

Problem 1: Comet

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Data of the problem:

```
clear; clc; close all;
muE = 398600; % mu Earth [km^3/s^2]
R_e = 6371; % Earth radius [km]
angular_rate = 2*pi/86400; % Angular rate of the Earth about its rotation axis [rad/s]
phi = deg2rad(40); % Latitude (40 deg)
r1 = [-735761, 506988, -540928]; % Position vector in S1 (SEZ) [km]
v1 = [25.3451, 62.8133, 29.5876]; % Velocity vector in S1 (SEZ) [km/s]
t_0 = 16200; % Time from 13:30 until 18:00 [s]
```

a) Initial position and velocity in ECI

The goal of this first section is to retrieve the position and velocity vector in ECI from the given position and velocity vectors observed from sea level and given in SEZ. In order to perform such computation, the function [SEZ2ECI.m](#) has been developed.

Inside this function, the first step consists on knowing the longitude of the observer with respect to the O_{xy} plane, considering the rotation of the Earth and the time passing (t_{pass}) from 13.30h to 18.00h., which is 16200 seconds.

$$\varphi = \Omega_{Earth} \cdot t_{pass}$$

Once the longitude φ is computed, the following step will consist on computing the position vector in SEZ frame.

$$\vec{r}|_2^P = R_e \vec{k}_2 + \vec{r}_1$$

After that, two rotations need to be performed: one about z-axis with angle φ , and one about x-axis with angle $-(90 - \phi)$, being ϕ the latitude of the observer (40°).

$$\vec{r}|_0 = R_1 \cdot R_2 \vec{r}|_2$$

The following step once computed the position vector, is to compute the velocity vector. In order to do that, Coriolis' theorem has been applied.

$$\vec{v}|_0 = \frac{d\vec{r}|_0}{dt} = \frac{d\vec{r}|_2}{dt} + \vec{\omega}_{20} \times \vec{r}|_2 = \vec{v}|_2 + \vec{\omega}_{20} \times \vec{r}|_2$$

Note that the units of the following results are km for the position vector, and km/s for the velocity vector.

```
[r0, v0] = SEZ2ECI(r1,v1,R_e, phi,t_0)
```

```

r0 = 3×1
105 ×
    -8.0609
    -6.2125
    2.2002
v0 = 3×1
    2.0545
    1.4087
    -0.3969

```

b) Classical orbital elements and hyperbola characteristics

Given the position and the velocity vector in ECI, classical orbital elements (COEs) can be easily computed. In order to do that, the function [stat2coe.m](#), which return a vector with the 6 COEs, has been implemented.

```
coe = stat2coe([r0,v0],muE);
```

```
a = coe(1)
```

```
a = -7.1212e+04
```

```
e = coe(2)
```

```
e = 1.5665
```

```
i = coe(3)
```

```
i = 0.8052
```

```
RAAN = coe(4)
```

```
RAAN = -2.6943
```

```
omega = coe(5)
```

```
omega = 2.4807
```

```
theta = coe(6)
```

```
theta = -2.1832
```

After computing the 6 COEs of the comet, we are asked to compute the impact parameter B , the turning angle δ , and the hyperbolic excess velocity v_h of the comet's trajectory. This calculations can be carried out by means of the following formulae set:

$$\cos(\theta_\infty) = \frac{-1}{e}$$

```
theta_inf = acos(-1/e) % True anomaly of the asymptotes [rad]
```

```
theta_inf = 2.2631
```

$$\beta = \pi - \theta_\infty$$

```
beta = pi-theta_inf % Related angle, angle of the asymptotes with the line of apses
```

beta = 0.8784

$$\delta = \pi - 2\beta$$

delta = pi - 2*beta % Turning angle [rad]

delta = 1.3847

$$B = e \cdot a \cdot \sin(\beta)$$

B = -e*a*sin(beta) % Impact parameter

B = 8.5870e+04

$$v_h = \sqrt{\frac{-\mu}{a^3}}$$

vH = sqrt(-muE/a) % Excess hyperbolic velocity [km/s]

vH = 2.3659

c) Time to pericenter and position and velocity after 5 h

This section is devoted to determine the time until the comet passes through the pericenter and also the position and velocity of the comet in ECI after 90h. The first part of the task is carried out by considering the concepts of 'Hyperbolic anomaly' (H), 'Mean anomaly' (M_h) and 'Mean angular rate' (n_h).

