Comp 330 (Fall 2024): Assignment 1 Solutions

Answers must be submitted online by September 30 (11:59 pm), 2024.

General instructions

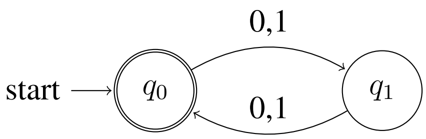
- **Important:** All of the work you submit must be done by only you, and your work must not be submitted by someone else. Plagiarism is academic fraud and is taken very seriously.
- To some extent, collaborations are allowed. These collaborations should not go as far as sharing code or giving away the answer. **You must indicate on your assignments the names of the people with whom you collaborated or discussed your assignments (including members of the course staff). If you did not collaborate with anyone, write "No collaborators". If asked, you should be able to orally explain your solution to a member of the course staff.**
- It is your responsibility to guarantee that your assignment is submitted on time. We do not cover technical issues or unexpected difficulties you may encounter. Last minute submissions are at your own risk.
- Multiple submissions are allowed before the deadline. We will only grade the last submitted file. Therefore, we encourage you to submit as early as possible a preliminary version of your solution to avoid any last minute issue.
- Late submissions can be submitted for 24 hours after the deadline, and will receive a flat penalty of 20%. We will not accept any submission more than 24 hours after the deadline. The submission site will be closed, and there will be no exceptions, except medical.
- In exceptional circumstances, we can grant a small extension of the deadline (e.g. 24h) for medical reasons only. However, such request must be submitted before the deadline, justified and approved by the instructors.
- Violation of any of the rules above may result in penalties or even absence of grading. If anything is unclear, it is up to you to clarify it by asking either directly the course staff during office hours, by email at or on the discussion board on Ed. Please, note that we reserve the right to make specific/targeted announcements affecting/extending these rules in class and/or on the website. It is your responsibility to monitor Ed for announcements.
- The course staff will answer questions about the assignment during office hours or in the online forum. We urge you to ask your questions as early as possible. We cannot guarantee that questions asked less than 24h before the submission deadline will be answered in time. In particular, we will not answer individual emails about the assignment that are sent the day of the deadline.
- Unless specified, **you must show your work and all answers must be justified.**

1. (10 points) Let $\Sigma = \{0, 1\}$. Draw a deterministic finite automaton (DFA) that accepts the language

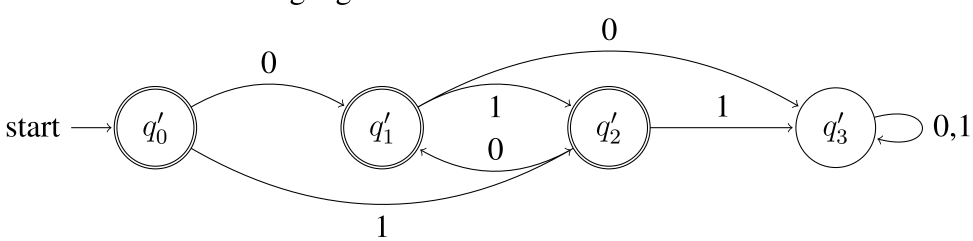
$$L_1 = \{w | w \text{ has an even number of letters, and}$$
$$w \text{ does not have two identical consecutive letters}\}$$

Hint: You can build a automaton for each language (i.e., "even number of letter" and "no two identical consecutive letters) then construct the product of the two's.

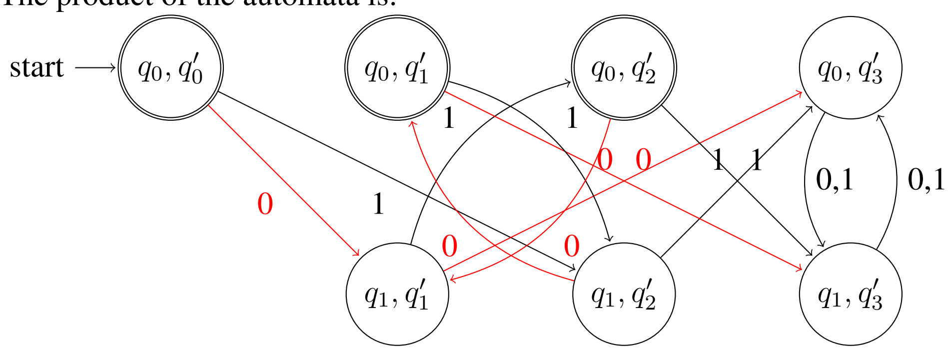
Solution: A DFA for the language of words with even number of letters is:



