

Universidad de San Carlos de Guatemala
Facultad de Ingeniería
Departamento de Matemática
Primer Semestre, 2025
Curso: Matemática Aplicada 1
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TAREA No. 1

PONDERACIÓN	
Presentación	/10
Tarea completa	/50
Ejercicios calificados	/40
TOTAL	/100

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Guatemala, 2___/05___/2025___

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#1

Ej 2

$$f(t) = \begin{cases} 4 & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^2 e^{-st} (4) dt + \int_2^{\infty} e^{-st} (0) dt$$

$$= \int_0^2 4e^{-st} dt \rightarrow \left[-\frac{4e^{-st}}{s} \right]_0^2 \rightarrow \boxed{\frac{-4e^{-2s}}{s} + \frac{4}{s}}$$

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#2

Ej 5

$$f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\pi} e^{-st} \sin t dt + \int_{\pi}^{\infty} e^{-st} (0) dt$$

$$v = \sin t \\ dv = \cos t dt$$

$$dv = e^{-st} \\ v = -\frac{e^{-st}}{s}$$

e^{-st}	$\sin t$
$-se^{-st}$	$\textcircled{+} \cos t$
$s^2 e^{-st}$	$\textcircled{-} \sin t$

$$\int u dv = uv - \int v du$$

$$\int \sin t e^{-st} dt = \frac{\sin t e^{-st}}{s} - \int \frac{e^{-st}}{s} \cos t dt$$

$$\int \frac{e^{-st}}{s} \cos t dt \Rightarrow v = \cos t \quad dv = -\sin t dt$$

$$\int \cos t \frac{e^{-st}}{s} dt = \cos t \left(-\frac{e^{-st}}{s^2} \right) - \int -\frac{e^{-st}}{s^2} (-\sin t) dt$$

$$\int e^{-st} \sin t dt = -\frac{1}{s} e^{-st} \sin t - \frac{1}{s^2} e^{-st} \cos t - \int \frac{1}{s^2} e^{-st} \sin t dt$$

$$\int e^{-st} \sin t dt = -\frac{1}{s} e^{-st} \sin t - \frac{1}{s^2} e^{-st} \cos t - \frac{1}{s^2} \int e^{-st} \sin t dt$$

$$\int e^{-st} \sin t dt + \frac{1}{s^2} \int e^{-st} \sin t dt = -\frac{1}{s} e^{-st} \sin t - \frac{1}{s^2} e^{-st} \cos t$$

$$1 + \frac{1}{s^2} \int e^{-st} \sin t dt = -\frac{1}{s} e^{-st} \sin t - \frac{1}{s^2} e^{-st} \cos t$$

$$\int e^{-st} \sin t dt = -\frac{\frac{e^{-st} \sin t}{s}}{\frac{s^2+1}{s^2}} - \frac{\frac{e^{-st} \cos t}{s^2}}{\frac{s^2+1}{s^2}}$$

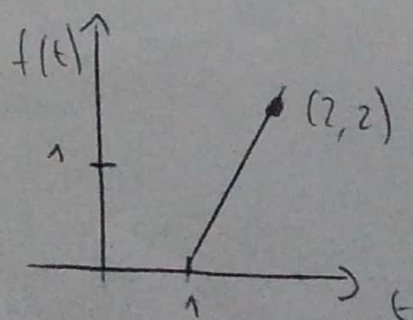
$$\int e^{st} \sin t dt = \frac{e^{-st} \sin t}{s^2+1} - \frac{e^{-st} \cos t}{s^2+1}$$

$$\left[\frac{-e^{-s\pi} \sin \pi}{s^2+1} - \frac{e^{-s\pi} \cos \pi}{s^2+1} \right] - \left[\frac{-e^{-s0} \sin 0}{s^2+1} - \frac{e^{-s0} \cos 0}{s^2+1} \right]$$

$$+ \frac{e^{-s\pi}}{s^2+1} + \frac{1}{s^2+1} \Rightarrow \boxed{\frac{e^{-s\pi} + 1}{s^2+1}}$$

