See back cover for an English translation of this cover

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91578M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

SUPERVISOR'S USE ONLY

Tuanaki, Kaupae 3, 2014

91578M Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga

9.30 i te ata Rātū 18 Whiringa-ā-rangi 2014
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pārōnaki hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pārōnaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mehemea e ōrite ana te Tau Ākonga ā-Motu (NSN) kei tō pepa whakauru ki te tau kei runga ake nei.

Whakautua e koe ngā pātai KATOA kei roto i te pukapuka nei.

Whakaaturia ngā mahinga KATOA.

Me mātua riro mai i a koe te pukaiti o ngā Tikanga Tātai me ngā Papatau L3–CALCMF.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te (ngā) whārangi kei muri i te pukapuka nei, ka āta tohu ai i ngā tau pātai.

Tirohia mehemea kei roto nei ngā whārangi 2–23 e raupapa tika ana, ā, kāore hoki he whārangi wātea.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

PĀTAI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria te pārōnaki o $y = 5\cos(3x)$.

(b) Kimihia te rōnaki o te rārangi hāngai ki te pānga $y = (3x^2 - 5x)^2$ i te pūwāhi (1,4).

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

(c) Mēnā ko $x = 2\sin t$, ā, ko $y = \cos 2t$, whakaaturia ko $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2\sin t$.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Differentiate $y = 5\cos(3x)$.

(b) Find the gradient of the normal to the function $y = (3x^2 - 5x)^2$ at the point (1,4).

Show any derivatives that you need to find when solving this problem.

(c) If $x = 2\sin t$ and $y = \cos 2t$ show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\sin t$.

(d) Kimihia te uara-x ko te pātapa ki te pānga $y = \dfrac{4}{e^{2x-2}} + 8x$ he whakarara ki te tuaka-x.

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

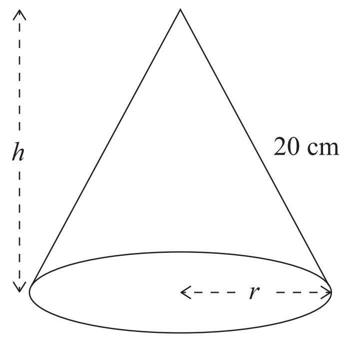
MĀ TE KAIMĀKA ANAKE

(d) Find the x-value at which the tangent to the function $y = \dfrac{4}{e^{2x-2}} + 8x$ is parallel to the x-axis.

Show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

(e) He aha te rōrahi mōrahi o tētahi koeko mēnā ko te roa tītaha o te koeko he 20 cm?

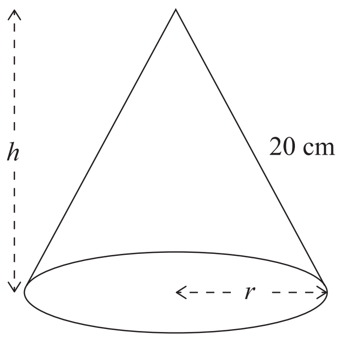


Kāore he tikanga kia parahautia e koe he mōrahi te rōrahi i tātaihia.

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(e) What is the maximum volume of a cone if the slant length of the cone is 20 cm?



You do not need to prove that the volume you have found is a maximum.

Show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

PĀTAI TUARUA

(a) Whiriwhirihia te pārōnaki o $f(x) = \dfrac{e^{4x}}{2x-1}$

Hei aha noa te whakarūnā i tō whakautu.

(b) Kimihia te rōnaki o te kōpiko $y = 8\ln(3x - 2)$ i te pūwāhi ko $x = 2$.

