Jorge Sánchez Polo

Individual Exercise 1

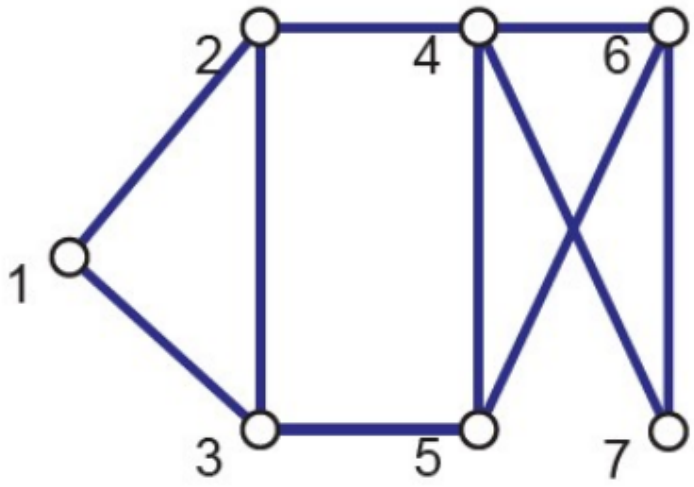

Figure 1

1. For the graph in the figure, complete the following:

(a) Determine a walk from vertex 2 to vertex 7 that is not a trail.
A walk that is not a trail is allowed to repeat edges, for instance, $\{2, e_4, 4, e_6, 6, e_5, 5, e_4, 4, e_6, 6, e_7, 7\}$.

(b) Determine a trail from vertex 2 to vertex 7 that is not a path.
A trail that is not a path is allowed to repeat vertices, for example, $\{2, e_4, 4, e_6, 6, e_5, 5, e_4, 4, e_7, 7\}$.

(c) Determine a path from vertex 2 to vertex 7.
We can take $\{2, e_4, 4, e_7, 7\}$.

(d) How many paths are there from vertex 1 to vertex 7? What is the geodesic distance between these two vertices? Is there a unique shortest path between them?
We have to look for all possible walks without repeated vertices or edges. These are:
$\{1, e_2, 2, e_4, 4, e_7, 7\}$
$\{1, e_2, 2, e_4, 4, e_6, 6, e_7, 7\}$
$\{1, e_2, 2, e_4, 4, e_5, 5, e_6, 6, e_7, 7\}$
$\{1, e_2, 2, e_3, 3, e_5, 5, e_4, 4, e_7, 7\}$
$\{1, e_2, 2, e_3, 3, e_5, 5, e_6, 6, e_4, 4, e_7, 7\}$
$\{1, e_2, 2, e_3, 3, e_5, 5, e_4, 4, e_6, 6, e_7, 7\}$
$\{1, e_2, 2, e_3, 3, e_5, 5, e_6, 6, e_7, 7\}$
$\{1, e_3, 3, e_2, 2, e_4, 4, e_7, 7\}$
$\{1, e_3, 3, e_2, 2, e_4, 4, e_6, 6, e_7, 7\}$
$\{1, e_3, 3, e_2, 2, e_4, 4, e_5, 5, e_6, 6, e_7, 7\}$

$\{1, e_3, 3, e_5, 5, e_4, 4, e_6, 6, e_7, 7\}$
$\{1, e_3, 3, e_5, 5, e_6, 6, e_4, 4, e_7, 7\}$
$\{1, e_3, 3, e_5, 5, e_6, 6, e_7, 7\}$
$\{1, e_3, 3, e_5, 5, e_4, 4, e_7, 7\}$
We see that there are 14 paths from 1 to 7 and the shortest path is $\{1, e_2, 2, e_4, 4, e_7, 7\}$ and it is unique, there is no path of the same length or smaller. So, the geodesic distance is 3.

(e) Obtain the adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$