Central Reserve Bank of Peru

LXXI ADVANCED ECONOMICS EXTENSION COURSE

A Bayesian Approach to Identification of Structural VAR Models

Grupo 2

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1 Part I

Consider the following **AR(2) model:**

$$y_t = c + b_1 y_{t-1} + b_2 y_{t-2} + \epsilon_t$$

where the **residuals are serially correlated** according to:

$$\epsilon_t = \rho \epsilon_{t-1} + v_t \quad with \quad v_t \sim N(0, \tau^2)$$

1.1 Gibbs Sampling

a. Write down the Gibbs sampler for this extended model (on paper). Describe each step, the prior specification, and the conditional posterior distributions including the formulas for their moments.

Having the correlated errors: $\epsilon_t = \rho \epsilon_{t-1} + v_t \quad with \quad v_t \sim N(0, \tau^2)$.
We can express in a general form as it follows: $e = E\phi + v$. So in our model, $E = e_{t-1}$, representing the first lag of error, where the roots of $(1 - \rho L) = 0$ lie outside the complex unit circle.

1. Conditional on b, the problem reduces to making inferences on ρ and τ^2 from the following model:

$$e_t^* = \rho e_{t-1}^* + v_t, \quad v_t \sim iid\, N(0, \tau^2),$$
$$(e^* = E^* \rho + v \quad v \sim N(0, \tau^2 I_{T-1})),$$

where $e_t^* = y_t - c - b_1 y_{t-1} - b_2 y_{t-2}$.

2. Conditional on ρ and τ^2, the problem reduces to making inferences on b from the following regression model with known variances

$$y_t^* = c + b_1 y_{t-1} + b_2 y_{t-2} + v_t, \quad with \quad v_t \sim iid\, N(0, \tau^2),$$

$$(Y^* = X^* b + v \quad v \sim N(0, \tau^2 I_{T-1})),$$

where $y_t^* = y_t - \rho y_{t-1}, x_{it}^* = x_t - \rho x_{t-1}$, i = 1, 2, 3, according to our right hand side variables of the AR(2) model (k=3, constant included), and $b_1^* = b_1(1 - \rho)$.

Conditional distribution of b given ρ and τ^2

Prior:

$$b \mid \rho, \tau^2 \sim N(b_0, P_0)$$

Posterior:

$$b \mid \rho, \tau^2, Y \sim N(b_1, P_1)$$

where,

$$b_1 = P_1 * (P_0^{-1} b_0 + \tau^{-2} X^{*'} Y^*)$$

$$P_1 = (P_0^{-1} + \tau^{-2}\ X^{*'}\ X^*)^{-1}$$

To sample a $kx1$ vector β from $N(\beta_0, A_0)$, generate $kx1$ draws β^0 from the standar normal distribution (randn in MatLab) and then apply the following transformation:

$$b = b_1 + [(b^0)'(P_1)^{(1/2)}]'$$

The Matlab code will be expressed as follows :

$$b = b_1 + [randn(1, k).Chol(P_1)]'$$

Conditional distribution of ρ given b and τ^2

Prior:

$$\rho \mid b, \tau^2 \sim N(h_0, M_0)_{I[s(\rho)]}$$

where h_0 and M_0 are known and $I[s(\rho)]$ is and indicator function used to denote that the roots of $\rho(L) = 0$ lie outside the unit circle.

Posterior:

$$\rho \mid b, \tau^2, Y \sim N(h_1, M_1)$$

where,

$$h_1 = (M_0^{-1} + \tau^2\ E^{*'}\ E^*\ )^{-1}\ (M_0^{-1}\ h_0 + \tau^{-2}\ E^{*'}\ e^*)$$
$$M_1 = (M_0^{-1} + \tau^2\ E^{*'}\ E^*\ )^{-1}$$

As we mention, to sample a $kx1$ vector ρ from $N(h_0, M_0)$, generate $kx1$ draws ρ^0 from the standar normal distribution (randn in MatLab) and then apply the following transformation:

$$\rho = h_1 + [(\rho^0)'(M_1)^{(1/2)}]'$$

The Matlab code will be expressed as follows :

$$\rho = h_1 + [randn(1, k).Chol(M_1)]'$$

Conditional distribution of τ^2 given ρ and b

Prior:

$$\frac{1}{\tau^2 \mid \rho, b} \sim \Gamma(t_0, \frac{1}{R_0})$$

$$\tau^2 \mid \rho, b \sim \Gamma^{-1}(t_0, R_0)$$

where Γ^{-1} refers to inverted Gamma distribution and t_0 and R_0 are known. The first component of the distribution is the sample lenght and the second component is the scale. Initially the values were imputed in Matlab.

Posterior:

$$\frac{1}{\tau^2 \mid \rho, b, Y} \sim \Gamma(t_1, \frac{1}{R_1})$$

$$\tau^2 \mid \rho, b, Y \sim \Gamma^{-1}(t_1, R_1)$$

where $t_1 = t_0 + t$ (t is the sample size) and $R_1 = R_0 + (Y^* - X^*b)'(Y^* - X^*b)$

In conclusion, we have set priors and initial guess for β, τ and ρ. Then we estimate the posteriors, using conditional distributions. Finally, an iteration process is , where in total was 100000 times and keep only the last 10000 draws. In order to sum up this Gibbs sampling algorithm, we expressed each step in the following list:

1. We can start the GS iteration with arbitrary starting values, $\rho = \rho^0$ and $\left\{\tau^2\right\}^0$.

2. Conditional on $\rho = \rho^{j-1}$ and $\tau^2 = \left\{\tau^2\right\}^{j-1}$, generate b^j from its corresponding posterior distribution.