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#3



$$f(t) = \begin{cases} 0 & 0 \leq t < 1 \\ 2t-2 & 1 \leq t < 2 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} (2t-2)e^{-st} dt$$

(2)

$$\lim_{b \rightarrow \infty} \int_a^b (2t-2) e^{-st} dt$$

$$\begin{aligned} 2t-2 & \quad 0 e^{-st} \\ 2 & \quad 0 \frac{e^{-st}}{s} \\ 0 & \quad 0 \frac{e^{-st}}{s^2} \end{aligned}$$

$$(2t-2) \left(+ \frac{e^{-st}}{s} \right) - 2 \frac{e^{-st}}{s^2}$$

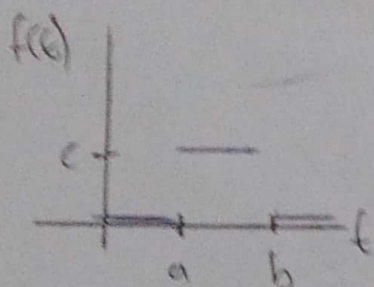
→ al ser infinito y bajar todo es 0

$$f(s) = \left(+ \frac{2t e^{-st}}{s} - 2 \frac{e^{-st}}{s} + 2 \frac{e^{-st}}{s^2} \right)$$

$$f(s) = -\frac{2e^{-s}}{s} + \frac{2e^{-s}}{s} + \frac{2e^{-s}}{s^2} \Rightarrow \boxed{\frac{2e^{-s}}{s^2}}$$

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#4



$$f(t) = \begin{cases} 0 & 0 \leq t < a \\ c & a \leq t < b \\ 0 & t \geq b \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_0^a e^{-st} \cdot 0 dt + \int_a^b e^{-st} \cdot c dt + \int_b^\infty e^{-st} \cdot 0$$

$$\mathcal{F}\{f(t)\} = \left[-\frac{c e^{-st}}{s} \right]_a^b = -\frac{c e^{-bs}}{s} + \frac{c e^{-as}}{s} = \boxed{\frac{c}{s} (-e^{-bs} + e^{-as})}$$

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#5

$$f(t) = t e^{4t} \Rightarrow \mathcal{F}\{f(t)\} = \int_0^\infty t e^{4t} e^{-st} dt = \int_0^\infty t e^{(4-s)t} dt$$

t	$e^{(4-s)t}$
1	$\frac{e^{(4-s)t}}{4-s}$
0	$\frac{e^{(4-s)t}}{(4-s)^2}$

$$f(s) = \left[\frac{t e^{(4-s)t}}{4-s} - \frac{e^{(4-s)t}}{(4-s)^2} \right]_0^\infty$$

$$f(s) = \left[\frac{\infty e^{\infty}}{4-s} - \frac{e^{\infty}}{(4-s)^2} \right] - \left[\frac{0 e^0}{4-s} - \frac{1}{(4-s)^2} \right]$$

$$F(s) = + \frac{1}{(4-s)^2}$$

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#6

$$f(t) = t^2 e^{-2t}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} t^2 e^{-2t} e^{-st} dt$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} t^2 e^{-(2+s)t} dt = \int_0^{\infty} t^2 e^{-t(2+s)} dt$$

t^2	$e^{-t(2+s)}$
$2t$	$0 - \frac{e^{-t(2+s)}}{-(2+s)}$
2	$0 + \frac{e^{-t(2+s)}}{(2+s)^2}$
0	$0 - \frac{e^{-t(2+s)}}{(2+s)^3}$

$$f(s) = \left[-\frac{t^2 e^{-t(2+s)}}{(2+s)} - \frac{2t e^{-t(2+s)}}{(2+s)^2} + \frac{2 e^{-t(2+s)}}{(2+s)^3} \right]_0^{\infty} = \frac{2}{(2+s)^3}$$

$$F(s) = \frac{2}{(2+s)^3}$$

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$$f(t) = e^{-t} \sin t \rightarrow \mathcal{L}\{f(t)\} = \int_0^{\infty} \sin t e^{-t} e^{-st} dt$$

