Piecewise Constant Curvature (PCC)

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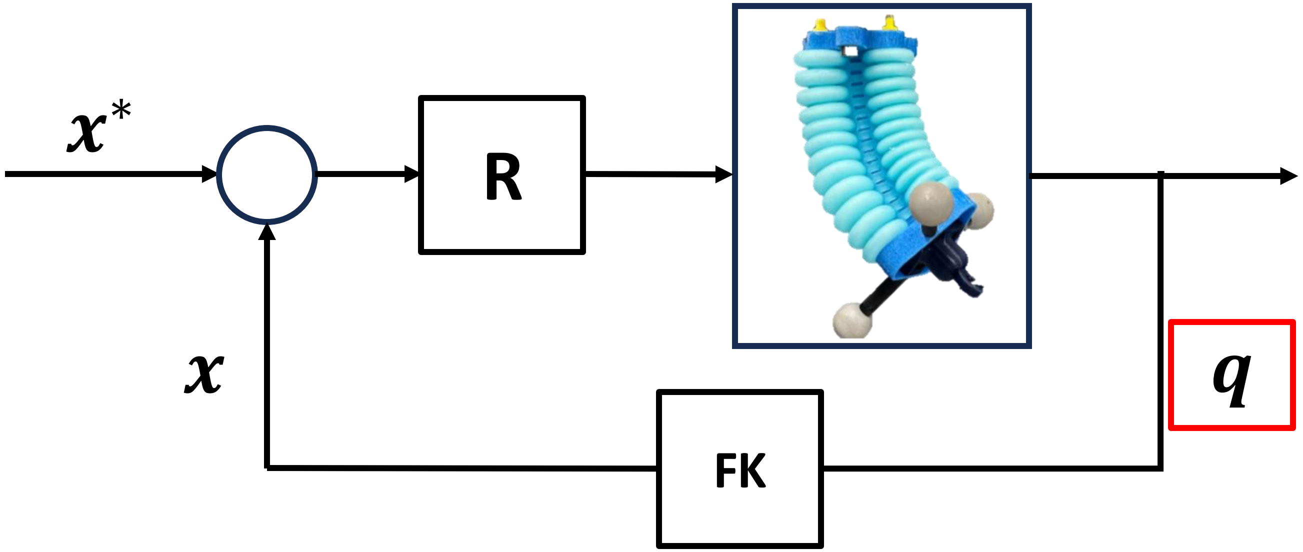
Soft Robot Configuration In Space

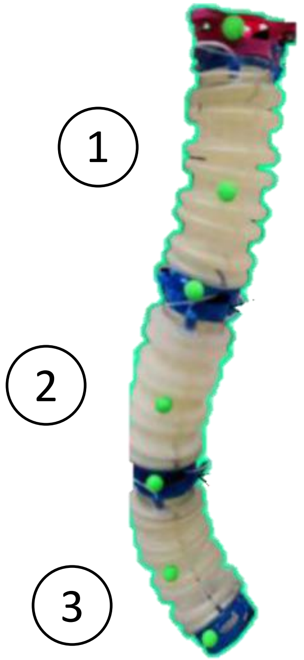


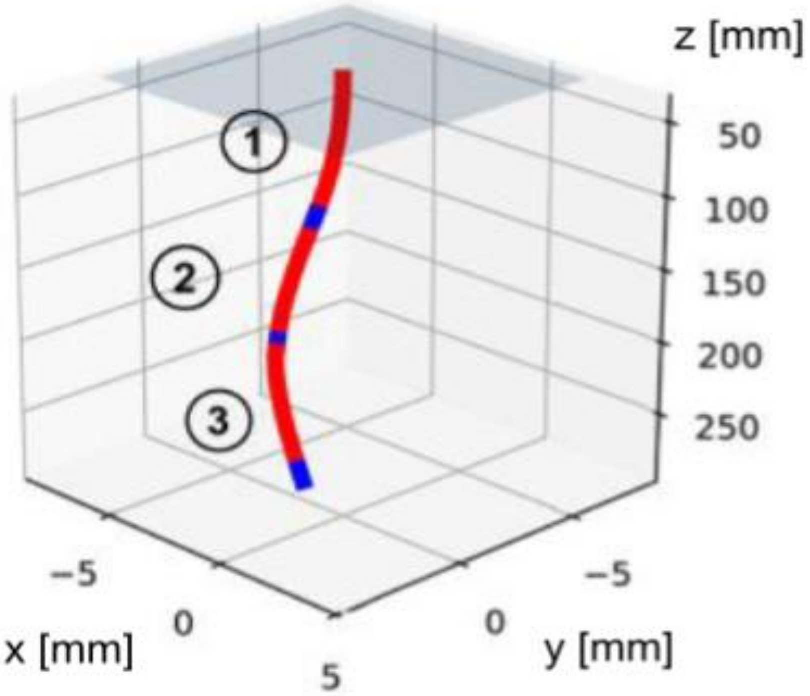
To model the configuration of a soft robot in space is **challenging**:

- Deformations
- Material Properties
- Physical Interactions with the enviornment

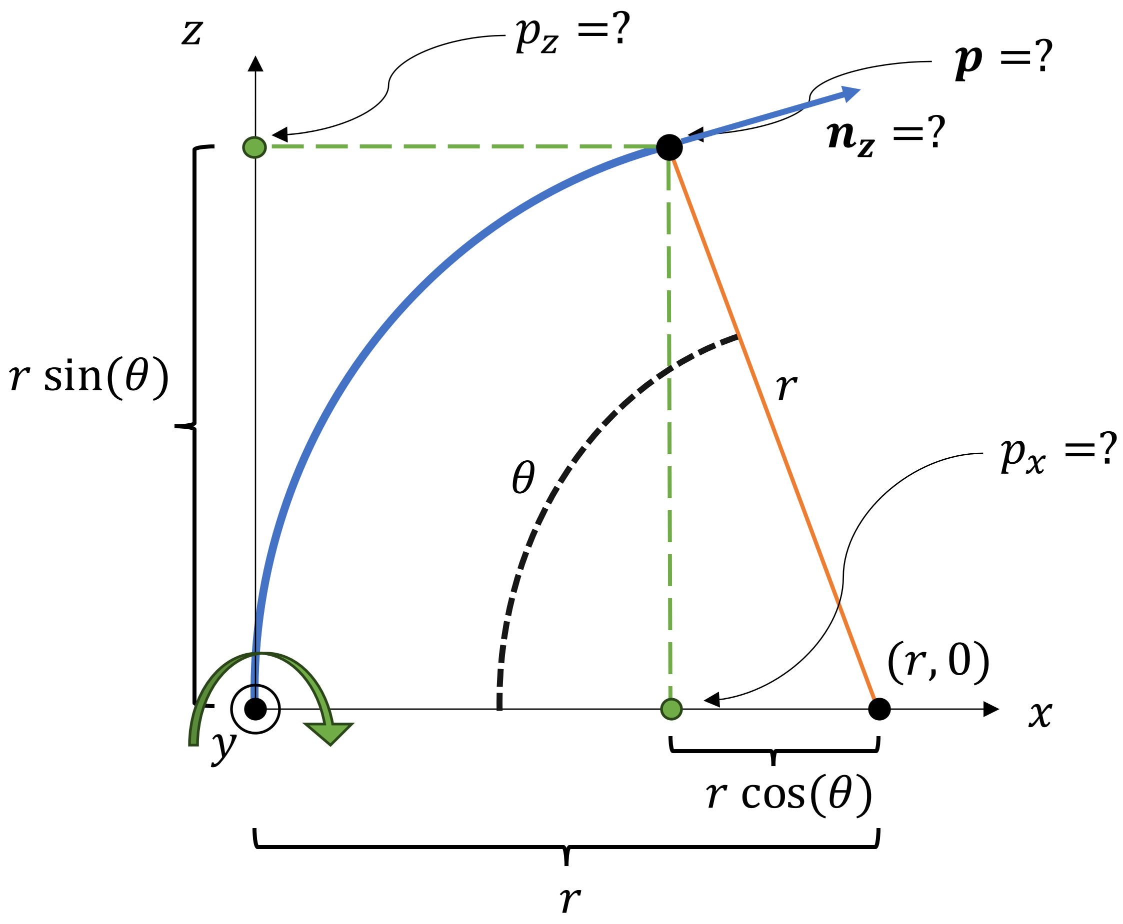
Piecewise Constant Curvature (PCC)