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

QUESTION TWO

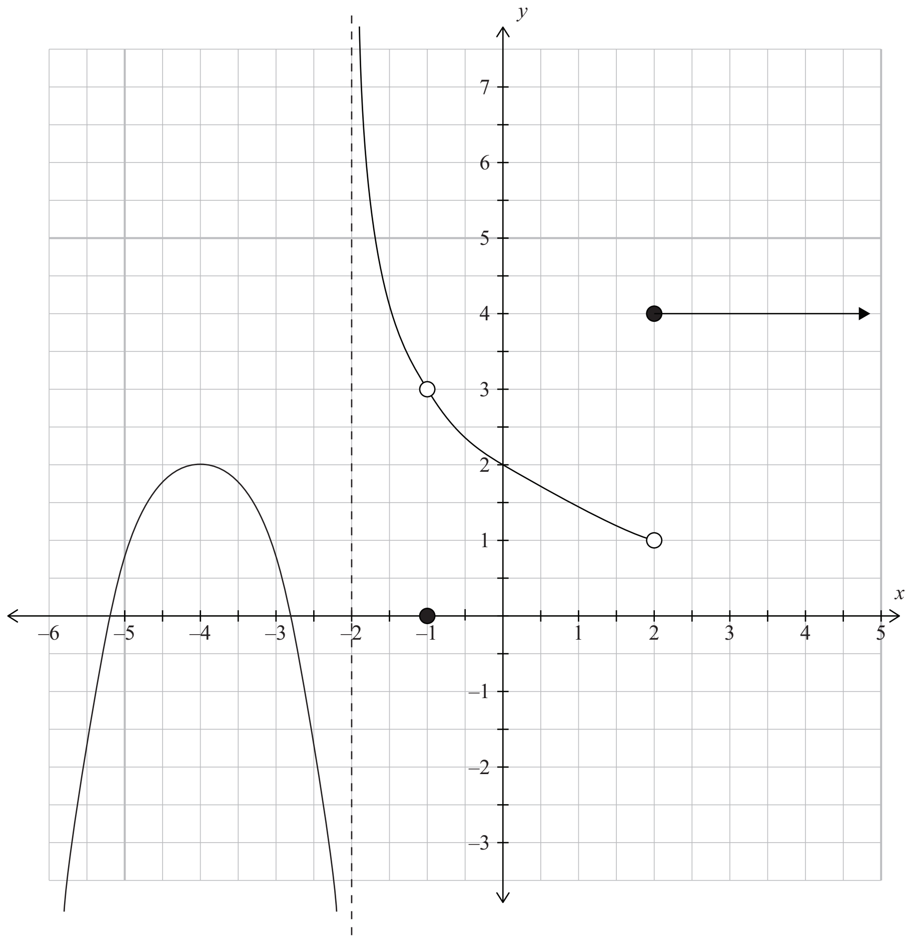
(a) Differentiate $f(x) = \dfrac{e^{4x}}{2x - 1}$.

You do not need to simplify your answer.

(b) Find the gradient of the curve defined by $y = 8\ln(3x - 2)$ at the point where $x = 2$.

Show any derivatives that you need to find when solving this problem.

(c) E tohu ana te kauwhata i raro nei i te pānga $y = f(x)$.



Mō te pānga $f(x)$ i runga ake:

(i) Kimihia te (ngā) uara mō x e ū ki ēnei whakaritenga e whai ake:

1. kāore e taea te kimi pārōnaki mō $f(x)$: __________

2. $f''(x) < 0$: __________

3. kāore i te tautohua a $f(x)$: __________

(ii) He aha te uara o $f(2)$? __________

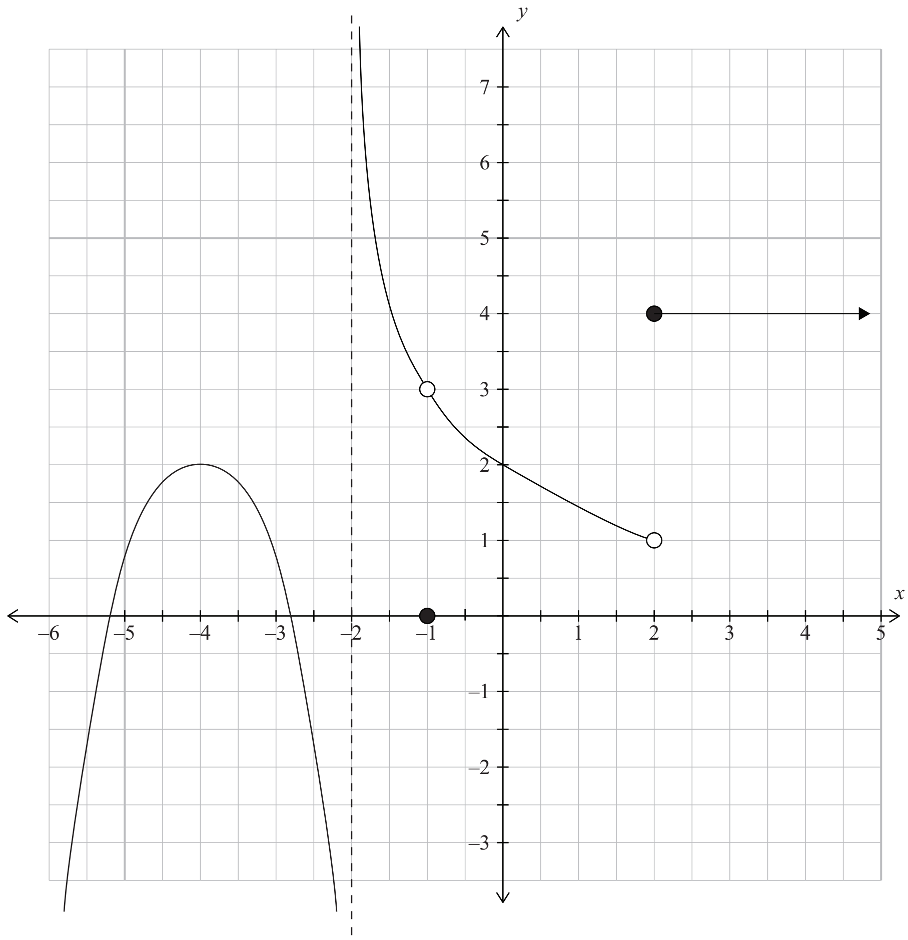
Āta kōrero mai mēnā kāore rawa he uara.

(iii) He aha te uara o $\lim\limits_{x \to -1} f(x)$? __________

Āta kōrero mai mēnā kāore rawa he uara.

MĀ TE KAIMĀKA ANAKE

(c) The graph below shows the function $y = f(x)$.



For the function $f(x)$ above:

(i) Find the value(s) for x that meet the following conditions:

1. $f(x)$ is not differentiable: ________________

2. $f''(x) < 0$: ________________

3. $f(x)$ is not defined: ________________

(ii) What is the value of $f(2)$? ________________

State clearly if the value does not exist.

(iii) What is the value of $\lim\limits_{x \to -1} f(x)$? ________________

State clearly if the value does not exist.

(d) Mā te rere o te waka rererangi ka mohīo ai he aha te utu mo te hāora kotahi.

Ka pēnei kē te wharite mo te waha rerangi

$$C = 4v + \frac{1\,000\,000}{v}, \;\; 200 \le v \le 800$$

ina ko C te utu ā-hāora o te whakahaere i te waka rererangi, ā-tāra i te hāora
ko v te tere 0 te hau o te waka rererangi, ā-kiromita i te hāora

Kimihia te utu ā-hāora mōkito e taea ai te whakarere i tēnei waka rererangi.

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

MĀ TE KAIMĀKA ANAKE

(d) The hourly cost of running an aeroplane depends on the speed at which it flies.

For a particular aeroplane this is given by the equation

$$C = 4v + \frac{1\,000\,000}{v}, \;\; 200 \le v \le 800$$

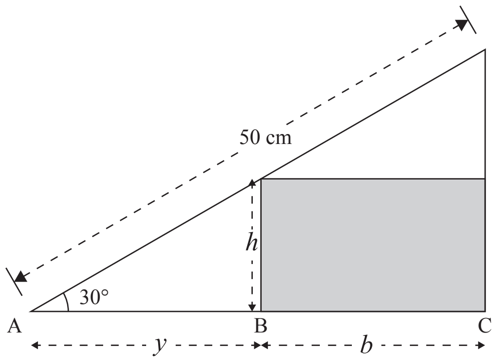
where C is the hourly cost of running the aeroplane, in dollars per hour
and v is the airspeed of the aeroplane, in kilometres per hour.

Find the minimum hourly cost at which this aeroplane can be flown.

Show any derivatives that you need to find when solving this problem.

(e) Ka tāngia tētahi tapawhā hāngai i roto i tētahi tapatoru hāngai, e ai ki te hoahoa i raro.

MĀ TE KAIMĀKA ANAKE



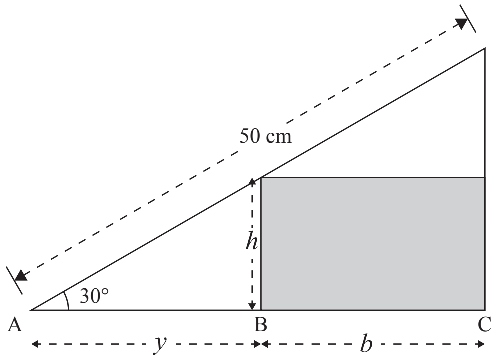
Ka neke te pūwāhi B i te pūtake o te tapatoru AC, e tīmata ana i te pūwāhi A, ki te tere aumou o te 3 cm s^{-1}.

He aha te tere e huri ai te horahanga o te tapatoru ina e 20 cm te tawhiti mai o te pūwāhi B ki te pūwāhi A?

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

ASSESSOR'S USE ONLY

(e) A rectangle is drawn inside a right angled triangle, as shown in the diagram below.



Point B moves along the base of the triangle AC, beginning at point A, at a constant speed of 3 cm s^{-1}.

At what rate is the area of the rectangle changing when point B is 20 cm from point A?

Show any derivatives that you need to find when solving this problem.

MĀ TE KAIMĀKA ANAKE

PĀTAI TUATORU

(a) Whiriwhirihia te pārōnaki o $y = \left(\sqrt[3]{x^2 + 4x}\right)^2$.

(b) Kimihia te (ngā) uara o x, he pūwāhi tūnoa tō te kauwhata o te pānga $y = x + \dfrac{32}{x^2}$.

