exist continuous and strictly monotonic functions , 1*2* , and *3* such that

*U,*

*1 1 U,*

*1" 12 13,1 12*

A,,i(u, v) **=** J-12'(1,(u ) + liCv)) ,

A*2* ,iu, v) **=** l2*3*'CliCu ) +liv)) .

By now setting

fg(u ) **=** {lg(u ) -l*g* (O) }/{li(0. 5) -liCo)} when g **=** 1, 2, 3,

fgh(*V)* **=** {lgh **O** </>*g* h(*V* ) -l*g* (O) -lh(O) }/{liC0. 5) -fi(O) }, we obtain

(9a) I(x,, x*2* ) **=** A(/*1* , 8f, 1*2* , 8f) **=** li*2*'U,(I,) +/*2*(/*2* )),

(9b) I(x*2* , x*3* ) **=** A(/*2* , 8f, /*3* , 8t) **=** JiiU*2*U2) +J;(/*3* )),

where

(10a) A is continuous and strictly increasing *(k* = 1, 2, 3, 12, 23),

(l Ob) /g(O) **=** 0 (g **=** 1, 2, 3) ,

(10c) /i{0. 5) **=** 1.

In general we may expect the functionsA to depend on the choice of 8f, 8f, 8!. Suppose 8***3*** changes to 8***3*** and denote the new functionsA with a tilde superscript. Then, from (9a)

1idU,U,) +!2U*2* )) **=** lidei,U,) +j*2*U2n

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and Lemma 1 combined with conditions (10) yields jk(u ) **=** fk(u ) when*k* **=** I, 2, 12.

Thus /*1* , /*2* , and/*12* do not depend on the choice ofOf. Similar reasoning applied to equation (9b) establishes that /*2* , 1*3* , and /*23* are independent of the choice of Of. Making explicit the dependence ofA on the values ofO = O(x*g* ) , and allowing X*g* to be chosen freely from the set X*2* **=** ES(O) IOE @} converts equation (9a) into

*g*

*{x*

(11) I(x*1* , x*2* ) =.JiiU1U(x*1* ), 0*1* , 0*2* ) +fi(I(x*2* ) , 0*2* ) , 0*1* , 0*2* ) for all x*1* , X*2* EX*2*

where inversion of/*12* is taken with respect to its first argument.***6*** But the symmetry of ***I*** also implies that

I(xi, X*2* ) = I(x*2* , X i) = JiiU1U(x*2* ) , 0*2* , 0*1* ) +fz(I(x*i* ), 0*1* ) , 0*2* , 0*1* )­

Applying Lemma 1 to the last two equations then gives fi(u, 0*1* , 0*2* ) **=** cif (u, 0*1* ),

fi(v, 0*2* ) **=** cfi(v, 0*2* , 0*1* ) ,

and (1Oc) ensures that

C **=** /*1* (0. 5, 0*1* , 0*2* ) = c(0*1* , 0*2* ) > 0

**=** /*1* (0.5, 0*2* , 0*1* ) - *1* **=** c(0**2*,*** 0**1 *)-* 1 *•***

Hence equation (11) can be rewritten in the form

(12) h(J(x*1* , x*2* ) , 0**1 *,*** 0**2*)* =** c(0*1* , 0*2* ) J(x*1* ) + J(x*2* ) for all X*i* , x*2* EX*2*

where J(x) = fi(I(x) , O(x)) , and his continuous and strictly increasing in its first argument.

Now consider any 0*1* , 0**2*,*** 0***3*** E 0 and choose X ES(O*g* ) . Using the earlier nota­

tion, define X h= (x , xh) , O = O(x ) , J = J(x ), *g* c = c(O, 0 ***).*** Then equation

*g g*

(12) implies

*k k k*

*k* and

*1c,* 1 *k* ***1***

h- *1* (c*12* ,*3* h- *1* (J*2* , 0*1* , 0*2* ) + 1*3* , 0*12* , 0***3 )* =** h- *1* (c*12* ,*3* l(i*1* , X*2* ) + 1*3* , 0*12* , 0*3* )

**=** J(i*1* , X*2* , X*3* ) **=** J(i1, X*23* )

**=** h- *1* (J(X*23* ) , 0*1* , 0*23* )

**=** h- *1* (h- *1* (c2*,3* 12 + 1*3* , 0**2*,*** 0*3* ), 0*1* , 0*23* )

= H(c*2*,*3* l*2* + 1*3* , 0*1* , 0*2* , 0*3* ) .

Applying Lemma 1 gives

c*12* ,*3* h- *1* (u, 0**1 *,*** 0**2*)*** = yc*2,3* ***u+a,***

*V* **=** *')'V* + {3,

*6* This convention with respect to inversion of functions of several variables is retained throughout this paper.

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*and setting* ***u****=****v****=0 establishes that {3 =0, y = 1,* ***a =*** *c12,3h-1 (0,* ***81,****8****2),*** *and*

*c(82,* ***83)***

*C2,3*

*(13)*

***h (u, 81, 82)-h (O, 81, 82) = -u* = (**

*C12,3 C* ***8***

*-I*

*-I*

***, 83* ) *u,***

*The left hand side of (13) is independent of 8****3,*** *so for any fixed 8\* c(8****2,*** *8****3 )*** *c(8****2,*** *8\*) c(82)*

*=*

***12***

*c(812, 8****3 )*** *c(81 2,8\*) c(812)*

*c(82,8****12)*** *c(* ***8 ,****8 )*

*c(8*

*1 2,8*

*1 2)*

***2***

*12*

*since* ***c(8,****8)* **= *c(8,****8)-****1* =** *1. Substituting into equations (12) and (13) then yields*

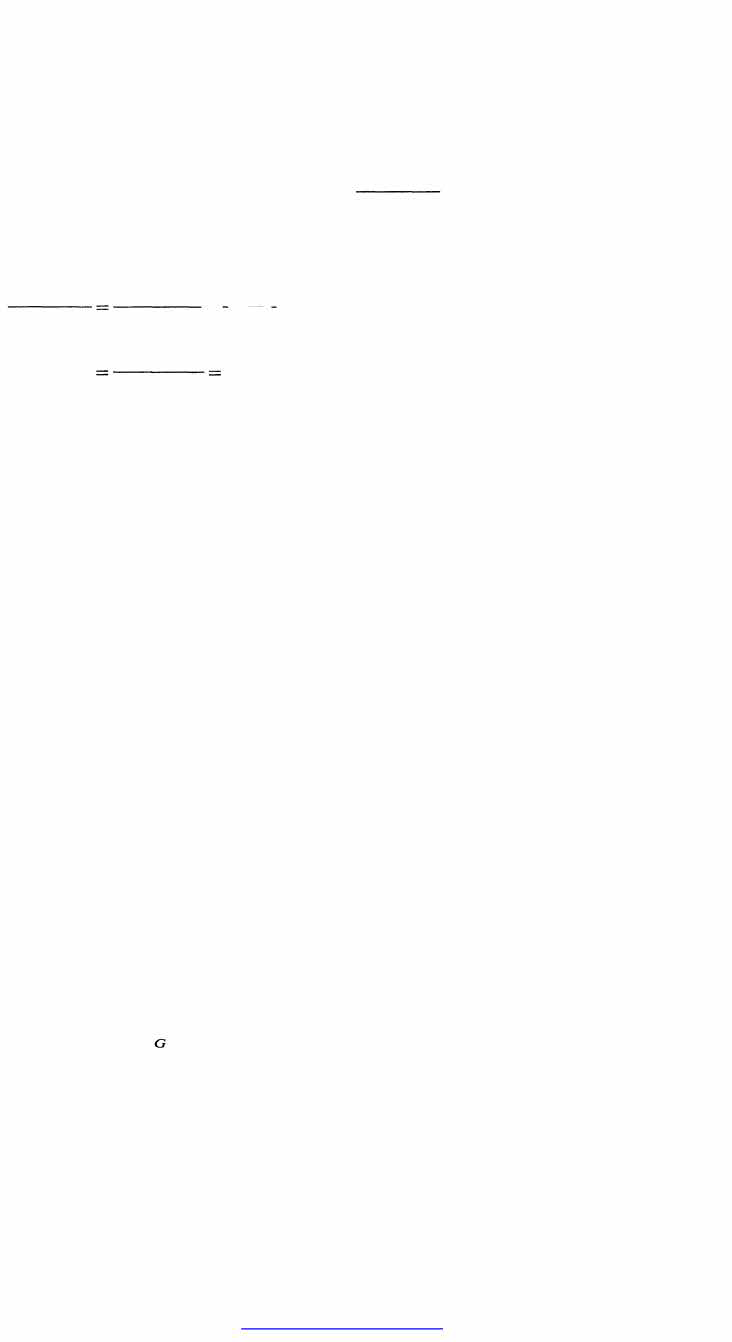
***J(xi , x2)****= h- 1 (c(81 ,* ***82)J(x1)+J(x2),****81 ,82)*

*C* ***812*** *C* ***82***

*and*

***J'(x i ,X2)*** = *c(81 2****)J(x 1, x2) =J'(x1)+J'(x2)+c(8n)h-1****(0,* ***81, 82)***

***=J'(x1)+J'(x2)+J'(x1 , x2),***

*The proof is completed by noting that the sequence of transformations* ***I = ijJ(I,8), J****=****fi(I,****8), and* ***J'****=c (8)1 can be combined into a single transformation* ***J'****=* ***F(I',****8) satisfying conditions (8).*

*Theorem 1 is a powerful result that provides the foundation for the remainder of this paper. The next theorem extends the analysis by demonstrating that the transformed index J has the properties of an inequality measure, and that its corresponding decomposition equation is applicable to any number of subgroups containing any positive number of persons.*

***For any decomposable inequality index I there exists a function F(I,µ.,, n) continuous in I and µ.,, and strictly increasing in I, such that J(x)*** =

*THEOREM 2:*

***F(I(x),µ.,(x), n(x))*** *is* ***another decomposable inequality index satisfying***

*(14)* ***J(x1,. . . , xG)* =** *I* ***J(xg)+J(xi , . . . , xG) for all*** *G-;;;,, 2* ***and all Xg*** E *X*1 •

g-1

***J* =***PROO****,****F****µ.,****:* ***,****F****n****irst consider G* **=** *2. From Theorem 1 there exists a transfo*E*rmation*

***F(I )*** *with the properties (8) such that (14) holds for all* ***xg*** *X*2• *By*

*defining* ***J(x)* =** *0 when* ***n(x)* =** *1, the index* ***J*** *is extended to single person distribu­ tions. Now (7) implies that*

***J(x, x)* = *J(x)+J(x)+J(x, x)* = *2J(x)*** *for all xEX2*

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and this result also holds for all xEX1 *,* since J(x, x) = 0= J(x) when n(x) = I. Therefore, for all X*i* , x*2* EX1 *,*

J(x*1* , X*2* ) = ½J(x*i* , X*2* , X*i* , X*2* ) = ½J(x*i* , X*i* , X*2* , X*2* )

since *J* inherits the symmetry of *I.* Applying (7) we obtain J(x*1* , X*2* ) = ½J(x*1* , X*1* ) + ½J(x*2* , X*2* ) + ½J(.f1, X*1* , X*2* , X*2* )

**=** J(x*1* ) + J(x*2* ) + J(x*1* , .f*2* )

demonstrating that (14 ) holds when G **=** 2.

