1.

$$p(t)^\top B(t) + b_v(t)^\top = 0$$
$$\dot{p}(t) = -A(t)^\top p(t) - a_z(t)$$
$$\dot{z}(t) = A(t)z(t) + B(t)v(t),$$

$$B(t) = -b(t)^T p(t)'$$

$$\begin{bmatrix} \dot{z}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^T \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} B(t)v(t) \\ -a_z(t) \end{bmatrix}$$

$$v(t) = \underbrace{\int_0^T \overbrace{D_1 \ell(x(t), u(t))}^{a_x(t)} \cdot z(t) + \overbrace{D_2 \ell(x(t), u(t))}^{b_u(t)} \cdot v(t)\, dt} + Dm(x(T)) \cdot z(t)$$

$$\underbrace{+ \int_0^T z(t)^T Q_z\, z(t) + v(t)^T R_v\, v(t)\, dt}$$

adding together

$$\int_0^T \Big(\underbrace{z(t)^T Q_z + \overbrace{D_1 \ell(x(t), u(t))}^{a_x(t)}}_{a_z(t)} \Big) z(t) + \Big(\underbrace{v(t)^T R_v + \overbrace{D_2 \ell(x(t), u(t))}^{b_u(t)}}_{b_v(t)} \Big) v(t)\, dt$$

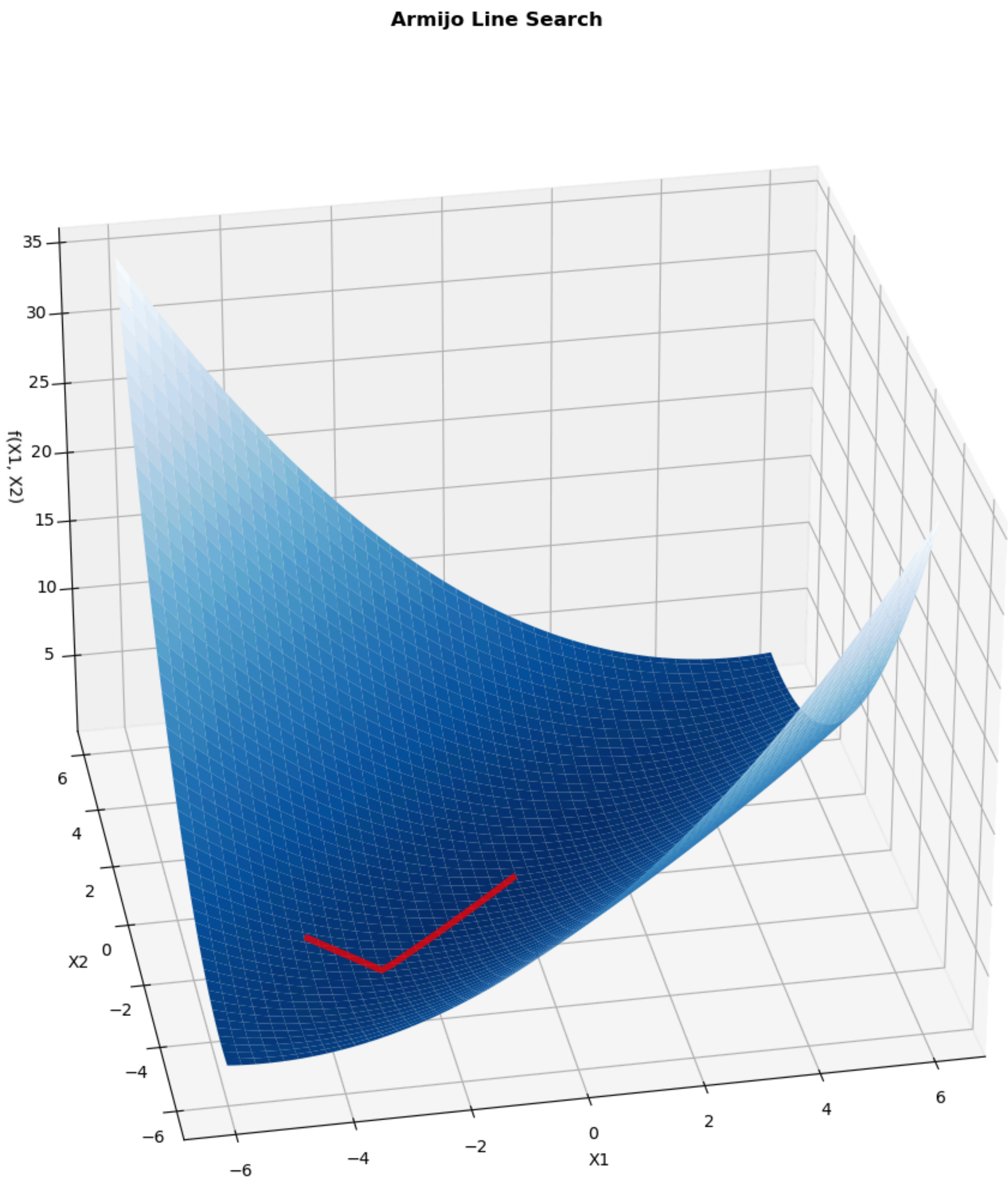
$$p(t)^T B(t) + b_v(t)^T = 0 \qquad b_v(t)^T = R_v(t)^T v(t) + D_2 \ell(x(t), u(t))^T$$

