91579M

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tohua tēnei pouaka mēnā kāore he tuhituhi i roto i tēnei pukapuka

Tuanaki, Kaupae 3, 2020

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

9.30 i te ata Rāhina 23 Whiringa-ā-rangi 2020
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–23 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int \left(x + 2 + \frac{3}{x} \right) \mathrm{d}x$.

(b) Mō $t \geq 0$, ka tohua te tere o tētahi ahanoa mā $v(t) = 0.6\sqrt{t}$
ina ko v te tere o te ahanoa i te cm s^{-1}
ā, ko t te wā ā-hēkona mai i te tīmatanga o te neke o te ahanoa.

Ko te peinga o te ahanoa he 5 cm i $t = 0$.

He aha te peinga o te ahanoa i muri i te 16 hēkona?

(c) Whiriwhiria $\int_4^8 \frac{5x - 11}{x - 3} \mathrm{d}x$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Find $\int \left(x + 2 + \frac{3}{x} \right) \mathrm{d}x$.

(b) For $t \geq 0$, the velocity of an object is given by $v(t) = 0.6\sqrt{t}$
where v is the velocity of the object in cm s^{-1}
and t is time in seconds from the start of the object's motion.

The object has a displacement of 5 cm at $t = 0$.

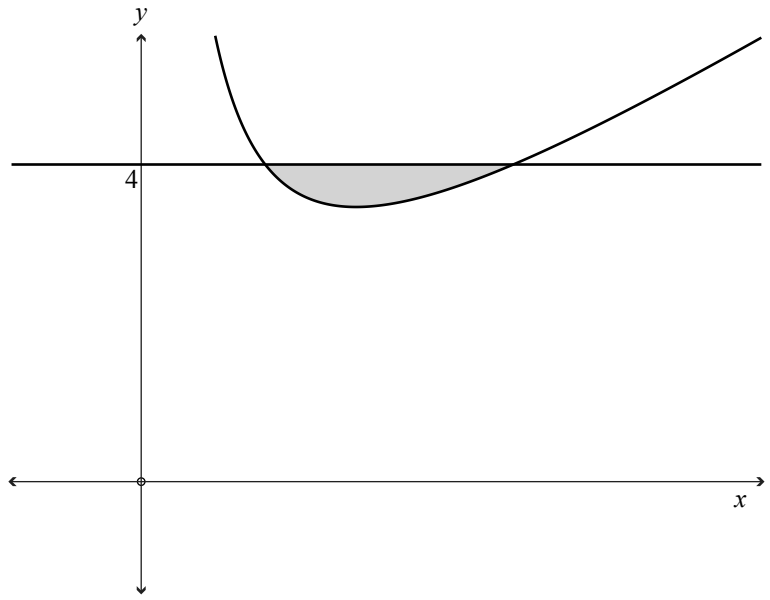
What will be the displacement of the object after 16 seconds?

(c) Find $\int_4^8 \frac{5x - 11}{x - 3} \mathrm{d}x$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaatu ana te kauwhata i raro i te kōpiko $y = x + \frac{3}{x}$ me te rārangi $y = 4$.

MĀ TE KAIMĀKA ANAKE

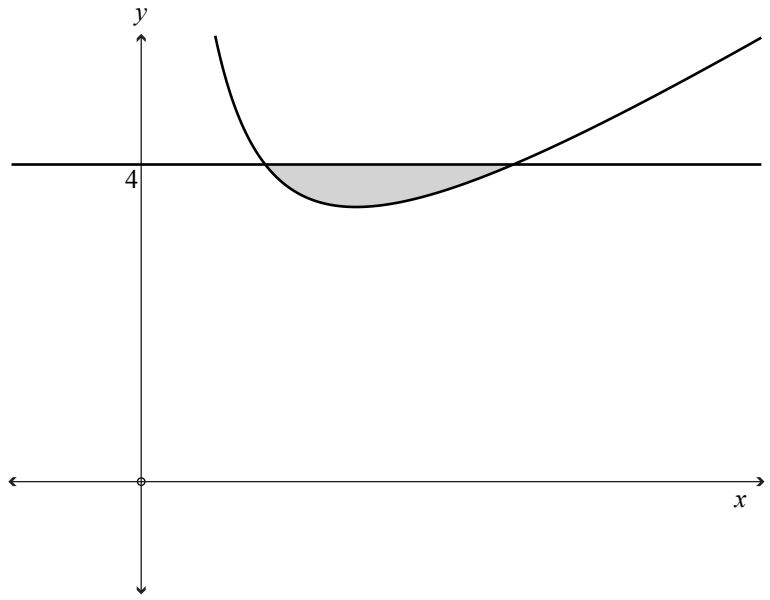


Tātaihia te horahanga o te wāhi kauruku.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(d) The graph below shows the curve $y = x + \frac{3}{x}$ and the line $y = 4$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

MĀ TE KAIMĀKA ANAKE

(e) Me whakaaro ki te whārite pārōnaki $\tan x \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sec^2 x}{y}, 0 < x < \dfrac{\pi}{2}$.

Mēnā ko $y = 2$ ina $x = \dfrac{\pi}{4}$, whiriwhiria te (ngā) uara o y ina ko $x = \dfrac{\pi}{3}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(e) Consider the differential equation $\tan x \dfrac{dy}{dx} = \dfrac{\sec^2 x}{y}, 0 < x < \dfrac{\pi}{2}$.

Given that $y = 2$ when $x = \dfrac{\pi}{4}$, find the value(s) of y when $x = \dfrac{\pi}{3}$.

You must use calculus and give the results of any integration needed to solve this problem.

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int\left(\pi-\frac{2}{x^2}\right)\mathrm{d}x$.

(b) Whakamahia ngā uara i te papatau i raro hei whiriwhiri i tētahi āwhiwhitanga ki $\int_0^3 f(x)\,\mathrm{d}x$, mā te whakamahi i te Ture a Simpson.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	1.1	1.8	2.1	2.4	2.7	1.8	1.3

(c) Whiriwhiria te uara o k kia $\int_1^k 9\sqrt{3x-2}\,\mathrm{d}x=126$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int\left(\pi-\frac{2}{x^2}\right)\mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_0^3 f(x)\,\mathrm{d}x$, using Simpson's Rule.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	1.1	1.8	2.1	2.4	2.7	1.8	1.3

(c) Find k such that $\int_1^k 9\sqrt{3x-2}\,\mathrm{d}x = 126$.

You must use calculus and give the results of any integration needed to solve this problem.

(d) Mēnā ko $\frac{dy}{dx} = \sqrt{y} \cdot \cos 4x$ ā, $y = 1$ ina $x = \frac{\pi}{8}$, whiriwhiria te uara o y ina $x = \frac{\pi}{4}$.

MĀ TE KAIMĀKA ANAKE

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

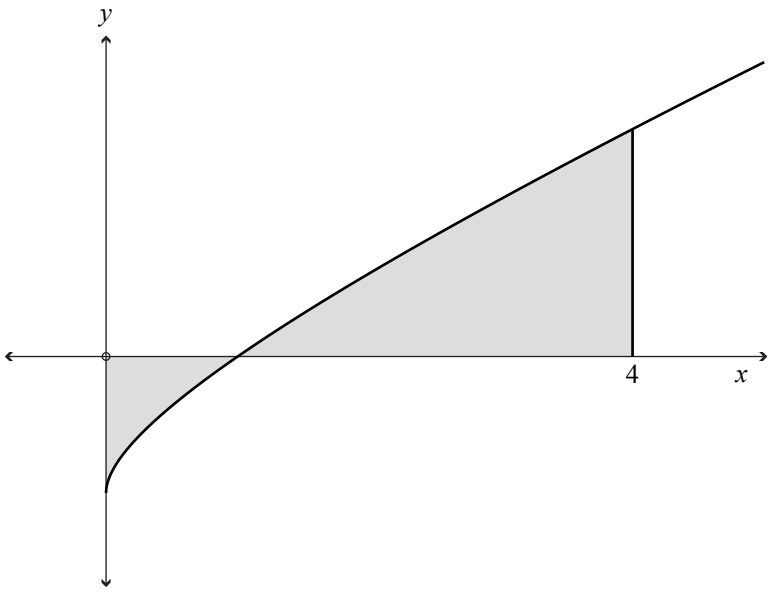
(d) If $\frac{dy}{dx} = \sqrt{y} \cdot \cos 4x$ and $y = 1$ when $x = \frac{\pi}{8}$, find the value of y when $x = \frac{\pi}{4}$.

