Software library for public use

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MultiNetX

multiNetX is a python package for the manipulation and study of multilayer networks. The core of this package is a MultilayerGraph, a class that inherits all properties from networkx.Graph().

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This allows for:

- Creating networks with weighted or unweighted links (only undirected networks are supported in this version)
- Analysing the spectral properties of adjacency or Laplacian matrices
- Visualizing dynamical processes by coloring the nodes and links accordingly

Contents

1 How to install multiNetX

multinetx does not need intallation. You simply download the source files and save them into your file system. Then you have to add that directory to your PYTHONPATH. In Unix/Linux you can do this by writting in the terminal the following command:

```
export PYTHONPATH=path_to_your_python_libraries/multinetx:$PYTHONPATH
```

2 How to use multiNetX

multiNetX is very easy to use. It is based on networkX package (https://networkx.github.io/) which is written in pure python and make use of the standard python packages numpy and scipy. Basic knowledge of python2.7 as well as of those packages is required in order to understand the following guide. A fundamental knowledge of network theory is also required.

Import standard python packages for numerics and plots

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Import the package multiNetX

```
import multinetx as mx
```

Create three Erd"os- R'enyi networks with N nodes for each layer

```
N = 5
g1 = mx.generators.erdos_renyi_graph(N,0.9,seed=218)
g2 = mx.generators.erdos_renyi_graph(N,0.9,seed=211)
g3 = mx.generators.erdos_renyi_graph(N,0.9,seed=208)
```

Create an 3Nx3N lil sparse matrix. It will be used to describe the layers interconnection

```
adj_block = mx.lil_matrix(np.zeros((N*3,N*3)))
```

Define the type of interconnection among the layers (here we use identity matrices thus connecting one-to-one the nodes among layers)

```
adj_block[0:  N,  N:2*N] = np.identity(N)    # L_12
adj_block[0:  N,2*N:3*N] = np.identity(N)    # L_13
adj_block[N:2*N,2*N:3*N] = np.identity(N)    # L_23

# use symmetric inter-adjacency matrix
adj_block += adj_block.T
```

Create an instance of the MultilayerGraph class

```
mg = mx.MultilayerGraph(list_of_layers=[g1,g2,g3],
                        inter_adjacency_matrix=adj_block)
```

Weights can be added to the edges

```
mg.set_edges_weights(intra_layer_edges_weight=2,
                     inter_layer_edges_weight=3)
```

3 Create a multiplex 2nd way

Create an empty multiplex network

```
mg = mx.MultilayerGraph()
```

Add layers

```
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.9,seed=218))
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.9,seed=211))
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.9,seed=208))
```

Create an instance of the MultilayerGraph class

```
mg.layers_interconnect(inter_adjacency_matrix=adj_block)
```

Weights can be added to the edges

```
mg.set_edges_weights(intra_layer_edges_weight=2,
                     inter_layer_edges_weight=3)
```

4 Take some information for the multiplex network

```
print 'MultiNetX name:\n', mg.name ,'\n', mg.info(),'\n'
```

```
MultiNetX name:
gnp_random_graph(5,0.9)
3-layer graph, intra_layer_edges:27, inter_layer_edges:15, number_of_nodes_in_layer:5
```

```
print 'MultilayerGraph edges:',\
        '\n\n intra-layer edges: ',mg.get_intra_layer_edges(),\
        '\n\n inter-layer edges: ',mg.get_inter_layer_edges(),'\n'
```

```
MultilayerGraph edges:

 intra-layer edges:  [(0, 1), (0, 2), (0, 4), (1, 2), (1, 4), (2, 3), (2, 4), (3, 4),
 (5, 6), (5, 7), (5, 8), (5, 9), (6, 7), (6, 8), (6, 9), (7, 8), (7, 9), (8, 9), (10, 11),
 (10, 12), (10, 13), (10, 14), (11, 12), (11, 13), (11, 14), (12, 13), (12, 14)]

 inter-layer edges:  [(5, 0), (6, 1), (7, 2), (8, 3), (9, 4), (10, 0), (10, 5),
 (11, 1), (11, 6), (12, 2), (12, 7), (13, 3), (13, 8), (14, 4), (14, 9)]
```

```
print 'intralayer edges of 1: ',mg.get_intra_layer_edges_of_layer(layer=0)
print 'intralayer edges of 2: ',mg.get_intra_layer_edges_of_layer(layer=1)
print 'intralayer edges of 3: ',mg.get_intra_layer_edges_of_layer(layer=2)
```

```
intralayer edges of 1:  [(0, 1), (0, 2), (0, 4), (1, 2), (1, 4), (2, 3), (2, 4), (3, 4)]
intralayer edges of 2:  [(5, 6), (5, 7), (5, 8), (5, 9), (6, 7), (6, 8), (6, 9), (7, 8),
(7, 9), (8, 9)]
intralayer edges of 3:  [(10, 11), (10, 12), (10, 13), (10, 14), (11, 12), (11, 13),
(11, 14), (12, 13), (12, 14)]
```