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

(c) Mō ēhea uara o x e piki ai te pānga $f(x) = 5x - x\ln x$?

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Differentiate $y = \left(\sqrt[3]{x^2 + 4x}\right)^2$.

(b) Find the value(s) of x for which the graph of the function $y = x + \dfrac{32}{x^2}$ has stationary points.

Show any derivatives that you need to find when solving this problem.

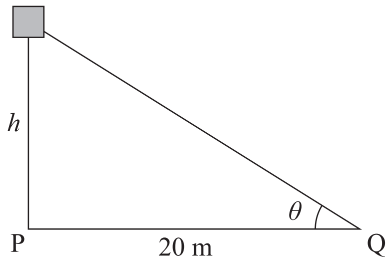
(c) For what values of x is the function $f(x) = 5x - x\ln x$ increasing?

Show any derivatives that you need to find when solving this problem.

(d) Ka kumea poutūhia tētahi paepae mai i te pūwāhi P ki te tere aumou o te 1.5 m s^{-1}.

Kei te mātakihia mai i te pūwāhi Q, he 20 m te tawhiti ā-huapae mai i te pūwāhi P.

Ko θ te koki rewa o te paepae mai i te pūwāhi Q.



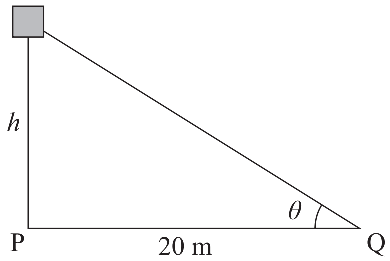
He aha te tere e piki ai te koki rewa ina eke te mea ki te 20 m i runga ake o te pūwāhi P?

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

(d) A container is winched up vertically from a point P at a constant rate of 1.5 m s^{-1}.

It is being observed from point Q, which is 20 m horizontally from point P.

θ is the angle of elevation of the container from point Q.



At what rate is the angle of elevation increasing when the object is 20 m above point P?

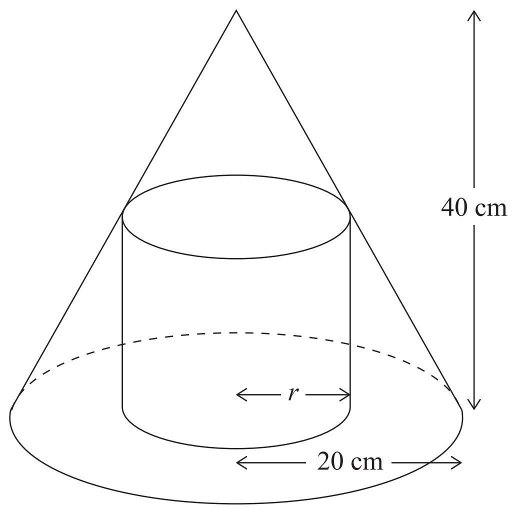
Show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

(e) He 20 cm te pūtoru o te koeko, ā, he 40 cm te teitei.

Ka uru tētahi rango ki te koeko, e ai ki te whakaaturanga i raro.



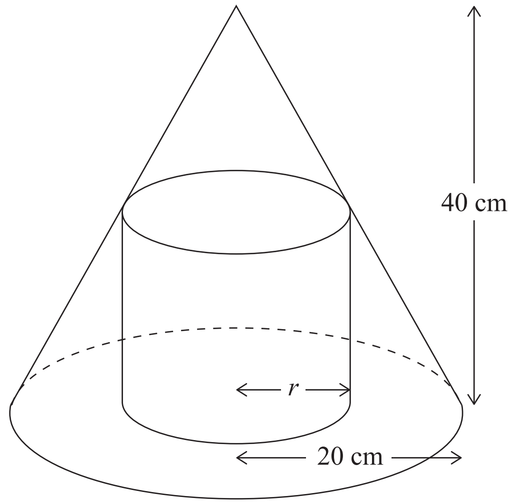
He aha te pūtoro o te rango kia whiwhi ai te rango i te rōrahi mōrahi?

Kāore he tikanga kia hāponotia e koe he mōrahi te rōrahi i tātaihia.

Whakaaturia ngā pārōnaki māmā e hiahiatia ana hei whakaoti i tēnei rapanga.

(e) A cone has a radius of 20 cm and a height of 40 cm.

A cylinder fits inside the cone, as shown below.



What must the radius of the cylinder be to give the cylinder the maximum volume?

You do not need to prove that the volume you have found is a maximum.

Show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

He puka anō mēnā ka hiahiatia.
Tuhia te (ngā) tāu pātai mēnā e hāngai ana.

TAU PĀTAI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2014

91578 Apply differentiation methods in solving problems

9.30 am Tuesday 18 November 2014
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

91578M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.