EXAM QUESTIONS for Advanced Economics Extension Course at the Central Reserve Bank of Peru:
"*A Bayesian Approach to Identification of Structural VAR Models*"

Students should prepare answers to the exam questions in teams of **6 students**. Please include the Matlab code together with the written solutions. Students have until **Saturday, February 15 (midnight)** to complete the exam.

PART I: Univariate Model

Consider the following **AR(2) model**:

$$y_t = c + b_1 y_{t-1} + b_2 y_{t-2} + \epsilon_t$$

where the **residuals are serially correlated** according to:

$$\epsilon_t = \rho\epsilon_{t-1} + v_t \quad \text{with} \quad v_t \sim N(0, \tau^2)$$

a. Write down the **Gibbs sampler** for this extended model (on paper). Describe each step, the prior specification, and the conditional posterior distributions including the formulas for their moments.
b. Write Matlab code to estimate this model for the **annual growth rate of real GDP** for the US (FRED code: GDPC1) over the period 1948Q1 to 2019Q4 using the Gibbs sampler described in (a). Provide the following graphs:
    i. Plot the annual growth rate of real GDP (in percent).
    ii. Plot the histogram of the (marginal) posterior distribution for ρ and report its posterior median.

NOTE: Chapter 7 of Kim and Nelson (1999) is a <u>very</u> useful reference for this task.

PART II: Multivariate Models

Collect from FRED the following variables for the US economy at quarterly frequency for the period 1985Q1-2006Q4: real GDP, the consumer price index (CPI), the effective federal funds rate, and the M2 money stock.

1. Transform the data to **quarter-on-quarter growth rates** where appropriate so that they have a useful economic interpretation. Plot the transformed data with appropriate labels.

2. Consider a bivariate VAR model for real GDP growth and CPI inflation.

a. Write Matlab code to estimate a reduced-form VAR(4) model with a constant term using OLS. Report the (point) estimates of the reduced-form covariance matrix (omega) and the $(k \times n)$ matrix of reduced-form coefficients where n is the number of endogenous variables.
b. Assume that there is a supply-demand model that determines the fluctuations in output and inflation but that you do not know the values of the contemporaneous structural parameters that characterize that model. Plot the **identified set** for all possible values that are compatible with the observed data.

3. Now add the federal funds rate to your bivariate model.
    a. Fit a VAR(4) to those data ordered as follows: output, inflation, interest rate. Check whether the model is stable. How can you tell? Show the output you use to determine stability of the system.
    b. Apply the Choleski decomposition for identification. Plot the impulse responses of the three variables after a monetary policy shock (just the point estimates, without error bands). What do you find? Briefly comment on your results.
    c. Compute and report the coefficients on output and inflation in the Taylor rule implied by the estimated model.

4. Now add money to your set of variables. Write down the **structural** equations that describe these 4 variables. Provide an economic interpretation for each equation and the corresponding contemporaneous structural parameters (**A** matrix).

5. Estimate this model using the Baumeister-Hamilton algorithm implementing the Choleski identification in that framework and uninformative priors for **B** and **D**.
    a. Write down (on paper) the prior for each element in **A** as well as the joint prior for **A**.
    b. Plot the impulse responses to the one-standard-deviation structural shocks (median together with 16^{th} and 84^{th} percentiles of the posterior distribution) for a horizon of 5 years.
    c. Plot the posterior distributions for the contemporaneous structural coefficients. Are the estimates (magnitudes and signs) consistent with the economic interpretation that you provided under (4)?

6. Suppose you wanted to identify the shocks underlying this 4-variable model using the traditional sign-restriction algorithm – but **without** imposing any signs.
    a. Provide a plot for the impact effect of a one-standard deviation shock using the <u>analytical</u> expression for the implicit prior distribution.
    b. Verify <u>empirically</u> what the impact effect for each variable looks like. Report plots of the impact effects and provide the numerical values for the cut-off points.

GOOD LUCK!!! ☺