A layer can be chosen: it is a networkx.Graph so it inherits all of its properties.

```
layer = 1
mg1 = mg.get_layer(layer-1)
print 'layer', layer, ' name is', mg1.name
```

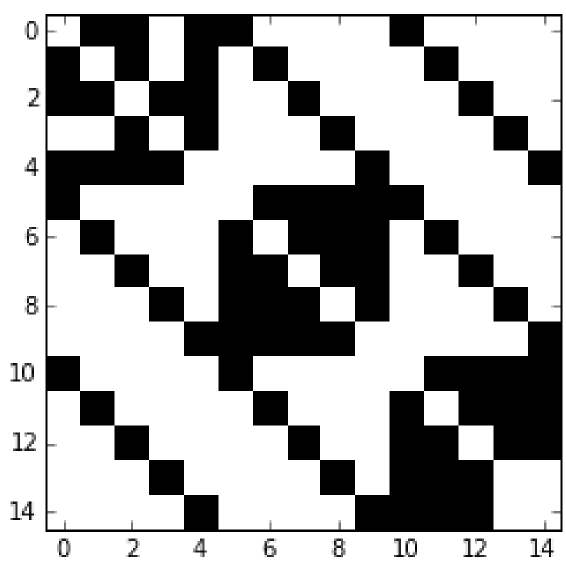
```
layer 1  name is gnp_random_graph(5,0.9)
```

```
print 'Adjacency matrix:\n', \
        mx.adjacency_matrix(mg,weight=None).todense(),'\n'
print 'Adjacency matrix (weighted):\n', \
        mx.adjacency_matrix(mg,weight="weight").todense(),'\n'
```

```
Adjacency matrix:
[[0 1 1 0 1 1 0 0 0 0 1 0 0 0 0]
 [1 0 1 0 1 0 1 0 0 0 0 1 0 0 0]
 [1 1 0 1 1 0 0 1 0 0 0 0 1 0 0]
 [0 0 1 0 1 0 0 0 1 0 0 0 0 1 0]
 [1 1 1 1 0 0 0 0 0 1 0 0 0 0 1]
 [1 0 0 0 0 0 1 1 1 1 1 0 0 0 0]
 [0 1 0 0 0 1 0 1 1 1 0 1 0 0 0]
 [0 0 1 0 0 1 1 0 1 1 0 0 1 0 0]
 [0 0 0 1 0 1 1 1 0 1 0 0 0 1 0]
 [0 0 0 0 1 1 1 1 1 0 0 0 0 0 1]
 [1 0 0 0 0 1 0 0 0 0 0 1 1 1 1]
 [0 1 0 0 0 0 1 0 0 0 1 0 1 1 1]
 [0 0 1 0 0 0 0 1 0 0 1 1 0 1 1]
 [0 0 0 1 0 0 0 0 1 0 1 1 1 0 0]
 [0 0 0 0 1 0 0 0 0 1 1 1 1 0 0]]

Adjacency matrix (weighted):
[[0 2 2 0 2 3 0 0 0 0 3 0 0 0 0]
 [2 0 2 0 2 0 3 0 0 0 0 3 0 0 0]
 [2 2 0 2 2 0 0 3 0 0 0 0 3 0 0]
 [0 0 2 0 2 0 0 0 3 0 0 0 0 3 0]
```

```
 [2 2 2 2 0 0 0 0 0 3 0 0 0 0 3]
 [3 0 0 0 0 0 2 2 2 2 3 0 0 0 0]
 [0 3 0 0 0 2 0 2 2 2 0 3 0 0 0]
 [0 0 3 0 0 2 2 0 2 2 0 0 3 0 0]
 [0 0 0 3 0 2 2 2 0 2 0 0 0 3 0]
 [0 0 0 0 3 2 2 2 2 0 0 0 0 0 3]
 [3 0 0 0 0 3 0 0 0 0 0 2 2 2 2]
 [0 3 0 0 0 0 3 0 0 0 2 0 2 2 2]
 [0 0 3 0 0 0 0 3 0 0 2 2 0 2 2]
 [0 0 0 3 0 0 0 0 3 0 2 2 2 0 0]
 [0 0 0 0 3 0 0 0 0 3 2 2 2 0 0]]
```

