Guía Resuelta

P1. Determine las siguientes integrales indefinidas:

$$a)\int(3x^2-2x+5)dx=3\cdot\frac{x^3}{3}-\frac{2x^2}{2}+5x+C=x^3-x^2+5x+C$$

$$\text{b)}\int\left(\frac{1}{x}-e^x+5x\right)dx=\ln(x)-e^x+\frac{5x^2}{2}+C$$

$$c)\int(5e^x-x-\sqrt{x})dx=\int\left(5e^x-x-x^{1/2}\right)dx=5e^x-x-\frac{x^{3/2}}{3/2}+C$$

$$d)\int\left(\frac{2}{x}+5x^{0,4}\right)dx=2\ln(x)+\frac{x^{1,4}}{1,4}+C$$

$$e)\int\left(\frac{2}{\sqrt{x}}+\sqrt[3]{x}\right)dx\int\left(\frac{2}{x^{1/2}}+x^{1/3}\right)dx=\int\left(2x^{-1/2}+x^{1/3}\right)dx=2\cdot\frac{x^{1/2}}{1/2}+\frac{x^{4/3}}{4/3}+C$$

$$f)\int\left(\frac{3}{x^2}-4x^{-5}\right)dx=\int(3x^{-2}-4x^{-5})dx=\frac{3x^{-1}}{-1}-\frac{4x^{-4}}{-4}+C=-3x^{-1}+x^{-4}+C$$

P2. Identifique el método de integración a utilizar y calcule las siguientes integrales:

$$a)\int 2(x+1)^5dx=\int 2u^5du=2\cdot\frac{u^6}{6}+C=2\cdot\frac{(x+1)^6}{6}+C$$

$u=x+1$
$du=dx$

$$b)\int\sqrt{3x+2}\;dx=\int(3x+2)^{1/2}dx=\int u^{1/2}\frac{du}{3}=\frac{1}{3}\cdot\int u^{1/2}du=\frac{1}{3}\cdot\frac{u^{3/2}}{3/2}+C=\frac{1}{3}\cdot\frac{(3x+2)^{3/2}}{3/2}+C$$

$u=3x+2$
$du=3dx$
$\frac{du}{3}=dx$

$$c)\int \frac{4x^2}{x^3+1}dx = \int \frac{4}{u}\cdot\frac{du}{3} = \frac{4}{3}\cdot\int \frac{1}{u}du = \frac{4}{3}\ln(u) + C = \frac{4}{3}\ln\left(x^3+1\right) + C$$

$u = x^3 + 1$
$du = 3x^2dx$
$\frac{du}{3} = x^2dx$

$$d)\int 2x^3e^{2x^4+5}dx = \int 2\cdot e^u\frac{du}{8} = \frac{2}{8}\cdot\int e^u du = \frac{1}{4}\cdot e^u + C = \frac{1}{4}\cdot e^{2x^4+5} + C$$

$u = 2x^4 + 5$
$\mathrm{d}u = 8x^3\,\mathrm{d}x$
$\frac{\mathrm{d}u}{8} = x^3\,\mathrm{d}x$

$$e)\int 3x^2\ln(x)dx = \ln(x)\cdot x^3 - \int x^3\cdot\frac{1}{x}dx = \ln(x)\cdot x^3 - \int x^2dx = \ln(x)\cdot x^3 - \frac{x^3}{3} + C$$

$u = \ln(x)$	$\mathrm{d}v = 3x^2dx$
$\mathrm{d}u = \frac{1}{x}\mathrm{d}x$	$v = 3\cdot\frac{x^3}{3} = x^3$

$$f)\int 3xe^{3x}dx = 3x\cdot\frac{e^{3x}}{3} - \int\frac{e^{3x}}{3}\cdot 3\,dx = x\cdot e^{3x} - \int e^{3x}dx = x\cdot e^{3x} - \frac{e^{3x}}{3} + C$$

$u = 3x$	$\int \mathrm{d}v = \int \mathrm{e}^{3x}\,\mathrm{d}x$
$\mathrm{d}u = 3\,\mathrm{d}x$	$v = \frac{\mathrm{e}^{3x}}{3}$

$$g)\int\ln\left(\sqrt[3]{x}\right)dx = \int\ln\left(x^{1/3}\right)dx = \int\frac{1}{3}\cdot\ln(x)dx = \frac{1}{3}\int\ln(x)dx = \frac{1}{3}\cdot\left(\ln(x)\cdot x - \int x\cdot\frac{1}{x}dx\right)$$

$$= \frac{1}{3}\cdot\left(\ln(x)\cdot x - \int dx\right)$$

$$= \frac{1}{3}\cdot\left(\ln(x)\cdot x - x\right) + C$$

$u = \ln(x)$	$\int\mathrm{d}v = \int\mathrm{d}x$
$\mathrm{d}u = \frac{1}{x}\mathrm{d}x$	$v = x$

$$h)\ \int xe^{-0,5x}dx = x\cdot\frac{e^{-0.5x}}{-0,5} - \int \frac{e^{-0,5x}}{-0,5}dx = x\cdot\frac{e^{-0.5x}}{-0,5} - \frac{1}{-0,5}\int e^{-0,5x}\,dx = x\cdot\frac{e^{-0.5x}}{-0,5} + \frac{1}{0,5}\cdot\frac{e^{-0,5x}}{-0,5} + C$$

$u = x$	$dv = e^{-0,5x}\,dx$
$du = dx$	$v = \dfrac{e^{-0.5x}}{-0,5}$

$$= x\cdot\frac{e^{-0.5x}}{-0,5} - \frac{1}{0,5}\cdot\frac{e^{-0,5x}}{0,5} + C$$

P3. Determinar la función Antiderivada considerando la condición dada:

a) $f'(x) = 2\cdot\dfrac{1}{x+1}\quad ; f(0) = 2$

$$\int f'(x)\,dx = \int 2\cdot\frac{1}{x+1}dx = \int 2\cdot\frac{1}{u}du = 2\cdot\ln(u) + C = 2\cdot ln(x+1) + C$$

$u = x+1$
$du = dx$

$$\boxed{f(x) = 2\cdot\ln(x+1) + C}$$

Aplicando condición inicial $f(0) = 2$

$f(0) = 2\cdot\ln(0+1) + C$

$2 = 0 + C$

$\boxed{2 = C}$

$$\boxed{f(x) = 2\cdot\ln(x+1) + 2}$$

$b)\ f'(x) = 1.000e^{-0.25x} \quad ; f(0) = 5.000$

$$\int f'(x)\,\mathrm{d}x = \int 1.000\mathrm{e}^{-0.25x}\,\mathrm{d}x = 1000\,\frac{\mathrm{e}^{-0.25x}}{-0,25} + C = -4000\cdot\mathrm{e}^{-925x} + C$$

$$\boxed{f(x) = -4000\mathrm{e}^{-0,25x} + C}$$

Aplicando condición inicial $f(0) = 5000$

$$f(0) = -4000\cdot\mathrm{e}^{-0,25\cdot 0} + C$$

$$5000 = -4000 + C$$

$$\boxed{9000 = C}$$

$$\boxed{f(x) = -4000\cdot\mathrm{e}^{-0,25x} + 9000}$$

$c)\ f'(x) = \sqrt{2x+1} \quad ; f(0) = \frac{1}{3}$

$$\int f'(x)\,\mathrm{d}x = \int \sqrt{2x+1}\,\mathrm{d}x = \int (2x+1)^{1/2}\,\mathrm{d}x = \int u^{1/2}\,\frac{\mathrm{d}u}{2} = \frac{1}{2}\cdot\int u^{1/2}\,\mathrm{d}u = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C = \frac{1}{2}\cdot\frac{(2x+1)^{3/2}}{3/2} + C$$

$$\boxed{\begin{array}{l} u = 2x+1 \\ \mathrm{d}u = 2\,\mathrm{d}x \\ \dfrac{\mathrm{d}u}{2} = \mathrm{d}x \end{array}}$$

