GSE15E030 Problem Set 1 (due by 11:59 p.m. May 1st 2018)

Question 1. Stationarity of AR(2) process. *(1 point)*
Consider the autoregressive process of order two:

$$y_t = 1 + 0.8y_{t-1} - 1.2y_{t-2} + \varepsilon_t$$

where ε_t is an *i.i.d.* $N(0, 0.5)$ process.

a. Rewrite the model in companion form

b. Find the eigenvalues of the companion matrix. Is the process stationary?

c. Compute the roots of the lag polynomial. Is the process stationary?

Question 2. Sum of covariance-stationary processes *(1 point)*
Consider the following two moving average models

$$\begin{aligned} y_{1,t} &= 1 + \varepsilon_t + 0.2\varepsilon_{t-1} + 0.4\varepsilon_{t-2} \\ y_{2,t} &= 0.5 + \xi_t + 0.8\xi_{t-1} \end{aligned}$$

where ε_t is an *i.i.d.* $N(0, 0.1)$.and ξ_t is an *i.i.d.* $N(0, 1)$ with $\varepsilon_t \perp \xi_t$. Define x_t the time series process obtained by summing the two moving average models $y_{1,t}$ and $y_{2,t}$:

$$x_t = y_{1,t} + y_{2,t}$$

a. Is x_t a covariance stationary process?

b. Calculate the autocovariance function for x_t, that is, calculate γ_0, γ_1, γ_2 and in general γ_j for $j > 2$.

Question 3. Matlab exercise: Simulation of Covariance-Stationary processes *(3 points)*

a. Generate $T = 100$ observations from the following time series processes:

   i) White Noise

$$y_t = \varepsilon_t$$

   ii) Moving Average

$$y_t = 1.2 + \varepsilon_t + 0.6\varepsilon_{t-1}$$

iii) Autoregressive Process AR(1)

$$y_t = 0.2 + 0.9y_{t-1} + \varepsilon_t$$

iv) Autoregressive Process AR(2)

$$y_t = 1 + 0.6y_{t-1} + 0.2y_{t-2} + \varepsilon_t$$

where ε_t is *i.i.d.* $N(0, 0.5)$ for all processes (i)-(iv).

b. For the generated samples:

i) plot the series in a graph

ii) compute the sample mean, variance and autocovariances up to order $j = 10$

iii) compare the sample moments in ii) with the population mean, variance and autocovariances

iv) compute and plot both the sample and the population autocorrelation function up to horizon 10 (ρ_j with $j = 0, .., 10$) -for the *sample* autocorrelation you can use the matlab function '*autocorr*';

c. For the autoregressive model in (iii):

i) Estimate the regression model $y_t = c + \phi y_{t-1} + \varepsilon_t$ and report the value of the estimated coefficients ($\hat{c}$ and $\hat{\phi}$ and $\hat{\sigma}^2$);

ii) Report the 95% confidence interval for $\hat{\phi}$ (write your own function, do not use the matlab one)

d. For the autoregressive model in (iv):

i) Estimate the regression model $y_t = c + \phi y_{t-1} + \varepsilon_t$ and report the value of the estimated coefficients ($\hat{c}$ and $\hat{\phi}$ and $\hat{\sigma}^2$);

ii) Compute the residuals $\hat{\varepsilon}_t = y_t - \left(\hat{c} + \hat{\phi} y_{t-1}\right) \quad t = 2, ..T$

iii) Test whether the residuals $\hat{\varepsilon}_t$ are autocorrelated of order one -you can use the matlab function '*dwtest*';

iv) Estimate the regression model $y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$ and report the value of the estimated coefficients ($\hat{c}$ and $\hat{\phi}_1, \hat{\phi}_2$ and $\hat{\sigma}^2$);

ii) Compute the residuals $\hat{\varepsilon}_t = y_t - \left(\hat{c} + \hat{\phi}_1 y_{t-1} + \hat{\phi}_2 y_{t-2}\right) \quad t = 2, ..T$

iii) Test whether the residuals $\hat{\varepsilon}_t$ are autocorrelated of order one;