Log Linearized Phillips Curve for Simple New Keynesian Model with No Capital

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Objective

- Obtain the log-linearized Phillips curve for New Keynesian model.
- Follows up on equilibrium conditions derived in handout, "Simple New Keynesian Model without Capital"
  - Work with the equilibrium conditions in which $G_t = 0$, so that $C_t = Y_t$.

Equilibrium Conditions

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \text{ (1)}, \; F_t = 1 + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \text{ (2)}$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)},$$

$$p_t^* = \left[(1-\theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right]^{-1} \quad \text{(4)}$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \text{ (5)}, \; C_t = p_t^* \exp(a_t) N_t \text{ (6)}$$

Steady State

Conditional on $\bar{\pi}$

$$K = \frac{\frac{\varepsilon}{\varepsilon - 1} s}{1 - \beta\theta\bar{\pi}^{\varepsilon}} \text{ (1)}, \; F = \frac{1}{1 - \beta\theta\bar{\pi}^{\varepsilon - 1}} \text{ (2)}$$

$$\frac{K}{F} = \left[\frac{1 - \theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad \text{(3)},$$

$$p^* = \left[(1 - \theta) \left(\frac{1 - \theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}^{\varepsilon}}{p^*} \right]^{-1} \quad \text{(4)}$$

$$1 = \beta \exp\left(-\Delta a\right) \frac{R}{\bar{\pi}} \text{ (5)}$$

Log Linearization

- Hat notation:

$$\hat{x}_t = \frac{dx_t}{x} = \frac{x_t - x}{x} \to dx_t = \hat{x}_t x.$$

- Log linearize equation (1) about steady state:

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1}$$

$$\hat{K}_t K = \frac{\varepsilon}{\varepsilon - 1} \hat{s}_t s + \beta\theta\varepsilon\bar{\pi}^{\varepsilon-1} \widehat{\bar{\pi}}_{t+1} \bar{\pi} K + \beta\theta\bar{\pi}^{\varepsilon} \hat{K}_{t+1} K$$

$$\hat{K}_t = \frac{\varepsilon}{\varepsilon - 1} \hat{s}_t \overbrace{\frac{s}{K}}^{=\frac{1-\beta\theta\bar{\pi}^{\varepsilon}}{\frac{\varepsilon}{\varepsilon-1}}} + \beta\theta\bar{\pi}^{\varepsilon} \left(\varepsilon\widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right)$$

$$\hat{K}_t = \left(1 - \beta\theta\bar{\pi}^{\varepsilon}\right) \hat{s}_t + \beta\theta\bar{\pi}^{\varepsilon} \left(\varepsilon\widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right)$$

Phillips Curve

- Linearizing (1), (2) and (3), about steady state,

$$\hat{K}_t = (1 - \beta\theta\bar{\pi}^{\varepsilon})\,\hat{s}_t + \beta\theta\bar{\pi}^{\varepsilon} E_t\left(\varepsilon\widehat{\bar{\pi}}_{t+1} + \hat{K}_{t+1}\right) \text{ (a)}$$

$$\hat{F}_t = \beta\theta\bar{\pi}^{\varepsilon-1} E_t\left((\varepsilon - 1)\,\widehat{\bar{\pi}}_{t+1} + \hat{F}_{t+1}\right) \text{ (b)}$$

$$\hat{K}_t = \hat{F}_t + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_t. \text{ (c)}$$

- Substitute out for $\hat{K}_t$ in (a) using (c) and then substitute out for $\hat{F}_t$ from (b) to obtain the equation on the next slide.