$$\boxed{f(x) = \frac{1}{2}\cdot\frac{(2x+1)^{3/2}}{3/2} + C}$$

Aplicando condición inicial $f(0) = \frac{1}{3}$

$$f(0) = \frac{1}{2}\cdot\frac{(2\cdot 0+1)^{3/2}}{3/2} + C$$

$$\frac{1}{3} = \frac{1}{3} + C$$

$$\boxed{0 = C}$$

$$f(x) = \frac{1}{2}\cdot\frac{(2x+1)^{3/2}}{3/2} + 0$$

d) $\frac{dy}{dx} = \frac{x}{x^2+1} \quad ; y(0) = 1$

$$\int dy = \int \left(\frac{x}{x^2+1}\right) dx = \int \frac{1}{u} \cdot \frac{du}{2} = \frac{1}{2} \cdot \int \frac{1}{u} du = \frac{1}{2} \cdot \ln(u) + C = \frac{1}{2} \cdot \ln(x^2+1) + C$$

$$\boxed{\begin{aligned} u &= x^2 + 1 \\ du &= 2x\, dx \\ \frac{du}{2} &= x\, dx \end{aligned}}$$

$$\boxed{y(x) = \frac{1}{2} \cdot \ln(x^2+1) + C}$$

Aplicando condición inicial $y(0) = 1$

$$y(0) = \frac{1}{2} \cdot \ln(0^2+1) + C$$

$$1 = 0 + C$$

$$\boxed{1 = C}$$

$$\boxed{y(x) = \frac{1}{2} \cdot \ln(x^2+1) + 1}$$

e) $\frac{dy}{dx} = \ln(x^2) \quad ; y(1) = 0$

$$\frac{dy}{dx} = 2 \cdot \ln(x)$$

$$\int dy = \int 2 \cdot \ln(x)\, dx = \ln(x) \cdot 2x - \int 2x \cdot \frac{1}{x} dx = \ln(x) \cdot 2x - \int 2\, dx = ln(x) \cdot 2x - 2x + C$$

$$\boxed{\begin{array}{ll} u = \ln(x) & \int dv = \int 2\, dx \\ du = \frac{1}{x} dx & v = 2x \end{array}}$$

$$\boxed{y(x) = ln(x) \cdot 2x - 2x + C}$$

Aplicando condición inicial $y(1) = 0$

$$y(1) = ln(1) \cdot 2 \cdot 1 - 2 \cdot 1 + C$$

$$0 = -2 + C$$

$$\boxed{2 = C}$$

$$\boxed{y(x) = ln(x) \cdot 2x - 2x + 2}$$

f) $f'(x) = 2xe^x \quad ; f(0) = 5$

$$\int f'(x)\,dx = \int 2xe^x\,dx = 2x \cdot e^x - \int e^x \cdot 2\,dx = 2 \cdot x \cdot e^x - 2 \cdot e^x + C$$

$$\boxed{\begin{array}{ll} u = 2x & \int dv = \int e^x\,dx \\ du = 2\,dx & v = e^x \end{array}}$$

$$\boxed{f(x) = 2 \cdot x \cdot e^x - 2 \cdot e^x + C}$$

Aplicando la condición inicial $f(0) = 5$

$f(0) = 2 \cdot 0 \cdot e^0 - 2 \cdot e^0 + C$

$5 = -2 + C$

$\boxed{7 = C}$

$$\boxed{f(x) = 2 \cdot x \cdot e^x - 2 \cdot e^x + 7}$$

P4. Se ha determinado que la población $P(t)$ de una comuna (en miles de habitantes) luego de t años a partir de este momento, aumentará a razón de $\frac{dP}{dt} = 4e^{0,05t}$ miles de habitantes por año.

Si actualmente la población es de veinticinco mil habitantes. ¿Cuál será la población en tres años más?

$P(0) = 25$

$$\int dP = \int 4 \cdot e^{0.05t}\,dt$$

$$P(t) = 4 \cdot \frac{e^{0,05t}}{0,05} + C = 80 \cdot e^{0,05 \cdot t} + C$$

$$\boxed{P(t) = 80 \cdot e^{0,05 \cdot t} + C}$$

Aplicando la condición inicial $P(0) = 25$

$P(0) = 80 \cdot e^{0,05 \cdot 0} + C$

$25 = 80 + C$

$\boxed{-55 = C}$

$$\boxed{P(t) = 80 \cdot e^{0,05 \cdot t} - 55}$$

$$\boxed{P(3) = 80 \cdot e^{0,05 \cdot 3} - 55 = 37{,}947\ miles\ de\ personas = 37.947\ personas}$$