Extension to any G > 2 is accomplished by induction, since

J(x*1* , . . . , x*O* ) = J(x*1* , , x*O*\_*2* , (x*O*\_ *1* , x*O* ))

**= G-2** J(x ) + J(xa , xa) + B(x , . . . , .fa)

L *g* - *1 1*

g-1

*G*

**=** L J(x*g* ) + J(xa - *1* , .fa) + B(x*1* , . . . , .fa)

g-1

and (14 ) is obtained by replacing x*g* by .f*g* throughout.

*F J*

The properties (8) of ensure that inherits the symmetry, normalization (4 b) and Schur-concavity (4 c) features of is also continuous over any set S(*µ,*, n) when *µ,* and n are fixed. However it remains to be shown that J is continuous over�:*+* . Consider any sequence x*<k)* E�:*+* with the limit x*<00)* E�:*+* . Select any value *µ,* such that *µ,* > *µ,* (x*<k)* ) for all and define u*<k)* **=** n(*µ,* -*µ,* (x*<k)* )) > 0. It then follows from (14 ) that

*k,*

*I. J*

J(x*<k)*' u*<k)* ) = J(x*<k)* ) + J(u*<k)* ) + J(x*<k*>, u*<k)* )

and, since J is continuous over the set S(*µ,*, n),

Lt J(x*<k)* ) = Lt {J(x*<k*>, u*<k)* ) -J(x*<k*>, u*<k)* ) }

k➔OO k➔OO

= J(x*<00*>, u*<00)* ) -J(x*<00*>, u*<*""*)* ) = J(x*<00)).*

Thus *J* is continuous over �:*+* for any *n,* and is therefore an inequality index.

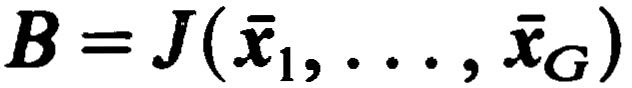
The primary implication of Theorem 2 is that any decomposable inequality measure can be transformed, via into an additively decomposable inequality measure satisfying the "normal form" equation (14 ) , which is a special case of equation (1) . As additive decomposability (together with differentiability) is known to lead to indices of the form given in (2) , the transition to decomposable measures can be expected to expand the class of admissible indices only by adding monotonic transformations of the type indicated. This introduces new numbering schemes for the index values, and different ranking possibilities for pairs of distributions with different means or population sizes, since the transfor­ mation can depend on *µ,* and n. However, any decomposable inequality index ***I*** will be constrained to agree with its corresponding additively decomposable index *J* when ranking distributions with similar sized populations and identical

*F*

*F,*

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# aggregate incomes. To this extent relaxing the additive element of the index aggregation provides little more than alternative, ordinally equivalent, versions of the same set of indices. This is sufficient to encompass the Atkinson [2] family of inequality measures, which are decomposable, but not additively decompos-

**. able ; but does not admit the Gini coefficient, which cannot be expressed as a suitable transformation of an additively decomposable index.**

*J*

**The normal form index corresponding to any decomposable index *I* indicates a particularly satisfactory way of assigning inequality contributions to the popula­ tion subgroups. For, when equation (1 4) holds, we can define the subgroup contributions**

**(1 5a)** *C8 = J(x8 )*

**(1 5b) =** *J(x1 , • • • , x0 ) - J(x1 , ... , x8 \_,, i8, X8 + 1 , . .. , x0 )*

**(I 5c) = J(i*1* , • • • ,** *i8\_ 1 , Xg, i8 + 1 , ... , i0 ) - J(ii , . .. ,* **i*0* )**

# and

**to obtain**

*G*

*J(x) = J(x , ... , x ) L C + B.*

*1 0 = 8*

g- 1

# Thus the overall inequality level can be expressed as the sum of the subgroup contributions plus the "between group" term *B.* Furthermore, *C8* is consistent with all the obvious interpretations of "the contribution to equality of subgroup g": either the inequality level of subgroup alone (15a) ; the amount by which the overall value falls if inequality within subgroup is eliminated (1 5b) ; or the amount by which the overall value increases if inequality within subgroup is introduced, starting from the position of equality within each subgroup ( 15c). Since these interpretations will not in general coincide for the untransformed index *I,* there is a good argument for using the normal form of the index, and its corresponding assignment of contributions, in the absence of any stroqg preference for one particular cardinalization of a decomposable inequality

***g***

*J*

*J g*

*g*

**measure.**

*J*

**Another interesting characteristic of the normal form index is that it almost satisfies the replication invariance property. Specifically, by choosing subgroups with identical distributions** *x,* **equation (14) implies**

*r*

**(16) *J( R,x)* =** *J(x, ... ,* **x) =** *rJ(x) + J(i, . . . ,* **i) =** *rJ(x).*

# Thus = n(x) is a replication invariant inequality index. Since replacing with *J'* in (14) still generates an equation of the form (1), is also additively decomposable. It therefore follows that any decomposable inequality index is a monotonic transformation of a replication invariant, additively decomposable

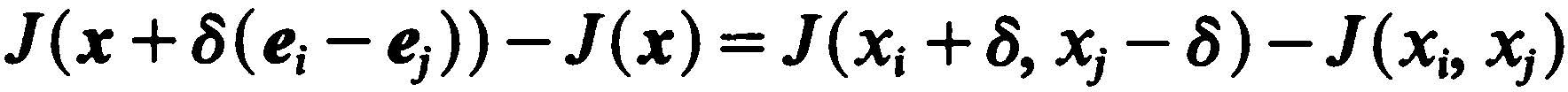
*J J'*

*J'(x) J(x)/*

**inequality index.**

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INEQUALI TY DECOMPOSI TION 1379



1. THE CLASS OF DECOMPOSABLE INEQUALI TY MEASURES

Hav ing establ ished the g eneral char act erist ics of dec om posabl e inequ al ity m easur es, we now tum tothe qu est ion of ident ify ing thefunctional re prese nt at ion of such indic es. The fol l owing theor em dem onstr at es, wit hout t he assum pt ion of diff er ent iabil ity, t hat dec om posabl e inequ al ity m easur es in norm al form nec essarily sat isfy a r estr ict ed v er sion of equ at ion (2).