Phillips Curve

- Performing the substitutions described on the previous slide:

$$
\begin{aligned}
&\beta\theta\bar{\pi}^{\varepsilon-1}E_t\left((\varepsilon-1)\,\widehat{\bar{\pi}}_{t+1}+\hat{F}_{t+1}\right)\\
&\qquad+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_t=(1-\beta\theta\bar{\pi}^{\varepsilon})\,\hat{s}_t\\
&+\beta\theta\bar{\pi}^{\varepsilon}E_t\left(\varepsilon\widehat{\bar{\pi}}_{t+1}+\hat{F}_{t+1}+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\widehat{\bar{\pi}}_{t+1}\right).
\end{aligned}
$$

Phillips Curve

- Collecting terms,

$$\overbrace{\hat{\bar{\pi}}_t = \frac{\left(1-\theta\bar{\pi}^{(\varepsilon-1)}\right)\left(1-\beta\theta\bar{\pi}^{\varepsilon}\right)}{\theta\bar{\pi}^{(\varepsilon-1)}}\hat{s}_t + \beta E_t\hat{\bar{\pi}}_{t+1}}^{\text{familiar Phillips curve}}$$

$$+\left(1-\bar{\pi}\right)\left(1-\theta\bar{\pi}^{(\varepsilon-1)}\right)\beta$$

$$\times E_t\left(\hat{F}_{t+1}+\left(\varepsilon+\frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)\hat{\bar{\pi}}_{t+1}\right).$$

- Don't actually get standard Phillips curve unless $\bar{\pi}=1$.
  - More generally, get standard Phillips curve as long as there are no price distortions in steady state.
- Going for the Phillips curve in terms of the output gap.

Linearized Marginal Cost

- Real Marginal Cost:

$$
\begin{aligned}
s_t &= \frac{(1-\nu)\exp(\tau_t)\,C_t N_t^{\varphi}}{A_t} \\
&\overbrace{=}^{N_t = C_t/(p_t^* A_t)} (1-\nu)\,\frac{\exp(\tau_t)}{(p_t^*)^{\varphi}}\left(\frac{C_t}{A_t}\right)^{1+\varphi} \\
&= (1-\nu)\,\frac{1}{(p_t^*)^{\varphi}}\left(\frac{C_t}{A_t \exp\left[-\frac{\tau_t}{1+\varphi}\right]}\right)^{1+\varphi} \\
&= (1-\nu)\,\frac{1}{(p_t^*)^{\varphi}} X_t^{1+\varphi},
\end{aligned}
$$

where

$$
X_t = \frac{\text{Actual consumption}}{\text{Natural consumption}} = \text{"output gap"}
$$

Linearized Marginal Cost, cnt'd

- Real Marginal Cost:

$$s_t = (1 - \nu) \frac{1}{(p_t^*)^{\varphi}} X_t^{1+\varphi},$$

- Let

$$x_t = dlog X_t = \frac{dX_t}{X} = \hat{X}_t$$

- Then,

$$\hat{s}_t = \frac{(1-\nu)(1+\varphi)\frac{1}{(p^*)^{\varphi}} X^{1+\varphi}}{s} x_t - \frac{\varphi(1-\nu)\frac{1}{(p^*)^{\varphi}} X^{1+\varphi}}{s} \hat{p}_t^*$$

$$= (1+\varphi) x_t - \varphi \hat{p}_t^*$$

Phillips Curve in Terms of Output Gap

- Collecting terms,

$$\widehat{\bar{\pi}}_t = \frac{\left(1 - \theta\bar{\pi}^{(\varepsilon-1)}\right)\left(1 - \beta\theta\bar{\pi}^{\varepsilon}\right)}{\theta\bar{\pi}^{(\varepsilon-1)}}\left[\left(1+\varphi\right)x_t - \varphi\hat{p}_t^*\right] + \beta E_t\widehat{\bar{\pi}}_{t+1}$$

$$+\left(1-\bar{\pi}\right)\left(1 - \theta\bar{\pi}^{(\varepsilon-1)}\right)\beta$$

$$\times E_t\left(\hat{F}_{t+1} + \left(\varepsilon + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\bar{\pi}^{(\varepsilon-1)}}\right)\widehat{\bar{\pi}}_{t+1}\right).$$

- Only looks like the familiar Phillips curve when $\bar{\pi} = 1$.