Tarea

1)

a.
$$\rho V(m) = \log(c_t) + \frac{E\,V(m)}{dt}$$

$$E\frac{V(m)}{dt} = V_m\,dm + V_{mm}(dm)^2$$

~~$V_m dm = V_m(-c_t\,dt + m(1-\theta_t^s)$~~

$$V_m\,dm = V_m\left(-c_t + m_t[(1-\theta)r^b + \theta^s r_s]\right)dt + m\sigma\theta^s dz)$$

$$V_{mm}(dm)^2 = V_{mm}(m\sigma\theta^s)^2\,dt$$

$$V_m\,dm = V_m(-c + m(1-\theta^s)r^b + \theta^s r^s)dt$$

Resuelvo

$$\rho V(m) = \log(c_t) + V_m(-c + m[1-\theta^s]r^b + \theta^s r^s)$$

$$+\frac{1}{2}V_{mm}(m\sigma\theta^s)^2$$

Escaneado con CamScanner

$$\rho V(m) = \max_{c} \left(\log(c) - V_m c \right) +$$

$$\max_{\theta^s} \left(V_m m \left[(1-\theta^s) r^b + \theta^s r^s \right] + \frac{1}{2} V_{mm} (m\sigma \theta^s)^2 \right)$$

ii) CPO

$$\frac{\partial V(m)}{\partial c} = \frac{1}{c} - V_m = 0$$

$$\frac{1}{c} = V_m \Rightarrow c = \frac{1}{V_m}$$

$$\frac{\partial V_m}{\partial \theta^s} = V_m \left(-m r^b + r^s m \right) + V_{mm} (m\sigma)^2 \theta^s = 0$$

$$\theta^s = \frac{m r^b - r^s m}{(m\sigma)^2} \left(\frac{V_m}{V_{mm}} \right)$$

iii)

$$c = a m$$

Asumo que en el óptimo

$$\frac{1}{c} = V_m$$

$$\frac{1}{a m} = V_m$$

$$V = \int V_m \, dm = \int \frac{1}{am} dm \Rightarrow \frac{1}{a}\log(m) + b$$

(iV) de los CPO

$$\theta^s = \frac{mr^b - r^s m}{m\sigma^2}\left(\frac{V_m}{V_{mm}}\right)$$

$$V_m = \frac{1}{am} \qquad V_{mm} = -\frac{1}{a}\left(\frac{1}{m^2}\right)$$

$$= \frac{\cancel{m}(r^b - r^s)}{\cancel{m}\sigma^2} \cdot \left(\frac{1}{a\cancel{m}}\right)\left(\frac{a\cancel{m^2}}{1}\right)$$

$$\theta^s = \frac{r^s - r^b}{\sigma^2}$$

V)

$$\rho\left(\frac{1}{a}\log(m) + \frac{b}{a}\right) = \log(am) - \frac{am}{am} +$$

$$\cancel{\frac{1}{am} m\left[1 - \left(\frac{r^s - r^b}{\sigma^2}\right)\right] r^b + \theta}$$

$$\frac{1}{a\cancel{m}}(\cancel{m})\left[r^b + (r^s - r^b)\cdot\left(\frac{r^s - r^b}{\sigma^2}\right)\right.$$

$$\frac{1}{a}\left(r^b(\ldots) + \frac{(r^s - r^b)^2}{\sigma^2}\right)$$

$$\frac{1}{a}\int\frac{1}{m}\,dm$$

$$\frac{1}{2}\left(\frac{1}{a}\right)\left(\frac{1}{m^2}\right)\cancel{m^2}\sigma^2\left(\frac{(r^b-r^s)^2}{\sigma^4}\right)$$

$$\rho\left(\frac{1}{a}\log(m)+b\right)=\log(am)-1$$

$$+\frac{1}{a}\left[r^b+\left(\frac{r^s-r^b}{\sigma}\right)^2\right]$$

$$+\frac{1}{a}\left[r^b\left(\ldots\right)+\left(\frac{r^s-r^b}{\sigma}\right)^2\right]$$

$$-\frac{1}{2a}\left(\frac{r^b-r^s}{\sigma^2}\right)^2$$

$$\rho\left(\frac{1}{a}\log(m)+b\right)=\log a+\log m-1+\frac{1}{2a}\left(\frac{r^b-r^s}{\sigma}\right)^2+\frac{1}{a}r^b$$

$$\rho b=\log a+\log m\left(1-\frac{\rho}{a}\right)-1+\frac{1}{2a}\left(\frac{r^b-r^s}{\sigma}\right)^2$$

$$V = \frac{1}{a}\log(m)\ t$$

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(1)

$$\rho b = \log(a) + \log(m)\left(1 - \frac{\rho}{a}\right) - 1 + \frac{1}{2a}\left(\frac{r^b - r^s}{\sigma}\right)^2 + \frac{1}{a}r^2$$