THEOREM 3: *If J(x) is any continuous and symmetricfunction satisfying*

(17) *J(x,* y) **=** *J(x) + J(y) + J(x, y) for all x, ye Xi , then there exists a continuous function </> : �*++ *➔ � suchthat*

(18) *J(x) =* **n ( x)** *{</>(x; } - <f> (µ, (x)) } for all xeX .*

L *1*

*i=I*

PROOF: L et x be any v ect or from *X3,* and e*i* be t he ith st andard basis v ect or (e*jj* = 1 ; e*if* = 0 for *i* 7': *j)* of the sam e dime nsion. Then (17) im pl ies

*J(x*+ S (e***1*** *-* e**2**)) - J(x) = *J(x1* + S, *x2 -* S, *x3, • • • , Xn ) - J(xi , x2, . . . ,* X*n* )

**=** *J(x1* + S, *X2* - *S) - J(xi ,* X*2*)

and m ore general ly, bec au se J is symm etric,

for al l /; such t hat x*i* + /; **>** 0 and *xi* - /; **>** 0. Ther efor e

*J(xj* + s, *Xk* -/;) - J(x*i*, *xk ) + J(xk, Xj - /;) - J(xk* - /;, *Xj)*

= *J(x* + S (e*i* - *ek )) - J(x) + J(x* + S (e*i* -*ei)) -J(x* + S(e*i* - e*k* ))

**=** *J(x* + /;( e*i* -*e))- J(x)*

= *J(xj* + s, *xi* -/;) - J(x*i*, *xi).*

Sett ing x*i* = *u* + 1 , *xi* = *v* + *w* + 1, *xk* = *v* + 1, and /; = *v,* we obt ain

*J(u+ v+l,* 1 ) - J(u +l , *v+ l) + J(v+ l,* w +l ) -J(l , *v+ w+l )*

*= J(u+ v+l, w + 1 )- J(u+ 1 , v+ w+l )*

and t his bec om es

(1 9) *.d(u + v,* w)+ "1 (u, v) = "1(u, *v+ w)+ "1 (v,* w)

when *.d(u,* v) *= J(u* + 1 , *v* + *1 ) -J(u* + 1 , *1 )- J(v* + 1 , 1 ). N ote that the pr opert ie s of *J* ensur e t hat "1 is cont inu ous and that "1 (*u,* v) = "1 (*v, u*).

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**Eq uat ion (19) has t he s ol ut ion (Erd os [8]}*7***

*.1(u,* ***v* )=** *1/J(u* **+ *v* ) -tfJ(u) -1/J (*v* ) for s ome c ont in uous funct ion** *1/J.* **Hence**

*J(u,* ***v* )= .1(u-1,** *v -l }+ J(u, l } + J(v,* **I )**

**=** *1/J(u* **+ *v* -2)** *+ l(u,* **l} -1/J(u)** *+l(v,* **1 ) -1/J(*v* )**

**=** *1/J(u* **+ *v* -2)** *+ q,(u) + q,(v)*

### wher e *q,(u}= l(u,* l } -1/J(u} . B ut *J(u,* u) = 0 = I/J(2u -2} + 2q,(u), so I/J(w -2)=

**-2q,(w / 2) and**

*u* ***v***

**.**

*J(u,* ***v* ) =** *q,(u)* **+ q,(*v* ) -24> ( ; )**

### This c onfir ms t hat (1 8) holds for al l *x*E X*i* s uch t hat *n(x)* = 2 (and als o, tr iv ial ly, when *n(x)* = 1 ) .

**The r es ult c an be ext end ed to al l** *x***e X*i* by ind uct ion on** *n(x) .* **S uppos e *µ,* =** *µ, (x)*

**and** *n* **=** *n(x) .* **Th en (17) impl ies**

*J(µ,,* x} **=** *J(x} +l(µ,,* **i)** *= l(x}*

### and als o

***J(x)* =** *f(Xi , X2) -J(µ,, Xi + X2 -µ, } + ](µ,, Xi + X2 -µ,,* **X*3*, . . . ,** *Xn )*

**=** *](Xi , X2) -J(µ,, Xi + X2 -µ, } +](Xi + X2 -µ,,* **X*3*, . • • ,** *Xn )*

**= q,(x*i* ) + q,(x*2*) -***q,(µ,* **) -***q,(Xi* **+** *X2* **-JJ, ) +** *q,(Xi* **+** *X2* **-J), )**

### " L q,(xJ -(n-l } q,(*µ,* }

***i=3***

### = I {<f, (xJ -4>(*µ,* ) }

***i=i***

### wher e it has been ass umed t hat *xi* + *x2* > *µ,* (wit hout l oss of gen eral ity, s inc e *J* is symmetr ic). T his compl et es the pr oof.