#7

$$\int_0^{\infty} \sin t e^{-t-st} dt \rightarrow \int_0^{\infty} \sin t e^{-t(1+s)} dt$$

$u = \sin t$ $dv = e^{-t(1+s)}$
 $du = \cos t$ $v = -\frac{e^{-t(1+s)}}{(1+s)}$

$$\int \sin t e^{-t(1+s)} dt = -\frac{e^{-t(1+s)}}{(1+s)} (\sin t) - \int -\frac{e^{-t(1+s)}}{(1+s)} (\cos t) dt$$

$$= -\frac{\sin t e^{-t(1+s)}}{(1+s)} + \frac{1}{(1+s)} \int e^{-t(1+s)} \cos t dt$$

$$\begin{aligned} u &= \cos t & dv &= e^{-t(1+s)} \\ du &= -\sin t & v &= -\frac{e^{-t(1+s)}}{(1+s)} \end{aligned}$$

$$\int \sin e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{(1+s)} + \frac{1}{(1+s)} \left[\cos t \left(-\frac{e^{-t(1+s)}}{(1+s)} \right) - \int -\frac{e^{-t(1+s)}}{(1+s)} (-\sin t) dt \right]$$

$$\downarrow$$

$$= \frac{1 \cdot \cos t (e^{-t(1+s)})}{(1+s)^2} - \frac{1}{(1+s)^2} \int e^{-t(1+s)} (\sin t) dt$$

$$\int \sin t e^{-t(1+s)} + \frac{1}{(1+s)^2} \int e^{-t(1+s)} (\sin t) dt = -\frac{\sin t e^{-t(1+s)}}{(1+s)} - \frac{\cos t (e^{-t(1+s)})}{(1+s)^2}$$

$$\frac{(1+s)^2 + 1}{(1+s)^2} \int \sin t e^{-t(1+s)} = -\frac{\sin t e^{-t(1+s)}}{(1+s)} - \frac{\cos t (e^{-t(1+s)})}{(1+s)^2}$$

$$\int \sin t e^{-t(1+s)} = \frac{-e^{-t(1+s)}}{(1+s)} \left(\sin t + \frac{\cos t}{(1+s)} \right)$$

$$= \frac{(1+s)^2 (-e^{-t(1+s)})}{(1+s)((1+s)^2 + 1)} \left(\sin t + \frac{\cos t}{(1+s)} \right) \rightarrow$$

$$-\left[\frac{(1+s)(1-e^{-s})}{(1+s)^2+1} \right] \left(0 + \frac{1}{(1+s)} \right)$$

$$F(s) = \frac{1}{s^2+2s+2}$$

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$$f(t) = t \sin t \Rightarrow \mathcal{L}\{f(t)\} = \int_0^\infty t \sin t e^{-st} dt$$

#19

$$\begin{aligned} v = t & \quad dv = 1 dt \\ dv = \sin t e^{-st} & \quad v = -\frac{e^{-st} \sin t}{s+1} - \frac{e^{-st} \cos t}{s+1} \end{aligned}$$

$$\begin{aligned} \int \sin t e^{-st} dt &= \sin t \left(-\frac{e^{-st}}{s} \right) - \int -\frac{e^{-st}}{s} \cos t dt \\ \int \sin t e^{-st} dt &= -\frac{\sin t e^{-st}}{s} + \int \frac{e^{-st}}{s} \cos t dt \end{aligned}$$

$$\int \sin t e^{-st} dt = -\frac{\sin t e^{-st}}{s} + \frac{1}{s} \int \cos t e^{-st} dt$$

$$\int \sin t e^{-st} dt = -\frac{\sin t e^{-st}}{s} - \frac{\cos t e^{-st}}{s^2} - \frac{1}{s^2} \int e^{-st} \sin t dt$$

$$\int \sin t e^{-st} dt + \frac{1}{s^2} \int e^{-st} \sin t dt = -\frac{\sin t e^{-st}}{s} - \frac{\cos t e^{-st}}{s^2}$$

$$\left(\frac{s^2+1}{s^2} \right) \int \sin t e^{-st} dt = -\frac{\sin t e^{-st}}{s} - \frac{\cos t e^{-st}}{s^2}$$

$$\int \sin t e^{-st} dt = -\frac{e^{-st} \sin t}{s^2+1} - \frac{e^{-st} \cos t}{s^2+1}$$