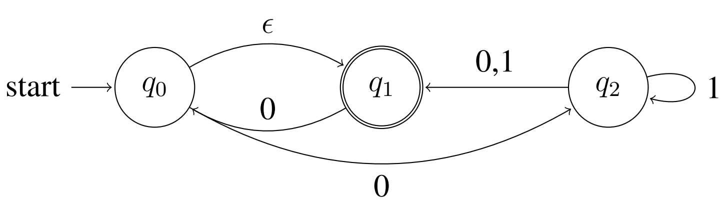
A DFA for the language of words that do not have two identical consecutive letters is:



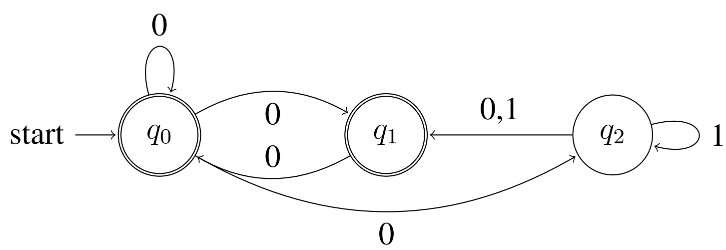
The product of the automata is:



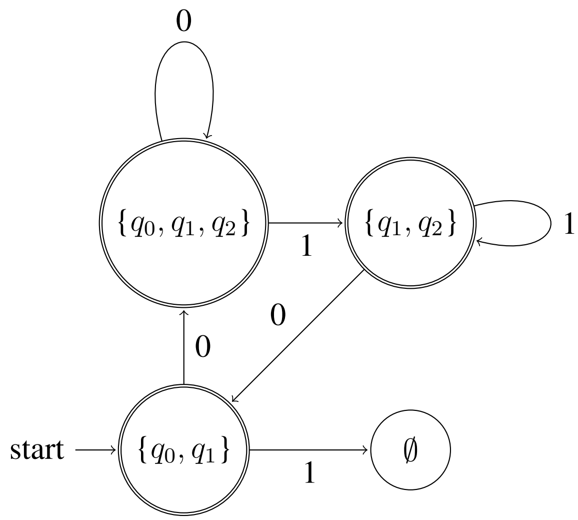
2. (a) (10 points) Convert the following nondeterministic finite automaton (NFA) into a DFA.



Solution: First, build a NFA from the ϵ-NFA.



Then, build a DFA from the NFA.



(b) (5 points) Use your solution to build another DFA that accepts the complementary language $\overline{L} = \{ \ w \mid w \notin L \ \}$

Solution: Swap accept and non-accept states.

3. Write a regular expression describing the set of strings over the alphabet $\Sigma = \{0, 1\}$ that:

(a) (10 points) Does not contain the string 110.

Solution: $(0 + 10)^*1^*$

(b) (10 points) Start and end with the same letter.

Solution: $1(0 + 1)^*1 + 0(0 + 1)^*0$

4. Let $\Sigma = \{0, 1\}$. Show that:

(a) (10 points) the language $L_2 = \{ \ 0^k s 0^k \mid k \geq 1 \text{ and } s \in \Sigma^* \ \}$ is regular.

Solution: The language is equivalent to $L_2' = 0(0 + 1)^*0$, which is regular. Indeed, if $w \in L_2$, then $\exists k$ such that $w = 0^k s 0^k$ for some $k \geq 1$ and $s \in \Sigma^*$. But we can also write w

as $w = 0s'0$ where $s' = 0^{k-1}s0^{k-1}$. Thus, $w \in L'_2$. Conversely, if $w \in L'_2$, then $w = 0s0$ with $s \in \Sigma^*$. Using $k = 1$, we have $w \in L_2$.

(b) (10 points) the language $L_3 = \{ \ 0^k1s0^k \mid k \geq 1 \text{ and } s \in \Sigma^* \ \}$ is **not** regular.

Solution: Similar than as in class. Let $n \geq 1$ be the integer associated with the pumping lemma. Let $w = 0^n1s0^n \in L_3$ for some $s \in \Sigma^*$. Then, we have $|w| \geq n$ and we can write w as $w = xyz$ where $|xy| \leq n$ and $|y| \geq 1$. Furthermore, we observe that y is only made of 0. Let $k = |w| + 1$ (or any other k such that $|xy^k| > |w|$). Using the pumping lemma, we should have $w = xy^kz \in {}_3$. But $|xy^k| \geq |z|$ (and 1 belong to z) and therefore there is clearly more zeros before the first one that after. Therefore, $w = xy^kz \notin L_3$ and L_3 is not regular.