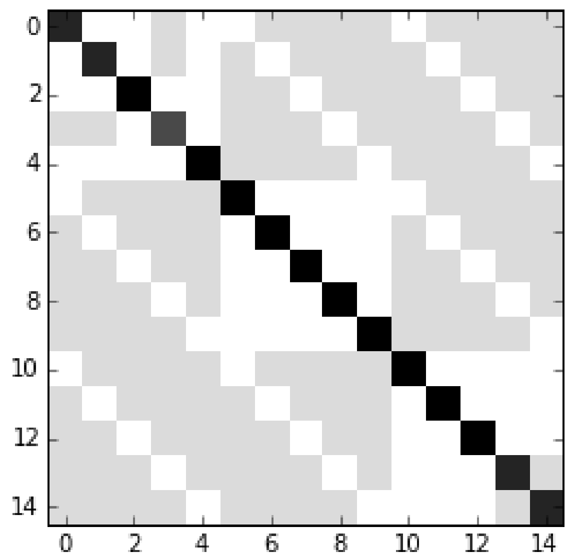
```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.imshow(mx.adjacency_matrix(mg,weight=None).todense(),
          origin='upper',interpolation='nearest',cmap=plt.cm.binary);
```




```
print 'Laplacian matrix:\n',\
        mx.laplacian_matrix(mg,weight=None).todense(),'\n'
print 'Laplacian matrix (weighted):\n', \
```

```
        mx.laplacian_matrix(mg,weight="weight").todense(),'\n'

Laplacian matrix:
[[ 5 -1 -1  0 -1 -1  0  0  0  0 -1  0  0  0  0]
 [-1  5 -1  0 -1  0 -1  0  0  0  0 -1  0  0  0]
 [-1 -1  6 -1 -1  0  0 -1  0  0  0  0 -1  0  0]
 [ 0  0 -1  4 -1  0  0  0 -1  0  0  0  0 -1  0]
 [-1 -1 -1 -1  6  0  0  0  0 -1  0  0  0  0 -1]
 [-1  0  0  0  0  6 -1 -1 -1 -1 -1  0  0  0  0]
 [ 0 -1  0  0  0 -1  6 -1 -1 -1  0 -1  0  0  0]
 [ 0  0 -1  0  0 -1 -1  6 -1 -1  0  0 -1  0  0]
 [ 0  0  0 -1  0 -1 -1 -1  6 -1  0  0  0 -1  0]
 [ 0  0  0  0 -1 -1 -1 -1 -1  6  0  0  0  0 -1]
 [-1  0  0  0  0 -1  0  0  0  0  6 -1 -1 -1 -1]
 [ 0 -1  0  0  0  0 -1  0  0  0 -1  6 -1 -1 -1]
 [ 0  0 -1  0  0  0  0 -1  0  0 -1 -1  6 -1 -1]
 [ 0  0  0 -1  0  0  0  0 -1  0 -1 -1 -1  5  0]
 [ 0  0  0  0 -1  0  0  0  0 -1 -1 -1 -1  0  5]]

Laplacian matrix (weighted):
[[12 -2 -2  0 -2 -3  0  0  0  0 -3  0  0  0  0]
 [-2 12 -2  0 -2  0 -3  0  0  0  0 -3  0  0  0]
 [-2 -2 14 -2 -2  0  0 -3  0  0  0  0 -3  0  0]
 [ 0  0 -2 10 -2  0  0  0 -3  0  0  0  0 -3  0]
 [-2 -2 -2 -2 14  0  0  0  0 -3  0  0  0  0 -3]
 [-3  0  0  0  0 14 -2 -2 -2 -2 -3  0  0  0  0]
 [ 0 -3  0  0  0 -2 14 -2 -2 -2  0 -3  0  0  0]
 [ 0  0 -3  0  0 -2 -2 14 -2 -2  0  0 -3  0  0]
 [ 0  0  0 -3  0 -2 -2 -2 14 -2  0  0  0 -3  0]
 [ 0  0  0  0 -3 -2 -2 -2 -2 14  0  0  0  0 -3]
 [-3  0  0  0  0 -3  0  0  0  0 14 -2 -2 -2 -2]
 [ 0 -3  0  0  0  0 -3  0  0  0 -2 14 -2 -2 -2]
 [ 0  0 -3  0  0  0  0 -3  0  0 -2 -2 14 -2 -2]
 [ 0  0  0 -3  0  0  0  0 -3  0 -2 -2 -2 12  0]
 [ 0  0  0  0 -3  0  0  0  0 -3 -2 -2 -2  0 12]]
```

```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.imshow(mx.laplacian_matrix(mg,weight=None).todense(),
          origin='upper',interpolation='nearest',cmap=plt.cm.binary);
```




```
print 'Laplacian spectrum:\n', \
        mx.laplacian_spectrum(mg,weight="weight"),'\n'
```

```
Laplacian spectrum:
[  7.29267473e-15   6.55428082e+00   8.90511420e+00   9.00000000e+00
   9.00000000e+00   9.22799813e+00   1.00000000e+01   1.51991214e+01
   1.73414836e+01   1.77720019e+01   1.90000000e+01   1.90000000e+01
   1.90000000e+01   1.90000000e+01   1.90000000e+01]
```

