Stochastic Optimization - 3

Particle Swarm Optimization, PSO

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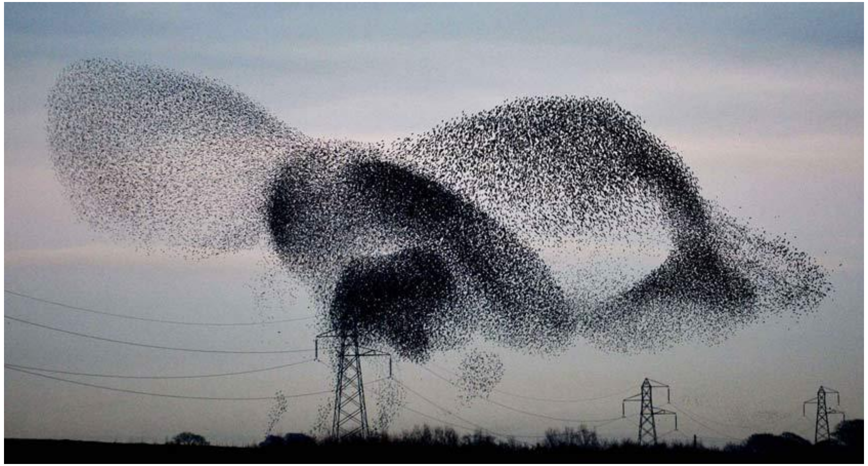
Cergy - Pau

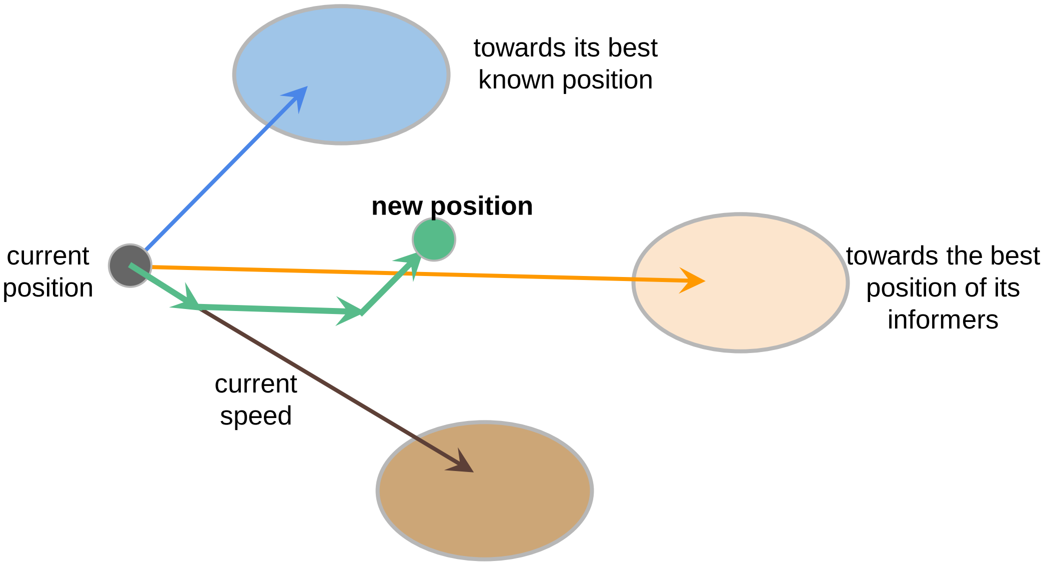
Particle Swarm Optimization (PSO)



A little history...

- Proposed "by chance" by James Kennedy (socio-psychologist) and Russel Eberhart (electrical engineer) in (1995);
- Multi-agent algorithm, adapted to **continuous** problems;
- Population of individuals who are not very intelligent (with a **small memory**) but who are highly **communicative**;
- Based on imitation and not on selection;;
- http://www.swarmintelligence.org/, http://www.particleswarm.info/, a book by Maurice Clerc on the subject.