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$$\int t \sin t e^{-st} = t \left(-\frac{e^{st} \sin t}{s^2+1} - \frac{e^{st} \cos t}{s^2+1} \right) - \int \left(-\frac{e^{st} \sin t}{s^2+1} - \frac{e^{st} \cos t}{s^2+1} \right)$$

$$\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$$

$$\mathcal{L}\{t f(t)\} = -f'(s)$$

$$f(s) = \frac{1}{s^2+1} \rightarrow \frac{-(2s+0)(1) + (s^2+1)(0)}{(s^2+1)^2} \rightarrow -\frac{2s}{(s^2+1)^2}$$

$$f(s) = \frac{2s}{(s^2+1)^2}$$

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#9

$$f(t) = 2t^4 \rightarrow \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \rightarrow \frac{4!}{s^5} \rightarrow \frac{24}{s^5}$$

$$f(s) = \frac{48}{s^5}$$

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#10

$$f(t) = t^2 + 6t - 3 \rightarrow \frac{2!}{s^3} + \frac{6}{s^2} - \frac{3}{s} \rightarrow \frac{2}{s^3} + \frac{6}{s^2} - \frac{3}{s}$$

$$f(t) = (2t-1)^3 \rightarrow (2t-1)^2(2t-1) \rightarrow (4t^2 - 4t + 1)(2t-1)$$

$$8t^3 - 8t^2 + 2t - 4t^2 + 4t - 1 \rightarrow 8t^3 - 12t^2 + 6t - 1$$

$$8\left(\frac{3!}{s^4}\right) - 12\left(\frac{2!}{s^3}\right) + 6\left(\frac{1}{s^2}\right) - \frac{1}{s} \rightarrow f(s) = \frac{48}{s^4} - \frac{24}{s^3} + \frac{6}{s^2} - \frac{1}{s}$$

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$$f(t) = (1 + e^{2t})^2 \rightarrow 1 + 2e^{2t} + e^{4t}$$

$$\frac{1}{s} + 2\left(\frac{1}{s-2}\right) + e\left(\frac{1}{s-4}\right) \rightarrow f(s) = \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$$

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$$f(t) = \cos 5t + \sin 2t$$

$$\frac{s}{s^2+5^2} + \frac{2}{s^2+4^2} \rightarrow f(s) = \frac{s}{s^2+25} + \frac{2}{s^2+16}$$

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$$f(t) = \sin 2t \cos 2t \rightarrow \frac{1}{2} \sin 2(2t) \rightarrow \frac{1}{2} \sin 4t$$

$$\frac{1}{2} \left(\frac{4}{s^2+4^2} \right)$$

$$f(s) = \frac{2}{s^2+16}$$

Section 7.2

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2} - \frac{40}{s^5} \right\} \rightarrow \mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} - \mathcal{L}^{-1} \left\{ \frac{40}{s^5} \right\}$$

$$f(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} - 2 \mathcal{L}^{-1} \left\{ \frac{40}{s^5} \right\}$$

$$f(t) = t - 2t^4$$

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$$\mathcal{L}^{-1} \left\{ \frac{(s+2)^2}{s^3} \right\} = \mathcal{L}^{-1} \left\{ \frac{(s+2)^2}{s^3} \right\} \cdot \mathcal{L}^{-1} \left(\frac{1}{s^3} \right) \rightarrow \frac{s+4s+4}{s^3}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} + \frac{4}{s^2} + \frac{4}{s^3} \right\} \rightarrow f(t) = 1 + 4t + 2t^2$$