P5. La intensidad de corriente eléctrica, en función del tiempo, que circula por un conductor está dada por la expresión $i(t) = 2t^2\left(t^3+1\right)^{-1}$ Amperes. Determine la cantidad de carga elécrica, en Coulombs, que circula en 20 segundos.

$$\int i(t) = Q(t)$$

$$Q(0) = 0$$

$$i(t) = 2 \cdot t^2 \cdot \left(t^3+1\right)^{-1} = \frac{2 \cdot t^2}{t^3+1}$$

$$\int i(t)\,\mathrm{dt} = \int \frac{2 \cdot t^2}{t^3+1}\,\mathrm{d}t = \int \frac{2}{u} \cdot \frac{\mathrm{du}}{3} = \frac{2}{3}\int \frac{1}{u}\,\mathrm{du} = \frac{2}{3} \cdot \ln(u) + C = \frac{2}{3}\ln\left(t^3+1\right) + C$$

$$u = t^3 + 1$$
$$\mathrm{du} = 3t^2 dt$$
$$\frac{\mathrm{du}}{3} = t^2 dt$$

$$\boxed{Q(t) = \frac{2}{3} \cdot \ln\left(t^3+1\right) + C}$$

$$\boxed{Q(20) = \frac{2}{3} \cdot \ln\left(20^3+1\right) = 5{,}99\ Coulomb}$$

Aplicando condición inicial $Q(0) = 0$

$$Q(0) = \frac{2}{3} \cdot \ln\left(0^3+1\right) + C$$

$$0 = 0 + C$$

$$\boxed{0 = C}$$

$$\boxed{Q(t) = \frac{2}{3} \cdot \ln\left(t^3+1\right)}$$

P6. Una Pyme dedicada a la producción de artesanías ha determinado que si se producen x artículos mensuales, el costo marginal se determina por $C'(x) = \frac{3}{\sqrt{x+1}}$ en miles de pesos por producto. Determine el costo total de producir 50 artesanías sabiendo con los costos fijos mensuales ascienden a 400 mil pesos.

$$\int C'(x)\,\mathrm{dx} = C(x)$$

$$c(0) = 400$$

$$C'(x) = \frac{3}{\sqrt{x+1}} = \frac{3}{(x+1)^{1/2}} = 3 \cdot (x+1)^{-1/2}$$

$$\int C'(x)\,\mathrm{dx} = \int 3 \cdot (x+1)^{-1/2}\,\mathrm{dx} = \int 3 \cdot u^{-1/2}\,\mathrm{du} = 3 \cdot \frac{u^{1/2}}{1/2} + k = 3 \cdot \frac{(x+1)^{1/2}}{1/2} + k = 6 \cdot (x+1)^{1/2} + k$$

$$\boxed{\begin{array}{l} u = x+1 \\ \mathrm{du} = \mathrm{dx} \end{array}}$$

$$\boxed{\mathrm{C(x)} = 6 \cdot (x+1)^{1/2} + k}$$

$$\boxed{\mathrm{C(x)} = 6 \cdot (x+1)^{1/2} + 394}$$

Aplicando condición inicial $C(0) = 400$

$$\mathrm{C(0)} = 6 \cdot (0+1)^{1/2} + k$$

$$400 = 6 + k$$

$$\boxed{394 = k}$$

$$\mathrm{C(50)} = 6 \cdot (50+1)^{1/2} + 394 = 436{,}849\ mil\ pesos$$

$$\boxed{C(50) = 436.849\ pesos}$$

P7. La potencia de un artefacto está definida por $P(t) = \dfrac{6t^2}{t^3+10}$ *watts*, donde t es el tiempo en segundos en que se mantiene en funcionamiento. Determine la energía consumida por el artefacto, en *joules*, si está encendido por 15 minutos.