3. Conditional on $\tau^2 = \left\{\tau^2\right\}^{j-1}$ and $b = b^j$, generate ρ^j from its corresponding posterior distribution.

4. Conditional on $b = b^j$ and $\rho = \rho^j$, generate $\left\{\tau^2\right\}^j$ from its corresponding posterior distribution.

5. Set $j = j - 1$, and go to the second step.

1.2 Code report

By using the posteriors established in the previous section in the Gibbs Sampling method, we obtain a series of results that are shown in the following table:

Table 1: Gibbs Sampling results

	Prior		Posterior	
	Mean	SD	Mean	SD
b_1	1.00	1.00	1.37	0.52
b_2	0.60	1.00	0.40	0.11
b_3	0.80	1.00	0.10	0.06
ρ	0.80	1.00	0.84	0.05
τ^2	-	-	0.81	-

Note: Prior distribution of τ^2 is improper. SD refer to standard deviation.

The reasoning behind the choice of the mean and standard deviation of the prior is based on the fact that informative priors based on AR models have been collected with the CPI variable in the analysis. In addition, use has been made of non-informative priors with a large variance that symbolizes the mistrust of the priors. Despite the different priors used, the data predominates and the posterior distribution is highly assimilated in both types of priors. These posteriors have been used as a simple average as priors and with a unit variance, demonstrating confidence in them.

Along these lines, Zhang et al. [2020] proposes a working document on the application of ARMA models to the G7 CPI under stochastic volatility models, in which they suggest small posterior standard deviations for the variable in question with a value of 5, while their posterior means also take small values

With this, the Gibbs sampling iteration is materialized, obtaining that the posterior distribution of b, which represent the lagged CPI coefficients, has a mean of 1.37, 0.40 and 0.10, respectively (constant included). In short, the rho that symbolizes the serial correlation with the first lag of the residual has a posterior mean of 0.84. The standard deviations also present similar results to the posteriors analyzed with the different priors used. Next, the results will be presented in the form of equations to finally materialize it in the histograms of the posterior distributions.

With these results the AR(2) model would be expressed as:

$$y_t = \underset{(\ 0.52)}{1.37} + \underset{(\ 0.11)}{0.40}\, y_{t-1} + \underset{(\ 0.06)}{0.10}\, y_{t-2} + \epsilon_t$$

Meanwhile, the correlated residuals would be expressed as:

$$\epsilon_t = \underset{(\ 0.05)}{0.84}\, \epsilon_{t-1} + \upsilon_t$$

Where y_t is the annual Consumer Price (CPI) inflation for United States over the period 1948Q1 to 2010Q4.

The exercise asks for the marginal posterior distribution for ρ. The distribution for this parameter is expressed as:

$$\rho \sim N(0.84, 0.09)$$

Consequently, the posterior distributions for b and τ^2 are:

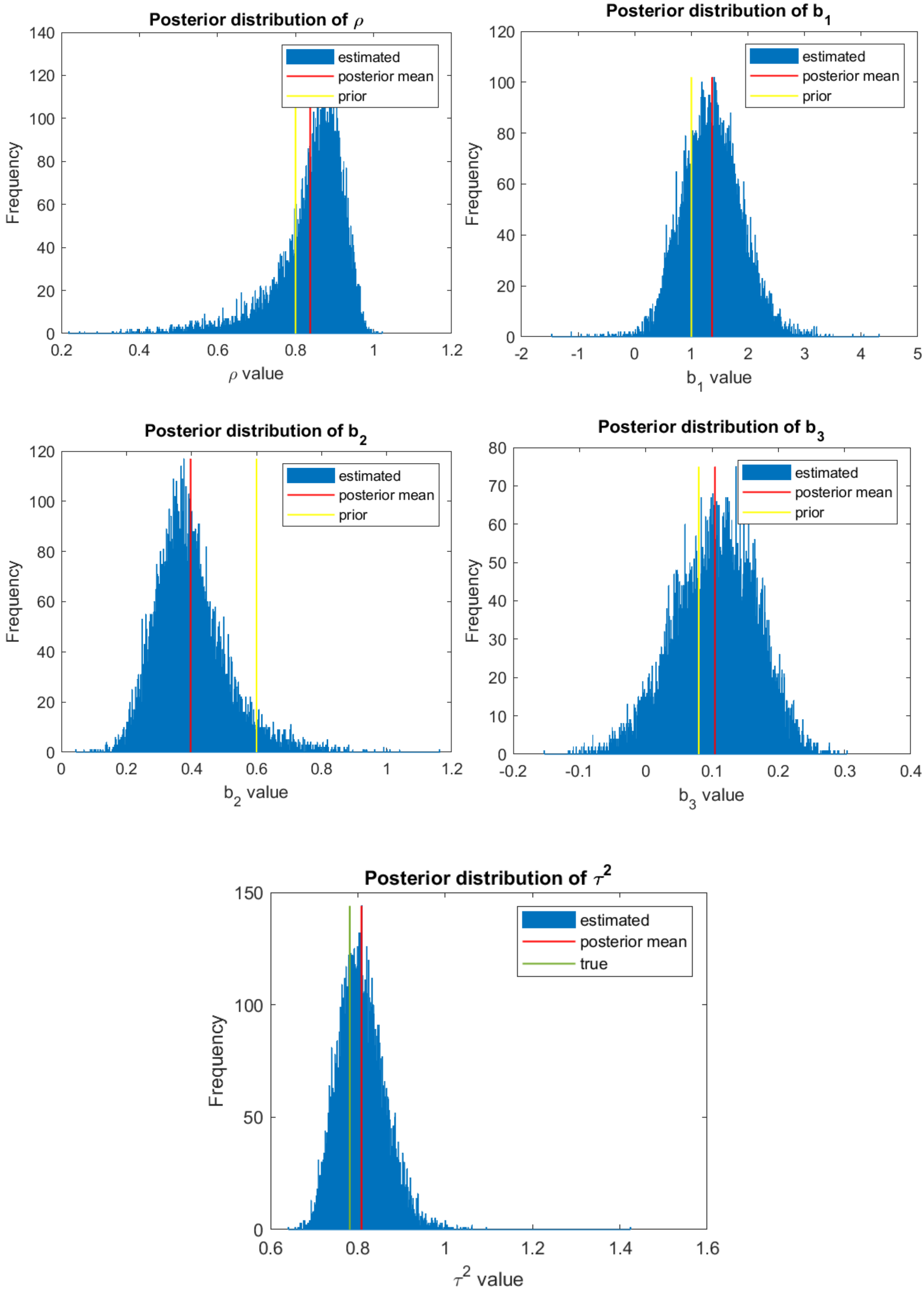
$$b_1 \sim N(1.37, 0.52)$$
$$b_2 \sim N(0.40, 0.11)$$
$$b_3 \sim N(0.10, 0.06)$$

The posterior distribution for τ^2 corresponds to an inverse gamma distribution and its parameters are the shape and the scale, respectively:

$$\tau^2 \sim \Gamma^{-1}(250, 0.0051)$$

And the graphic for its distribution:

Figure 1: Marginal Posterior Distributions

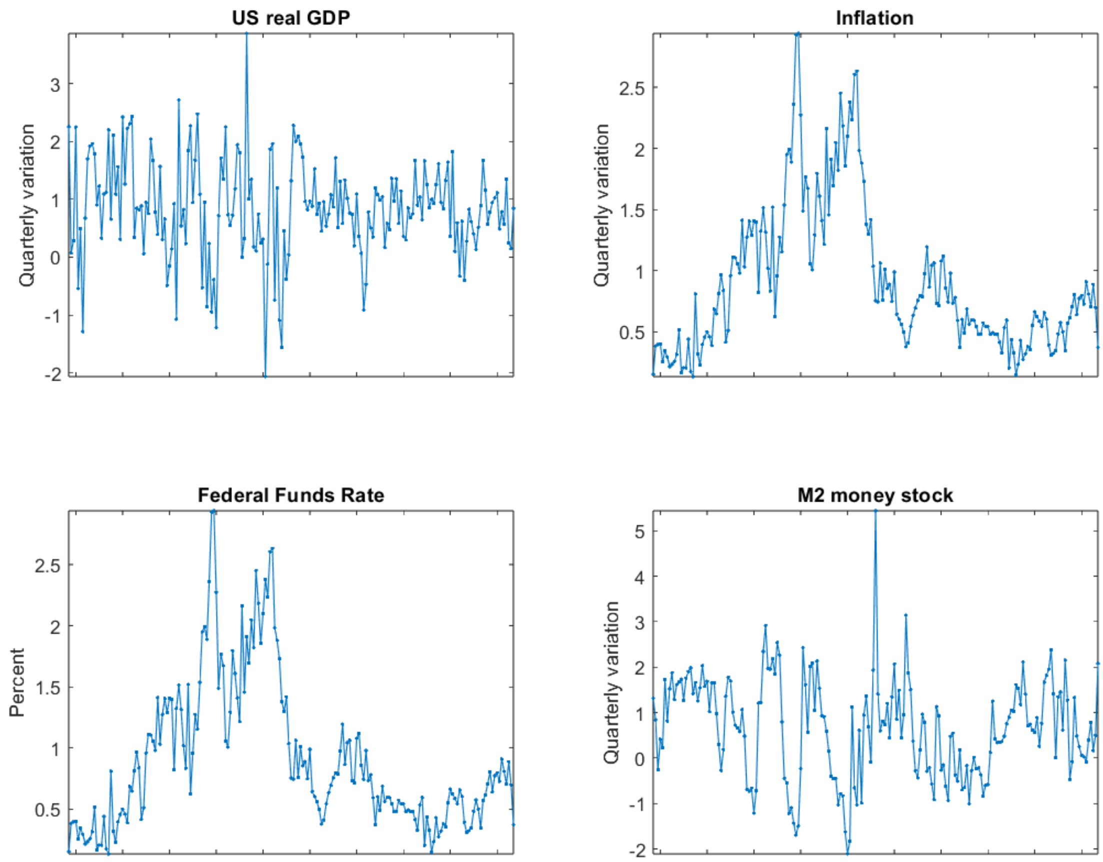


2 Part II

2.1 Collect quarterly data for US

a. Transform the data to quarterly growth rates where appropriate so that they have a useful economic interpretation. Plot the transformed data with appropriate labels.

Figure 2: Transformed Data



b. Write Matlab code to estimate a reduced-form $VAR(4)$ model with a constant term using OLS. Report the estimates of the reduced-form covariance matrix and the (kxn) matrix of reduced-form coefficients where $n = 4$ is the number of endogenous variables.