5 Plot Multiplex

5.0.1 Edge colored nertwork (no inter-connected layers)

Create a multiplex network with three random layers

```
mg = mx.MultilayerGraph()


N = 50
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.07,seed=218))
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.07,seed=211))
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.07,seed=208))
```

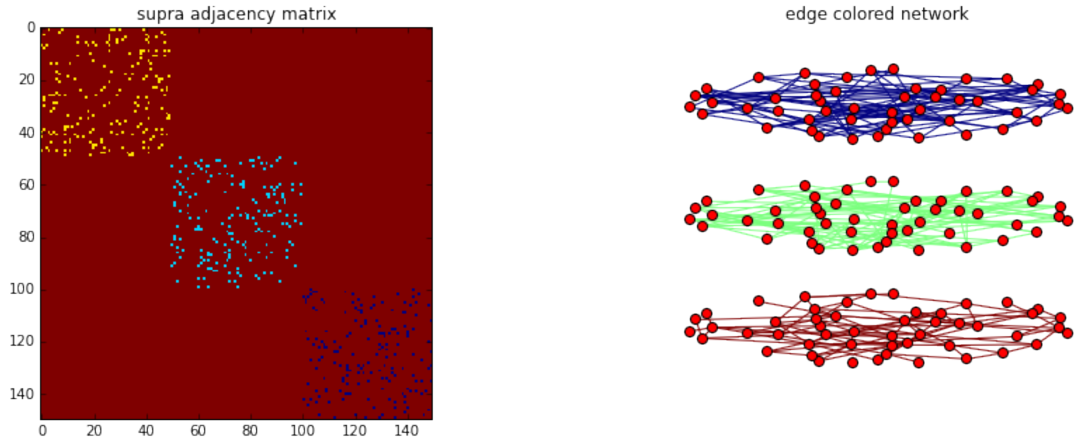
Set weights to the edges

```
mg.set_intra_edges_weights(layer=0,weight=1)
mg.set_intra_edges_weights(layer=1,weight=2)
mg.set_intra_edges_weights(layer=2,weight=3)


fig = plt.figure(figsize=(15,5))
ax1 = fig.add_subplot(121)
ax1.imshow(mx.adjacency_matrix(mg,weight='weight').todense(),
          origin='upper',interpolation='nearest',cmap=plt.cm.jet_r)
ax1.set_title('supra adjacency matrix')

ax2 = fig.add_subplot(122)
ax2.axis('off')
ax2.set_title('edge colored network')
pos = mx.get_position(mg,mx.fruchterman_reingold_layout(mg.get_layer(0)),
                      layer_vertical_shift=0.2,
                      layer_horizontal_shift=0.0,
                      proj_angle=47)
mx.draw_networkx(mg,pos=pos,ax=ax2,node_size=50,with_labels=False,
                 edge_color=[mg[a][b]['weight'] for a,b in mg.edges()],
                 edge_cmap=plt.cm.jet_r)
plt.show()
```

