

L3-CALCMF



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tuanaki, Kaupae 3, 2019

9.30 i te ata Rātū 26 Whiringa-ā-rangi 2019

TE PUKAPUKA O NGĀ TIKANGA TĀTAI ME NGĀ TŪTOHI mō 91577M, 91578M me 91579M

Tirohia tēnei pukapuka hei whakatutuki i ngā tūmahi o ō Pukapuka Tūmahi, Tuhiinga hoki.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

KA TAEA TĒNEI PUKAPUKA TE PUPURI HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE PĀNGARAU – ĒTAHI TURE WHAI HUA

TE TAURANGI

Ngā Whārite Pūrua

$$\text{Mēnā } ax^2 + bx + c = 0$$

$$\text{kāti } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ngā Taupū Kōaro

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Ngā Tau Matatini

$$z = x + iy$$

$$= r \operatorname{cis} \theta$$

$$= r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy$$

$$= r \operatorname{cis}(-\theta)$$

$$= r(\cos \theta - i \sin \theta)$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$$

$$\theta = \arg z$$

$$\text{ina } \cos \theta = \frac{x}{r}$$

$$\bar{a}, \sin \theta = \frac{y}{r}$$

Te Ture a De Moivre

Mēnā he tau tōpū a n , kāti,

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$$

TE ĀHUAHANGA TAUNGA

Te Rārangi Torotika

$$\text{Whārite } y - y_1 = m(x - x_1)$$

TE TUANAKI

Kimi Pārōnaki

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Ngā Tikanga Pāwhaitua

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1} + c$ ($n \neq -1$)
$\frac{1}{x}$	$\ln x + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$

Te Pānga Tawhā

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

MATHEMATICS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n \log_b x$

$\log_b x = \frac{\log_a x}{\log_a b}$

Complex numbers

$z = x + iy$

$= r \operatorname{cis} \theta$

$= r(\cos \theta + i \sin \theta)$

$\bar{z} = x - iy$

$= r \operatorname{cis}(-\theta)$

$= r(\cos \theta - i \sin \theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos \theta = \frac{x}{r}$

and $\sin \theta = \frac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

CALCULUS

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1} + c$ ($n \neq -1$)
$\frac{1}{x}$	$\ln x + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$

Parametric Function

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$

Te Ture mō te Otinga Whakarau¹

$$(f \cdot g)' = f \cdot g' + g \cdot f' \quad \text{mēnā rānei } y = uv \quad \text{kāti } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Te Ture mō te Otinga Wehe

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad \text{mēnā rānei } y = \frac{u}{v} \quad \text{kāti } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Te Ture Pānga Hiato, te Ture Mekameka rānei

$$\left(f(g)\right)' = f'(g) \cdot g'$$

$$\text{mēnā rānei } y = f(u) \quad \bar{a}, u = g(x) \quad \text{kāti } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

NGĀ TIKANGA TAU**Te Ture Taparara**

$$\int_a^b f(x) \, dx \approx \frac{1}{2} h \left[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1}) \right]$$

$$\text{ina } h = \frac{b-a}{n} \quad \bar{a}, y_r = f(x_r)$$

Te Ture a Simpson

$$\int_a^b f(x) \, dx \approx \frac{1}{3} h \left[y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) \right]$$

$$\text{ina } h = \frac{b-a}{n}, y_r = f(x_r), \bar{a}, \text{ he taurua te } n.$$

¹ whakarea

Product Rule

$$(f \cdot g)' = f \cdot g' + g \cdot f' \quad \text{or if } y = uv \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad \text{or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Composite Function or Chain Rule

$$(f(g))' = f'(g) \cdot g'$$

$$\text{or if } y = f(u) \text{ and } u = g(x) \text{ then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

NUMERICAL METHODS**Trapezium Rule**

$$\int_a^b f(x) \, dx \approx \frac{1}{2} h [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r)$$

Simpson's Rule

$$\int_a^b f(x) \, dx \approx \frac{1}{3} h [y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{where } h = \frac{b-a}{n}, y_r = f(x_r) \text{ and } n \text{ is even.}$$

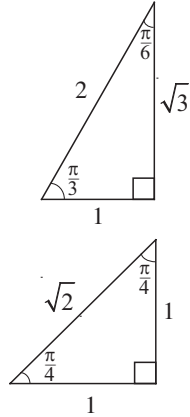
TE PĀKOKI

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Te Ture Aho**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Te Ture Whenua

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Ngā Whārite ka Pono Ahakoa
ngā Uara Ka Whakaurua Atu**

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

Ngā Otinga Whānui

$$\text{Mēnā } \sin \theta = \sin \alpha \text{ kāti } \theta = n\pi + (-1)^n \alpha$$

$$\text{Mēnā } \cos \theta = \cos \alpha \text{ kāti } \theta = 2n\pi \pm \alpha$$

$$\text{Mēnā } \tan \theta = \tan \alpha \text{ kāti } \theta = n\pi + \alpha$$

ko te n , he tau tōpū ahakoa

Ngā Koki Hiato

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Ngā Koki Rearua

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

Ngā Otinga Whakaraui

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Ngā Otinga Tāpiri

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

TE INE**Te Tapatoru**

$$\text{Horahanga} = \frac{1}{2} ab \sin C$$

Te Taparara

$$\text{Horahanga} = \frac{1}{2} (a+b)h$$

Te Pewanga

$$\text{Horahanga} = \frac{1}{2} r^2 \theta$$

$$\text{Te roa o te pewa} = r\theta$$

Te Rango

$$\text{Rōrahi} = \pi r^2 h$$

$$\text{Horahanga mata kōpiko} = 2\pi rh$$

Te Koeko

$$\text{Rōrahi} = \frac{1}{3} \pi r^2 h$$

$$\text{Horahanga mata kōpiko} = \pi r l \text{ ina ko te } l \text{ te teitei o te tītaha}$$

Te Poi

$$\text{Rōrahi} = \frac{4}{3} \pi r^3$$

$$\text{Horahanga mata} = 4\pi r^2$$

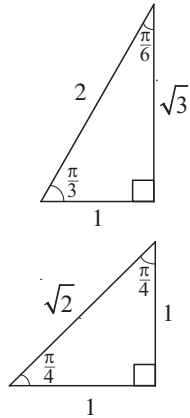
TRIGONOMETRY

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$



Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

General Solutions

$$\text{If } \sin \theta = \sin \alpha \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$\text{If } \cos \theta = \cos \alpha \text{ then } \theta = 2n\pi \pm \alpha$$

$$\text{If } \tan \theta = \tan \alpha \text{ then } \theta = n\pi + \alpha$$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

Products

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

MEASUREMENT

Triangle

$$\text{Area} = \frac{1}{2} ab \sin C$$

Trapezium

$$\text{Area} = \frac{1}{2} (a+b)h$$

Sector

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$\text{Arc length} = r\theta$$

Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi r h$$

Cone

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi r l \text{ where } l = \text{slant height}$$

Sphere

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$

English translation of the wording on the front cover

L3-CALCMF

Level 3 Calculus, 2019

9.30 a.m. Tuesday 26 November 2019

FORMULAE AND TABLES BOOKLET for 91577, 91578 and 91579

Refer to this booklet to answer the questions in your Question and Answer booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.