Note:

Your name and student ID needs to be prominently displayed on the first page of your answer book. Please submit your answer book as a single PDF file in the format 'firstname-lastname.pdf' by 9 AM, October 20, 2022. You will also need to append the numerical codes (including Simulink/Simscape block diagrams) used to generate solutions for the questions in this assignment. All problems need to be attempted, this assignment would count for 25% towards the final grade.

1 Problem 1

In this problem, you will be attempting to achieve position and attitude tracking control of a quadrotor in the presence of external perturbation such as wind gusts. The quadrotor position dynamics for such a system is given by

$$\begin{aligned} m\ddot{x} &= T(\sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta) - F_{x_g} \\ m\ddot{y} &= T(-\cos\psi\sin\phi + \sin\psi\cos\phi\sin\theta) - F_{y_g} \\ m\ddot{z} &= T\cos\phi\cos\theta - mg. \end{aligned} \tag{1}$$

where F_{x_g}, F_{y_g} are the planar perturbations induced by the external gust. The attitude dynamics in the presence of wind gusts can now be written as

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{\tau} - \boldsymbol{\omega} \times I\boldsymbol{\omega} - \boldsymbol{\tau}_g, \tag{2}$$

where $\boldsymbol{\omega} = [p,\, q,\, r]^\top$ are the angular velocity components in the body-fixed reference frame, $\boldsymbol{\tau} = [\tau_x,\, \tau_y,\, \tau_z]^\top$ are the roll, pitch and yaw moment inputs respectively, and $\boldsymbol{\tau}_g = [L_g,\, M_g,\, N_g]^\top$ represent the external torque induced by the gust. Note that the kinematic model remains the same as that discussed in class. You may assume the motor mixing algorithm as

$$\boldsymbol{\tau} = \begin{bmatrix} 0 & k_f L & 0 & -k_f L \\ -k_f L & 0 & k_f L & 0 \\ k_m & -k_m & k_m & -k_m \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}, \tag{3}$$

where k_f and k_m are calibration constants identified as $k_f = 0.005022$, $k_m = 1.858 \times 10^{-5}$, so that the thrust and torque produced by each motor satisfy the relationship $T_i = 0.005022\,\omega_i^2$ N and $\tau_i = 1.858 \times 10^{-5}\,\omega_i^2$ N-m, where ω_i represents the speed (RPM) of the i^{th} motor. Furthermore, assume that the inertia matrix is diagonal, given by $I = \text{diag}[2.3951,\, 2.3951,\, 3.2347] \times 10^{-5}$ kg-m^2. You will observe that the inplane values of the inertia tensor are the same, which attests to the fact that the quadrotor geometry is symmetric along the axes in the horizontal plane. Finally, assume that the vehicle mass is $m = 34$ g, the moment arm is $L = 32$ mm, and the external perturbations are given by $\boldsymbol{\tau}_g = [\cos t, \sin t, 2\sin t]^\top$ N-m, and $F_{x_g} = F_{y_g} = 0.5$ N.
(a) You are expected to design the PID control architecture to achieve accurate position tracking of a rectangle and a circle, where the rectangle is described by the vertices at (1,1,1) m, (2,1,1) m, (2,2,1) m and (1,2,1) m, and the circle is described by the time-varying coordinates $(\cos t,\, \sin t,\, 1)$ m.
You can assume the initial and final locations to be the same, which are given by (2,2,0) m for the rectangle and (1,0,0) m for the circle. In essence, you need to accomplish takeoff, reach an altitude of 1 m and track each of the two figures (the rectangle and the circle), and accomplish landing back in the position you started from.
(Hint: You may want to decouple this system and the design of the inner PID loop for attitude control first, and then subsequently move to the design of PID controller for the outer loop for position tracking). (10 points)
(b) For this choice of control gains, how do the results vary in the absence and the presence of wind gusts? What conclusion can you arrive at regarding the suitability of using PID control for achieving position tracking in the presence of gusts? (5 points)
(c) Synthesize an optimal set of gains for the system without external gusts and compare the results for this case with the case of using manual tuning to achieve perfect tracking. (10 points)