#17

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2} \right\} \Rightarrow f(t) = t - 1 + \frac{1}{s-2}$$

$$f(t) = t - 1 + e^{2t}$$

#18

$$\mathcal{L}^{-1} \left\{ \frac{4s}{4s^2 + 1} \right\} \rightarrow \left\{ \frac{4s}{(2s)^2 + 1^2} \right\} \rightarrow \left\{ \frac{s}{s^2 + \frac{1}{4}} \right\}$$

$$f(t) = \cos\left(\frac{1}{2}\right)t$$

#19

$$\mathcal{L}^{-1} \left\{ \frac{s}{s^2 + 2s - 3} \right\} \rightarrow \frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$$

$$s = A(s-1) + B(s+3) \rightarrow s = As - A + Bs + 3B$$

$$1 = A + B \rightarrow B = 1 - A$$

$$\frac{s}{(s+3)(s-1)} = \frac{3/4}{s+3} + \frac{1/4}{s-1}$$

$$0 = -A + 3B$$

$$0 = -A + 3(1-A)$$

$$0 = -A + 3 - 3A$$

$$A = 3/4$$

$$B = 1/4$$

$$\frac{s}{(s+3)(s-1)} = \frac{3}{4} \left(\frac{1}{s+3} \right) + \frac{1}{4} \left(\frac{1}{s-1} \right)$$

$$f(t) = \frac{3}{4} e^{-3t} + \frac{1}{4} e^t$$

#20

$$\mathcal{L}^{-1} \left\{ \frac{s}{(s-3)(s-3)(s-6)} \right\} \rightarrow \frac{A}{s-3} + \frac{B}{s-3} + \frac{C}{s-6}$$

$$s = A(s-3)(s-6) + B(s-2)(s-6) + C(s-2)(s-3)$$

$$6 = -4(4)(3) \rightarrow C = \frac{1}{2}$$

$$2 = A(-1)(-4) \rightarrow A = \frac{1}{2}$$

$$B = -1$$

$$\frac{s}{(s-2)(s-3)(s-6)} = \frac{A/2}{(s-2)} - \frac{1}{s-3} + \frac{A/2}{(s-6)}$$

$$= \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t} \Rightarrow f(s) = \frac{e^{2t}}{2} - e^{3t} + \frac{e^{6t}}{2}$$

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$$\mathcal{L}^{-1} \left\{ \frac{2s-4}{(s^2+s)(s^2+1)} \right\} \Rightarrow \frac{2s-4}{s(s+1)(s^2+1)} \Rightarrow \frac{A}{s} + \frac{B}{s+1} + \frac{C \cos t + D \sin t}{s^2+1}$$

$$2s-4 = A(s^2+1)(s+1) + B s(s^2+1) + (s \cos t + D \sin t)(s+1)$$

$$2s-4 = A(s^3+s^2+s+1) + B s^3 + B s + (s^2 + C s^2 + D s^2 + D s)$$

$$2 = A + B + D$$

$$-4 = A + C$$

$$0 = A + B + C$$

$$0 = A + C + D$$

$$0 = -4 + C + 3$$

$$C = 1$$

$$2 + 4 - D = 0$$

$$0 = -4 + 2 + 4 - D + C$$

$$D = 3$$

$$4 - 6 + D = C$$

$$0 = -4 + 4 - 6 + D + D$$

$$D = 3$$

$$= -4 \left(\frac{1}{s} \right) + 3 \left(\frac{1}{s+1} \right) + \frac{s}{s^2+1} + \frac{3}{s^2+1}$$

$$f(s) = -4 + 3e^{-t} + \cos t + 3 \sin t$$

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$$y' + 6y = e^{4t}, y(0) = 2$$

$$\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = \mathcal{L}\{e^{4t}\}$$

$$sY(s) - y(0) + 6Y(s) = \frac{1}{s-4}$$

$$Y(s)(s+6) - 2 = \frac{1}{s-4} \Rightarrow Y(s) = \frac{\frac{1}{s-4} + 2}{s+6}$$

