Problem 1: Comet

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Data of the problem:

```
clear; clc; close all;
muE = 398600;                                   % mu Earth [km^3/s^2]
R_e = 6371;                                     % Earth radius [km]
angular_rate = 2*pi/86400;                      % Angular rate of the Earth about its ro
phi = deg2rad(40);                              % Latitude (40 deg)
r1 = [-735761,506988,-540928];                  % Position vector in S1 (SEZ) [km]
v1 = [25.3451, 62.8133, 29.5876];               % Velocity vector in S1 (SEZ) [km/s]
t_0 = 16200;                                    % Time from 13:30 until 18:00 [s]
```

a) Initial position and velocity in ECI

The goal of this first section is to retrieve the position and velocity vector in ECI from the given position and velocity vectors observed from sea level and given in SEZ. In order to perform such computation, the function SEZ2ECI.m has been developed.

Inside this function, the first step consist on knowing the longitude of the observer with respect to the O_{xy} plane, considering the rotation of the Earth and the time passing (t_{pass}) from 13.30h to 18.00h., which is 16200 seconds.

$$\varphi = \Omega_{Earth} \cdot t_{pass}$$

Once the longitude φ is computed, the following step will consist on computing the position vector in SEZ frame

$$\vec{r}\,|_2^P = R_e \vec{k_2} + \vec{r_1}$$

After that, we must perform two rotations (one about z-axis with angle φ and one about x-axis with angle $-(90-\phi)$, being ϕ the latitude of the observer (40°).

$$\vec{r}\,|_0 = {}_0R_1 \cdot {}_1R_2 \vec{r}\,|_2$$

The following step once computed the position vector is to compute the velocity vector. In order to do that, Coriolis' theorem has been applied

$$\vec{v}\,|_0 = \frac{\mathrm{d}\vec{r}\,|_0}{\mathrm{d}t}|_2 = \frac{\mathrm{d}\vec{r}\,|_2}{\mathrm{d}t}|_2 + \vec{\omega_{20}} \times \vec{r}\,|_2 = \vec{v}\,|_2 + \vec{\omega_{20}} \times \vec{r}\,|_2$$

```
[r0, v0] = SEZ2ECI(r1,v1,R_e, phi,t_0)
```

```
r0 = 3×1
10^5 ×
   -8.0609
   -6.2125
    2.2002
v0 = 3×1
    2.0545
    1.4087
   -0.3969
```

b) Classical orbital elements and hyperbola characteristics

Given the position and the velocity vector in ECI, classical orbital elements (COEs) can be easily computed. In order to do that computation, the function stat2coe.m, which return a vector with the 6 COEs, has been implemented.

```
coe = stat2coe([r0,v0],muE);

a = coe(1)
```

```
a = -7.1212e+04
```

```
e = coe(2)
```

```
e = 1.5665
```

```
i = coe(3)
```

```
i = 0.8052
```

```
RAAN = coe(4)
```

```
RAAN = -2.6943
```

```
omega = coe(5)
```

```
omega = 2.4807
```

```
theta = coe(6)
```

```
theta = -2.1832
```

After computing the 6 COEs of the comet, we are asked to compute the impact parameter B, the turning angle δ, and the hyperbolic excess velocity v_h of the comet's trajectory. This calculations can be carried out by means of the following formulae set:

$$cos(\theta_\infty) = \frac{-1}{e}$$

```
theta_inf = acos(-1/e)          % True anomaly of the asymptotes
```

```
theta_inf = 2.2631
```

$$\beta = \pi - \theta_\infty$$

```
beta = pi-theta_inf                    % [rad]
```

```
beta = 0.8784
```

$$\delta = \pi - 2\beta$$

```
delta = pi -2*beta                 % Turning angle
```

```
delta = 1.3847
```

$$B = e \cdot a \cdot sin(\beta)$$

```
B = -e*a*sin(beta)                 % Impact parameter
```

```
B = 8.5870e+04
```

$$v_h = \sqrt{\frac{-\mu}{a^3}}$$

```
vH = sqrt(-muE/a)                  % Excess hyperbolic velocity [km/s]
```

```
vH = 2.3659
```

c) Time to pericenter and position and velocity after 5 h

This section is devoted to determine the time until the comet passes through the pericenter and also the position and velocity of the comet in ECI after 90h.

The first part of the task is carried out by considering the concepts of 'Hyperbolic anomaly' (H), '*Mean anomaly'* (M_h) and *'Mean angular rate'* (n_h).