Particle Swarm Optimization - The Particle



PSO - The particle

A particle knows:

- Its current position and evaluation;
- Its speed;
- The best position by which it has passed and its evaluation;
- The composition of its informant neighborhood;
- The best position of her informant neighborhood and her evaluation.

PSO - The detailed metaphor

The underlying concepts of metaphor:

- A particle: a solution of the problem and a little more ...;
- The swarm: the particle population;
- The speed of a particle: the displacement vector (disturbance of the solution);
- The neighborhood of a particle: all the particles exchanging information with it.

PSO - Principle

The algorithm is based on two iterations that we can parallelize:

- For each particle, information relevant to the displacement is updated;
- For each particle, the new velocity is calculated and the displacement is performed.

PSO - Algorithm

```
initialization of the swarm of N particles
Repeat
    iter + 1
    For each particle Do
         update of his best position
         update of the best position of the neighborhood
    End For
    For each particle Do
         calculating the new velocity
         displacement of the particle
         evaluation of the new position
    End For
    update the best global position
Until a convergenceCriterium()
```

PSO - Speed and Displacement

At each iteration, we compute the new velocity V_{k+1}, compromise between the 3 behaviors and we move the particle X_k according to:

- $V_{k+1} = \psi\, V_k + \rho_1\, (X_{pbest} - X_k) + \rho_2\, (X_{vbest} - X_k)$
  - ψ : inertia coefficient, refines the convergence of the method;
  - ρ_i : coefficients of confidence, modify the instinct of the particle, $\rho_i = r_i.c_i$, with ri follows a uniform law on [0,1] and $c_1 + c_2 \leq 4$;
- $X_{k+1} = X_k + V_{k+1}$

PSO - Coefficients

Defining efficient values of coefficients is the sensitive parameterization of the method. They define several types of behavior:

- **coefficient of inertia**: I trust my displacement;
- **social coefficient**: I trust my informants;
- **cognitive coefficient**: I trust my memory;
- **contrition factor**: coefficient introduced to avoid divergence out of the search space;
- There are many research papers on this topic..

PSO - Coefficients

The equation of speed can be expressed more simply in the form :

- $V_{k+1} = \psi\, V_k + c_{max}*\mathit{rand(0,1)}\,(X_{pbest} - X_k) + c_{max}*\mathit{rand(0,1)}\,(X_{vbest} - X_k)$
  - $c_1 = c_2 = c_{max}$
  - avec $(\psi;\ c_{max}) = (0.7;\ 1.47)$ or $(0.8;\ 1.62)$: performing values of couple

Using the contrition factor[1], a speed calculation making consensus :

- $V_{k+1} = 0.7298844\, V_k + 2.05*\mathit{rand(0,1)}\,(X_{pbest} - X_k) + 2.05*\mathit{rand(0,1)}\,(X_{vbest} - X_k)$

[1] Clerc et Kennedy. 2002, “The particle swarm - explosion, stability, and convergence in a multidimensional complex space”, Transactions on Evolutionary Computation, vol. 6, n o 1, p. 58–73.

PSO - Characteristics of the swarm

The number of particles composing the swarm and their organization within it are two parameters of the method:

- A too low swarm size prevents appropriate exploration of the search space;
- Too large a size does not necessarily improve the convergence and increases the calculation time;
- A *moderate* size, coupled with a maximum number of judicious iterations, leads to an efficient behavior of the algorithm;;
- The size of the problem nevertheless influences the number of particles ($10+2\sqrt{D}$: SPSO 2006, SPSO 2007).

