MAASAI MARA UNIVERSITY

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF SCIENCE IN ECONOMICS AND STATISTICS

THEORY OF ESTIMATION

APRIL 2020 **TIME: 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY THREE QUESTIONS.

SECTION A (25 MARKS)

QUESTION ONE

a) Derive the distribution of $\bar{x} = \frac{I}{n}\sum_{1=1}^{n} x_i$ if $x \sim N(\mu_1\ \sigma^2)$ (2 marks)

b) Given the uniform distribution $f\ (x, a, b) = \begin{cases} \frac{1}{b-a} & , a \leq x \leq b \\ o, & elsewhere \end{cases}$

Obtain moment estimator of the parameters A and D if the Uniform distribution is on the interval. $(A,\ \ A + D)$ (3 marks)

c) Find the maximum likelihood estimate of the parameter θ for the population those

Probability function. $f(x, \theta) = \begin{cases} \theta^x & (-\theta)^{1-x} \\ 0 & elsewhere \end{cases}, x = 0, 1$ (3 marks)

d) Let $x_1\ x_2 ....\ x_n$ be a random samples from a population with mean θ and variance θ^2, define the sample mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and examine it for unbiasedness. (5 marks)

e) Suppose that $X_1, X_2 ....\ X_n$ from a random sample for size n from a normal population, having mean μ and variance σ^2. Let $\bar{x} = \frac{1}{n}\ \sum_{i=1}^{n} xi$ and $s^2 = \frac{1}{n}\sum_{i=1}^{n}(xi - \bar{x})^2$ be the sample mean variance respectively, show that:

$\bar{x}_n;\ n = 1,2,3, \dots\}\ and\ S_n\ ;\ n = 1,2,3 \dots\dots\dots$ MSE consistent estimate of μ and σ^2 respectively. (6 marks)

f) Suppose that there is a random sample $X_1 X_2$ ....X_n from a Poisson distribution with parameters verify that $T_1 = \bar{x}$ and $T_2 = \sum_{i=1}^{n}(xi - \bar{x})^2$

Are jointly sufficient for μ and σ^2 (6 marks)

SECTION B

QUESTION TWO [15 MARKS]

a) Suppose that the proportion θ of detecting item in a large manufacturer lot is unknown and that the Prior distribution of θ is a uniform distribution in the interval [0,1]. Suppose that a random sample is taken from this lot. Determine the posterior distribution of θ given the sampling is done in such a way that for

$$x_i = \begin{cases} 1 & \text{if the item is detective} \\ 0 & \text{if the item is not detective} \end{cases}$$ (8 marks)

b) State and proof the Raw-Blackwell-Theory. (7 marks)

QUESTION THREE [15 MARKS]

a) Let $X_1 X_2$.............. X_n be a random sample from a Bernoulli distribution with probability of success θ. Let $T = \sum_{1^2=1}^{n} x_i$ Whereas x_i is to be unbiased for θ, use this information to construct a better estimator for θ. (7 marks)

b) Let $X_1 X_2$........................ X_n be a random sample from a Bernoulli distribution with parameter θ, find the unique UMVUE of

(i) θ (2 marks)

(ii) $\theta(1-\theta)$ (6 marks)

QUESTION FOUR [15 MARKS]

a) Define the term "Pivotal quantity" [2 marks]

b) The following data are the income per year in million shillings for 30 companies selected within a given city.

14.7	13.0	14.9	12.9	16.0	15.2
15.3	15.1	12.6	14.8	14.9	15.1
15.3	12.4	17.2	16.0	12.5	14.3

15.4	15.1	15.8	15.4	15.5	12.4
12.0	12.5	16.9	14.4	16.3	15.6

Assuming normality with unknown parameters;

i. Construct a 90% confidence interval for the true population mean. [6 marks]

ii. Construct a 99% confidence interval for the true population variance. [7 marks]

QUESTION FIVE [15 MARKS]

a) Let $X_1, X_2 \ldots . X_n$ be i.i.d random sample from a population with p.d.f

$$f(x,\theta) = \begin{cases} \dfrac{1}{\theta} & 0 < x < \theta \\ 0 & elsewhere \end{cases}$$

Suppose T is an estimator of the parameter θ and T has a density;

$$g(t) = \begin{cases} \dfrac{nt^{n-1}}{\theta^n} & 0 < t < \theta \\ 0 & elsewhere \end{cases}$$

i. Investigate unbiasness of T for the parameter θ. (5 marks)

ii. Find the cramer-Rao lowerbound for the variance of unbiased estimator of the parameter θ. (5 marks)

b) X~ $N(u, \sigma^2)$ where u is known. Let $x_1, x_2 \ldots . x_n$ be a random sample from x. Find a minimum variance best unbiased estimator of the variance σ^2.(5 marks)