Reduced-form Covariance Matrix

Table 2: Reduced-form Covariance Matrix

GDP	**Inflation**	**FFR**	**M2**
0.5122	-0.0091	0.0831	0.0317
-0.0091	0.0594	0.0480	-0.0489
0.0831	0.0480	0.7191	-0.2460
0.0317	-0.0489	-0.2460	0.5501

Reduced-form coefficients

$$\beta = \begin{pmatrix} 0.5698 & 0.0497 & -0.4607 & 0.4115 \\ 0.0899 & -0.0173 & 0.3271 & -0.1317 \\ 0.2377 & 0.5569 & 0.4793 & -0.4432 \\ 0.0883 & 0.0670 & 1.1736 & -0.3180 \\ 0.1862 & -0.0257 & 0.1754 & 0.5048 \\ 0.1999 & -0.0130 & 0.2565 & 0.0860 \\ 0.3146 & 0.1180 & 0.7337 & 0.1969 \\ -0.3736 & -0.0691 & -0.5842 & 0.3144 \\ 0.0578 & 0.0157 & -0.2158 & -0.0178 \\ -0.0574 & 0.0202 & 0.0556 & 0.0759 \\ -0.5554 & 0.1553 & -0.5455 & -0.0106 \\ 0.3161 & -0.0053 & 0.4824 & -0.0127 \\ 0.0611 & -0.0042 & 0.1082 & 0.2475 \\ 0.0283 & 0.0456 & 0.0075 & -0.1132 \\ 0.1232 & 0.1442 & -0.2497 & 0.1661 \\ -0.0721 & -0.0011 & -0.1393 & 0.0121 \\ -0.0204 & 0.0094 & -0.1486 & -0.0690 \end{pmatrix} \tag{1}$$

2.2 Recursive ordering

Estimate this model using the Baumeister-Hamilton algorithm implementing the Choleski identification in that framework and uninformative priors for B and D.

a. Use the same 4-variable dataset that you assembled for (1). Assign a structural interpretation to this 4-variable VAR by making an economically-motivated case for a recursive ordering (Choleski identification). In other words, what ordering would you choose and why? Explain.

The matrix A would take the following form:

$$A = \begin{pmatrix} 1 & -\theta_1 & -\theta_2 & -\theta_3 \\ -\alpha_1 & 1 & -\alpha_2 & -\alpha_3 \\ -\beta_1 & -\beta_2 & 1 & -\beta_3 \\ -\delta_1 & -\delta_2 & -\delta_3 & 1 \end{pmatrix} \tag{2}$$

In the next section, the order of the variables will be justified based on the literature supporting the relationships between variables through empirical evidence. Therefore, we will continue to follow the convention that the most exogenous variable is the product, followed by the inflation rate, the interest rate, and monetary mass (as the most endogenous variable).

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\alpha_1 & 1 & 0 & 0 \\ -\beta_1 & -\beta_2 & 1 & 0 \\ -\delta_1 & -\delta_2 & -\delta_3 & 1 \end{pmatrix} \begin{pmatrix} Y_t \\ \pi_t \\ r_t \\ M_t \end{pmatrix} \tag{3}$$

2.3 Structural equations

Translate this model into a structural model and write down the structural equations. Provide an economic interpretation for the structural parameters (A matrix).
The structural equations will be:

$$Y_t = c_Y + \theta_1 \pi_t + \theta_2 r_t + \theta_3 M_t + lags + u_t^Y \tag{4}$$

Following the convention, we will have that inflation will be defined by:

$$\pi_t = c_\pi + \alpha_1 Y_t + \alpha_2 r_t + \alpha_3 M_t + lags + u_t^\pi \tag{5}$$

There is substantial empirical evidence on the impact of inflation on economic growth. Barro [1995] found a negative relationship between the two variables in his long-term analysis, while Bruno, Easterly, and Fischer (1996) concluded that inflationary crises have lasting effects on long-term growth rates, supporting the earlier work of Stanley [1993] which used panel data to find the well-established relationship between inflation and growth. Khan and Ssnhadji [2001] later showed that this relationship varies with the level of inflation.

$$r_t = c_r + \beta_1 Y_t + \beta_2 \pi_t + \beta_3 M_t + lags + u_t^r \tag{6}$$

In accordance with Taylor [1993] He found that U.S. monetary policy during the 1980s was consistent with a Taylor Rule, which states that the nominal interest rate should adjust in response to fluctuations in inflation and output.

Forero [2015] compares the transition of monetary policy shocks in countries that employ inflation targeting schemes. For this, he uses a hierarchical penalty VAR where monetary shocks are identified using an agnostic procedure that imposes zero and sign restrictions. Thus, as is standard in the literature, it assumes that Gross Domestic Product (Y) and the Consumer Price Index (P) are slow variables, so they do not react to monetary shocks contemporaneously.

One of the articles that helps to understand and emphasizes the importance of the relationship between interest rates and economic growth for optimal policy formulation is Gali [1992], which highlighted the importance of developing more sophisticated macroeconomic models by using econometric techniques that serve to evaluate the fit of the IS-LM model to historical time series data, whose results showed that the IS-LM model had relatively poor fits.

Subsequently, Bernanke and Gertler [1995] made significant contributions to the understanding of how monetary policy affects the real economy through the credit channel. The study argues that the availability of credit affects investment and consumption spending, which in turn affects the level of production and employment. The authors provide empirical evidence through a VAR model that shows that monetary policy affects the supply of credit, which impacts spending and investment, which in turn affects the level of production.