The hyperbolic anomaly H is related to the true anomaly θ by means of the two following expressions, implemented in the function True2EccH.m:

$$sinh(H) = \frac{\sqrt{e^2 - 1} \cdot sin(\theta)}{1 + e \cdot cos(\theta)} \qquad cosh(H) = \frac{e + cos(\theta)}{1 + e \cdot cos(\theta)}$$

```
H = True2EccH(theta, e)                % Eccentric anomaly
```

```
H = -2.9904
```

The mean anomaly M_h is defined as the fraction of an orbital period that has ellapsed since the orbiting body passed the periapsis, expressed as an angle. In the hyperbolic case it is expressed as

$$M_h = e \cdot sinh(H) - H$$

```
M_h = e*sinh(H) - H                    % Mean anomaly
```

```
M_h = -12.5523
```

The mean angular rate is the result of dividing a full period angle (2π) by the total period of the orbit. Its expression in a hyperbolic orbit is given by

$$n_h = \sqrt{-\frac{\mu}{a^3}}$$

```
n_h = sqrt(-muE/(a^3))                    % Mean angular rate [rad/s]
```

```
n_h = 3.3223e-05
```

Finally, we can apply the relation between M_h and n_h to obtain the time until passing the periapsis

$$M_h = n_h \cdot (t - t_0)$$

where t_0 is the time until passing the periapsis, and t is set to 0. Solving the equation, the result obtained is $t_0 = 104.95h$.

```
t_periapsis = -M_h/n_h                    % Time until passing the periapsis [s]
```

```
t_periapsis = 3.7782e+05
```

With the expressions detailed above, one may find the true anomaly after 90 hours from the observation. Note that the function Ecc2TrueH.m has been implemented to pass from the hyperbolic anomaly H and the eccentricity e to the true anomaly θ. The function Ecc2TrueH.m makes use of the following expressions to obtain θ

$$sin(\theta) = \frac{-\sqrt{e^2-1} \cdot sinh(H)}{1 - e \cdot cosh(H)} \qquad cos(\theta) = \frac{cosh(H) - e}{1 - e \cdot cosh(H)}$$

We also must note that, in order to retrieve $H$from the expression for M_h, newton method (see newton.m) has been implemented in function Mean2EccH.m since the relation between both variables is not linear. Results are the following

```
t_90 = 90*3600                                    % 90 hours to seconds
```

```
t_90 = 324000
```

```
M_h90 = n_h*(t_90 - t_periapsis)                  % Mean anomaly at 90h
```

```
M_h90 = -1.7880
```

```
H_90 = Mean2EccH(M_h90,e)                         % Hyperbolic anomaly at 90h
```

```
Iteration: 0  x= -1.78797  f(x) = 9.74855E-01
Iteration: 1  x= -1.53229  f(x) = 1.36103E-01
Iteration: 2  x= -1.48359  f(x) = 4.02626E-03
Iteration: 3  x= -1.48206  f(x) = 3.83359E-06

Solution converged
Iteration: 4  x= -1.48206  f(x) = 3.48432E-12
H_90 = -1.4821
```

```
theta_90 = Ecc2TrueH(H_90,e)                      % True anomaly at 90h
```

```
theta_90 = -1.8597
```

Since the rest of the COEs are the same as before except θ, one may retrieve position and velocity vectors in ECI.

```
X = coe2stat([a,e,i,RAAN,omega,theta_90],muE);
r_90 = X(1:3)                                        % Position vector in ECI
```

```
r_90 = 3×1
10^5 ×
   -1.0451
   -1.3376
    0.7845
```

```
v_90 = X(4:6)                                        % Velocity vector in ECI
```

```
v_90 = 3×1
    2.4814
    1.8289
   -0.5989
```

Finally, we can transform these vectors in ECI into the same vectors in SEZ by means of ECI2SEZ.m.

```
[r_90_1,v_90_1] = ECI2SEZ(r_90,v_90,R_e,phi,t_0 + t_90)
```

```
r_90_1 = 3×1
10^5 ×
   -0.8925
   -1.6357
    0.0930
v_90_1 = 3×1
   -6.1635
    5.9382
   -8.2772
```

d) COE of fragments A and B

In this section we need to analyze a controlled explosion that splits the comet into two pieces A and B 90 hours after the observation. We know that part A enters a elliptic orbit with $e = 0.7$ and $\theta = -110^{\circ}$ coplanar with the initial hyperbolic orbit, and we need to determine the rest of the COEs of the orbit A, as well as all the COEs of orbit B.