5.0.2 Regular interconnected multiplex

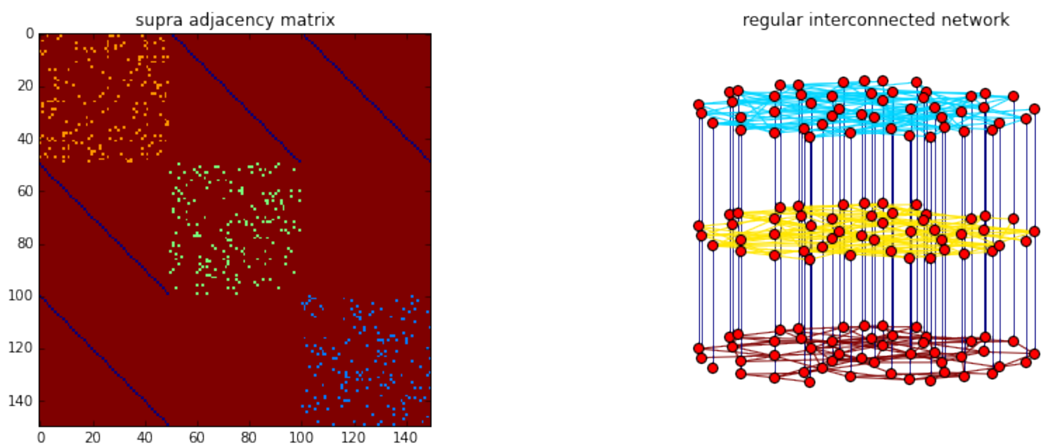
Define the type of interconnection between the layers

```
adj_block = mx.lil_matrix(np.zeros((N*3,N*3)))
```




```
adj_block[0:  N,  N:2*N] = np.identity(N)    # L_12
adj_block[0:  N,2*N:3*N] = np.identity(N)    # L_13
#adj_block[N:2*N,2*N:3*N] = np.identity(N)    # L_23
adj_block += adj_block.T


mg.layers_interconnect(inter_adjacency_matrix=adj_block)

mg.set_edges_weights(inter_layer_edges_weight=4)

mg.set_intra_edges_weights(layer=0,weight=1)
mg.set_intra_edges_weights(layer=1,weight=2)
mg.set_intra_edges_weights(layer=2,weight=3)


fig = plt.figure(figsize=(15,5))
ax1 = fig.add_subplot(121)
ax1.imshow(mx.adjacency_matrix(mg,weight='weight').todense(),
          origin='upper',interpolation='nearest',cmap=plt.cm.jet_r)
ax1.set_title('supra adjacency matrix')

ax2 = fig.add_subplot(122)
ax2.axis('off')
ax2.set_title('regular interconnected network')
```

```
pos = mx.get_position(mg,mx.fruchterman_reingold_layout(mg.get_layer(0)),
                      layer_vertical_shift=1.4,
                      layer_horizontal_shift=0.0,
                      proj_angle=7)
mx.draw_networkx(mg,pos=pos,ax=ax2,node_size=50,with_labels=False,
                 edge_color=[mg[a][b]['weight'] for a,b in mg.edges()],
                 edge_cmap=plt.cm.jet_r)
plt.show()
```



5.0.3 General multiplex

```
adj_block = mx.lil_matrix(np.zeros((N*4,N*4)))

adj_block[0   :  N ,   N:2*N] = np.identity(N)   # L_12
adj_block[0   :  N , 2*N:3*N] = np.random.poisson(0.005,size=(N,N))   # L_13
adj_block[0   :  N , 3*N:4*N] = np.random.poisson(0.006,size=(N,N))   # L_34
adj_block[3*N:4*N , 2*N:3*N] = np.random.poisson(0.008,size=(N,N))   # L_14
adj_block += adj_block.T
adj_block[adj_block>1] = 1
```

Add one more layer

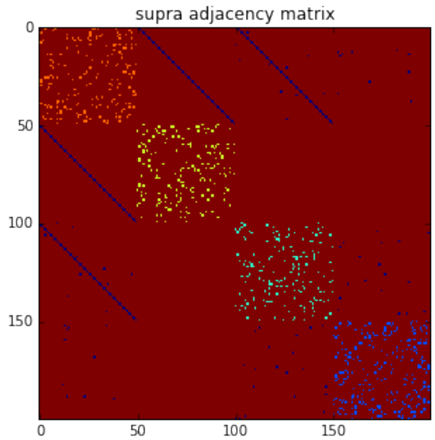
```
mg.add_layer(mx.generators.erdos_renyi_graph(N,0.1,seed=218))



mg.layers_interconnect(inter_adjacency_matrix=adj_block)
```

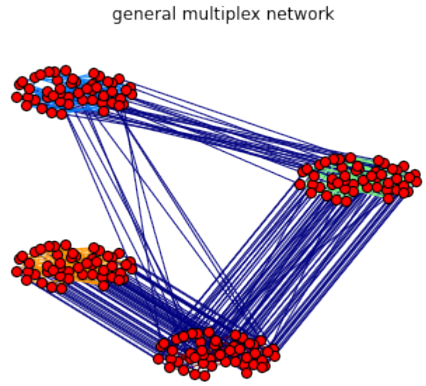
```
mg.set_edges_weights(inter_layer_edges_weight=5)

mg.set_intra_edges_weights(layer=0,weight=1)
mg.set_intra_edges_weights(layer=1,weight=2)
mg.set_intra_edges_weights(layer=2,weight=3)
mg.set_intra_edges_weights(layer=3,weight=4)


fig = plt.figure(figsize=(15,5))
ax1 = fig.add_subplot(121)
ax1.imshow(mx.adjacency_matrix(mg,weight='weight').todense(),
          origin='upper',interpolation='nearest',cmap=plt.cm.jet_r)
ax1.set_title('supra adjacency matrix')

ax2 = fig.add_subplot(122)
ax2.axis('off')
ax2.set_title('general multiplex network')
pos = mx.get_position(mg,mx.fruchterman_reingold_layout(mg.get_layer(0)),
                      layer_vertical_shift=.4,
                      layer_horizontal_shift=1.2,
                      proj_angle=.2)
mx.draw_networkx(mg,pos=pos,ax=ax2,node_size=50,with_labels=False,
                 edge_color=[mg[a][b]['weight'] for a,b in mg.edges()],
                 edge_cmap=plt.cm.jet_r)
plt.show()
```

6 Dynamical processes on top of multiplex networks

Import libraries

```
import numpy as np
import matplotlib.pylab as plt

import multinetx as mx

from scipy.integrate import ode
```





