

PPGEE2249 – Aprendizado de Máquina

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Assignment 1

- 1) X is a continuous-valued random variable with uniform density in $(-1, +1)$.
 - a) Draw its probability density function. What is the area under the curve? Justify your answer.
 - b) Draw its cumulative distribution function.
 - c) Calculate the probability of the event $X \in (-0.2, 0.2)$.
 - d) Calculate the expected value $E[X]$, the second $E[X^2]$ and the fourth moment $E[X^4]$ of the random variable. Calculate its variance $Var[X]$, as well.
- 2) X is a discrete-valued random variable with uniform distribution over the set $\{-2, -1, 0, 1, 2\}$. Draw its probability mass function and calculate $E[X]$ and $Var[X]$.
- 3) Consider the normal random variables $X_1 \sim \mathcal{N}(-2, 2)$, $X_2 \sim \mathcal{N}(1, 4)$, with $Cov(X_1, X_2) = -0.8$. Calculate the joint pdf of $\mathbf{X} = (X_1, X_2)^T$.
- 4) Based on the following estimators:
 - Sample mean: $m = \frac{1}{N} \sum_{t=1}^N x^t$
 - Sample variance: $s^2 = \frac{1}{N} \sum_{t=1}^N (x^t - m)^2$
 - Sample standard deviation: $s = \sqrt{\frac{1}{N} \sum_{t=1}^N (x^t - m)^2}$
 - Sample covariance: $s_{ij} = \frac{1}{N} \sum_{t=1}^N (x_i^t - m_i)(x_j^t - m_j)$
 - Sample correlation coefficient: $r_{ij} = \frac{s_{ij}}{s_i s_j}$

Calculate r_{ij} between two features of a dataset. Choose features that you expect to be interconnected. Present the results of all the aforementioned metrics, as well. Use a numerical software of your own choice and explain the steps of your solution.