$\int P(t) = E(t)$

$E(0) = 0$

$$\int P(t)\,\mathrm{d}t = \int \frac{6t^2}{t^3+10}\mathrm{d}t = \int \frac{6}{u}\cdot\frac{\mathrm{d}u}{3} = \int \frac{2}{u}\mathrm{d}u = 2\ln(u) + C = 2\ln(t^3+10) + C$$

$$\boxed{\begin{array}{l} u = t^3 + 10 \\ \mathrm{d}u = 3t^2\,\mathrm{d}t \\ \dfrac{\mathrm{d}u}{3} = t^2\,\mathrm{d}t \end{array}}$$

$$\boxed{E(t) = 2\ln(t^3+10) + C}$$

$$\boxed{E(15) = 2\ln(15^3+10) - 4{,}61 = 11{,}64 \text{ Joule}}$$

Aplicando condición inicial $E(0) = 0$

$$E(0) = 2\ln(0^3+10) + C$$

$$0 = 4{,}61 + C$$

$$\boxed{-4{,}61 = C}$$

$$\boxed{E(t) = 2\ln(t^3+10) - 4{,}61}$$

P8. La intensidad de corriente eléctrica que circula por un conductor está dada por la expresión $i(t) = xe^{0.25-0.5x}$ Amperes. Determine la cantidad de carga elécrica, en Coulombs, que circula en 25 segundos.

$$\mathrm{i}(t) = t\cdot \mathrm{e}^{0.25-0.5\cdot t} = t\cdot \mathrm{e}^{0.25}\cdot \mathrm{e}^{-0.5\cdot t} = 1{,}28\cdot t\cdot \mathrm{e}^{-0{,}5\cdot t}$$

$$\int \mathrm{i}(t)\,\mathrm{d}t = \int 1{,}28\cdot t\cdot \mathrm{e}^{-0{,}5t}\,\mathrm{d}t = 1{,}28\cdot t\cdot\frac{\mathrm{e}^{-0{,}5\cdot t}}{-0.5} - \int \frac{\mathrm{e}^{-0.5t}}{-0{,}5}\cdot 1{,}28\,\mathrm{d}t = 1{,}28\cdot t\cdot\frac{\mathrm{e}^{-0{,}5\cdot t}}{-0.5} - \frac{1{,}28}{-0{,}5}\cdot\int \mathrm{e}^{-0{,}5t}\,\mathrm{d}t$$

$$\boxed{\begin{array}{ll} u = 1{,}28\cdot t & \int \mathrm{d}v = \int \mathrm{e}^{-0{,}5t}\,\mathrm{d}t \\ \mathrm{d}u = 1{,}28\,\mathrm{d}t & v = \dfrac{\mathrm{e}^{-0{,}5t}}{-0{,}5} \end{array}}$$

$$= -2{,}56\cdot t\cdot e^{-0.5t} + 2{,}56\cdot\frac{\mathrm{e}^{-0{,}5t}}{-0.5} + C$$

$$= -2{,}56\cdot t\cdot e^{-0.5t} - 5{,}12\cdot \mathrm{e}^{-0{,}5t} + C$$

$$\boxed{Q(t) = -2{,}56\cdot t\cdot e^{-0{,}5t} - 5{,}12\cdot \mathrm{e}^{-0{,}5t} + C}$$

Aplicando condición inicial $Q(0) = 0$

$$Q(0) = -2{,}56\cdot 0\cdot e^{-0{,}5\cdot 0} - 5{,}12\cdot \mathrm{e}^{-0{,}5\cdot 0} + C$$

$$\boxed{Q(t) = -2{,}56\cdot t\cdot e^{-0{,}5t} - 5{,}12\cdot \mathrm{e}^{-0{,}5t} + 5{,}12}$$

$$Q(25) = -2{,}56\cdot 25\cdot e^{-0{,}5\cdot 25} - 5{,}12\cdot \mathrm{e}^{-0{,}5\cdot 25} + 5{,}12$$

$$\boxed{Q(25) = 5{,}11\ Coulomb}$$

$$Q(0) = -2{,}56 \cdot 0 \cdot e^{-0{,}5 \cdot 0} - 5{,}12 \cdot \mathrm{e}^{-0{,}5 \cdot 0} + C$$

$$0 = -5{,}12 + C$$

$$\boxed{5{,}12 = C}$$

$$Q(25) = -2{,}56 \cdot 25 \cdot e^{-0{,}5 \cdot 25} - 5{,}12 \cdot \mathrm{e}^{-0{,}5 \cdot 25} + 5{,}12$$

$$\boxed{Q(25) = 5{,}11\ Coulomb}$$