**The r es ults of T heor ems 2 and** 3 **c an be combin ed to pr ov ide a gen er al st at ement c onc ern ing the funct ion al for m of d ecompos abl e in eq ual ity ind ic es . G iv en t he d isc uss ion of r epl ic at ion inv ar ianc e fol lowin g eq uat ion (16), it is als o a s impl e matt er to inc or por at e t his pr operty .**

**T HE OREM 4: *is a decomposable inequality index there exist functions F and***

*J ijf*

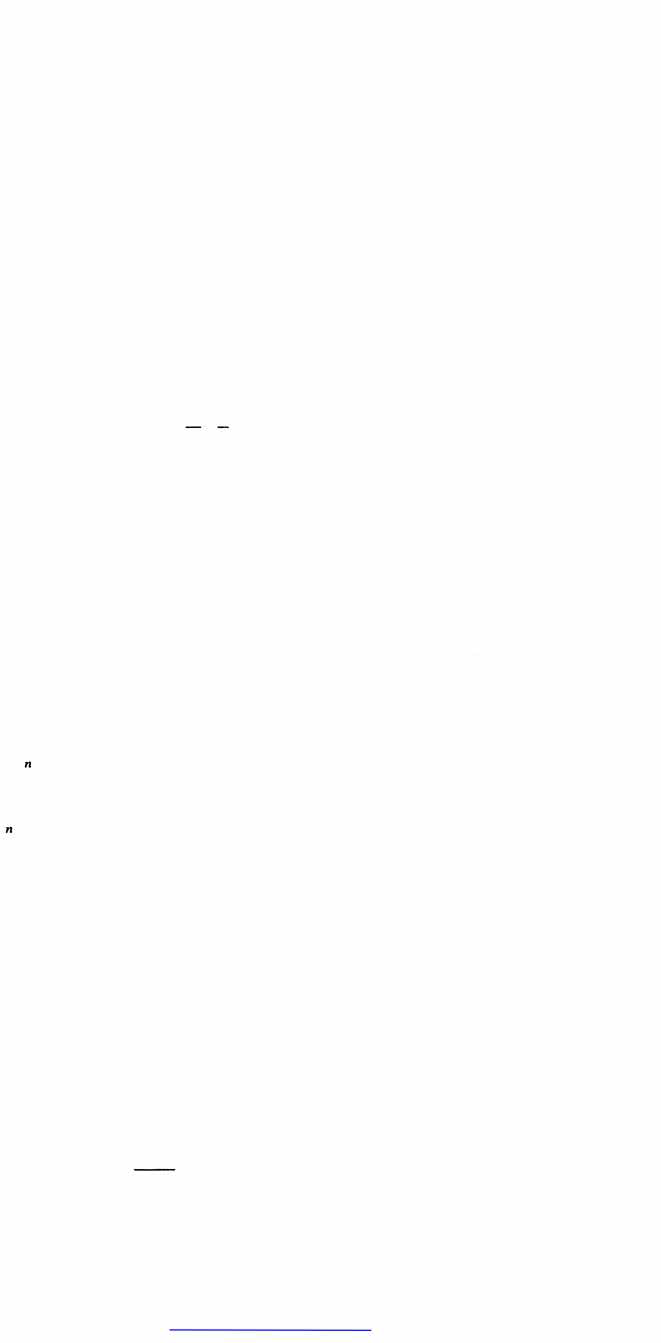
*q,* ***such that***

(20}

*F(I(x),* ***µ,(x),* n(x} } =**

*1* **n (.r)** *{q, (xJ -q, (*

*(x)) }*

*n � µ,*

*(x) i i*

**7 I am grateful to Professor Aczel for bringing this reference to my attention, and to Dr. C. T. Ng for suggesting to me how the problem in hand could be transformed to make use of this result.**

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INEQ UAL ITY DEC OMP OS ITION **1 381**

***where tp(x) is continuous and strictly convex ; and where F(I, µ,, n) is continuous in I and µ,, strictly increasing in I, with F(0, µ,,* n) = 0. *In addition, I is replication invariant ijf F is independent of n.***

PROOF: From Theor em 2 t her e exist s a funct ion ***fa*** wit h the st at ed pr opert ies, such that *J =* ***fa(I, µ,,*** n) is an ineq ual ity index sat isfy ing eq uat ion (1 4). Theor em 3 shows that *J* can be wr itt en as

***J(x)= L { tjJ(x;) - tjJ(µ,)}***

*i=l*

wher e <p is cont inuous and str ictly convex, since sat isfies 4 c Necessity of

*J* ).(

(20) then fol l ows by defining ***F(I, µ,,* n)** = ***fa(J, µ,, n)/ n*** *=* / ***n.*** Suffi ciency of (20)

*J*

is demonstr at ed by checking t hat ***J(x),*** as given by (20), sat isfies condit ions (4) and (5 ). The eq uival ence bet ween ***I*** being r epl icat ion invar iant and ***F*** being independent of ***n*** fol l ows immediat ely from the fact that the right hand side of

(20) is invariant to a repl icat ion of the distr ibut ion.

We final ly consider t he pr operty of scal e invariance which, when combined wit h decomposabil ity, pr oduces th e fol l owing r esult:

THE ORE M 5: ***I is a scale invariant, decomposable inequality index ijfthere exists a parameter c*** E *[1/1,* ***and a function F(I, n), continuous and strictly increasing in I, with F(0,* n) =** 0, ***such that***

**{(X;)c**

*l 1 �*

(21 )

