Foundations for the New Keynesian Model II

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Standard New Keynesian Model (NK model of last time, with Taylor rule)

- Taylor rule: designed, so that in steady state, inflation is zero ($\bar{\pi} = 1$)

- Employment subsidy extinguishes monopoly power in steady state:

$$(1-\nu)\frac{\varepsilon}{\varepsilon-1} = 1$$

Equations of the NK Model

$$\frac{1}{C_t} - E_t \frac{\beta}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} = 0 \qquad 1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta\theta F_{t+1} - F_t = 0$$

$$C_t - p_t^* e^{a_t} N_t = 0 \qquad F_t \left(\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t = 0$$

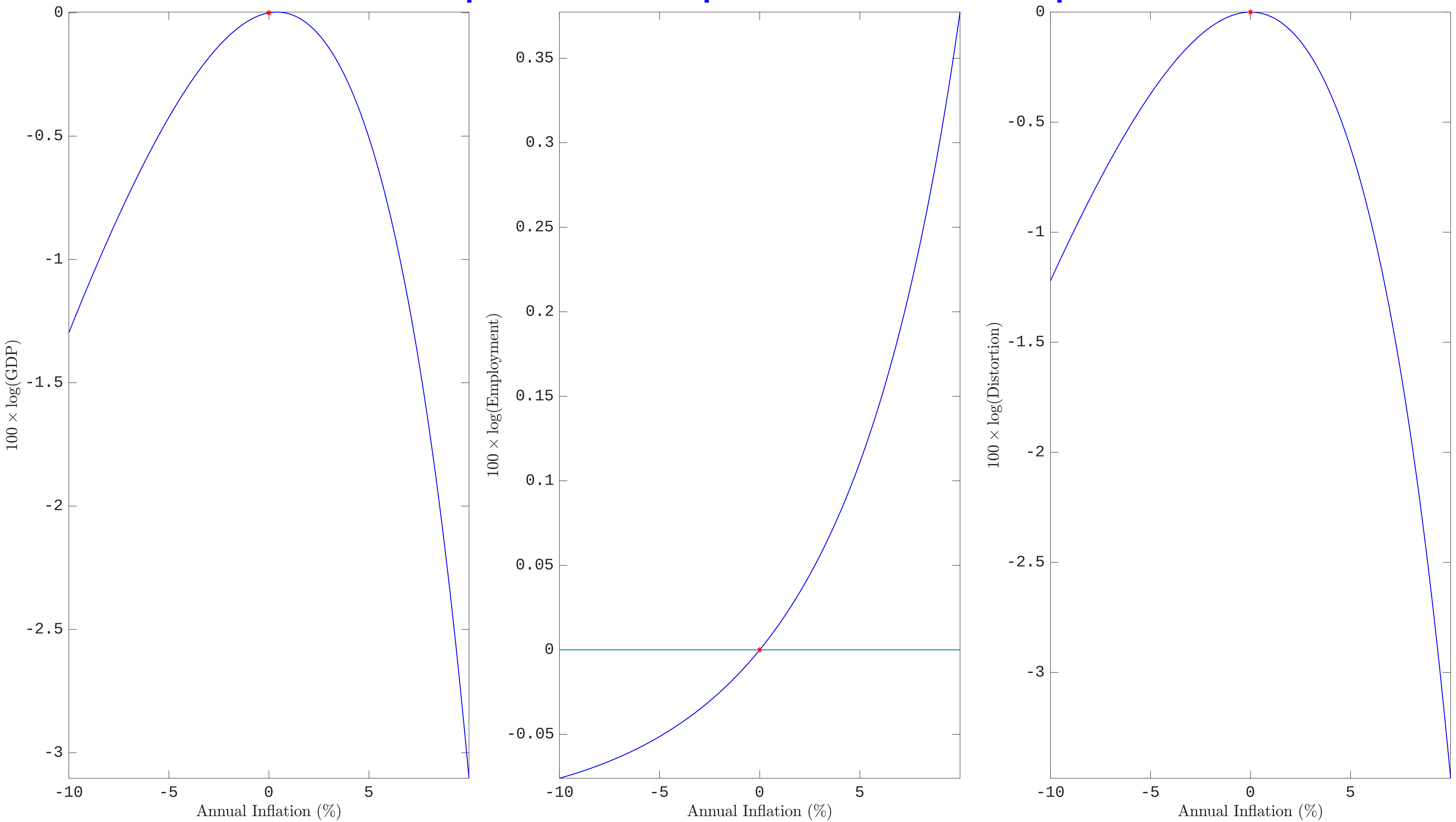
$$(1-\nu)\frac{\varepsilon}{\varepsilon-1} \frac{C_t \exp(\tau_t) N_t^{\varphi}}{e^{a_t}} + E_t \beta\theta\bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t = 0$$

$$\frac{1}{p_t^*} - \left((1-\theta)\left(\frac{1-\theta(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}_t^{\varepsilon}}{p_{t-1}^*} \right) = 0$$

