Properties of New Keynesian Model that Can be Derived Analytically

Lawrence J. Christiano

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Outline

- Fisherian vs anti-Fisherian Debate:
  - How do you get inflation down (or, up)?
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- How does the Taylor Principle work to stabilize inflation in the equilibrium local to steady state?

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- *Anti-Fisherian effect*
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  - $cov\left(\pi_t, r_t\right) > 0$

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- Undetermined coefficients method, a_1, a_2, a_3 :

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- Substitute the solution into the equations and require that they hold for all possible $\bar{\pi}_t$:

$$\begin{aligned} a_3 &= a_1 + \phi\left(a_1 - 1\right) \\ a_1 &= \beta\delta a_1 + \kappa a_2 \\ a_2 &= a_2\delta - \left[a_3 - a_1\delta\right]. \end{aligned}$$

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- Want to know: a_1, a_3 when $\delta = 0$ and $\delta = 1$.

Solving the Model: Getting the a's

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- Now start rearranging stuff

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a_3 &= (1 + \phi)\,a_1 - \phi \\
a_1 &= \frac{\kappa}{1 - \beta\delta} a_2 \\
a_2 &= a_2\delta - (1 + \phi - \delta)\,a_1 + \phi
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$$a_1 = \frac{\phi}{\left[\frac{1-\beta\delta}{\kappa} + 1\right](1-\delta) + \phi}, \qquad a_3 = (1+\phi)\,a_1 - \phi.$$

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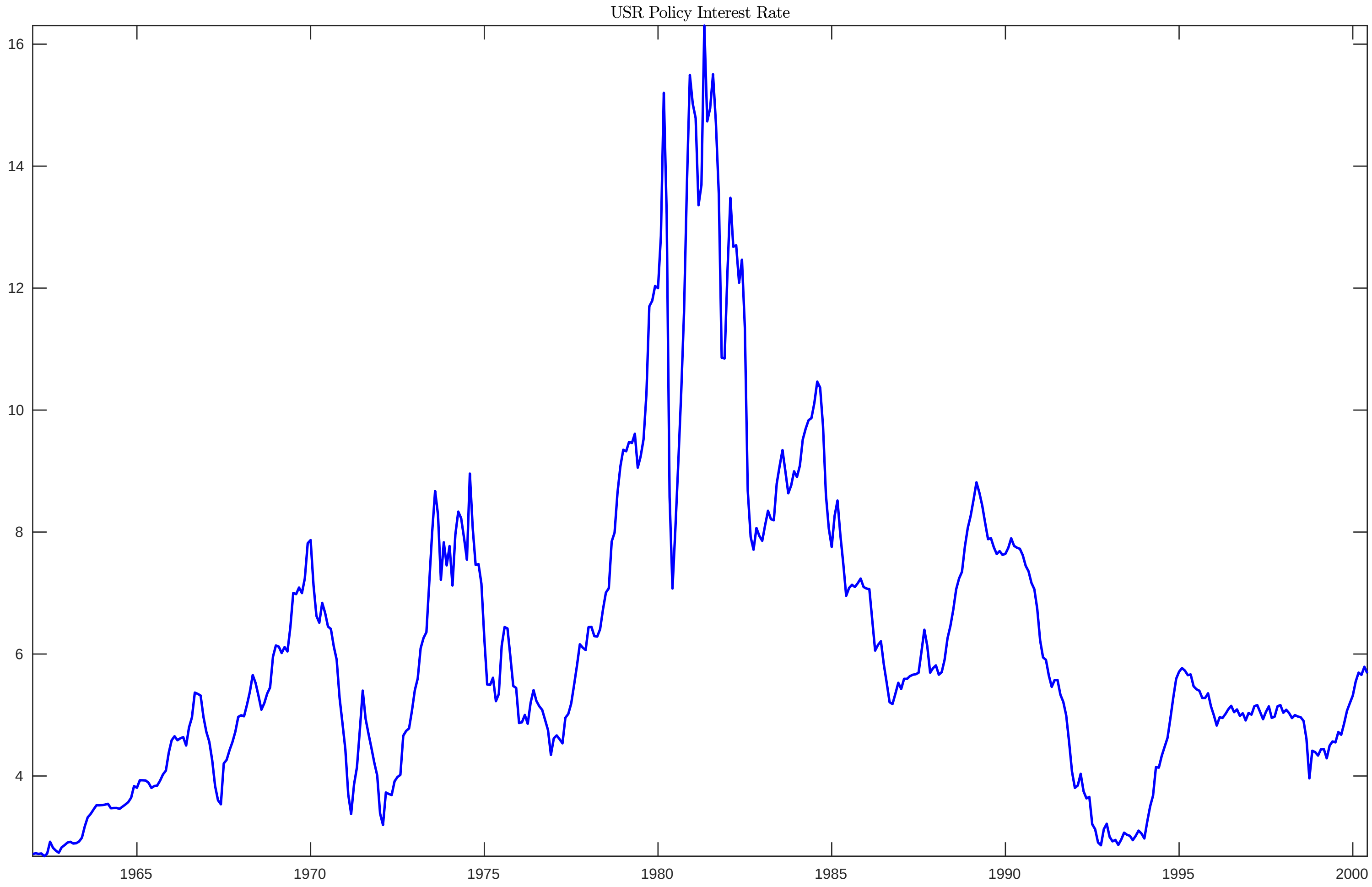
  - Temporary case, $\delta = 0$:
    - $a_1 = \frac{\phi}{\frac{1}{\kappa}+1+\phi} > 0, \qquad a_3 = -\frac{\phi/\kappa}{\frac{1}{\kappa}+1+\phi} < 0.$