$$p(t)^T B(t) + R_v(t)^T v(t) + D_2 \ell(x(t), u(t))^T = 0$$

$$v(t) = [-D_2 \ell(x(t), u(t)) - p(t)^T B(t)]\, R_v(t)^T$$

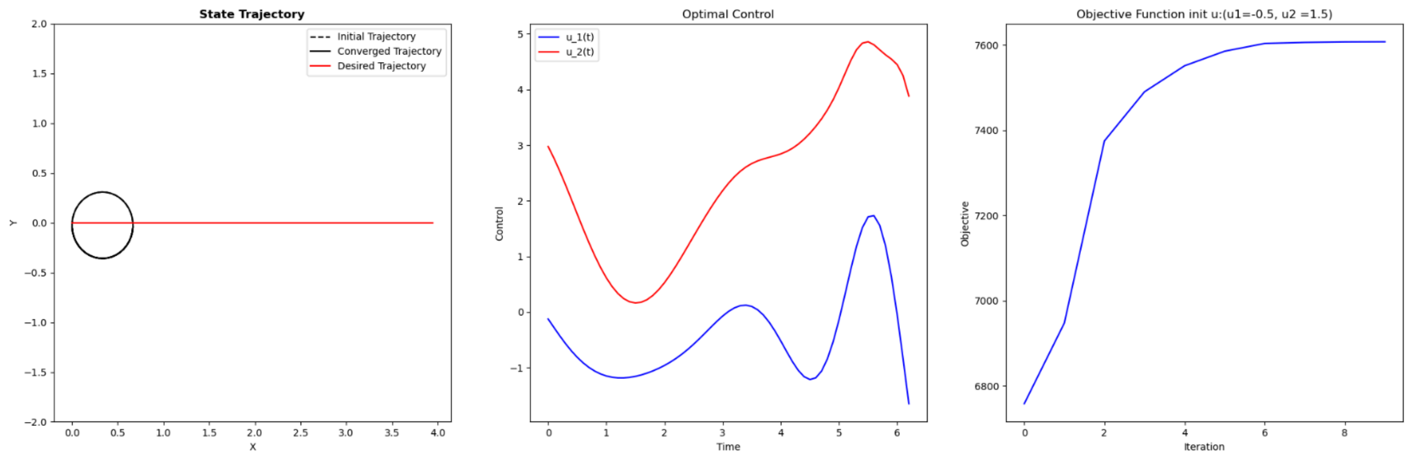
$$\begin{bmatrix} \dot{z}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^T \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} B(t)[-D_2 \ell(x(t), u(t)) - p(t)^T B(t)] R_v(t)^T \\ z(t)^T Q_z + D_1 \ell(x(t), u(t)) \end{bmatrix}$$

2.



Armijo Line Search
f(X1, X2)
35
30
25
20
15
10
5
X2
6
4
2
0
−2
−4
−6
X1
−6
−4
−2
0
2
4
6

3.



State Trajectory
Initial Trajectory
Converged Trajectory
Desired Trajectory
X
Y
Optimal Control
u_1(t)
u_2(t)
Time
Control
Objective Function init u:(u1=-0.5, u2 =1.5)
Iteration
Objective