n c(c-1 } ;*f..,*- 1 ***µ,***

***F(I(x), n(x))=*** .!.***n*** I � l og (�)

*l }* ifc� 0, 1,

ifc*=* l,

*i~I Ji,* ( *Ji,*

*1 n*

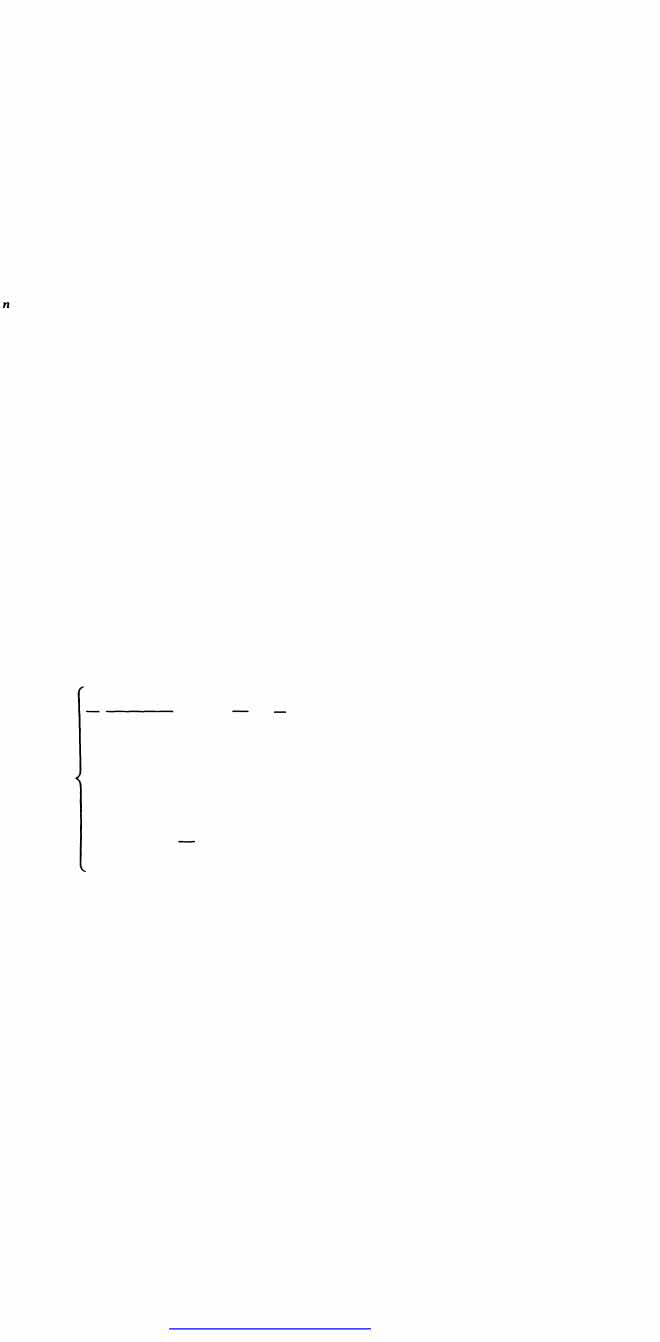
*-;; ;�1*

l og

***µ,)***

*X;*

***ifc* = 0.**

***I is also replication invariant ijf F is independent of n.***

PROOF: Since ***I*** is decomposabl e it sat isfies (20) for some suit abl e ***F*** and <p. Nor mal ize <p so t hat <p (1 ) = 0 and define ***fa(I, µ,,*** n) **= *nF ( I, µ,, n).*** Choose any X*i* , x*2* E X***2*,** and wr it e x*12 =* (x*i* , x**2*),*** h = ***I(xd, nk*** *=* ***n(xd*** and ***JLk*** *=* ***µ,(xd.*** Then

I**fa-let**

***l(X1 2)* = *tjJ(xl i)+ tjJ(x2J - n12tfJ(µ, 1 2), JJ, 12, n12)***

**= *fa-1(fa(Ii , µ, i , n1)*** + ***fa(I2, JL2, n2)*** + ***n1 tjJ(µ, 1)*** + ***n2 tfJ(µ,2)***

***- n12tfJ(µ, 1 2), JJ, 12, n1 2)***

***= H(fa(Ii , µ, i , n1)*** + ***fa(I2, JJ,2, n2), µ, i , JJ,2, ni , n2)***

**= *J(A x 1 2)***

**= *H(fa(Ii , Aµ, 1 , n1)*** + ***fa(I2, Aµ,2, n2), Aµ, 1 , Aµ,2, ni , n2).***

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***Applying Lemma I gives***

***F(I*i *, A*µ, *1, n*1*) = cF(I****1 ,* **µ, i *, n*1*),***

***F(I****2,* ***A*µ,*2, n*2 *) = cF(I****2,* **µ,***2 ,* ***n*2 *),***

***from which it follows that***

***c = F(I, >..*µ,*, n) = c(A)'***

***independent of I,***

***, and n.***

**µ,**

***But***

***F(I,***

**µ,*, n)***

***F(I, >..*µ,*, n) = c(A)F'(J,* µ,*, n) = c(A )c(*µ,*)F(J, I, n)***

***= c(A*µ,*)F(J, I, n).***

***So c(A*µ,*) = c(A )c(*µ,*) > 0 for all A,* µ, *> 0, and continuity of c ensures that c(*µ,*) =* µ, *c***

***[l, p. 144, Theorem 4]. Hence***

***(22) F(I(x), I, n) = F(I(x), I, n)/ n =* µ, *-*c*fa(I(x),* µ,*, n)/ n***

**c**

***=* µ,**

***-*c*F(I(x),***

**µ,*, n)= I { </>(xJ - </>(*µ,**

*i=l*

***)}/ n*µ, *.***

***Now choose x = (u, v) and define 1/J(u, A) = </>(A u)-A* c*</>(u). Then 0 = F(I(>.. x), Aµ,(x), 2) -A* c*fa(J(x), µ,(x), 2)***