$$Y(s) = \frac{2s-8+1}{s+6} \rightarrow Y(s) = \frac{2s-7}{s+6}$$

$$2s-7 = A(s-4) + B(s+6)$$

$$2(4)-7 = B(10)$$

$$\frac{1}{10} = B$$

$$2(-6)-7 = A(-6-4)$$

$$-19 = A(-10) \rightarrow A = \frac{19}{10}$$

$$= \frac{19}{10} + \frac{1}{10}$$

$$\rightarrow \boxed{f(s) = \frac{19}{10} e^{-6t} + \frac{1}{10} e^{4t}}$$

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#19

$$y'' + 5y' + 4y = 0 \quad y(0) = 1 \quad y'(0) = 0$$

$$\mathcal{L}\{y''\} + 5\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = 0$$

$$s^2(Y(s)) - sy(0) - y'(0) + 5sY(s) - 5f(0) + 4Ys = 0$$

$$Ys[s^2 + 5s + 4] - s + 5 = 0 \rightarrow Ys = \frac{s+5}{s^2 + 5s + 4}$$

$$Ys = \frac{s+5}{(s+4)(s+1)} \rightarrow \frac{A}{s+4} + \frac{B}{s+1} \rightarrow s+5 = A(s+1) + B(s+4)$$

$$-1+5 = B(3) \rightarrow B = \frac{4}{3}$$

$$-4+5 = A(-3) \rightarrow A = -\frac{1}{3}$$

$$= -\frac{1}{3} \left(\frac{1}{s+4} \right) + \frac{4}{3} \left(\frac{1}{s+1} \right)$$

$$\boxed{y(t) = -\frac{1}{3} e^{-4t} + \frac{4}{3} e^{-t}}$$

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$$2y''' + 3y'' - 3y' - 2y = e^{-t} \quad y(0)=0 \quad y'(0)=0 \quad y''(0)=1$$

$$2s^3 Y(s) - 2s^2 y(0) - 2s y'(0) - 2y''(0) + 3s^2 Y(s) - 3s y(0) - 3y'(0) - 3s Y(s) + 3y(0) - 2Y(s) = \frac{1}{s+1}$$

$$-3s Y(s) + 3y(0) - 2Y(s) = \frac{1}{s+1}$$

$$2s^3 Y(s) - 2 + 3s^2 Y(s) - 3s Y(s) - 2Y(s) = \frac{1}{s+1}$$

$$Y(s) (2s^3 + 3s^2 - 3s - 2) = \frac{1}{s+1} + 2$$

$$Y(s) = \frac{\frac{1}{s+1} + 2}{2s^3 + 3s^2 - 3s - 2} \rightarrow \frac{\frac{1}{s+1} + 2}{(s+1)(2s-1)(s+2)}$$

$$\downarrow$$

$$\frac{2s+3}{(s+1)^2(2s-1)(s+2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{2s-1} + \frac{D}{s+2}$$

$$2s+3 = A(s+1)^2(2s-1)(s+2) + B(s+1)(2s-1)(s+2) + C(s+1)^2(2s-1) + D(s+1)^2(s+2)$$

$$2(-1)+3 = B(-1)(-1) \rightarrow B = -\frac{1}{7}$$

$$2(-2)+3 = D(-2+1)^2/2(-2-1)$$

$$D = \frac{1}{9}$$

$$A = -\frac{6}{9}$$

$$2\left(\frac{1}{2}\right)+3 = \left(\left(\frac{1}{2}+1\right)^2/\frac{1}{2}+2\right)$$

$$4 = C\left(\frac{9}{4}\right)/\left(\frac{1}{2}\right) \rightarrow C = \frac{8}{9}$$

$$\frac{2s+3}{(s+1)^2(s-1)(s+2)} = \frac{1/2}{s+1} - \frac{8/9}{(s+1)^2} + \frac{5/18}{s-1} + \frac{1/9}{s+2}$$

$$y(t) = -\frac{8}{9} e^{-\frac{t}{2}} + \frac{1}{9} e^{-2t} + \frac{1}{2} e^{-t} + \frac{5}{18} e^t$$