Logistic Regression

Step 1: Function Set

We want to find $P_{w,b}(C_1|x)$

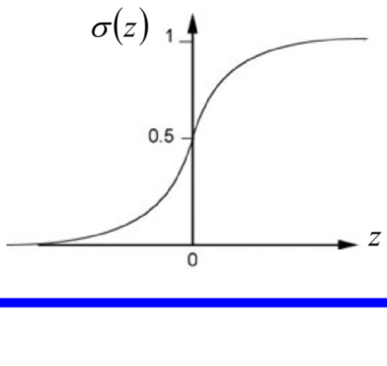
If $P_{w,b}(C_1|x) \geq 0.5$, output C_1

Otherwise, output C_2

$$P_{w,b}(C_1|x) = \sigma(z)$$

$$z = w \cdot x + b$$

$$\sigma(z) = \frac{1}{1 + exp(-z)}$$



Function set:

$$f_{w,b}(x) = P_{w,b}(C_1|x)$$

Including all different w and b

Step 2: Goodness of a Function

Training Data

x^1	x^2	x^3	……	x^N
C_1	C_1	C_2		C_1

Assume the data is generated based on $f_{w,b}(x) = P_{w,b}(C_1|x)$

Given a set of w and b, what is its probability of generating the data?

$$L(w,b) = f_{w,b}(x^1)f_{w,b}(x^2)\left(1 - f_{w,b}(x^3)\right)\cdots f_{w,b}(x^N)$$

The most likely w^* and b^* is the one with the largest $L(w,b)$.

$$w^*, b^* = arg\max_{w,b} L(w,b)$$

Step 2: Goodness of a Function

$$L(w,b) = f_{w,b}(x^1)f_{w,b}(x^2)\left(1 - f_{w,b}(x^3)\right)\cdots f_{w,b}(x^N)$$

$$-lnL(w,b) = lnf_{w,b}(x^1) + lnf_{w,b}(x^2) + ln\left(1 - f_{w,b}(x^3)\right)\cdots$$

$\hat{y}^n$: 1 for class 1, 0 for class 2

$$= \sum_n -\underbrace{\left[\hat{y}^n lnf_{w,b}(x^n) + (1 - \hat{y}^n)ln\left(1 - f_{w,b}(x^n)\right)\right]}_{\text{Cross entropy between two Bernoulli distribution}}$$

Distribution p:

$p(x = 1) = \hat{y}^n$

$p(x = 0) = 1 - \hat{y}^n$

$\longleftrightarrow$ cross entropy

Distribution q:

$q(x = 1) = f(x^n)$

$q(x = 0) = 1 - f(x^n)$

$$H(p,q) = -\sum_x p(x)ln\left(q(x)\right)$$

Step 3: Find the best function

$$\frac{-\ln L(w,b)}{\partial w_i} = \sum_n -\left[\hat{y}^n \overset{\left(1-f_{w,b}(x^n)\right)x_i^n}{\boxed{\frac{\ln f_{w,b}(x^n)}{\partial w_i}}} + (1-\hat{y}^n)\overset{f_{w,b}(x^n)x_i^n}{\boxed{\frac{\ln\left(1-f_{w,b}(x^n)\right)}{\partial w_i}}}\right]$$

$$= \sum_n -\left[\hat{y}^n \left(1-f_{w,b}(x^n)\right)x_i^n - (1-\hat{y}^n) f_{w,b}(x^n)x_i^n\right]$$

$$= \sum_n -\left[\hat{y}^n \cancel{- \hat{y}^n f_{w,b}(x^n)} - f_{w,b}(x^n) \cancel{+ \hat{y}^n f_{w,b}(x^n)}\right] x_i^n$$

$$= \sum_n -\left(\hat{y}^n - f_{w,b}(x^n)\right) x_i^n$$

Larger difference, larger update

$$w_i \leftarrow w_i - \eta \sum_n -\left(\hat{y}^n - f_{w,b}(x^n)\right) x_i^n$$

	Logistic Regression	***Linear Regression***
Step 1:	$f_{w,b}(x) = \sigma\left(\sum_i w_i x_i + b\right)$ Output: between 0 and 1	$f_{w,b}(x) = \sum_i w_i x_i + b$ Output: any value
Step 2:	Training data: $(x^n, \hat{y}^n)$ $\hat{y}^n$: 1 for class 1, 0 for class 2 $L(f) = \sum_n C(f(x^n), \hat{y}^n)$	Training data: $(x^n, \hat{y}^n)$ $\hat{y}^n$: a real number $L(f) = \frac{1}{2}\sum_n (f(x^n) - \hat{y}^n)^2$

Step 3:

Logistic regression: $w_i \leftarrow w_i - \eta \sum_n -\left(\hat{y}^n - f_{w,b}(x^n)\right) x_i^n$

Linear regression: $w_i \leftarrow w_i - \eta \sum_n -\left(\hat{y}^n - f_{w,b}(x^n)\right) x_i^n$