Ideal Alternatives

- Function (Model):

$f(x)$

$x \rightarrow$

$g(x) > 0$	Output = class 1
$else$	Output = class 2

- Loss function:

$$L(f) = \sum_n \delta(f(x^n) \neq \hat{y}^n)$$

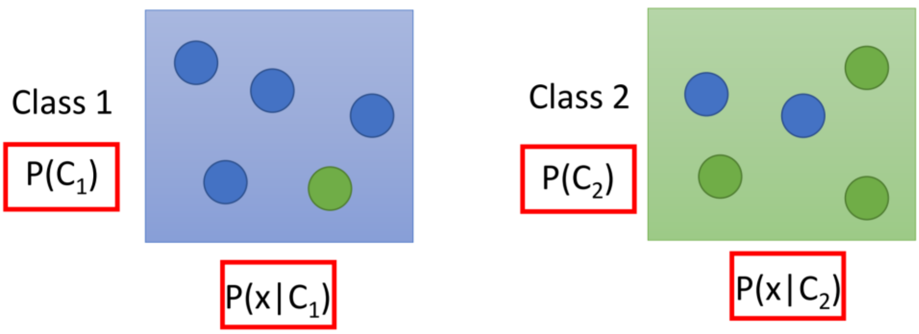
The number of times f get incorrect results on training data.

- Find the best function:
    - Example: Perceptron, SVM

Not Today

Two Classes

Estimating the Probabilities
From training data



Given an x, which class does it belong to

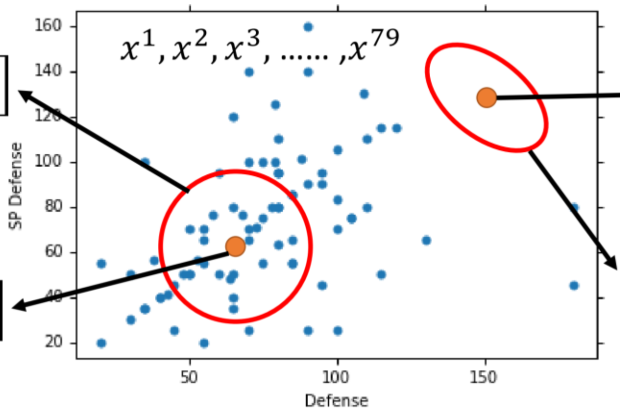
$$P(C_1|x) = \frac{P(x|C_1)P(C_1)}{P(x|C_1)P(C_1) + P(x|C_2)P(C_2)}$$

Generative Model $P(x) = P(x|C_1)P(C_1) + P(x|C_2)P(C_2)$

Maximum Likelihood

$$f_{\mu,\Sigma}(x) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma|^{1/2}} exp\left\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right\}$$

$$\Sigma = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \qquad \mu = \begin{bmatrix} 75 \\ 75 \end{bmatrix}$$



$$\mu = \begin{bmatrix} 150 \\ 140 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

The Gaussian with any mean μ and covariance matrix Σ can generate these points. ⇒ Different Likelihood

Likelihood of a Gaussian with mean μ and covariance matrix Σ

= the probability of the Gaussian samples $x^1, x^2, x^3, \ldots\ldots, x^{79}$

$$L(\mu,\Sigma) = f_{\mu,\Sigma}(x^1) f_{\mu,\Sigma}(x^2) f_{\mu,\Sigma}(x^3) \ldots\ldots f_{\mu,\Sigma}(x^{79})$$

Maximum Likelihood

We have the "Water" type Pokémons: $x^1, x^2, x^3, \ldots\ldots, x^{79}$

We assume $x^1, x^2, x^3, \ldots\ldots, x^{79}$ generate from the Gaussian (μ^*, Σ^*) with the ***maximum likelihood***

$$L(\mu,\Sigma) = f_{\mu,\Sigma}(x^1) f_{\mu,\Sigma}(x^2) f_{\mu,\Sigma}(x^3) \ldots\ldots f_{\mu,\Sigma}(x^{79})$$

$$f_{\mu,\Sigma}(x) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma|^{1/2}} exp\left\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right\}$$

$$\mu^*, \Sigma^* = arg \max_{\mu,\Sigma} L(\mu,\Sigma)$$

$$\mu^* = \underbrace{\frac{1}{79}\sum_{n=1}^{79} x^n}_{\text{average}} \qquad \Sigma^* = \frac{1}{79}\sum_{n=1}^{79} (x^n - \mu^*)(x^n - \mu^*)^T$$

Now we can do classification ☺

$$f_{\mu^1,\Sigma^1}(x) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma^1|^{1/2}} exp\left\{-\frac{1}{2}(x-\mu^1)^T(\Sigma^1)^{-1}(x-\mu^1)\right\}$$

$$\mu^1 = \begin{bmatrix} 75.0 \\ 71.3 \end{bmatrix} \quad \Sigma^1 = \begin{bmatrix} 874 & 327 \\ 327 & 929 \end{bmatrix}$$

P(C1)
= 79 / (79 + 61) =0.56

$$P(C_1|x) = \frac{P(x|C_1)P(C_1)}{P(x|C_1)P(C_1) + P(x|C_2)P(C_2)}$$

$$f_{\mu^2,\Sigma^2}(x) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma^2|^{1/2}} exp\left\{-\frac{1}{2}(x-\mu^2)^T(\Sigma^2)^{-1}(x-\mu^2)\right\}$$

$$\mu^2 = \begin{bmatrix} 55.6 \\ 59.8 \end{bmatrix} \quad \Sigma^2 = \begin{bmatrix} 847 & 422 \\ 422 & 685 \end{bmatrix}$$

P(C2)
= 61 / (79 + 61)
=0.44

If $P(C_1|x) > 0.5$ ➡ x belongs to class 1 (Water)