PPGEE2249 - Aprendizado de Máquina.

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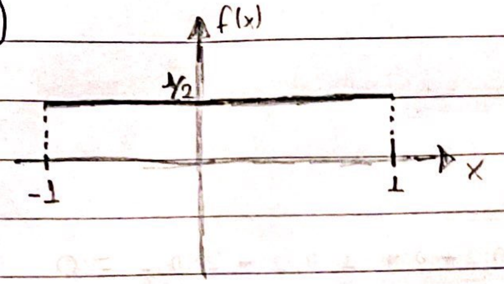
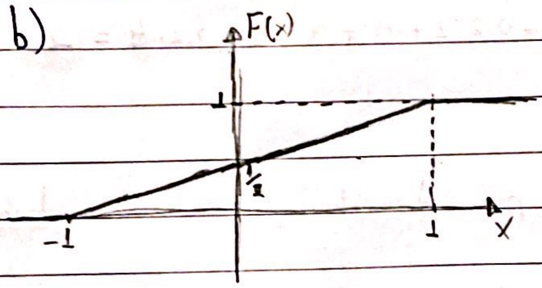
① a)



Como é uniforme a probabilidade é constante em qualquer ponto. Assim, como a área de uma PDF precisa ser igual a 1, de -1 a 1 $f(x) = 1/2$.

b)



$$F(x) = \begin{cases} 0, & x < -1 \\ (x+1)/2, & -1 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

c) $P(-0{,}2 < X < 0{,}2)$

$= F(0{,}2) - F(-0{,}2)$

$= \dfrac{0{,}2+1}{2} - \left(\dfrac{-0{,}2+1}{2}\right)$

$= 0{,}6 - 0{,}4$

$= 0{,}2 = 20\%$

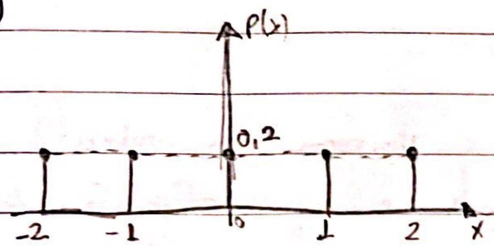
d) $E[x] = \int_{-\infty}^{\infty} x f(x)\, dx = \dfrac{x^2}{4}\Big|_{-1}^{1} = \dfrac{1}{4} - \dfrac{1}{4} = 0$

$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \dfrac{x^3}{6}\Big|_{-1}^{1} = \dfrac{1}{6} - \dfrac{(-1)}{6} = \dfrac{1}{3}$

$E[x^4] = \int_{-\infty}^{\infty} x^4 f(x)\, dx = \dfrac{x^5}{10}\Big|_{-1}^{1} = \dfrac{1}{10} - \dfrac{(-1)}{10} = \dfrac{1}{5}$

Var $E[(x - E(x))^2] = \int_{-\infty}^{\infty} (x-0)^2 f(x)\, dx = E[x^2] = \dfrac{1}{3}$

②



$p(x) = 0,2$

$$E[X] = \sum_{i=1}^{\infty} x_i p(x_i) = -2\cdot 0,2 - 1\cdot 0,2 + 0 + 1\cdot 0,2 + 2\cdot 0,2 = 0$$

$$Var[X] = \sum_{i=1}^{\infty} p_i (x_i - \mu)^2 = 0,2\cdot 4 + 0,2\cdot 1 + 0 + 0,2\cdot 1 + 0,2\cdot 4 = 2$$

③ PDF Conjunta, pode ser expressa pela distribuição Gaussiana bivariada

$$f_{X_1X_2}(X_1, X_2) = \frac{1}{2\pi\sigma_{x_1}\sigma_{x_2}\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right)^2 - 2\rho\left(\frac{x_1-\mu_{x_1}}{\sigma_{x_1}}\right)\left(\frac{x_2-\mu_{x_2}}{\sigma_{x_2}}\right) + \left(\frac{x_2-\mu_{x_2}}{\sigma_{x_2}}\right)^2\right]\right)$$

ρ = Pearson correlation coeficient $= \dfrac{Cov(X_1, X_2)}{\sigma_{x_1}\sigma_{x_2}}$ $\quad \mu_1 = -2 \quad \mu_2 = 1 \quad \sigma_1 = \sqrt{2} \quad \sigma_2 = 2$

$$\rho = \frac{-0,8}{\sqrt{2}\cdot 2} \qquad \rho^2 = 0,08$$

Voltando

$$= \frac{1}{2\pi\cdot\sqrt{2}\cdot 2\sqrt{1-0,08}} \exp\left(\frac{-1}{2(1-0,08)}\left[\frac{(x_1+2)^2}{2} + \frac{2\cdot 0,8}{\sqrt{2}\cdot 2}\left(\frac{x_1+2}{\sqrt{2}}\right)\left(\frac{x_2-1}{2}\right) + \frac{(x_2-1)^2}{4}\right]\right)$$

$$_{x_2}(X_1,X_2) = \frac{1}{4\pi\sqrt{1,84}} \exp\left(-\frac{1}{3,68}\left[(x_1+2)^2 + 0,4(x_1+2)(x_2-1) + \frac{(x_2-1)^2}{2}\right]\right)$$