The Mimura-Murray ecological model

```
def mimura_murray_duplex(t, y0, G):
    f = y0.copy()
    u = y0[:G.N] # activator
    v = y0[G.N:] # inhibitor
    sum_lapl_u = G.diff[0] * u * G.lapl[0:G.N,0:G.N]
    sum_lapl_v = G.diff[1] * v * G.lapl[G.N:2*G.N,G.N:2*G.N]
    # activator
    f[:G.N] = ( (G.a + G.b * u - u * u) / G.c - v) * u + sum_lapl_u
    # inhibitor
    f[G.N:] = (u - 1.0 - G.d * v) * v + sum_lapl_v
    return f
```

Define the integrate method

```
def integrate(G, dt=0.5, tmax=200, method='dopri5', store_solution=True):
    '''This function integrates the MM model'''
    ## Setup the integrator
    t0 = 0.0
    sol = [G.species]
    solver = ode(G.rhs).set_f_params(G)
    int_pars = dict(atol=1E-8, rtol=1E-8, first_step=1E-2,
                    max_step=2E1, nsteps=2E4)
    solver.set_integrator(method,**int_pars)
    solver.set_initial_value(G.species,t0)
    ## Integrate the system
```

```
    while solver.successful() and solver.t < tmax:
        solver.integrate(solver.t+dt)
        sol.append(solver.y)
    return np.array(sol)
```

Create the activator-inhibitor multiplex

```
G = mx.MultilayerGraph()



N = 350
G.add_layer(mx.barabasi_albert_graph(n=N, m=5,   seed=812))   # activators
G.add_layer(mx.barabasi_albert_graph(n=N, m=200, seed=812))   # inhibitors
```

Laplacian matrices of the multiplex

```
G.lapl = (-1.0) * mx.laplacian_matrix(G,weight=None)
```

Right-hand-side of the Mimura-Murray model

```
G.rhs = mimura_murray_duplex
```

Define the parameters of the model (They correspond to the uniform steady state)

```
G.a = 35.0
G.b = 16.0
G.c = 9.0
G.d = 0.4



G.N = G.get_number_of_nodes_in_layer()
G.diff = [0.12, 0.12]
```

Initial conditions and perturbation

```
activators = np.empty(G.N,dtype=np.double)
inhibitors = np.empty(G.N,dtype=np.double)
activators[:] = 5.0
```

```
inhibitors[:] = 10.0
activators[10] += 10E-5
G.species = np.concatenate((activators,inhibitors),axis=0)
```

Integrate the system

```
sol = integrate(G,dt=0.5,tmax=200,method='dopri5')
```

Sort the solution according to decreasing degree of the activator layer

```
deg_act = G.get_layer(0).degree().values()
sdeg_act = np.argsort(deg_act)[::-1]
sdeg_sol = np.append(sdeg_act,G.N+sdeg_act)
ssol = sol[:,sdeg_sol]
```

```
def plot_sol(ax, insol, NN, t=0):
    ax.plot(insol[t],':',color='green',lw=1.1,alpha=1)
    sc = ax.scatter(np.arange(NN),insol[t],
                    c=insol[t],s=25,marker='o',lw=1.2,
                    vmin=min(insol[t]),vmax=max(insol[t]),
                    cmap=plt.cm.YlOrBr)
    ax.set_xlim(-5,NN)
```

```
%matplotlib inline
```

Development of Turing pattern (activator layer is shown)