Additionally, Clarida et al. [2002] found that the interest rate is an important predictor of both inflation and output. According to the authors, monetary policy (which could well be originated by changes in the reference interest rate) influences the economy through two main channels: the price channel, where monetary policy affects inflation through prices, and the output channel, where monetary policy affects output and employment. Additionally, they observed that the impact of monetary policy through the output channel is stronger in the United States than in other countries studied, suggesting that the effect of monetary policy may vary depending on the economic context and characteristics of each country. The authors also found that monetary policy is more effective when it is clearly communicated to economic agents and is based on clear and transparent objectives.

$$M_t = c_M + \delta_1 Y_t + \delta_2 \pi_t + \delta_3 r_t + lags + u_t^M \tag{7}$$

Since the argument by Friedman [1968] on the nature of inflation as a phenomenon that could be controlled by maintaining a stable relationship between monetary supply and demand, there has been much debate about the adoption of this constant growth rule by central banks around the world as a guide for conducting economic policy. Subsequently, other studies analyzed the relationship between these variables, one of them being the study by Sims [1980], who examined this relationship with data from the United States and concluded that the evidence did not support the idea of a mechanical and stable relationship in the short term. Sims argued that causality between monetary supply and inflation could be bidirectional, and that other factors such as fiscal policy and exogenous shocks also played an important role in determining inflation. These are the approaches to the relationship between monetary supply and inflation levels in an economy.

There are several studies in the literature that explore the relationship between interest rate and money supply. One of them is the work of Mishkin [1978], which analyzes how the interest rate affects household consumption and investment, and how this influences the money supply during the Great Depression in the United States. On the other hand, Hamilton, J. D. (1988) uses a real business cycle model to analyze the relationship between interest rate and money supply measured by M_2, concluding that the interest rate is an important factor in determining the supply. short-term money. Likewise, McCallum [1989] proposes a theory of the demand for money in which the interest rate is a relevant factor, and discusses the relationship between monetary policy and the money supply through the interest rate. All of these approaches provide an empirical and less dogmatic view of the relationship between the interest rate and the money supply.

Additionally Christiano et al. [1999], suggest when carrying out the Cholesky identification, the variables should be ordered considering production and prices before the monetary policy instrument, in our case when working with quarterly frequency data, and considering the lag of monetary policy to have an impact on variables such as production and prices, it makes more economic sense to follow the monetary literature and consider production and prices before the monetary policy instrument.

2.4 Using Baumeister-Hamilton algorithm

Estimate this model using the Baumeister-Hamilton algorithm implementing the Choleski identification in that framework and uninformative priors for B and D.

a. Write down (on paper) the prior for each element in A as well as the joint prior for A.

$$p\left(\beta_1\right)=\frac{\Gamma((v+1)/2)}{\sqrt{\pi v},\Gamma(v/2)}\left(1+\frac{\beta_1^2}{v}\right)^{-(v+1)/2} \sim \text{Student } t(0,100,3) \tag{8}$$

$$p\left(\theta_1\right)=\frac{\Gamma((v+1)/2)}{\sqrt{\pi v},\Gamma(v/2)}\left(1+\frac{\theta_1^2}{v}\right)^{-(v+1)/2} \sim \text{Student } t(0,100,3) \tag{9}$$

$$p\left(\theta_2\right)=\frac{\Gamma((v+1)/2)}{\sqrt{\pi v},\Gamma(v/2)}\left(1+\frac{\theta_2^2}{v}\right)^{-(v+1)/2} \sim \text{Student } t(0,100,3) \tag{10}$$

$$p\left(\delta_1\right)=\frac{\Gamma((v+1)/2)}{\sqrt{\pi v},\Gamma(v/2)}\left(1+\frac{\delta_1^2}{v}\right)^{-(v+1)/2} \sim \text{Student } t(0,100,3) \tag{11}$$

$$p\left(\delta_2\right)=\frac{\Gamma((v+1)/2)}{\sqrt{\pi v},\Gamma(v/2)}\left(1+\frac{\delta_2^2}{v}\right)^{-(v+1)/2} \sim \text{Student } t(0,100,3) \tag{12}$$

$$p\left(\delta_3\right)=\frac{\Gamma((v+1)/2)}{\sqrt{\pi v},\Gamma(v/2)}\left(1+\frac{\delta_3^2}{v}\right)^{-(v+1)/2} \sim \text{Student } t(0,100,3) \tag{13}$$

Joint prior (A):

$$p(A)=p\left(\alpha_1\right)p\left(\theta_1\right)p\left(\theta_2\right)p\left(\delta_1\right)p\left(\delta_2\right)p\left(\delta_3\right) \sim T(0,100,3) \tag{14}$$