- *Kinematic* model
- The robot is assumed to be composed of flexible segments exhibiting continuous bending
- Segments are approximated using a *constant* curvature assumption





Forward Kinematics – Plane

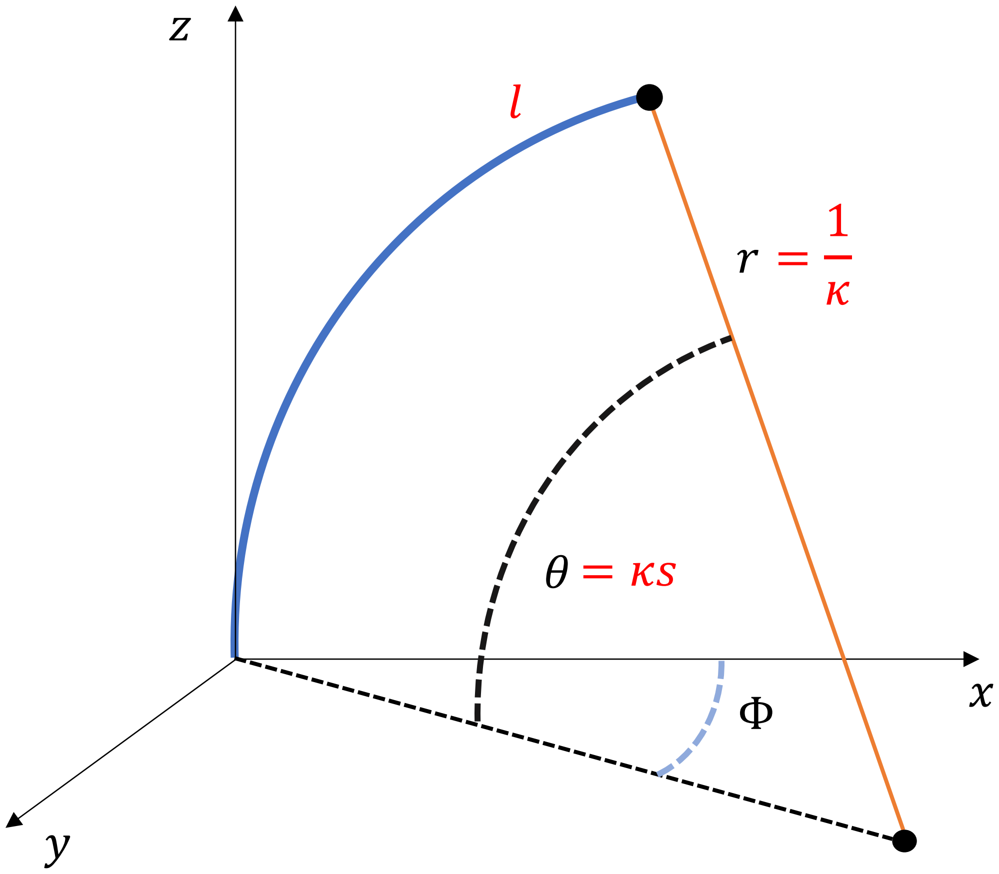


$$\boldsymbol{p}(r, \theta) = [r(1 - \cos\theta), 0, r\sin\theta]^T$$

$$[\boldsymbol{n_x}, \boldsymbol{n_y}, \boldsymbol{n_z}] = \boldsymbol{R}_y(\theta)$$

$$\boldsymbol{T}(r, \theta) = \begin{bmatrix} \boldsymbol{R_y}(\theta) & \boldsymbol{p}(r, \theta) \\ \boldsymbol{0} & 1 \end{bmatrix}$$

Forward Kinematics – Out of plane rotation



$$\boldsymbol{T}(r,\theta,\Phi) = \underbrace{\begin{bmatrix} \boldsymbol{R_z}(\Phi) & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix}}_{\text{Rotation}} \underbrace{\begin{bmatrix} \boldsymbol{R_y}(\theta) & \boldsymbol{p}(r,\theta) \\ \boldsymbol{0} & 1 \end{bmatrix}}_{\text{In-plane transformation}}$$

The forward kinematics is usually defined in terms of *arc parameters*, thus:

$$\boldsymbol{T}(\kappa, s, \Phi) = \begin{bmatrix} \boldsymbol{R_z}(\Phi) & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R_y}(k,s) & \boldsymbol{p}(k,s) \\ \boldsymbol{0} & 1 \end{bmatrix}$$

Curvature

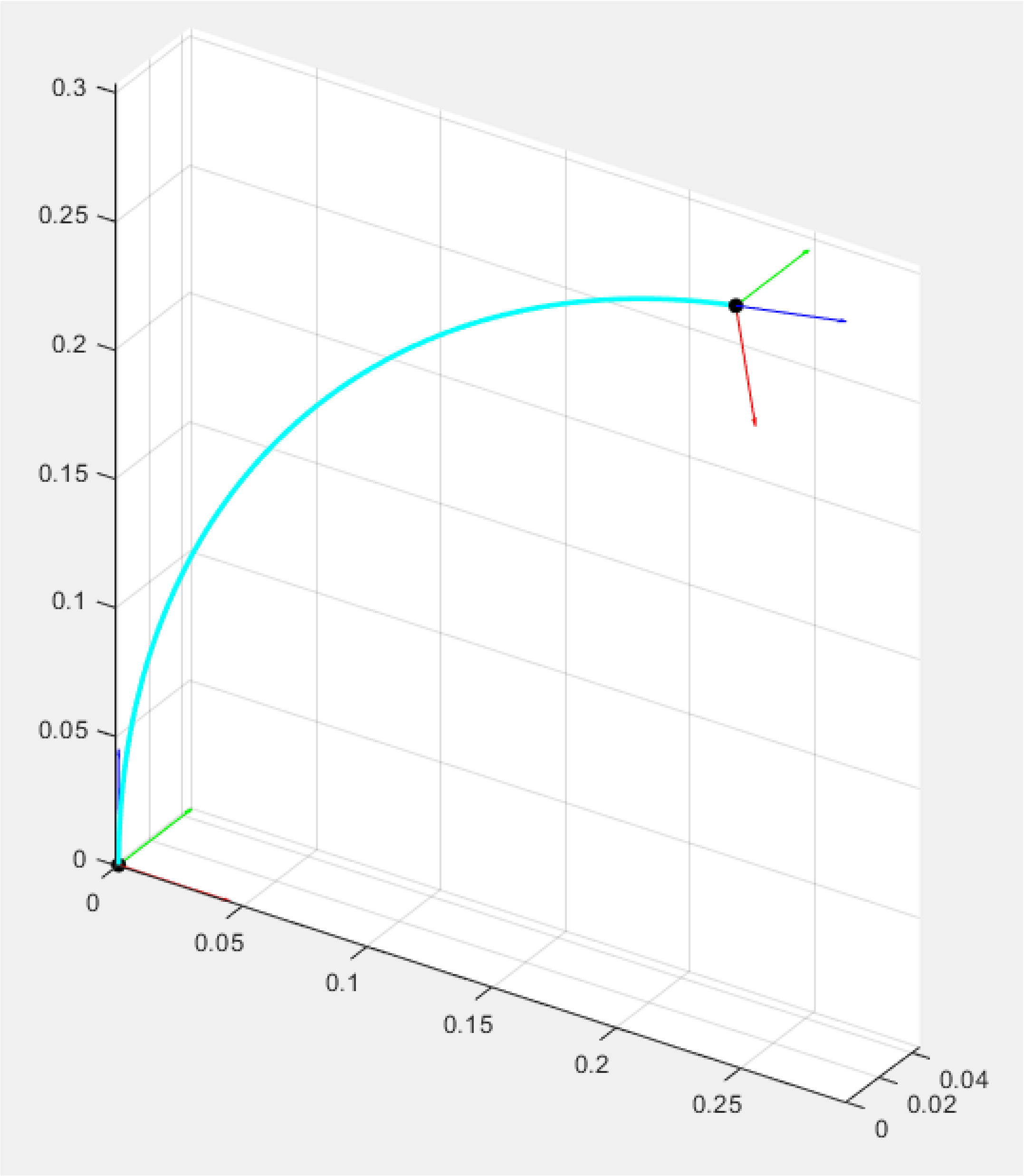
$s \in [0, l]$ with l the length of the arc

Rotation angle

Exercise 1

Draw points of the arc defined by $\theta = 80°$ and $r = 0.3\ m$

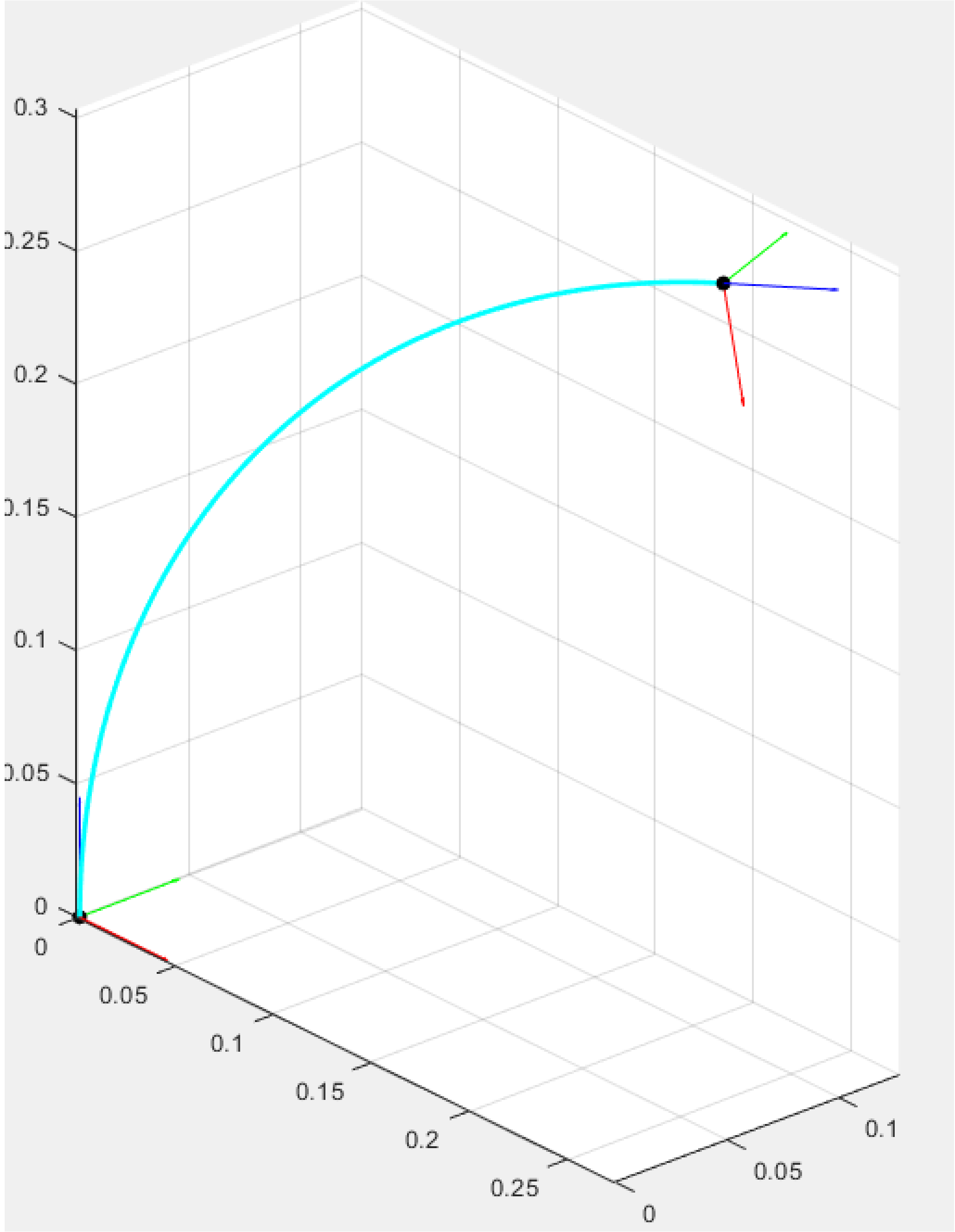
- Write the transformation matrix. Draw both the points and the orientation of the arc's tip
- Functions to draw points and frames are already provided
- Result should look like the one in the figure



Exercise 2.a

Extend the previous exercise considering the out of plane rotation $\Phi = 20°$

- Write the new transformation matrix. Draw both the points and the orientation of the arc’s tip
- Result should look like the one in the figure



Exercise 2.b

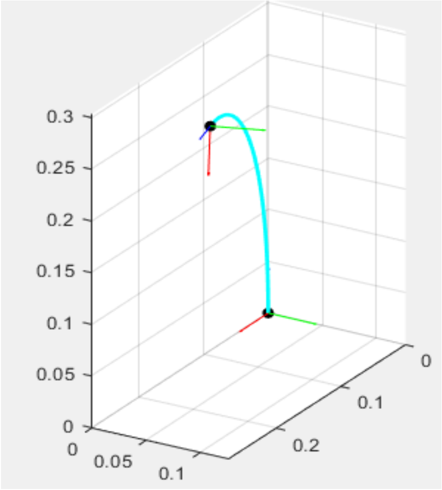
Rotating the z-axis by Φ, cause also the rotation of the frame at the tip of the arc.

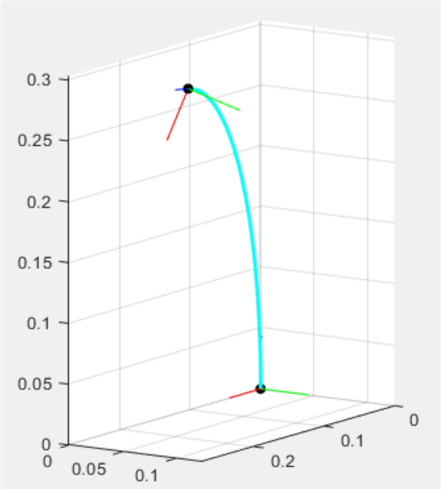
Can you align the frame with the axes of the base as if the frame *slides* along the arc?

Differences are highlighted in the figure

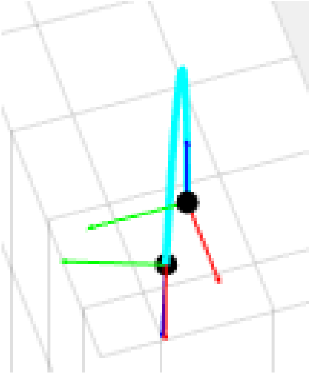
Exercise 2.a **Exercise 2.b**

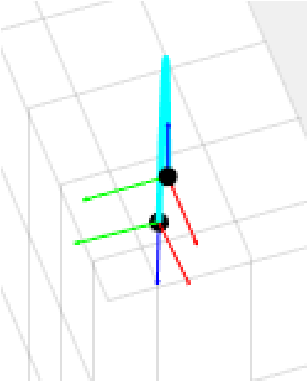
Front View





Bottom View





Exercise 2.c

For the same Φ, θ and r of the previous exercise, draw the arc using the *arc parameters.*

Rewrite the transformation matrix as

$$\boldsymbol{T}(\kappa, s, \Phi) = \begin{bmatrix} \boldsymbol{R_z}(\Phi) & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R_y}(k, s) & \boldsymbol{p}(k, s) \\ \boldsymbol{0} & 1 \end{bmatrix}$$

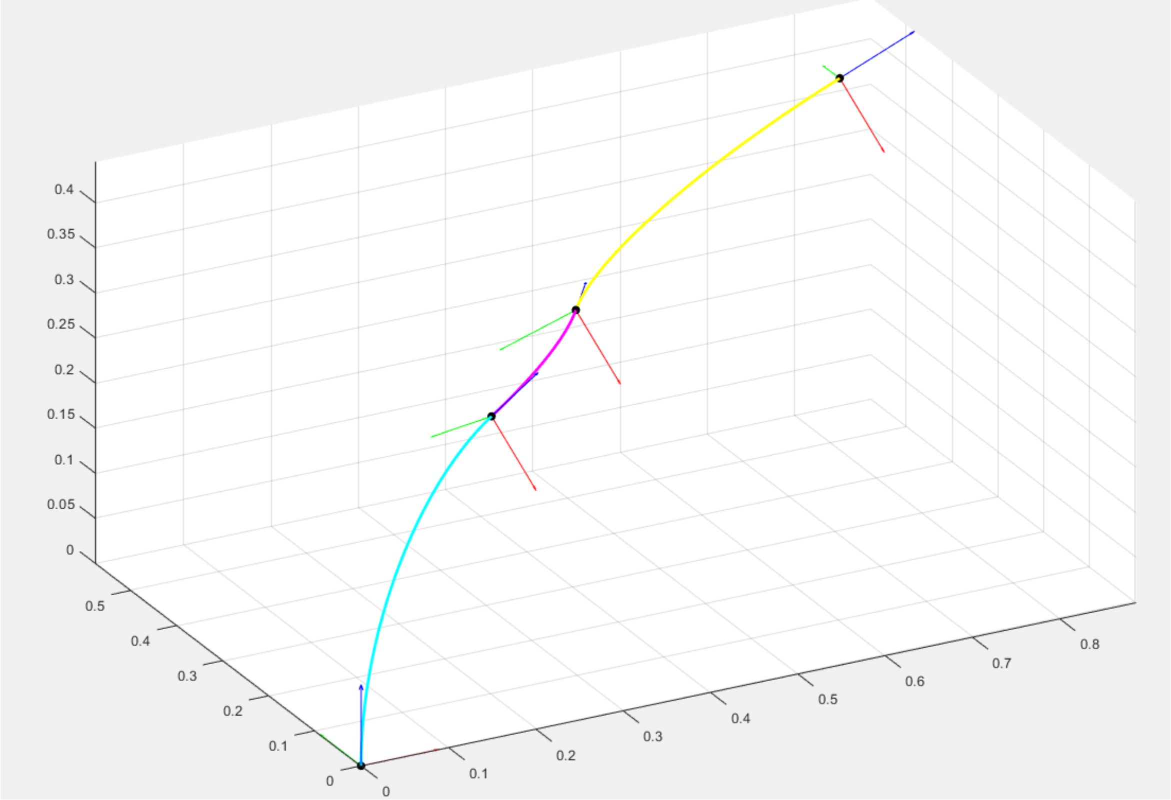
where

- $\kappa = 1/r$
- $\theta = \kappa s,$ with $s \in [0, l]$

Exercise 3

Extension to multi-segments.

- Draw the arc shown in the figure
- Draw the frames at the tip of each section
- The parameters of each segment are provided in the Matlab file



Exercise 4

The position of markers on the robot can be retrieved with a motion capture system.

Given the position and orientation of each segment's tip, find the arc parameters κ, l, Φ

- **Exercise 4.a** – Find κ, l given the position of the tip and the orientation obtained in Exercise 1 (2D case)

- **Exercise 4.b** – Find κ, l and Φ given the position of the tip and the orientation obtained in Exercise 2.c