***= </>(A u) + </>(Av)- 2</>(A(u + v)/ 2)-A* c *{ </>(u) + </>(v)- 2<f>((u + v)/ 2)}***

***= 1/1(u, A)+ 1/1(v, A) - 21/1(½(u + v), A).***

***This is a Pexider equation, whose solution [l, p. 142] is (23) 1/J(u, A) = a(A )u + b(A).***

***But***

***1/J(u, A )= </>(u*µ,*A)-A* c c*<f>(u)***

**µ, µ,**

***= 1/1(u*µ,*, A) + A* c*l/I(u,* µ,*) = 1/1(u>.., )+* c*l/I(u, A).***

**µ, µ,**

***Substituting (23) and equating coefficients of u produces***

***a(A***

**µ,*) = a(A )*µ,**

***" A* c*a(*µ,**

***)= a(*µ,*)A +* µ, c*a(>..),***

***b(A***

***) = b(A) +A* c*b(*µ,**

***} = b(*µ,**

***} +* µ, c*b(A),***

***for which the solutions are***

**µ,**

***c = I,***

***/3(*µ, *C*** - ***I), � ¥- 0,***

***b(*µ,*) = {/3 log***

***c - 0.***

**µ,*,***

***a(*µ, *c*** *-* **µ,*),*** *c* ***¥- I,***

***a(*µ,*) = { a***

***log***

***,***

**µ, µ,**

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Henc e

**INEQUALITY DECOMPOSITION**

***cf,(x)* =** 1/J(l, ***x)* = *a(x)+b(x),*** *c �* 0, 1, (a ***+/3)xc - ax -/3,*** c� 0, 1,

**=** { ***ax*** l og ***x*** + ***f3x*** -/3, *c* **=** 1,

/3 l og ***x*** -***ax*** + ***a,*** *c* = 0.

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Subst it ut ing int o (22) and inc or por at ing a suit abl e const ant into ***F*** pr oduc es (21 ). The conver se r esult is trivial ly acc ompl ished by c onfir ming t hat any funct ion

***I*** defined by (21 ) sat isfies sc al e invar ianc e and c ondit ions (4) and (5). Simil arly, t hat r epl ic at ion invarianc e of ***I*** is eq uival ent to ***F*** being independent of ***n*** fol l ows immediat ely from the fact t hat t he General ized Entropy index for m on the r ight hand side of (21 ) is repl ic at ion invar iant .

Theor em 5 showst hat the c ombinat ion of dec omposabil ity and sc al e invar ianc e for an ineq ual ity index l eads us inexor ably t owar ds t he G ener al ized Entropy family given earl ier in eq uat ion (3). In fact, given the addit ional undemanding r eq uir ement of r epl ic at ion invar ianc e, the cl ass of admissibl e indic es ***I*** ar e just simpl e incr easing transfor mat ions of a G ener al ized Entropy measur e ***J.*** Thus ***I*** is or dinal ly eq uival ent to *J* and wil l rank any pair of distribut ions in the same way. There may be c irc umst anc es in whic h, say, the At kinson [2] index or the coeffi cient of var iat ion is pr eferr ed to it s c orr esponding G ener al ized Entropy form. But the gener al conc l usion appears to be t hat, when dec omposabil ity is desir ed, and sc al e and repl ic at ion invar iance ar e acc ept ed, not hing subst ant ial is l ost by foc ussing excl usively on the Gener al ized Entr opy indic es.

*F(J)*

1. **CONCLUDING REMARKS**

This paper has examined the impl ic at ions for ineq ual ity measur es of imposing a weak aggr egat ion pr operty whic h r eq uir es t hat over all ineq ual ity can al way s be calcul at ed from the size, mean, and ineq ual ity val ue of eac h popul at ion subgr oup. In Sect ion 3 it was shown (Theor ems 1 and 2) t hat t his dec omposabil ity req uir ement impl ies t hat the index must be c apabl e of being transformed into an ***additively*** dec omposabl e index. The gener al form of a dec omposabl e index was der ived in Sect ion 4 (Theor em 4) and the impact of assuming r epl ic at ion invar ianc e and sc al e invar iance (homogeneity) wer e examined in Theor ems 4 and 5 . I mposing al l t hr ee restr ictions l eads to simpl e monot onic transfor mat ions of the Gener al ized Entr opy family . Thus r epl acing the ***additively*** dec omposabl e c ontr aint consider ed in Shorr oc ks [1 2] wit h a very weak aggregat ion pr operty expands the admissibl e set of indic es by incl uding monot onic transfor mat ions, but ot her wise cannot be said to al l ow any new perc ept ions of ineq ual ity to be inc or por at ed.

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In the context of inequality measurement there are two direc tions in whic h the result s may be ext ended. The first is to impose const raints on the permissibl e partit ions of the popul at ion. For inst anc e, the Gini coeffi cient is known to be dec omposabl e, in the sense of equat ion (5 ), when t he inc omes in one subgroup are al l l ess than those in t he ot her subgroup.**8** So the cl ass of inequal ity measures t hat are dec omposabl e under al l ***non-overlapping*** partit ions of the inc ome dist ribu­ t ion certainly cont ains indic es that are not covered by, say, equat ion (20). A sec ond pot ent ial l ine of devel opment is to rel axthe assumpt ion that t he subgroup ***means*** are rel evant in the aggregat ion, and subst it ute a variabl e rel at ing to a more general concept of represent at ive inc ome. This may not present t oo many diffic ul ­ t ies. For the mean inc ome paramet er pl ays a rel at ively minor rol e in Theorem 1, being used primarily to ensure that the sets S(8) have suit abl e properties, l ike c onnect edness. Consequently Theorems 1 and 2 may remain subst ant ial ly int act . To ext end the analy sis to indic es ot her than inequal ity measures, assumpt ion

(4) wil l cl early need to be amended. However, cont inuity, sy mmet ry and some kind of normal izat ion condit ion are st il l l ikely to be appropriat e. So it is only Sc hur-c onc avity that may have no general anal ogue, and t his again does not appear to be c rit ical ly important in the derivat ion of the result s. It woul d al so appear possibl e to repl ac e t he sc al ar-val ued individual observat ions wit h vect ors: for exampl e, t he pric e-quant ity pairs that might be needed for a general analysis of pric e and quant ity indic es.9