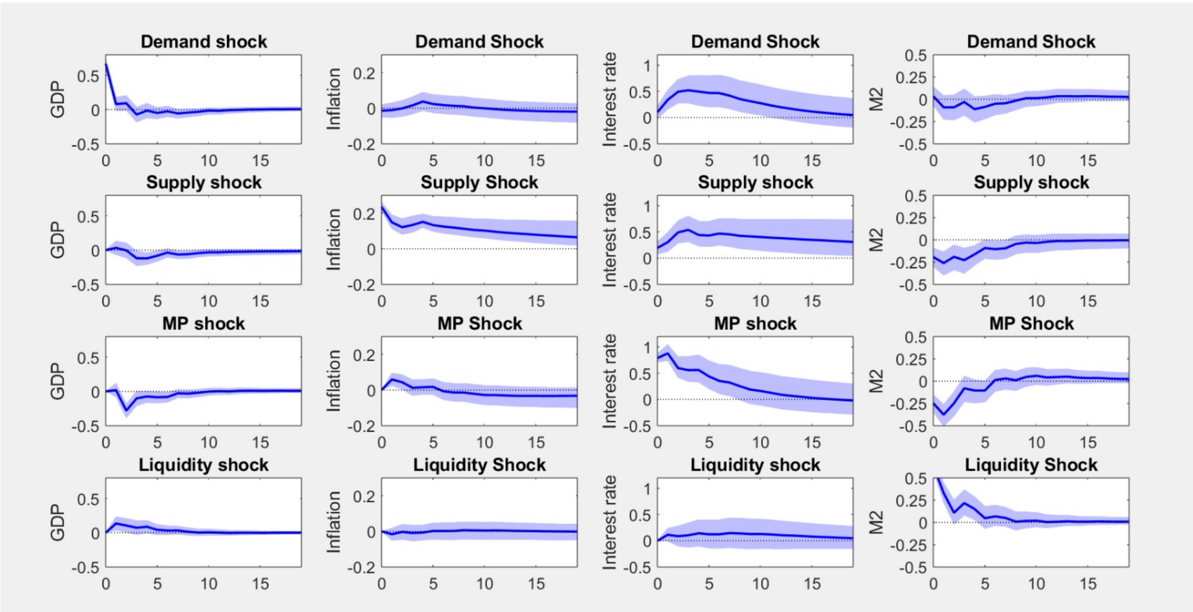
b. Plot the impulse responses to the one-standard-deviation structural shocks (median together with 16th and 84th percentiles of the posterior distribution) for a horizon of 5 years.

Figure 3 presents all impulse response functions in the VAR model. First, a demand shock have a positive but no significant impact on inflation and the response of Fed Funds rate is strictly positive in the first 10 periods, whose effect overcomes the increase of inflation. Then, this explains why demand shocks have no significant impact on inflation because Federal Reserve responds effectively to demand shocks.

Second, supply shock generates a persistant increase in Fed Funds rate that reduces liquidity in the model in order to reduce the increase of inflation. As a result, output falls significantly after 4 periods of the initial shock. Results suggests that Monetarity authority responds to supply shocks even though Central Banks do not responds in a contemporaneous way to this type of shock.

Third, we find an inverse relation between monetary policy shocks and M2 which relies in the idea that Central Bank fix a desired level of interest rate and controls liquidity in the economy to achieve this objective. Furthermore, output falls due to tightening of financial conditions and the highest response of US real GDP is achieve after 3 quarters where the lag of monetary policy is less than results in the empirical literature; see Christiano et al. [1998], Uhlig [2005], among others. Median response of inflation to monetary policy shock is positive and related to the presence of an empirical "price puzzle" which can be explained theoretically in the case where the working capital channel of inflation if higher than demand channel in the short run.

Figure 3: Impulse response functions



Finally, liquidity shock in this model have a positive impact on PBI that lasts for the first two periods. However, inflation and Fed Funds rate does not respond to liquidity shock as monetary authority uses interest rate as policy variable instead of M2. Therefore, if liquidity shock do not have a significant impact on interest rate, there is not a trasmission mechanism for output and inflation.

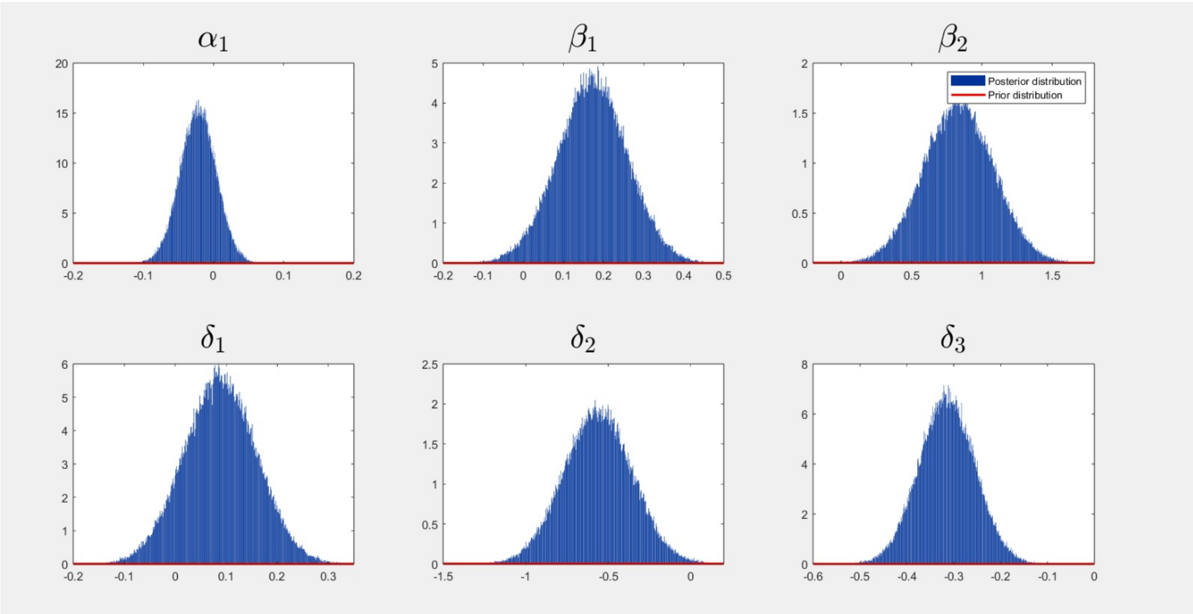
c. Plot the prior and posterior distributions for the contemporaneous structural coefficients. Are the estimates (magnitudes and signs) consistent with the economic interpretation that you provided under (3)?