5. We introduce the rotation operation on languages $rot(L) = \{xy \mid yx \in L\ \}$.

(a) (10 points) Show that $rot(L) = rot(rot(L)))$. Start to show that $rot(L) \subseteq rot(rot(L))$. Then, show $rot(L) \supseteq rot(rot(L))$.

Solution: For any language L, we have $L \subseteq rot(L)$. Thus, for any language L, we have $rot(L) \subseteq rot(rot(L))$.
Conversely, let Σ be the alphabet, for every $w \in \Sigma^*$, if $w \in rot(rot(L))$, then $w \in rot(L)$. Suppose $w \in rot(rot(L))$. Let $w = yx$ for some $x, y \in \Sigma^*$ such that $xy \in rot(L)$. For $xy \in rot(L)$ to hold, either $xy = x_1x_2y$ and $x_2yx_1 \in L$ for some $x_1, x_2 \in \Sigma^*$ or $xy = xy_1y_2$ and $y_2xy_1 \in L$ for some $y_1, y_2 \in \Sigma^*$. In the first case where $x_2yx_1 \in L$, we have $yx_1x_2 \in rot(L)$ and hence $w = yx = yx_1x_2 \in rot(L)$. The second case is similar.

(b) (15 points) Show that a regular language L is closed under the operation $rot()$. Let M_L be a DFA that recognizes L. Show how to build a NFA N_L that recognizes $rot(L)$.

Solution: Let L be an arbitrary regular language and $M_L = (Q_L, \Sigma, \delta_L, q_L, F_L)$ be a DFA that recognizes L. To prove that $rot(L)$ is also regular, we build from M_L a new NFA N_L that recognizes $rot(L)$.
Suppose N is given an input $w = yx$ for some $x, y \in \Sigma^*$ such that $xy \in L$.

- Let q_x be the state in which M_L ends up after reading x.
- Starting from q_x, M_L should end at some final state after reading y.
- To accept w, we let N_L simulate M_L from q_x.
- Then, after reading y and reaching a final state, we add in N_L an epsilon transition (which needs to be added to M_L) to the initial (starting) state q_L of M_L and continue simulating M_L with the rest of the input.

If N_L ends up at q_x, then the input w is of the correct form of yx such that $xy \in L$. Any state of M_L may act as q_x. For N to start and finish the simulation at the same state, we need $|Q_L|$ copies of M_L, one for each state in Q_L, with an epsilon transition added from every final state to the initial state. To start the simulation of M_L from any state, N has an epsilon transition from its initial state to every state of M_L.

6. (10 points) Using the k-path induction method, write a regular expression representing the language accepted by the following DFA. Show your work. Simplify the regular expression as much as you can.

$$\begin{tikzcd} \text{start} \arrow[r] & q_1 \arrow[r, bend left, "0"] \arrow[rr, bend left=40, "1"] & q_2 \arrow[l, bend left, "0"] \arrow[r, "1"] & q_3 \arrow[ll, bend left=40, "1"] \arrow[loop right, "0"] \end{tikzcd}$$

Solution:

$$R^3_{12} = R^2_{12} + R^2_{13}(R^2_{33})^* R^2_{32} \quad (1)$$
$$R^3_{13} = R^2_{13} + R^2_{13}(R^2_{33})^* R^2_{33} \quad (2)$$
$$R^2_{12} = 0(00)^* \quad (3)$$
$$R^2_{13} = 1 + (00)^*1 + 0(00)^*1 = 0^*1 \quad (4)$$
$$R^2_{32} = R^1_{32} + R^1_{32}(R^1_{22})^* R^1_{22} = (10) + (10)(00)^*00 = (10)(00)^* \quad (5)$$
$$R^2_{33} = R^1_{33} + R^1_{32}(R^1_{22})^* R^1_{23} = (11) + (10)(00)^*(1 + 01) = 10^*1 \quad (6)$$
$$R^3_{12} = 0(00)^* + 0^*1(10^*1)^*(10)(00)^* \quad (7)$$
$$R^3_{13} = 0^*1 + 0^*1(10^*1)^*10^*1 \quad (8)$$
$$R = 0(00)^* + 0^*1(10^*1)^*(10)(00)^* + 0^*1 + 0^*1(10^*1)^*10^*1 \quad (9)$$
$$\ldots \quad (10)$$

Question:	1	2	3	4	5	6	Total
Points:	10	15	20	20	25	10	100
Score:							