A useful and comprehensive type of index aggregat ion that coul d be invest i­ gat ed al ong t he l ines of t his paper takes the form of a rec ursive aggregat ion st ruct ure. The l owest l evel "indic es" woul d be t hose for whic h there is a simpl e and obvious rel at ion ship bet ween the subgroup index val ues and the popul at ion ind ex. The subgroup sizes (number of observat ion vect ors), or the subgroup sum (for exampl e, t ot al subgroup inc ome), are two possibl e candidat es for l evel 1 indic es. Level 1 index val ues then bec ome t he subgroup paramet ers for l evel 2 aggregat ion. Thus, for inst ance, the represent at ive inc ome l evel forthe popul at ion may be a funct ion of t he subgroup represent at ive inc omes ***and*** sizes. This woul d al l ow a general izat ion of the mean inc ome paramet er used earl ier, whil e ret aining it s aggregat ion properties. Aggregat ion of l evel 3 ind ic es woul d then t reat l evel

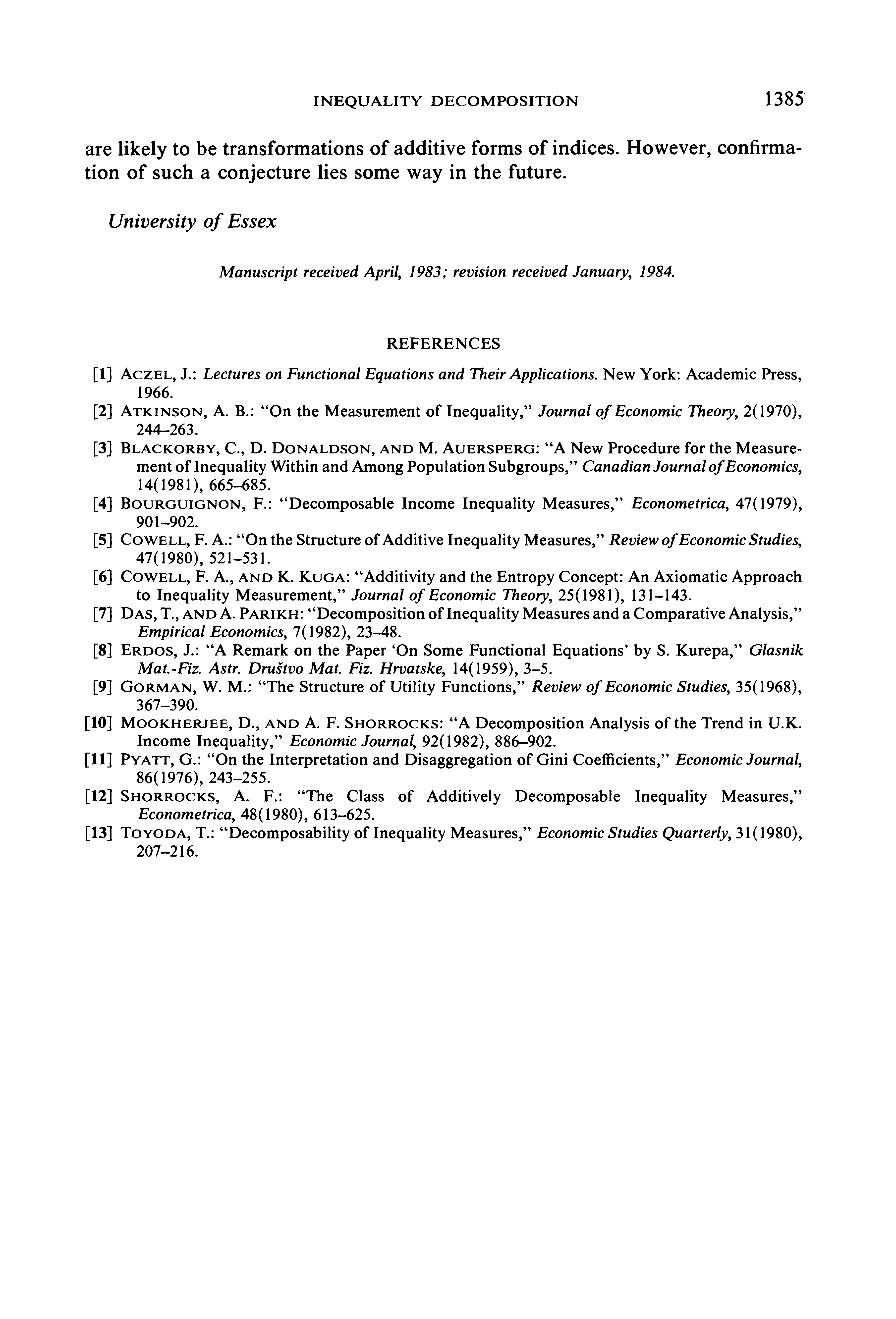
1 and 2 indic es as paramet ers in the same way that the aggregat ion of inequal ity measures in t his paper empl oys the subgroup means and sizes. In princ ipl e, t his st ruct ure coul d be ext ended indefinit ely, wit h each aggregat ion l evel treat ing al l t he l ower l evel indic es as paramet ers inthe aggregati on proc ess. But most pract ic al int erest is l ikely to reside int he few l owest l evel s, if only bec ausethe informat ional advant ages of aggregat ive indic es wil l be dil ut ed if l arge numbers of paramet ers are required in t he comput at ion. The proof of Theorem 1 indic at es an approach t hat might be appl ied to rec ursive aggregat ion st ruct ures, using an induct ive argument . The result s oft his paper al so suggest that recursively aggregabl e indic es

**8 *See, for example, Pyatt [11] or Mookherjee and Shorrocks [10].***

**9 *This could be viewed as a case in which the permitted partitions of the set of price and quantity observations is constrained, so that price-quantity pairs are never split up.***

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are l ikely to be transformat ions of addit ive forms of indic es. However, c onfirma­ t ion of suc h a conj ect ure l ies some way in the fut ure.

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***Manuscript received April, 1983 ; revision received January, 1984.***

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