Figure 4 presents prior and posterior distributions for the contemporaneous coefficients in A matrix. In this excercise we assume a non-informative prior distribution with large variance that is represented with a red line. As a result, posterior distributions are dominated by the information contained in time series under analysis.

Results shows that median of contemporaneous demand shocks on M2 and interest rate have the correct sign predicted by theory, except for inflation which response es near zero. Size of median impact are small in the case of demand shocks on account of Fed Funds rate overeaction. In the case of supply shocks, we found a strong contemporaneous response of interest rate which differs from stylized facts that shows that monetary authority do not respond inmediatly to supply shocks.

Finally, we find a negative relation between M2 and monetary policy shocks on impact. This represents the fact that Central Bank manipulates liquidity in the economy in order to achieve a monetary policy objective. Interest rate is the policy variable wherears M2 is the operational instrument and posterior median sign and magnitude represents this relation.

Figure 4: Prior and posterior distributions for parameters in matrix A



2.5 Sign restrictions

Suppose you wanted to identify the shocks underlying this model by means of sign restrictions

$$\underbrace{\begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 & -\alpha_3 \\ -\beta_1 & 1 & -\beta_2 & -\beta_3 \\ -\theta_1 & -\theta_2 & 1 & -\theta_3 \\ -\delta_1 & -\delta_2 & -\delta_3 & 1 \end{bmatrix}}_{A} \cdot \begin{bmatrix} Y_t \\ \pi_t \\ r_t \\ M_t \end{bmatrix} \tag{15}$$

We apply sign restriction to matrix A

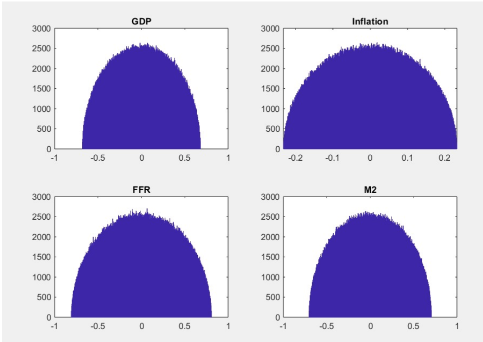
$$A = \begin{bmatrix} + & ? & - & + \\ ? & + & - & + \\ ? & ? & + & - \\ + & + & - & + \end{bmatrix} \tag{16}$$

- To α_1, β_1, θ_1 y θ_2 we assume that sign restrictions are undefined since we do not know what is generating this shock. For expample, por the case of α_1, it may be due to a negative productivity shock that leads to an increase in inflation and a fall in GDP; or by a preference shock that generates both an increase in inflation and a higher demand.
- To α_2 we impose a negative sign restriction based on empirical evidence that shows that a reduction in the interest rate of the monetary policy increases GDP.
- To impose the α_3 (positive), β_3 (positive) and δ_1 (positive) sign restrictions we rely on the Quantity Theory of Money, which tells us that if the quantity of money in circulation increases, the price level is expected to increase as well, and this leads to an increase in output and therefore an increase in GDP.
- To β_2 we impose a negative sign constraint based on empirical evidence that shows that an increase in the interest rate leads to a reduction in inflation.
- To θ_3 and δ_3 we impose a negative sign constraint because a Fed rate hike can make it more expensive for banks to borrow, which reduces the amount of money they have in reserve.
- To δ_2 we impose a negative sign constraint because when inflation rises, the Fed responds by raising the interest rate what reduces the demand for credit and the supply of money in the economy.

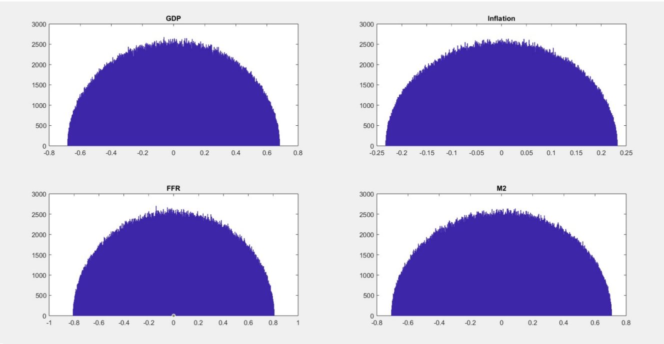
a. Provide a plot for the impact effect of a one-standard deviation shock using the analytical expression for the implicit prior distribution.

On figure 5, we can see the impact effects of one-standard deviation shocks on the four variables of the model without imposing any sign restrictions. All the four shocks generate similar plots and that is because of the implicit distribution for the Q matrix that comes along with the algorithm.