PSO - Neighborhood

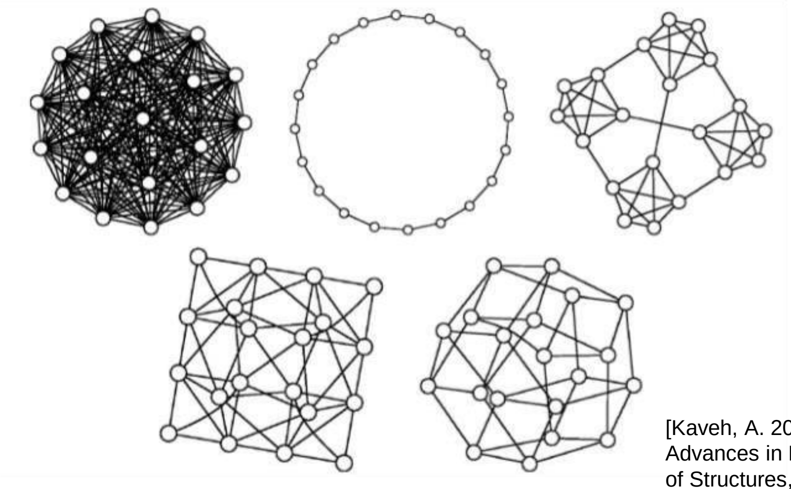
Several questions to ask:

- Who communicates with whom?
- What information is exchanged?
- What to do with the information?
- When does the exchange take place?

PSO- Neighborhood Topology

- Global Neighborhood: all particles communicate with each other;
- Neighborhood "local / social": each particle communicates with a subgroup, organized in a particular topology, for example:

Topologies Gbest, in ring, four-clusters, Von Neumann and pyramid



[Kaveh, A. 2014, ≪ Particle swarm optimization ≫ , dans Advances in Metaheuristic Algorithms for Optimal Design of Structures, Springer]

PSO - Initialization of the swarm

A multi-agent metaheuristic requires initialization that provides good coverage of the search space:

- **Random**: the particles are initialized by a random draw of their position on the search space (pseudorandom, chaotic ...);
- **Guided**: There are several strategies to "disperse" solutions as much as possible across the entire search space:
    - division of the research area;
    - opposition-based learning technique
    - use of initialization sequences (Halton, Sobol ...);
    - use of Latin hypercubes;....

PSO - Problem to the limits

There are several strategies to avoid going beyond the boundaries of the search space:

- **Limiter**: a parameter Vmax is introduced which controls the maximum displacement of the speed;
- **Ignorance**: the particle is left out of the research space and its evaluation is not done (or heavily penalized);
- **Stop**: the particle is stopped at the limits of space, we cancel or oppose the speed;
- **Rebound**: The particle bounces inward of the search space.

PSO - Convergence Criteria

Allows you to complete the algorithm according to several conditions:

- The number of evaluations reaches a limit. This number can be adjusted according to the number of particles composing the swarm;
- There has been no improvement since a number of iterations..

PSO - Discretization

Particle swarm optimization is a metaheuristic adapted to continuous problems. Nevertheless, it is "adaptable" to problems with discrete variables by solving the following problems:

- How do you define a particle?
- What does speed represent?
- What is the distance between two particles?
- What are the operations *, +, - ?

PSO - Other improvements

There are many improvements to this algorithm for each of its features, especially:

- *TRIBES*: the swarm is divided into communicating sub-swarms whose number of particles composing them evolves over time;
- *FIPS*: The displacement takes into account the position of all the particles of the swarm and not only the best;
- *Bare Bones PSO*: The displacement is carried out according to a probability law according to a Gaussian distribution whose parameters depend on the best personal values and the neighborhood;
- Many others!

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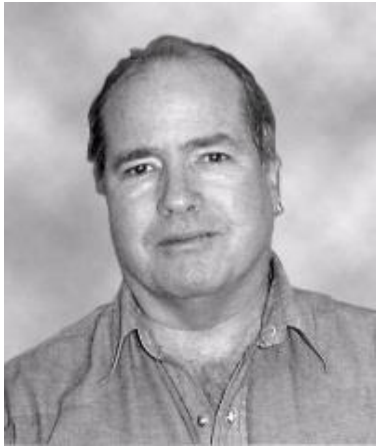
Russell C. Eberhart

James Kennedy



An American social psychologist. He received his Ph.D. in 1992 from the University of North Carolina, and works for the US Department of Labor in Washington, DC.

That’s all folks !