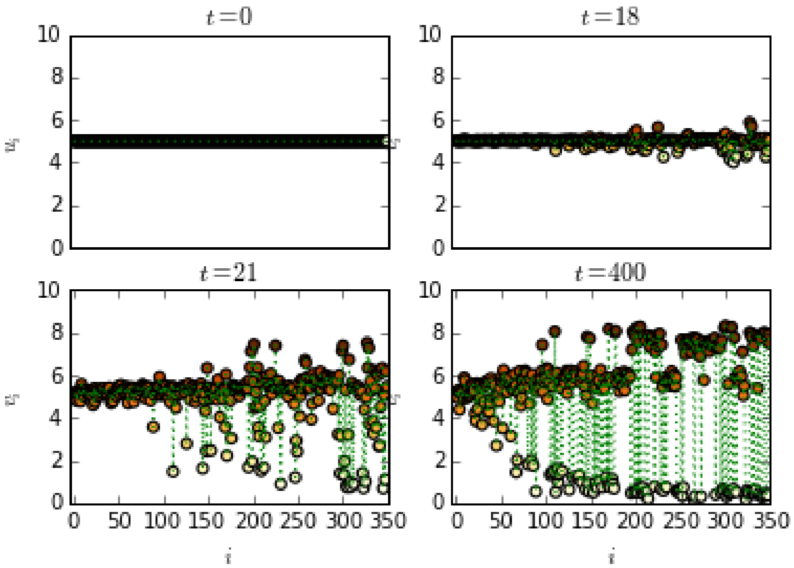
```
sol_act = ssol[:,:G.N]
```

```
fig = plt.figure()
ax1 = fig.add_subplot(221)
ax1.set_ylabel(r'$u_i$')
ax1.set_title(r'$t=0$')
ax1.set_ylim(0,10)
ax1.set_xticks([])
plot_sol(ax1,sol_act,G.N,t=0)
```

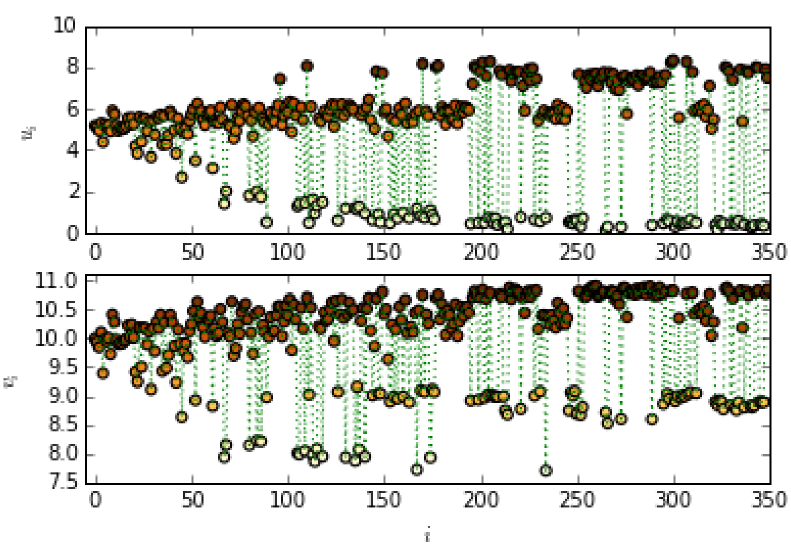
```
ax2 = fig.add_subplot(222)
ax2.set_ylabel(r'$v_i$')
ax2.set_title(r'$t=18$')
ax2.set_ylim(0,10)
ax2.set_xticks([])
plot_sol(ax2,sol_act,G.N,t=18)

ax3 = fig.add_subplot(223)
ax3.set_xlabel(r'$i$')
ax3.set_ylabel(r'$v_i$')
ax3.set_title(r'$t=21$')
ax3.set_ylim(0,10)
plot_sol(ax3,sol_act,G.N,t=21)

ax4 = fig.add_subplot(224)
ax4.set_xlabel(r'$i$')
ax4.set_ylabel(r'$v_i$')
ax4.set_title(r'$t=400$')
ax4.set_ylim(0,10)
plot_sol(ax4,sol_act,G.N,t=400)

plt.show()
```

Turing pattern shown in activator and inhibitor layer

```
sol_act = ssol[:,:G.N]
sol_inh = ssol[:,G.N:]
tsnap = 400

fig = plt.figure()
ax1 = fig.add_subplot(211)
ax1.set_ylabel(r'$u_i$')
ax1.set_ylim(0,10)
plot_sol(ax1,sol_act,G.N,t=tsnap)
```




```
ax2 = fig.add_subplot(212)
ax2.set_xlabel(r'$i$')
ax2.set_ylabel(r'$v_i$')
ax2.set_ylim(7.5,11.1)
plot_sol(ax2,sol_inh,G.N,t=tsnap)

plt.show()
```



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