First, as the orbit is coplanar, the angles i and Ω are the same as in the initial orbit

```
e_A = 0.7                          % Eccentricity
```

```
e_A = 0.7000
```

```
theta_A = deg2rad(-110)            % True anomaly
```

```
theta_A = -1.9199
```

```
i_A = i                            % Incidence angle
```

```
i_A = 0.8052
```

```
RAAN_A = RAAN                              % Right ascension of the ascending node
```

```
RAAN_A = -2.6943
```

In order to obtain the new argument of periapsis, we will apply the following relation $\omega_A = \omega + \Delta\theta$

```
omega_A = omega + (theta_90 - theta_A) % Argument of periapsis
```

```
omega_A = 2.5409
```

Hence, the only COE left is the semi-major axis. In order to compute the semi-major axis, we will apply the geometric relation between the semi-major axis, the semi-latus rectum and the eccentricity in an elliptic orbit. We can compute the semi-latus rectum if we know the radius of the orbit for the specified true anomaly. Note that the radius of the orbit of body A will be the same as the norm of the position vector of the initial comet after 90h ($r_A = |\vec{r}|$). Hence, the semi-latus rectum may be calculated by

$$p_A = r_A \cdot (1 + e \cdot cos(\theta_A))$$

```
r_A = norm(r_90);                           % Radius of the orbit at the time of the crash [km]
p_A = r_A*(1+e_A*cos(theta_A));             % Semi-latus rectum of the elliptic orbit [km]
```

Once these parameters are computed, we are able to compute the semi-major axis, which is the only COE that we did not calculate before

$$a_A = \frac{p_A}{1 - e_A^2}$$

```
a_A = p_A/(1-e_A^2)                         % Semi-major axis [km]
```

```
a_A = 2.7887e+05
```

Once the COEs for orbit of body A are computed, we can make use again of the coe2stat.m function in order to compute the state vector (i.e., the position and velocity vectors for body A)

```
X_A = coe2stat([a_A,e_A,i_A,RAAN_A, omega_A,theta_A],muE);

% Unpack the state vector into position and velocity
r_A_0 = X_A(1:3)                          % Position vector of body A in ECI [km]
```

```
r_A_0 = 3×1
10^5 ×
   -1.0451
   -1.3376
    0.7845
```

```
v_A_0 = X_A(4:6)                          % Velocity vector of body A in ECI [km/s]
```

```
v_A_0 = 3×1
    1.5938
    0.4612
    0.2847
```

At the moment of the crash, the position vector of bodies A and B must coincide ($\vec{r} = \vec{r_A} = \vec{r_B}$), but the velocity of the body B must be calculated by conservation of linear momentum

$$\vec{v} = \frac{1}{2}\vec{v_A} + \frac{1}{2}\vec{v_B} \rightarrow \vec{v_B} = 2\vec{v} - \vec{v_A}$$

```
r_B_0 = r_A_0                                  % Position vector of body B in ECI [km]
```

```
r_B_0 = 3×1
10^5  ×
   -1.0451
   -1.3376
    0.7845
```

```
v_B_0 = 2*v_90 - v_A_0                         % Velocity vector of body B in ECI [km/s]
```

```
v_B_0 = 3×1
    3.3689
    3.1965
   -1.4825
```

When the position and the velocity vector are known, one may use the previously mentioned stat2coe.m to compute the COEs of the orbit B.

```
coeB = stat2coe([r_B_0,v_B_0],muE);

a_B = coeB(1)                                 % Semi-major axis
```

```
a_B = -2.0439e+04
```

```
e_B = coeB(2)                                 % Eccentricity
```

```
e_B = 2.1150
```

```
i_B = coeB(3)                                 % Inclination angle
```

```
i_B = 0.8052
```

```
RAAN_B = coe(4)                               % RAAN
```

```
RAAN_B = -2.6943
```

```
omega_B = coe(5)                              % Argument of periapsis
```

```
omega_B = 2.4807
```

```
theta_B = coe(6)                              % True anomaly
```

```
theta_B = -2.1832
```

e) ΔV to circularize the orbit of fragment A

The last section of this first homework consist on computing the ΔV required to circularize the orbit of body A around its periapsis.

$$\Delta V = \sqrt{\frac{\mu}{r_F}} - \sqrt{\frac{2\mu}{r_F} - \frac{2\mu}{r_A + r_F}}$$

where r_A is the radius at the periapsis and r_F is the radius of the apoapsis

$$r_A = a(1 - e)$$

$$r_F = a(1 + e)$$

```
rF = a_A*(1-e_A)
```

```
rF = 8.3662e+04
```

```
rA = a_A*(1+e_A)
```

```
rA = 4.7409e+05
```

```
deltaV = sqrt(muE/rF) - sqrt((2*muE/rA)-(2*muE/(rF+rA)))
```

```
deltaV = 1.6805
```