Mentor Help
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(Optional) Closed form solution math

In this optional section, we'll develop the math of the closed form solution, which we introduced in the last video. First, we'll do it for the 2-dimensional case, and then for the general case.

2-Dimensional solution

Our data will be the values $x_1, x_2, \ldots, x_m$, and our labels will be the values $y_1, y_2, \ldots, y_n$. Let's call our weights w_1, and w_2. Therefore, our predictions are $\hat{y}_i = w_1 x_i + w_2$. The mean squared error is

$$E(w_1, w_2) = \frac{1}{2m} \sum_{i=1}^{m} (\hat{y} - y)^2$$

We need to minimize this error function. Therefore, the factor of $\frac{1}{m}$ can be ignored. Now, replacing the value of $\hat{y}$, we get

$$\begin{aligned} E(w_1, w_2) &= \sum_{i=1}^{m} (\hat{y} - y)^2 \\ &= \frac{1}{2} \sum_{i=1}^{m} (w_1 x_i + w_2 - y_i)^2 \end{aligned}$$

Now, in order to minimize this error function, we need to take the derivatives with respect to w_1 and w_2 and set them equal to 0.

Using the chain rule, we get

$$\begin{aligned} \frac{\partial E}{\partial w_1} &= \sum_{i=1}^{m} (w_1 x_i + w_2 - y_i) x_i \\ &= w_1 \sum_{i=1}^{m} x_i^2 + w_2 \sum_{i=1}^{m} x_i - \sum_{i=1}^{m} x_i y_i \end{aligned}$$

and

$$\begin{aligned} \frac{\partial E}{\partial w_2} &= \sum_{i=1}^{m} (w_1 x_i + w_2 - y_i) \\ &= w_1 \sum_{i=1}^{m} x_i + w_2 \sum_{i=1}^{m} 1 - \sum_{i=1}^{m} y_i \end{aligned}$$

Setting the two equations to zero gives us the following system of two equations and two variables (where the variables are w_1 and w_2).

$$\begin{aligned} w_1 \left(\sum_{i=1}^{m} x_i^2\right) + w_2 \left(\sum_{i=1}^{m} x_i\right) &= \sum_{i=1}^{m} x_i y_i \\ w_1 \left(\sum_{i=1}^{m} x_i\right) + w_2 (m) &= \sum_{i=1}^{m} y_i \end{aligned}$$

We can use any method to solve 2 equations and 2 variables. For example, if we multiply the first equation by $\sum_{i=1}^{m} x_i$, the second one by m, subtract them to obtain a value for w_1, and then replace this value in the first equation, we get the following: