LINEAR MODELS WITH DOMINANT VARIABLES

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1.0 INTRODUCTION

Most researchers experienced in applied regression analysis have encountered problems with dominant variables. This is the paradoxical situation where one (or more) independent variables fits the model too well, or as Kennedy (1980) describes it, "...(the dominant) accounts for so much of the variation in a dependent variable that the influence of other variables cannot be estimated." Typically, the dominant has an obvious and very strong relationship with the dependent variable, so that the problem lies with estimating the equation rather than with theoretical difficulties.

The statistical symptoms of the dominant variable problem are usually clear: a high multiple correlation coefficient (R^2); a very efficient estimate of one parameter - the dominant - in contrast to estimators with large standard errors for the remaining explanatory variables; and parameter estimates often with inappropriate magnitudes and incorrect signs. Such results are virtually useless when the main purpose of the model is to provide specific parameter estimates.

Traditional approaches to the problem include simply doing nothing, omitting the dominant, or redefining the dependent (Rao and Miller, 1971). These are relatively drastic solutions based on avoiding the dominant variable or ignoring its influence. New estimation strategies may be forthcoming as it is understood that collinearity between the dominant variable and the other explanatory variables is the critical reason for the effect of the dominant. In this paper we demonstrate that such collinearity is largely responsible for the dominant variable problem, and discuss one alternative estimation method based on orthogonalizing the dominant.

Attempting to estimate equations with dominant variables can lead to spurious conclusions regarding variable selection. Explanatory variables actually relevant to an equation may seem superfluous because of switched signs and high standard errors. This is often the case in models where outputs are regressed on raw materials. For example, Rao and Miller discuss the following equation of textile production; where (Q) is the value of output, (L) is the wage bill, (R) represents fixed capital, and (M) is the value of raw materials.

$$\log Q = -.408 - .059 \log K - .002 \log L + 1.094 \log M$$
$$(1.59) \quad (1.37) \quad (.039) \quad (16.8)$$

$R^2 = .997$

Although the model succeeds in explaining almost all of the variation in the dependent, only the raw material parameter has a significant t value (in parentheses). The labor and capital parameters have the wrong signs and their low t scores indicate their explanatory power is questionable. The analyst ignorant of the obvious relevance of the labor and capital variables might even drop them from the equation.

We can take a more detailed view of the problem with a hospital output model explaining variations in the total days of care provided by hospitals. The equation is specified so that total patient days (DAYS) is a function of hospital location (URBAN), complexity of the caseload (MIX), rates of surgery (SURG), seasonal trends (SUM), and a size variable - the number of beds (BEDS). A dummy variable (REV) has been included to measure the impact of a physican review program intended to reduce unnecessarily long hospital stays. The equation is estimated for a large sample of short-stay, non-teaching hospitals. The results (Table 1, model 1) show the same confusion of signs and low significance evident in the textiles example. We expected, for instance, that urban location (URBAN) would have a positive impact on total days (DAYS), and knew from past studies that case mix (MIX) and surgical rates (SURG) have a considerable effect. Only URBAN and BEDS have significant parameters, despite the theoretical importance of the other explanatory variables.

2.0 COLLINEARITY WITH THE DOMINANT VARIABLE

The sources of the dominant variable problem are evident when we examine collinearities in the hospital model. Referring to Appendix 1, we find that BEDS is the dominant variable: the covariance between DAYS and BEDS is over ten times higher than for any other variables; and the correlation of BEDS with DAYS is .93. BEDS is uncorrelated with the program variable REV.

The impact of collinearity with the dominant variable (BEDS) is apparent when the dominant is dropped from the equation (Table 1, model 2); Parameters change for each variable correlated with BEDS - the coefficients for URBAN and MIX both have the correct sign, and precision for URBAN and SURG coefficients have increased; parameters remain unchanged for variables uncorrelated with the dominant - the REV coefficient changes very little. The general consequences of omitting a relevant variable are noted in Appendix 2 and discussed in most econometric texts. See for example, Kmenta, (1971) and Maddala, (1977).

The impact of collinearity between individual variables is emphasized when a dominant variable is present. Even weak correlations with the dominant led to dramatic changes in the estimated parameters: a correlation of .25 with BEDS caused the URBAN parameter to increase by a magnitude of 4, and change signs when BEDS was dropped from the equation; the casemix parameter (MIX) flipped signs, also with a correlation with BEDS of .25; the SURG variable has the strongest

correlation with BEDS (.55) and its parameter increased from 2.5 to 61.6.

These results underscore the role collinearity among the explanatory variables plays in the dominant variable problem. Without such collinearity, dominant variables like BEDS would have no effect on the other parameters, irregardless of their strong covariation with the dependent. This is a fairly simple fact which has gone unnoticed in most discussions of dominant variables.

3.0 COLLINEARITY DIAGNOSTICS IN PROC REG

Recognizing that collinearity is at the heart of the matter, we can attempt to more rigorously assess its effects using the TOL, VIF, and COLLIN diagnostics available in the SAS procedure REG. These were estimated for the hospital model and are reported in Appendix 3.

The Variance Inflation Factor (VIF) and the closely related Tolerance (TOL) measures are based on auxiliary regressions of each independent variable of the remaining independents. An R^2 indicative of a strong linear dependency, say .9, would be reflected in a VIF of 10 (calculated as $1/1-R^2$). TOL is simply $1-R^2$, so that for the same example it would equal .1. Referring again to Appendix 3, neither of these statistics indicate serious collinearities.

COLLIN invokes a useful diagnostic procedure described by Belsley, Kuh and Welsh (1980). This method provides two types of information: The condition index indicates the presence of a near dependency among the columns of the data matrix; and the variance-decomposition proportions identify the variables involved. Belsley et.al. suggest, as a rough rule of thumb, that a condition index greater than 30 accompanied by variance proportions of at least .5 for two or more coefficients, would indicate a collinear relationship with the potential to degrade the estimated parameters (p.112-113). The highest condition index for the hospital model is 23, which suggests a moderate linear dependency. Reading across the variance proportion table we find that the intercept and MIX are highly related, with over 90% of the variation in their estimated parameters associated with the alarming condition index. This is due to the relatively small variation of MIX around a mean of 1. We do not find, however, any indication of degrading collinearities involving the dominant BEDS, despite the evidence discussed in the previous section.

This discussion makes it clear that the collinearity diagnostics supplied by PROC REG have limited usefulness for identifying harmful near dependencies in dominant variable equations. There are a number of reasons for this. First, the VIF and TOL do not reflect the extra sensitivity of the parameters to collinearity with a dominant variable. The linear dependencies indicated by these measures for BEDS were mild, but nevertheless capable of severely affecting parameter estimates. Second, a more general shortcoming of VIF and TOL is that even when serious collinearity is evident, they do not help to identify the specific variables involved. And third, the variance-decomposition method may not work when a dominant variable is involved. The procedure is sensitive to "essential" scaling imbalances, where the variation introduced by one variable is much greater than the variation contributed by the other independents (Belsley, Kuh, Welsh, p.154). In this situation potential near dependencies involving the dominant may be difficult to detect using the variance-decomposition methodology.

More useful information is provided by the parameter estimates of auxiliary regression of the dominant against the other independent variables. Once the covariance matrix has indicated a probable dominant variable, this regression can identify the variables most strongly affected.

BEDS=45.8 +21.6 URBAN −20.1 MIX +2.50 SURG
(3.8) (6.3) (1.5) (20)

−1.53 Sum −.703
(.43) (.11) R^2 = .33

The auxiliary regression for BEDS is reported above. As the correlation matrix suggested, SURG is most strongly related to BEDS, with URBAN also significant. Coefficients of this equation not only indicate the strength of collinearity, but also show its direction and provide an estimate of the bias to the remaining parameters when BEDS is omitted.

4.0 ESTIMATING EQUATIONS WITH DOMINANT VARIABLES

The dominant variable is only a problem for certain models, depending on their intended use. If the analyst is primarily concerned with predictions instead of parameter estimates, a dominant variable model can provide accurate forecasts. But if specific parameter estimates are important, as in descriptive models or evaluation models, then the influence of the dominant should be recognized and possibly ameliorated.

There are at least four strategies for estimating dominant variable equations: ignore the issue; omit the dominant variable; redefine the dependent in a form less susceptible to the influence of the dominant variable; or substitute a proxy for the dominant. Again, selection of the appropriate approach depends heavily on the purpose of the model, e.g. prediction, description, or evaluation.

IGNORE THE DOMINANT: This is a reasonable approach when the model is used for forecasting. The dominant variable may degrade the estimates of specific parameters, but the model can still be capable of accurate prediction. This is true only if any collinear relationships remain stable into the prediction period.

Ignoring the influence of the dominant may also be appropriate when variables of particular interest are independent of the dominant variable. As we noted earlier, the coefficients for variables unrelated to the dominant are unaffected by including or omitting the dominant from the equation. For instance, the program variable REV in the hospital model is nearly independent of the dominant BEDS. If we were interested solely in the parameter of REV, then the problems with

coefficients for the other variables would be irrelevant. Often in evaluation models explanatory variables serve only as statistical controls. If the intervention variable is orthogonal to the dependent then there is no problem.

However, it is rare that any two explanatory variables are completely independent, and even very slight collinearity can cause considerable effects to estimators and their variances when a dominant is involved. Thus, it may not be appropriate to "ignore the dominant" for any equation where variables of interest are even slightly collinear with the dominant.

OMIT THE DOMINANT VARIABLE: In some situations omitting the dominant variable is the simplest way to improve estimates of the remaining coefficients. For example, an economist's production function often does not include a variable for raw materials. Consider the re-estimate of the textile model discussed earlier:

$$\text{Log } Q = -.206 + .413 \log K + .708 \log L \qquad R^2 = .93$$
$$(.02) \quad (2.5) \qquad (5.1)$$

After the dominant (raw materials) has been dropped, the parameters for labor (L) and capital (K) have the proper signs and magnitudes (Rao and Miller,p.42). The R^2 indicates that relatively little explanatory power has been lost by the deletion; and overall the model is much more useful than its predecessor containing the dominant, raw materials.

However, there are reservations to this equation. First, the remaining parameters are biased to the extent that labor and capital are collinear with materials. But perhaps more important, the equation leaves out a critical variable in its description of the production process. This is a theoretical problem: One might legitimately ask how textiles can be made without raw materials.

Aside from theoretical difficulties, excluding the dominant can often reduce the explanatory power of a model to the extent that the estimates of the remaining parameters are suspect. This may not be the case for models with other strong explanatory variables like the textile equation, but it is a definite problem for equations like the hospital model.

As previously discussed in Table 1, model 2, the mean squared error (MSE)of the hospital equation increased almost sixfold when BEDS was dropped. The parameters of the remaining variables, like the textile model, now have appropriate signs and magnitudes, but are also biased and inconsistent. The standard error of REV has almost doubled. Examining a scatter plot of the standardized residuals (Appendix 4) makes it clear that without BEDS the error term is no longer distributed around mean 0 and that the equation is misspecified.

Omitting the dominant variable from the equation is a strategy that should be followed with considerable caution. It is inappropriate for forecast models because of the increased MSE and potential bias which comes from excluding a relevant variable. It is not very helpful for evaluation models because significant tests for intervention parameters (like REV, for example) are understated, even if the intervention variable is independent of the dominant. But for descriptive models omitting the dominant does provide at least rough estimates of parameters that were otherwise masked by collinearity with the dominant. In models where the dominant adds little new information, such as the textile example, estimates of the "omitted dominant" equation may be acceptable.

REDEFINE THE DEPENDENT: The influence of the dominant variable can be diminished by using a rate, rather than a total measure, as the dependent variable. This is an intuitively appealing way to include the dominant in the equation, but reduce its almost lock-step covariance with the dependent. For example, the hospital model was re-estimated using average days of care instead of total days of care. The results are reported in Table 1, model 3.

In this model the impact of BEDS is indeed reduced, but so is the overall power of the model-the rate equation explains less than 10% of the variation of the average length of stay. The standardized residuals are not centered on zero but are distributed in a linear pattern indicating that this equation is seriously misspecified. It is clear that variables important for explaining variation in the dependent are missing.

As an alternative to the original equation, the rate model can be extremely useful, given two rather obvious conditions. First, the model must still be meaningful; does it make a difference to the analyst that he or she is forecasting (describing, evaluating) averages instead of totals? Second, it is important that the independent variables have sufficient explanatory power relative to the new dependent. If the available independents do provide an adequately specified model then the rate strategy is the optimal solution to the dominant variable problem: the corrupting influence of the dominant has been diminished without dropping the dominant and biasing the remaining coefficients. Thus, redefining the dependent should be the first alternative when the equation is adequately specified and relevant to the analyst's needs. This is especially true for descriptive and evaluative models because of the importance of accurate parameter estimates.

Unfortunately, the rate model may not always be sufficiently specified. This is true for the hospital model, where the low explained variation and the skewed distribution of the residuals clearly indicate a misspecified equation. (See Appendix 4).

PROXY FOR THE DOMINANT: A final approach for estimating a dominant variable equation involves selecting a proxy for the dominant. This solution is appropriate for problems like the hospital equation, where the other alternatives we have discussed are impractical. Consider the hospital model: the original equation provides unreasonable parameter estimates for variables collinear with the dominant; omitting the dominant eliminates its influence on the remaining parameters, but leaves them biased, inconsistent and in-

efficient; redefining the dependent as a rate takes away most of the model's explanatory power leaving the equation obviously misspecified. Replacing BEDS with a suitable proxy could at least improve the efficiency of the parameters previously estimated by dropping the dominant.

Ideally, a proxy for the dominant can be found that minimizes collinearity with the other independents, yet is still strongly related to the dependent. The trick, of course, is to find such a variable. A logical choice for the proxy is an alternative form of the dominant variable, purged of its collinearity--and thus its dominance--with the other independent variables. This is done by regressing the dominant on the other independents and retrieving the residuals. These residuals define the proxy that can then be substituted into the original equation.

An equation using this type of proxy has four interesting characteristics, the last one being the most important. A more detailed discussion of these characteristics can be found in Appendix 2.

1. The parameter estimates are not different from those in the "omitted dominant" equation. This reminds us that, in general, adding or deleting an orthogonal variable (such as the proxy) will not affect the parameter estimates of the other independents.

2. The parameter estimates have the same biases and inconsistencies as the "omitted dominant" equation. The strength and direction of the bias can be explored by regressing the dominant on the other independents (the "auxiliary" regression described in an earlier section).

3. The coefficient for the proxy is identical to the coefficient for the dominant variable in the original equation.

4. The residual variance is less than for the equation that omitted the dominant. We have the same biased parameters, but they are now more precise. Other things being equal, the standard errors of parameters will be smaller, and their t values larger.

The hospital model was re-estimated using a proxy for BEDS, and the results are reported in Table 1, model 4. As was expected, the estimated parameters have not changed from the "omitted dominant"equation. BEDS has the same coefficient as in the original equation (Table 1, model 1). The greatest change brought about by the proxy was the improvement in the precision of the estimators. The standard errors for all of the parameter estimates are smaller than for any of the other hospital equations. This result is especially important for models where the significance of specific estimators, such as intervention effects like REV, are critical. When redefining the dependent as a rate is not a viable alternative, the proxy model will at least provide more efficient estimates than the "omitted dominant" strategy.

5.0 SUMMARY

In this paper we have taken an applied approach to estimating dominant variable equations. Using a simple economic production model and a hospital output model it was demonstrated that collinearity among the independent variables is necessary for a dominant variable problem to exist. If there is no such collinearity then the dominant is not "dominant" at all, and is just a very significant explanatory variable.

It was also shown that some of the collinearity diagnostics in Proc Reg fail to indicate the potential harm of collinearity with the dominant. This is because normally moderate collinearities can, nevertheless, seriously impact parameter estimates when a dominant variable is involved. The effects of collinearity seem emphasized in the dominant variable situation.

The variance-decomposition methodology was particularly disappointing in diagnosing collinearity with the dominant. We suggest that correlation and covariance matrices are less sophisticated, but more useful indicators of the dominant variable and its correlates. An auxiliary regression of the dominant on the other regressors will estimate the strength and direction of these collinearities.

Four approaches to estimating dominant variable equations were discussed. Of the three traditional approaches--ignoring the problem, omitting the dominant, and redefining the dependent--the latter was considered the optimal choice. By redefining the dependent variable as a rate instead of a total the severe covariance of the dominant with the dependent is diminished, without dropping the dominant and biasing the remaining parameters. When the rate model was not feasible, as in the case of the hospital model, a proxy for the dominant can be used to improve the efficiency of the estimators provided by the "omitted dominant" equation.

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TABLE 1. ALTERNATIVE ESTIMATES OF THE HOSPITAL DAYS OF CARE MODEL (N = 1239)

MODEL	INTERCEPT	URBAN	MIX	SURG	SUM	REV	BEDS	
1. Full Model, including the dominant, BEDS	-129 (1.0)	-91.9* (2.4)	249 (1.7)	2.56 (1.6)	-56.3 (1.4)	-135 (2.0)	23.5* (77.1)	R^2 = .88 MSE = 329455
2. Omit the Dominant	950 (3.1)	419* (4.7)	-224 (.65)	61.6* (19)	-92.5 (1.0)	-151 (.95)		R^2 = .31 MSE = 1919620
3. Redefine Dependent as a Rate: Days Per Admission	.562 (11)	-.020 (1.4)	.300* (5.3)	-.0007 (1.2)	.030 (2.0)	.004 (.17)	.0009* (7.7)	R^2 = .09
4. Original Model, with Proxy for Dominant, BEDS**	950 (7.4)	419* (11.3)	-224 (1.5)	61.6* (46)	-92.5* (2.4)	-151 (2.3)	23.5* (77.1)	R^2 = .88 MSE = 329455

t values included in parentheses.

* p less than .01.

** Proxy Defined: $\widehat{BEDS} = BEDS - (b_1 + b_2\ URBAN + b_3\ MIX + b_4\ SURG + b_5\ SUM + b_6\ REV)$

APPENDIX 1

COVARIANCE MATRIX

	DAYS	URBAN	MIX	SURG	SUMMER	REVIEW	BEDS
DAYS	2717047	160.831	57.1954	12970.7	-12.2487	-19.7962	100029
URBAN	160.831	0.206376	.0099568	1.29745	-6.2E-04	.0028518	7.4038
MIX	57.1954	.0099568	.0169834	0.91033	-.003443	-8.4E-04	2.16368
SURG	12970.7	1.29745	0.91033	204.429	-.010357	-0.19568	522.149
SUMMER	-12.2487	-6.2E-04	-.003443	-.010357	0.188035	-.006873	-.055191
REVIEW	-19.7962	.0028518	-8.4E-04	-0.19568	-.006873	.0632152	-.497636
BEDS	100029	7.4038	2.16368	522.149	-.055191	-.497636	4190.94

CORRELATION COEFFICIENTS / PROB > |R| UNDER HO:RHO=0 / NUMBER OF OBSERVATIONS

	DAYS	URBAN	MIX	SURG	SUMMER	REVIEW	BEDS
DAYS	1.00000 0.0000 1284	0.21467 0.0001 1284	0.26626 0.0001 1284	0.54493 0.0001 1240	-0.01714 0.5395 1284	-0.04777 0.0871 1284	0.93739 0.0001 1284
URBAN	0.21467 0.0001 1284	1.00000 0.0000 1284	0.16809 0.0001 1284	0.20031 0.0001 1240	-0.00313 0.9108 1284	0.02495 0.3716 1284	0.25162 0.0001 1284
MIX	0.26626 0.0001 1284	0.16809 0.0001 1284	1.00000 0.0000 1284	0.48315 0.0001 1240	-0.06447 0.0209 1284	-0.02568 0.3579 1284	0.25646 0.0001 1284
SURG	0.54493 0.0001 1240	0.20031 0.0001 1240	0.48315 0.0001 1240	1.00000 0.0000 1240	-0.00146 0.9533 1240	-0.05474 0.0540 1240	0.55842 0.0001 1240
SUMMER	-0.01714 0.5395 1284	-0.00313 0.9108 1284	-0.06447 0.0209 1284	-0.00146 0.9533 1240	1.00000 0.0000 1284	-0.06304 0.0239 1284	-0.00197 0.9437 1284
REVIEW	-0.04777 0.0871 1284	0.02495 0.3716 1284	-0.02568 0.3579 1284	-0.05474 0.0540 1240	-0.06304 0.0239 1284	1.00000 0.0000 1284	-0.03057 0.2734 1284
BEDS	0.93739 0.0001 1284	0.25162 0.0001 1284	0.25646 0.0001 1284	0.55842 0.0001 1240	-0.00197 0.9437 1284	-0.03057 0.2734 1284	1.00000 0.0000 1284

APPENDIX 2
THE LINEAR MODEL AND ISSUES RELATED TO DOMINANT VARIABLES

Consider the following equation, where X_2 is the dominant variable

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon \qquad (1)$$

If we drop X_2 and estimate

$$y = X_1\beta_1 + u \qquad (2)$$

Then the estimate of β_1 will be biased, as in the general case of omitting a relevant variable. This is shown with the familiar proof

$$\hat{\beta}_1 = (X_1'X_1)^{-1}X_1'y$$

$$= (X_1'X_1)^{-1}X_1'(X_1\beta_1 + X_2\beta_2 + \varepsilon)$$

$$E(\hat{\beta}_1) = \beta_1 + (X_1'X_1)^{-1}X_1'X_2\beta_2$$

or $E(\hat{\beta}_1) = \beta_1 + P\beta_2$ where $P = (X_1'X_1)^{-1}X_1X_2$ is the matrix of auxiliary regression coefficients of X_2 on X_1. These parameters are useful as a measure of collinearity between X_2 and X_1, and they provide estimates of the direction and magnitude of the bias of $\hat{\beta}_1$ if X_2 is dropped from the equation.

If X_1 and X_2 are independent, $P=\emptyset$, then $\hat{\beta}_1$ will be unbiased. However, the estimate of the residual variance S_2 will still include the variance of the omitted variable, X_2. Thus S^2 will be greater than the true residual σ^2 from (1) and will bias the estimated variance of the parameters upward, so that tests of significance and confidence intervals for β_1 will be overly conservative.

The effects of the dominant variable on estimates of β_1 can be eliminated by "purging" X_2 of its collinearity with X_1. The new values of X_2^* would replace X_2 in the original equation

$$y = X_1\beta_1 + X_2^*\beta_2 \qquad (3)$$

Where X_2^* is the residual of the regression of X_2 on X_1, such that $X_2^* = X_2 - PX_2$. Phillips and Wickens (1978) discuss some of the properties of estimators for this model:

First, the estimated coefficients for the proxy model (3) are identical to (2), where the dominant was dropped. To see this remember that now $X_1'X_2^* = \emptyset$ so that the solution set for $\hat{\beta}_1$ and $\hat{\beta}_2$ has become

$$\begin{bmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{bmatrix} = \begin{bmatrix} X_1'X_1 & \emptyset \\ \emptyset & X_2^{*\prime}X_2^* \end{bmatrix}^{-1} \begin{bmatrix} X_1'y \\ X_2^{*\prime}y \end{bmatrix}$$

$$= \begin{bmatrix} (X_1'X_1)^{-1}X_1'y \\ (X_2^{*\prime}X_2^*)X^{*\prime}y \end{bmatrix}$$

thus $\hat{\beta}_2 = (X^{*\prime}_2X_2^*)^{-1}X^{*\prime}y$

Second, the estimates of β_1 have the same bias $P\beta_2$ as (2). This can be shown using the "omitted relevant variable" proof, above.

Third, purging X_2 of its relation with X_1 does not change the estimated parameter $\hat{\beta}_2$ from its original value in equation (1). Consider the following: X_2^* can be defined as $X_2^* = Q_1X_2$ where $Q_1 = I - X_1(X_1'X_1)^{-1}X_1'$. We can estimate the original equation (with X_2) using Q_1 and the inverse of

the partitioned $X'X$ matrix:

$$\begin{bmatrix} \bar{\beta}_1 \\ \bar{\beta}_2 \end{bmatrix} = \begin{bmatrix} (X_1'X_1)^{-1}+(X_1'X_1)^{-1}X_1'X_2(X_2'Q_1X_2)^{-1}X_2'X_1(X_2'X_2)^{-1} & -(X_1'X_1)X_1'X_2(X_2Q_1X_2)^{-1} \\ -(X_2'Q_1X_2)^{-1}X_2'X_1(X_2'X_2)^{-1} & (X_2'Q_1X_2)^{-1} \end{bmatrix} \begin{bmatrix} X_1'y \\ X_2'y \end{bmatrix}$$

solving for $\bar{\beta}_2$ we find

$$\bar{\beta}_2 = (X_2'Q_1'X_2)^{-1}X_2'Q_1y$$

and since Q_2 is idempotent,

$$\bar{\beta}_2 = (X_2'Q_1'Q_1X_2)^{-1}X_2Q_1y$$
$$= (X_2^{*\prime}X_2^*)^{-1}X_2^{*\prime}y$$

which is identical to the expression for $\hat{\beta}_2$, where $X_2^* = Q_1X_2$.

Fourth, the estimate of the residual variance for the proxy model (3) will be less than for the omitted variable equation (2), because X_2 is no longer left in the error term. Thus, while there is no change in the parameter estimates between (2) and (3), the variances of the estimates are lower for the proxy model (3).

APPENDIX 3

COLLINEARITY DIAGNOSTICS

DEP VARIABLE: SAVE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	3027547173	756886829	1331.408	0.0001
ERROR	1233	486217759	329453		
C TOTAL	1237	3433764932			

ROOT MSE	573.982	R-SQUARE	0.8817
DEP MEAN	1892.709	ADJ R-SQ	0.8811
C.V.	30.32593		

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	TOLERANCE	VARIANCE INFLATION
INTERCEP	1	-129.372	127.706	-1.013	0.3112		0.000000
URBAN	1	-91.754134	37.474474	-2.494	0.0143	0.917464	1.090384
MIX	1	249.705	142.491	1.754	0.0797	0.754162	1.326076
SURG	1	2.362084	1.323049	1.682	0.0928	0.560713	1.783438
SUMMER	1	-54.348637	37.438729	-1.497	0.1344	0.989481	1.010427
REVIEW	1	-133.074	45.548545	-2.661	0.0395	0.990103	1.009997
BEDS	1	28.377244	0.305301	77.182	0.0001	0.446148	1.501169

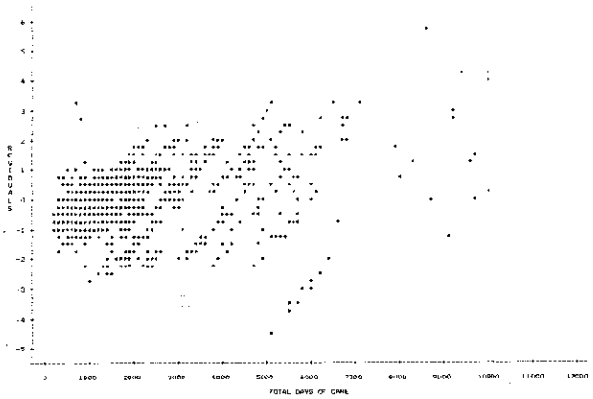
COLLINEARITY DIAGNOSTICS — VARIANCE PROPORTIONS

NUMBER	EIGENVALUE	CONDITION INDEX	PORTION INTERCEP	PORTION URBAN	PORTION MIX	PORTION SURG	PORTION SUMMER	PORTION REVIEW	PORTION BEDS
1	4.205	1.000	0.0008	0.0170	0.0007	0.0100	0.0134	0.3045	0.0133
2	0.952729	2.101	0.0000	0.0006	0.0000	0.0018	0.0563	0.5847	0.0012
3	0.761514	2.350	0.0001	0.1199	0.0060	0.0098	0.4855	0.0384	0.0144
4	0.573037	2.709	0.0010	0.8224	0.0009	0.0281	0.1248	0.0667	0.8194
5	0.318905	3.631	0.0141	0.0053	0.0059	0.1080	0.1042	0.0452	0.1285
6	0.181323	4.814	0.0011	0.0048	0.0060	0.6855	0.3010	0.0032	0.4850
7	0.067704	23.360	0.9830	0.0068	0.9883	0.1567	0.3128	0.0011	0.0070

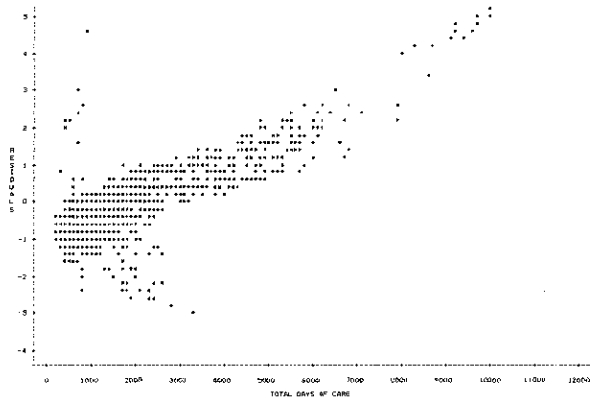
APPENDIX 4

SCATTER PLOTS OF RESIDUALS BY THE DEPENDENT FOR ALTERNATIVE ESTIMATIONS OF THE HOSPITAL MODEL

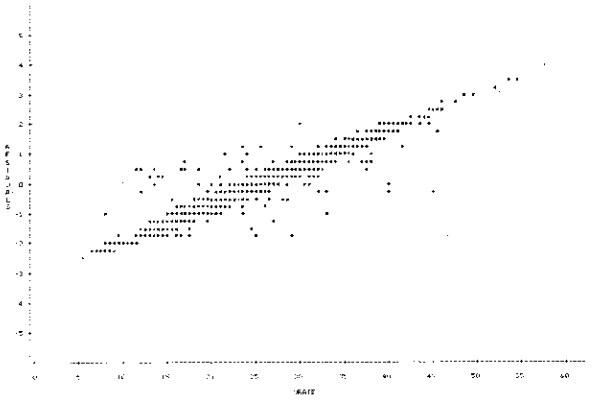
MODEL 1: Full Model, includes the Dominant



MODEL 2: Omit the Dominant



MODEL 3: Redefine the Dependent as a Rate



NOTE: The explained variation and the scatter plots for the proxy equation (model 4) and the full model equation (model 1) are identical.