Christiano
Exam for the course, ‘Analysis and Solution of the New Keynesian Model’
February 28, 2022

Instructions:

Points for each section are given in parentheses. This is a closed-book, no notes exam. If a question seems ambiguous, explain why, reformulate it and proceed. Duration: 9-11 a.m. (two-hour exam) + 30 minutes for uploading to the platform. Students’ answers in pdf will be handwritten in either Spanish or English. SPANGLISH IS NOT ALLOWED.

1. (20) Shorter questions:
   (a) (4) Explain why a rise in the one period interest rate, R_t, that persists into the future is expected to have a bigger impact on period t consumption than a rise in R_t that lasts only one period. You can answer this question under the assumption that there is no uncertainty, all variables eventually converge to steady state and inflation is unaffected by whether the jump in R_t is temporary or persistent.
   (b) (4) We discussed how it is that if monetary policy is governed by an interest rate rule, then we can ignore money demand in the simple New Keynesian model. Explain in intuitive terms why this is so. Feel free to use the IS-LM model to explain this, if you wish.
   (c) (4) Explain why it is that with Calvo-sticky prices resources across intermediate goods are misallocated when inflation is higher in the New Keynesian model. Explain why this mechanism implies that the New Keynesian model incorporates a concept of Total Factor Productivity that is partially endogenous.
   (d) (8) Consider a scenario in the simple New Keynesian model in which news arrives that technology will improve in the future.
      i. Explain why the efficient thing is for consumption and employment to remain unchanged in the current period, and to accomplish this requires a rise in the interest rate. Be sure to explain how the principle of consumption smoothing is useful for understanding that the interest rate should be increased.
      ii. Why might it be that a monetary authority focusing on inflation only might cut the interest rate and create an inefficient output boom in the current period.

2. (35) Consider a firm that has the following production function:

$$Y = K^{\alpha} N^{1-\alpha},\ 0 < \alpha < 1,$$

where $K \geq 0$ and $N \geq 0$ denote the amount of capital and labor that the firm uses. The firm hires N and K in competitive markets at price w and r, respectively. The firm treats r and w as beyond its control. The firm is a monopolist in the production of Y and sets the price, P, subject to a demand curve:

$$Y = HP^{-\varepsilon},$$

where $\varepsilon > 1$ and $H > 0$ are parameters.

(a) (5) Let $C(Y)$ denote the cost of producing $Y > 0$, given the production function and w, r. Formally, $C(Y) = \min_{K,N} rK + wN$, subject to $Y = K^{\alpha}N^{1-\alpha}$. In Lagrangian form, $C(Y)$ can be represented as follows:

$$C(Y) = \min_{K,N} rK + wN + \lambda \left[Y - K^{\alpha}N^{1-\alpha}\right],$$

where λ denotes the Lagrange multiplier. Explain informally the link between the formal definition of $C(Y)$ and the above Lagrangian problem. Explain, informally, which of the following three properties holds for the Lagrange multiplier: $\lambda = 0$, $\lambda > 0$ or $\lambda < 0$.

(b) (5) Consider the function, $Z = F(x, w)$. This is said to be a linear homogeneous function if for any $\mu > 0$, $\mu Z = F(\mu x, \mu w)$. Prove that

$$Z = F_1(x, w)\, x + F_2(x, w)\, w,$$

where F_i denotes the partial derivative of F with respect to its i^{th} argument

(c) (10) Note that the firm production function satisfies linear homogeneity. Derive the first order conditions associated with K and N for the Lagrangian representation of the firm problem. (You may assume that both first order conditions hold as strict equalities.) Show that under linear homogeneity the first order conditions imply:

$$C(Y) = \lambda Y,$$

where λ is the Lagrange multiplier.

(d) (10) Derive an explicit expression for λ in terms of r, w, α. (Hint: the two first order conditions of the Lagrangian problem involve only the two firm variables, λ and K/N, so each has a simple solution.) Explain why λ is the marginal cost of production.

(e) (5) Write out the firm optimization problem. Show that the solution involves setting P as a markup over marginal cost. Derive an explicit expression for the markup. How are K, N and Y chosen, given P?

3. (30) Here, we embed the model with capital and labor in the previous question into the Dixit-Stiglitz framework in the New Keynesian model. We show that the presence of capital (as long as we have linear homogeneity in production) does not affect Tack Yun's result. Thus, suppose there is a competitive final good producer with the following production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1,$$

where $Y_{i,t}$ is produced by a monopolist. The final good producer takes the aggregate price index, P_t, as given, as well as the price of each intermediate good, $P_{i,t}$, $0 \leq i \leq 1$. In class, we derived the first order condition for the final good producer:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t,$$

which the i^{th} intermediate good firm treats as a demand curve for its output. The intermediate good firm's production function is:

$$Y_{i,t} = e^{a_t} K_{it}^{\alpha} N_{i,t}^{1-\alpha}, 0 < \alpha \leq 1,$$

like in the previous question.

(a) (5) Show that if each final good firm satisfies its first order condition and the final good production function is satisfied, then

$$P_t = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}.$$

(b) (9) Suppose each intermediate good firm goes to the same competitive capital market and so faces the same rental rate on capital. Similarly, each intermediate good firm faces the same wage rate. Show that each intermediate good firm, no matter how much it produces, uses the same capital to labor ratio. That is, $K_{i,t}/N_{i,t} = K_{j,t}/N_{j,t}$ for all i, j. (Hint: think about the first order condition of the firm in its cost minimization problem.)

(c) (8) Let Y_t^* denote the simple 'sum' over all intermediate good output, $Y_t^* = \int_0^1 Y_{i,t} di$. Show that

$$Y_t^* = e^{a_t} K_t^{\alpha} N_t^{1-\alpha},$$

where $K_t = \int_0^1 K_{i,t} di$, and $N_t = \int N_{i,t} di$.

(d) (8) Show that

$$Y_t = p_t^* e^{a_t} K_t^{\alpha} N_t^{1-\alpha},$$

and provide the formula for p_t^*, the 'Tack Yun distortion'. (Hint: substitute out for $Y_{i,t}$ in the expression for Y_t^* using the demand curve.)

4. (15) The linearized New Keynesian Phillips curve is expressed as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

where x_t is the output gap and $\kappa > 0$. To a first order approximation, the output gap is proportional to firm marginal cost in the model.

(a) (2) Provide intuition into why it is that x_t is proportional to marginal cost.

(b) (10) The New Keynesian Phillips curve at time t implies that time t inflation is a function of current and time t expected future marginal costs. Derive this fact from the Phillips curve. (Hint: In this demonstration you must use two things - (i) the Law of Iterated Mathematical Expectations (LIME) and (ii) you must take a position on $\beta^j E_t \pi_{t+j}$ as j gets larger and larger. In fact, it is possible to show that

$$\beta^j E_t \pi_{t+j} \to 0 \text{ as } j \to \infty,$$

because $0 < \beta < 1$. You need not explain this result for $j \to \infty$ in your answer. However, for your answer to be complete you must say what LIME is and show how (i) and (ii) are used.)

(c) (3) Provide intuition for the property of the model that current inflation is a function of current and future marginal cost.