The hyperbolic anomaly H is related to the true anomaly θ by means of the two following expressions, implemented in the function [True2EccH.m](#):

$$\sinh(H) = \frac{\sqrt{e^2 - 1} \cdot \sin(\theta)}{1 + e \cdot \cos(\theta)} \quad \cosh(H) = \frac{e + \cos(\theta)}{1 + e \cdot \cos(\theta)}$$

H = True2EccH(theta, e) % Eccentric anomaly

H = -2.9904

The mean anomaly M_h is defined as the fraction of an orbital period that has elapsed since the orbiting body passed the periapsis, expressed as an angle. In the hyperbolic case it is expressed as:

$$M_h = e \cdot \sinh(H) - H$$

M_h = e*sinh(H) - H % Mean anomaly

M_h = -12.5523

The mean angular rate is the result of dividing a full period angle (2π) by the total period of the orbit. Its expression in a hyperbolic orbit is given by:

$$n_h = \sqrt{-\frac{\mu}{a^3}}$$

```
n_h = sqrt(-muE/(a^3))
```

```
% Mean angular rate [rad/s]
```

```
n_h = 3.3223e-05
```

Finally, the relation between M_h and n_h can be applied to obtain the time until passing the periapsis

$$M_h = n_h \cdot (t - t_0)$$

where t_0 is the time until passing the periapsis, and t is set to 0. Solving the equation, the result obtained is $t_0 = 104.95h$.

```
t_periapsis = -M_h/n_h
```

```
% Time until passing the periapsis [s]
```

```
t_periapsis = 3.7782e+05
```

With the expressions detailed above, one may find the true anomaly after 90 hours from the observation. Note that the function [Ecc2TrueH.m](#) has been implemented to pass from the hyperbolic anomaly H and the eccentricity e to the true anomaly θ . The function [Ecc2TrueH.m](#) makes use of the following expressions to obtain θ

$$\sin(\theta) = \frac{-\sqrt{e^2 - 1} \cdot \sinh(H)}{1 - e \cdot \cosh(H)} \quad \cos(\theta) = \frac{\cosh(H) - e}{1 - e \cdot \cosh(H)}$$

It is also important to note that, in order to retrieve H from the expression for M_h , newton method (see [newton.m](#)) has been implemented in function [Mean2EccH.m](#) since the relation between both variables is not linear. Results are the following:

```
t_90 = 90*3600
```

```
% 90 hours to seconds
```

```
t_90 = 324000
```

```
M_h90 = n_h*(t_90 - t_periapsis)
```

```
% Mean anomaly at 90h
```

```
M_h90 = -1.7880
```

```
H_90 = Mean2EccH(M_h90,e)
```

```
% Hyperbolic anomaly at 90h
```

```
Iteration: 0 x= -1.78797 f(x) = 9.74855E-01
```

```
Iteration: 1 x= -1.53229 f(x) = 1.36103E-01
```

```
Iteration: 2 x= -1.48359 f(x) = 4.02626E-03
```

```
Iteration: 3 x= -1.48206 f(x) = 3.83359E-06
```

```
Solution converged
```

```
Iteration: 4 x= -1.48206 f(x) = 3.48432E-12
```

```
H_90 = -1.4821
```

```
theta_90 = Ecc2TrueH(H_90,e)
```

```
% True anomaly at 90h
```

```
theta_90 = -1.8597
```

Since the rest of the COEs are the same as before except θ , one may retrieve position and velocity vectors in ECI.

```
X = coe2stat([a,e,i,RAAN,omega,theta_90],muE);
```

```
r_90 = X(1:3) % Position vector in ECI
```

```
r_90 = 3×1  
105 ×  
-1.0451  
-1.3376  
0.7845
```

```
v_90 = X(4:6) % Velocity vector in ECI
```

```
v_90 = 3×1  
2.4814  
1.8289  
-0.5989
```

Finally, these vectors in ECI can be transformed into the same vectors in SEZ by means of [ECI2SEZ.m](#).

```
[r_90_1,v_90_1] = ECI2SEZ(r_90,v_90,R_e,phi,t_0 + t_90)
```

```
r_90_1 = 3×1  
105 ×  
-0.8925  
-1.6357  
0.0930  
v_90_1 = 3×1  
-6.1635  
5.9382  
-8.2772
```

d) COE of fragments A and B

In this section we need to analyze a controlled explosion that splits the comet into two pieces A and B 90 hours after the observation. It is known that part A enters an elliptic orbit with $e = 0.7$ and $\theta = -110^\circ$, coplanar with the initial hyperbolic orbit. It is required to determine the rest of the COEs of the orbit A, as well as all the COEs of orbit B.