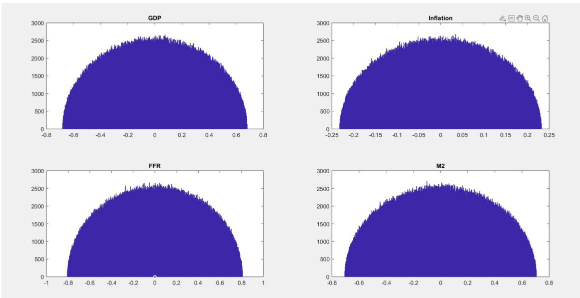
Figure 5: **Impact Effect of One-Standard Deviation Shocks**



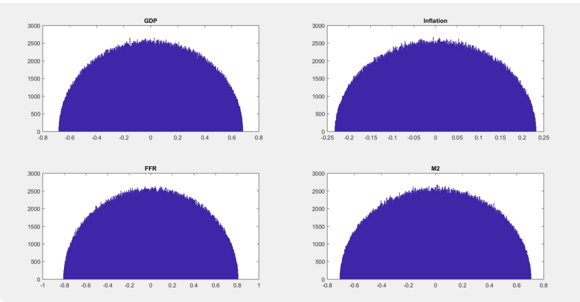
(a) Impact Effect of Demand Shock



(b) Impact Effect of Supply Shock



(c) Impact Effect of MP Shock



(d) Impact Effect of M2 Shock

b. Verify empirically what the impact effect for each variable looks like. Report plots of the impact effects and provide the numerical values for the cut-off points.

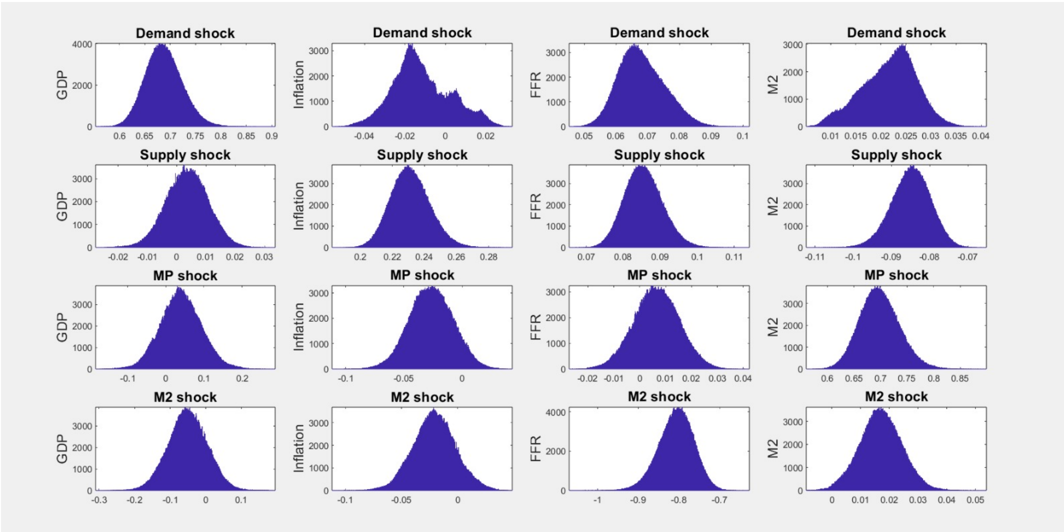
On figure 6, we can observe the impact effects of each variable in the model. Firstly, we can observe an expansive demand shock that generates an increase in GDP. On the other hand, inflation increases because the positive shock in GDP could be increasing the demand for goods and services, which could lead to an increase in prices. Moreover, in response to the expansive demand shock, the central bank may increase interest rates to cool down the demand increase and prevent an excessive increase in prices. Finally, regarding the money stock (M2), we can see that it increases, which could be reflecting the fact that with the expansion of demand for goods and services, people and companies spend more.

Secondly, a positive shock in inflation is observed (called ”supply shock”). Regarding GDP, a positive response is obtained, implying that the increase in inflation is caused by an increase in aggregate demand. As for the policy rate, a positive reaction is observed, as the central bank would increase interest rates in response to higher levels of inflation to slow down spending and investment and, consequently, reduce inflation. Finally, regarding the money stock (M2), it is observed that a higher level of inflation leads to a decrease in the amount of money in the economy, as people and companies may spend less due to higher prices.

Thirdly, we can observe a contractive monetary policy shock (the FED policy rate is raised). On the one hand, a reduction in both GDP and inflation is observed, as increases in the policy rate lead to more expensive borrowing costs, which can slow down spending and investment and, consequently, decrease GDP and inflation. On the other hand, regarding the money stock (M2), a negative reaction is observed, which could be because increases in the policy rate discourage banks from lending, as it becomes more expensive for them to obtain the funds they need to finance their operations.

Finally, we can observe a positive shock in the money stock (M2). On the one hand, a positive response in GDP is observed, which could be because the increase in the money stock encourages aggregate demand and increases the production of goods and services in the economy. In this same sense, it could lead to an increase in inflation, especially if the supply of goods and services cannot meet the additional demand. However, as shown in the figure, the effect on inflation is not positive, indicating that the positive effect of the shock on GDP prevails. Lastly, a negative effect is observed on the FED policy rate.

Figure 6: Impact Effect



To finish, table 3 shows cut-off points of the impact effects.

Table 3: Cut-off points

Cut-off points		Demand shock	Supply shock	MP shock	M2 shock
Demand	Max	0.02936	-0.01132	0.94860	0.02885
	Min	0.01015	-0.02864	0.55091	0.00795
Inflation	Max	0.10805	0.03366	0.01753	-0.64055
	Min	0.05767	0.01174	-0.01372	-1.08647
FFR	Max	0.03871	0.29393	0.01636	0.04300
	Min	-0.00347	0.18004	-0.13435	-0.10844
M2	Max	0.91956	0.01146	-0.00002	0.21783
	Min	0.54308	-0.00437	-0.22348	-0.00390

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