- In steady state:

$$R = \frac{1}{\beta},\ p^* = 1,\ F = K = \frac{1}{1-\beta\theta},\ N = \exp\left(-\frac{0}{1+\varphi} \right).$$

theta = 0.75, epsilon = 5, phi = 1, nu = 1/epsilon, beta = 0.99



inflation, annual percent400*(pibar - 1)

Natural Rate of Interest

- Intertemporal euler equation in natural equilibrium:

$$\overbrace{a_t - \frac{1}{1+\varphi}\tau_t}^{y_t^*} = -[r_t^* - rr] + E_t \overbrace{\left(a_{t+1} - \frac{1}{1+\varphi}\tau_{t+1}\right)}^{y_{t+1}^*}$$

- Back out the natural rate:

$$r_t^* = rr + \rho\Delta a_t + \frac{1}{1+\varphi}(1-\lambda)\tau_t$$

- Shocks:

$$\tau_t = \lambda\tau_{t-1} + \varepsilon_t^\tau, \; \Delta a_t = \rho\Delta a_{t-1} + \varepsilon_t$$

NK *IS* Curve

- Euler equation in two equilibria:

$$\text{Taylor rule equilibrium: } y_t = -[r_t - E_t\pi_{t+1} - rr] + E_t y_{t+1}$$

$$\text{Natural equilibrium: } y_t^* = -[r_t^* - rr] + E_t y_{t+1}^*$$

- Subtract:

Output gap

$$x_t = -[r_t - E_t\pi_{t+1} - r_t^*] + E_t x_{t+1}$$

Output in NK Equilibrium

- Agg output relation:

$$y_t = \log p_t^* + n_t + a_t, \ \log p_t^* = \begin{cases} = 0 & \text{if } P_{i,t} = P_{j,t} \text{ for all } i,j \\ \leq 0 & \text{otherwise} \end{cases} .$$

- To first order approximation,

$$\hat{p}_t^* \approx \theta \hat{p}_{t-1}^* + 0 \times \bar{\pi}_t, \ (\to p_t^* \approx 1)$$

Price Setting Equations

- Log-linearly expand the price setting equations about steady state.

$$1 + E_t \bar{\pi}_{t+1}^{\varepsilon-1} \beta \theta F_{t+1} - F_t = 0 \qquad F_t \left(\frac{1-\theta \bar{\pi}_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} - K_t = 0$$

$$(1-\nu) \frac{\varepsilon}{\varepsilon-1} \frac{C_t \exp(\tau_t) N_t^{\varphi}}{e^{a_t}} + E_t \beta \theta \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} - K_t = 0$$

- Log-linearly expand about steady state:

$$\hat{\bar{\pi}}_t = \frac{(1-\beta\theta)(1-\theta)}{\theta} (1+\varphi) x_t + \beta \hat{\bar{\pi}}_{t+1}$$

- See http://faculty.wcas.northwestern.edu/~lchrist/course/solving_handout.pdf

Taylor Rule

- Policy rule

$$r_t = \alpha r_{t-1} + (1-\alpha)[rr + \phi_\pi \pi_t + \phi_x x_t] \qquad , \; x_t \equiv y_t - y_t^*.$$

Equations of Actual Equilibrium Closed by Adding Policy Rule

$$\beta E_t \pi_{t+1} + \kappa x_t - \pi_t = 0 \text{ (Phillips curve)}$$

$$-[r_t - E_t \pi_{t+1} - r_t^*] + E_t x_{t+1} - x_t = 0 \text{ (IS equation)}$$

$$\alpha r_{t-1} + (1-\alpha)\phi_\pi \pi_t + (1-\alpha)\phi_x x_t - r_t = 0 \text{ (policy rule)}$$

$$r_t^* - \rho \Delta a_t - \frac{1}{1+\varphi}(1-\lambda)\tau_t = 0 \text{ (definition of natural rate)}$$

Solving the Model

$$s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^\tau \end{pmatrix}$$

$$s_t = P s_{t-1} + \epsilon_t$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \\ r_{t+1} \\ r^*_{t+1} \end{pmatrix} + \begin{bmatrix} -1 & \kappa & 0 & 0 \\ 0 & -1 & -\frac{1}{\sigma} & \frac{1}{\sigma} \\ (1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ rr^*_t \end{pmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \\ r_{t-1} \\ r^*_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} s_{t+1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\sigma\psi\rho & -\frac{1}{\sigma+\varphi}(1-\lambda) \end{pmatrix} s_t$$

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

Solving the Model

$$E_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0$$

$$s_t - P s_{t-1} - \epsilon_t = 0.$$

- Solution:

$$z_t = A z_{t-1} + B s_t$$

- As before:

$$\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,$$

$$F = (\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0$$

$$\phi_x = 0,\ \phi_\pi = 1.5,\ \beta = 0.99,\ \varphi = 1,\ \rho = 0.2,\ \theta = 0.75,\ \alpha = 0,\ \delta = 0.2,\ \lambda = 0.5.$$