USR Policy Interest Rate
16
14
12
10
8
6
4
1965
1970
1975
1980
1985
1990
1995
2000

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  - Forecasts of inflation remained stubbornly high.
  - Eventually, everyone realized that $\bar{\pi}_t$ was down permanently.
    - Fisherian effects kicked in and both interest rates and inflation fell.
    - Output returned to potential.

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Characterizing the Puzzle

- Phillips curve and output gap:

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    - $r_{t+s} = 0$ for $s = 0, ..., j-1$, $r_{t+j} = \theta$.

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  - Immediate policy:
    - $r_t = \theta$, $r_{t+s} = 0$, for $s = 1, 2, ..., j$.

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$$r_t = \overbrace{\phi}^{\text{Taylor principle, } \phi > 1} \pi_t.$$

- Result: impact on date t variables greater from forward guidance than from immediate policy.

One-period Forward Guidance ($j = 2$)

- Announcement at time t : $r_t = 0$, $r_{t+1} = \theta$, Taylor rule thereafter.

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- Because (i) there are no shocks, (ii) the model is purely forward looking and (iii) Taylor rule with $\phi > 1$ in place after $t + 1$:

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- What happens in period t?

One-period Forward Guidance ($j = 2$)

- Effect, in the period $t+1$, of $t+1$ policy action announced in t:

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$$\begin{aligned} r_t &= 0 \\ x_t &= x_{t+1} - [r_t - \pi_{t+1}] = -\left(\overbrace{1}^{\text{direct effect}} + \overbrace{\kappa}^{\text{indirect effect}} \right) r_{t+1} \end{aligned}$$

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so,

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\rightarrow \ \pi_t &= -[1+\beta+\kappa]\,\kappa\theta, \quad x_t = -(1+\kappa)\,\theta
\end{aligned}$$

Immediate Policy

- Announcement at time $t : r_t = \theta \neq 0$, $r_{t+1} = 0$ and Taylor rule thereafter.
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which is smaller than with one-period forward guidance:

$$\pi_t = -\left[1 + 2\kappa\right] \kappa\theta, \quad x_t = -\left(1 + \kappa\right) \theta$$

Intuition

- Consider j Period Forward Guidance.
  - Announcement at time t : $r_{t+j} = \theta \neq 0$ and $r_{t+s} = 0$ for $s = 0, 1, ..., j-1$. Switch to Taylor rule after $t+j$.

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&\vdots \\
x_t &= -\left(r_t - \pi_{t+1}\right) - \left(r_{t+1} - \pi_{t+2}\right) \\
&\quad -... - \left(r_{t+j-1} - \pi_{t+j}\right) - r_{t+j}
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- Change in r_{t+j} has a *direct* effect on x_t and an *indirect* effect.

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    - This channel holds fixed the real interest rates, $\left(r_{t+s} - \pi_{t+s+1}\right)$, $s = 0, ..., j-1$.
  - Indirect: change in r_{t+j} affects $\left(r_{t+s} - \pi_{t+s+1}\right)$, $0 \leq s \leq j-1$ in each date between now and $t+j$ by reducing inflation in each date.
    - The impact on x_t of the indirect effect is the *cumulative sum* (increasing in j) of the changes in the real interest rate.

Forward Guidance: Conclusion

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    - This offers what is perhaps the simplest resolution: in practice, announcements about policy actions far in the future have little impact on behavior because they are not credible.
    - In my presentation, I assumed 100% credibility.

How Does the Taylor Principle Work to Stabilize Inflation?