First, as the orbit is coplanar, the angles i and Ω are the same as in the initial orbit.

```
e_A = 0.7 % Eccentricity
```

```
e_A = 0.7000
```

```
theta_A = deg2rad(-110) % True anomaly
```

```
theta_A = -1.9199
```

```
i_A = i % Incidence angle
```

```
i_A = 0.8052
```

```
RAAN_A = RAAN % Right ascension of the ascending node
```

```
RAAN_A = -2.6943
```

In order to obtain the new argument of periapsis, the following relation $\omega_A = \omega + \Delta\theta$ is applied.

```
omega_A = omega + (theta_90 - theta_A) % Argument of periapsis
```

```
omega_A = 2.5409
```

Hence, the only COE left is the semi-major axis. In order to compute the semi-major axis, we will apply the geometric relation between the semi-major axis, the semi-latus rectum and the eccentricity in an elliptic orbit. We can compute the semi-latus rectum knowing the radius of the orbit for the specified true anomaly. Note that the radius of the orbit of body A will be the same as the norm of the position vector of the initial comet after 90h

($r_A = |\vec{r}|$). Hence, the semi-latus rectum may be calculated by:

$$p_A = r_A \cdot (1 + e \cdot \cos(\theta_A))$$

```
r_A = norm(r_90); % Radius of the orbit at the time of the crash [km]
p_A = r_A*(1+e_A*cos(theta_A)); % Semi-latus rectum of the elliptic orbit [km]
```

Once these parameters are computed, we are able to compute the semi-major axis, which is the only COE that we did not calculate before:

$$a_A = \frac{p_A}{1 - e_A^2}$$

```
a_A = p_A/(1-e_A^2) % Semi-major axis [km]
```

```
a_A = 2.7887e+05
```

Once the COEs for orbit of body A are computed, we can make use again of the [coe2stat.m](#) function in order to compute the state vector (i.e., the position and velocity vectors for body A).

```
X_A = coe2stat([a_A,e_A,i_A,RAAN_A, omega_A,theta_A],muE);
```

```
% Unpack the state vector into position and velocity
r_A_0 = X_A(1:3) % Position vector of body A in ECI [km]
```

```
r_A_0 = 3x1
10^5 x
-1.0451
-1.3376
0.7845
```

```
v_A_0 = X_A(4:6) % Velocity vector of body A in ECI [km/s]
```

```
v_A_0 = 3x1
1.5938
0.4612
0.2847
```

At the moment of the crash, the position vector of bodies A and B must coincide ($\vec{r} = \vec{r}_A = \vec{r}_B$), but the velocity of the body B must be calculated by conservation of linear momentum:

$$\vec{v} = \frac{1}{2}\vec{v}_A + \frac{1}{2}\vec{v}_B \rightarrow \vec{v}_B = 2\vec{v} - \vec{v}_A$$

```
r_B_0 = r_A_0 % Position vector of body B in ECI [km]
```

```

r_B_0 = 3×1
105 ×
-1.0451
-1.3376
0.7845

```

```

v_B_0 = 2*v_90 - v_A_0 % Velocity vector of body B in ECI [km/s]

```

```

v_B_0 = 3×1
3.3689
3.1965
-1.4825

```

When the position and the velocity vector are known, one may use the previously mentioned [stat2coe.m](#) to compute the COEs of the orbit B.

```

coeB = stat2coe([r_B_0,v_B_0],muE);

```

```

a_B = coeB(1) % Semi-major axis

```

```

a_B = -2.0439e+04

```

```

e_B = coeB(2) % Eccentricity

```

```

e_B = 2.1150

```

```

i_B = coeB(3) % Inclination angle

```

```

i_B = 0.8052

```

```

RAAN_B = coe(4) % RAAN

```

```

RAAN_B = -2.6943

```

```

omega_B = coe(5) % Argument of periapsis

```

```

omega_B = 2.4807

```

```

theta_B = coe(6) % True anomaly

```

```

theta_B = -2.1832

```

e) to circularize the orbit of fragment A

The last section of this first homework consist on computing the ΔV required to circularize the orbit of body A around its periapsis.

$$\Delta V = \sqrt{\frac{\mu}{r_F}} - \sqrt{\frac{2\mu}{r_F} - \frac{2\mu}{r_A + r_F}}$$

where r_A is the radius at the periapsis and r_F is the radius of the apoapsis.

$$r_A = a(1 - e)$$

$$r_F = a(1 + e)$$

$$rF = a_A(1-e_A)$$

$$rF = 8.3662e+04$$

$$rA = a_A(1+e_A)$$

$$rA = 4.7409e+05$$

$$\text{deltaV} = \sqrt{\mu E / rF} - \sqrt{(2 * \mu E / rA) - (2 * \mu E / (rF + rA))}$$

$$\text{deltaV} = 1.6805$$