(a) $\log(m)\cdot\left(1 - \frac{\rho}{a}\right) = 0$ } si esto se cumple la expresión se cumple para todo $m>0$

$$1 - \frac{\rho}{a} = 0$$

$$a = \rho$$

(2)

$$\rho(b) = \log(\rho) - 1 + \frac{1}{2\rho}\left(\frac{r^b - r^s}{\sigma}\right)^2 + \frac{1}{2\rho}r^b$$

$$b = \frac{1}{\rho}\log(\rho) - \frac{1}{\rho} + \frac{1}{2\rho^2}\left(\frac{r^b - r^s}{\sigma}\right)^2 + \frac{1}{2\rho^2}r^3$$

$$\log(m) = \left(\frac{a}{a-\rho}\right)\left[\log(a) - 1 + \frac{1}{2a}\left(\frac{r^b - r^s}{\sigma}\right)^2 + \frac{1}{a}r^b - \rho b\right]$$

Justus

2)

ii)

$$H = e^{-\rho t}(\log(c)) + \xi^m(m)(\mu^T) + Tr(\sigma^{m,\xi})^T \sigma m$$

~~$dm = -\frac{c}{\theta} + m(1-\theta$~~

$$dm = \left(-c + m\left(r^b + (r^s - r^b)\theta^s\right)\right)dt + m\sigma\theta^s dz_t$$

$$= m\left(r^b + (r^s - r^b)\theta^s - \frac{c}{m}\right)dt + m\sigma\theta^s dz_t$$

$$H = e^{-\rho t}\log(c) + \xi m\left(r^b + (r^s - r^b)\theta^s - \frac{c}{m}\right) + Tr(\xi\sigma^{m,\xi})^T m\sigma\theta^s$$

CPO

$$H_c = e^{-\rho t}\left(\frac{1}{c}\right) - \xi = 0$$

$$c = \frac{e^{-\rho t}}{\xi_t}$$

$$H_{\sigma s} = \xi\, n\,(r^s - r^b) + \cancel{\xi}\, \sigma^{\xi} \cdot n \cdot \sigma = 0$$

$$r^s - r^b = -\frac{(\sigma^{n,\xi} \cdot \sigma)}{\xi}$$

$\cancel{r^b}\ r^s$

Guess: $c_t = a\,n$

$a \qquad\qquad at$

III) Gauss

$$a m = \frac{e^{-\rho t}}{\xi} \Rightarrow \xi = \frac{e^{-\rho t}}{a m}$$

$$d\xi = \mu^{\xi} dt + \sigma^{\xi} dz$$

$$d\xi = -\frac{\rho e^{-\rho t}}{a m} dt - \frac{e^{-\rho t}}{a m^2}(dm) + \frac{e^{-\rho t}}{a m^3}(dm)^2$$

$$d\xi = -\frac{\rho e^{-\rho t}}{a m} dt - \frac{e^{-\rho t}}{a m^2}\left(m\left(r^b + (r^s - r^b)\theta^s - \frac{c}{m}\right)dt + m\sigma\theta^s dz\right) + \frac{e^{-\rho t}}{a m}\left(m\sigma\theta^s\right)^2 dt$$

$$\sigma^{\xi} = -\frac{e^{-\rho t}}{a m}\sigma\theta^s \Rightarrow \sigma^{\xi} = -\xi\sigma\theta^s$$

$$\mu = \frac{e^{-\rho t}}{a m}\left(-\rho - r^b - (r^s - r^b)\theta^s + \frac{c}{m} + (\sigma\theta^s)^2\right)$$

$$\frac{e^{-\rho t}}{a m}\left(-\rho - r^b - \cancel{\frac{(r^s - r^b)^2}{\sigma^2}} + a + \cancel{\frac{(r^s - r^b)^2}{\sigma^2}}\right)$$

$$\mu = \frac{e^{-\rho t}}{a m}\left(-\rho + a - r^b\right)$$

$$\mathcal{V}^{\xi} = \frac{e^{-\rho t}}{dm}\left(-p - \xi r^b + (r^s - r^b)\theta^s - \frac{c}{m}\right) + \sigma\theta^{\xi}$$

$$M_m = \xi_1\left(r^b + (r^s - r^b)\theta^s\right) + \xi_1\left(\sigma^{m,\xi}\cdot\sigma\cdot\theta^s\right)$$

$$\xi_1\left(r^b + \left(\sigma^{m,\xi}\cdot\sigma\right)\theta^s\right) + \xi_1\sigma^{n,\xi}\sigma\theta^s$$

$$M_m = \xi_1\left(r^b\right)$$

$$\mathcal{V}^{\xi_1}_{\theta} = \xi r^b$$

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