- Model

$$
\begin{aligned}
x_t &= E_t x_{t+1} - \left[r_t - E_t \pi_{t+1} - r_t^*\right] \\
r_t &= \phi_\pi \pi_t, \quad \phi_\pi > 1 \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
\Delta a_t &= \rho \Delta a_{t-1} + \varepsilon_t \\
r_t^* &= E_t \left(a_{t+1} - a_t\right) = \rho \Delta a_t.
\end{aligned}
$$

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x_t &= E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r_t^*] \\
r_t &= \phi_\pi \pi_t, \quad \phi_\pi > 1 \\
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\Delta a_t &= \rho \Delta a_{t-1} + \varepsilon_t \\
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\end{aligned}
$$

- Unique non-explosive solution:

$$
\pi_t = \gamma_1 \Delta a_t, x_t = \gamma_2 \Delta a_t, r_t = \gamma_3 \Delta a_t
$$

  – γ_i's ˜ undetermined coefficients.

Solving the Model

- Model and solution

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- Substitute solution into model:

$$
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\gamma_2 &= \rho \gamma_2 - \gamma_3 + \rho \gamma_1 + \rho \\
\gamma_3 &= \phi_\pi \gamma_1 \\
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\end{aligned}
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$$

- Real rate: $\tilde{r}_t = r_t - E_t \pi_{t+1} = \gamma_4 \Delta a_t$,

$$
\gamma_4 = \gamma_3 - \gamma_1 \rho.
$$

Solving the Model

- Each to verify:

$$r_t - E_t \pi_{t+1} = \overbrace{\psi}^{=\gamma_4} \Delta a_t, x_t = \overbrace{\frac{(1-\beta\rho)}{\kappa\left(\phi_\pi - \rho\right)}\psi}^{=\gamma_2} \Delta a_t, \pi_t = \overbrace{\frac{\psi}{\phi_\pi - \rho}}^{=\gamma_1} \Delta a_t$$

where

$$\psi \equiv \frac{\rho}{\frac{(1-\beta\rho)(1-\rho)}{\kappa(\phi_\pi - \rho)} + 1}.$$

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- For ϕ_π sufficiently large,

$$\psi \simeq \rho,\ r_t - E_t \pi_{t+1} \simeq r_t^*,\ \pi_t \simeq 0,\ x_t \simeq 0.$$

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- Big value of ϕ stabilizes equilibrium around first best.
  - However, requires very large value of ϕ_π.
  - For practical values, Taylor rule too weak, $\psi < \rho$ and $\gamma_2 > 0$.

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  - However, requires very large value of ϕ_π.
  - For practical values, Taylor rule too weak, $\psi < \rho$ and $\gamma_2 > 0$.
- Taylor principle:
  - real rate of interest increases when π_t high ($\psi > 0$ and $\phi > \rho$).
  - effects bigger with bigger ϕ_π.

Solving the Model

- The equations:

$$
\begin{aligned}
r_t &= \pi_t + \phi\left(\pi_t - \bar{\pi}_t\right) \\
\pi_t &= \beta \pi_{t+1} + \kappa x_t \\
x_t &= x_{t+1} - \left[r_t - \pi_{t+1}\right].
\end{aligned}
$$

- Substitute the solution in here:

$$
\begin{aligned}
a_3 &= a_1 + \phi\left(a_1 - 1\right) \\
a_1 &= \beta \delta a_1 + \kappa a_2 \\
a_2 &= a_2 \delta - \left[a_3 - a_1 \delta\right].
\end{aligned}
$$

- Rearranging:

$$
\begin{aligned}
a_3 &= (1+\phi)\, a_1 - \phi \\
a_1 &= \frac{\kappa}{1-\beta\delta} a_2 \\
a_2 &= a_2 \delta - \left[a_3 - a_1 \delta\right] = a_2 \delta - (1 + \phi - \delta)\, a_1 + \phi \\
\rightarrow a_2 &= -\frac{1+\phi-\delta}{1-\delta} a_1 + \frac{\phi}{1-\delta}
\end{aligned}
$$

Solving the Model

- Working on the second equation,

$$a_1 \frac{1-\beta\delta}{\kappa} = -\frac{(1+\phi-\delta)}{1-\delta} a_1 + \frac{\phi}{1-\delta}$$

then,

$$a_1 = \frac{\frac{\phi}{1-\delta}}{\frac{1-\beta\delta}{\kappa} + \frac{1+\phi-\delta}{1-\delta}} = \frac{\frac{\phi}{1-\delta}}{\frac{1-\beta\delta}{\kappa} + 1 + \frac{\phi}{1-\delta}} = \frac{\phi}{\left[\frac{1-\beta\delta}{\kappa} + 1\right](1-\delta) + \phi}$$

- Then,

$$a_3 = \frac{(1+\phi)\,\phi}{\left[\frac{1-\beta\delta}{\kappa} + 1\right](1-\delta) + \phi} - \phi$$

- So, when $\delta = 1 : a_1 = a_3 = 1$. When $\delta = 0$, get formulas for a_1, a_3 in main presentation. Go Back