You must use calculus and show the results of any integration needed to solve the problem.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

(e) E whakaaturia ana te kauwhata i raro i te kōpiko $y = x + 2\sqrt{x} - 3$.

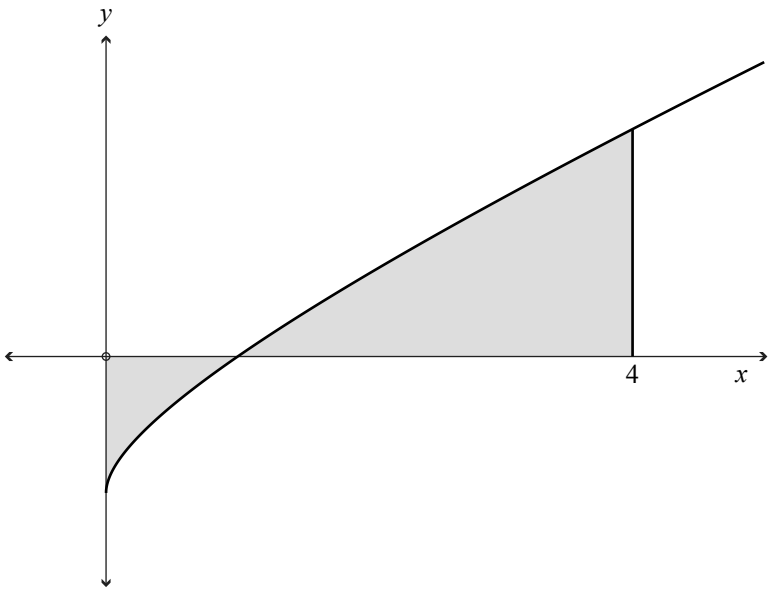


Tātaihia te horahanga o te wāhi kauruku.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(e) The graph below shows the curve $y = x + 2\sqrt{x} - 3$.



Find the shaded area.

You must use calculus and give the results of any integration needed to solve this problem.

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int \sec 2x \tan 2x \, \mathrm{d}x$.

(b) Mēnā $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos 2x$, ā, $y = 1$ ina $x = \frac{\pi}{12}$, whiriwhiria te uara o y ina ko $x = \frac{\pi}{4}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int \sec 2x \tan 2x \, dx$.

(b) If $\frac{dy}{dx} = \cos 2x$ and $y = 1$ when $x = \frac{\pi}{12}$, find the value of y when $x = \frac{\pi}{4}$.

You must use calculus and give the results of any integration needed to solve this problem.

(c) Ka ohorere te whakatere a tētahi ahanoa, i te neke ki tētahi tere aumou i te tuatahi. Mai i te tīmatanga o tana whakatere ka taea te whakatauira te nekehanga o te ahanoa mā te whārite pārōnaki

MĀ TE KAIMĀKA ANAKE

$$\frac{dv}{dt} = t + e^{0.2t} \text{ mō } 0 \le t \le 15$$

ina ko v te tere o te ahanoa i te m s^{-1}
ā, ko t te wā ā-hēkona i muri mai i te whakaterenga o te ahanoa.

Ina ko $t = 0$, ko te tere o te ahanoa he 8 m s^{-1}.

Whiriwhiria te terenga o te ahanoa ina ko $t = 10$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) An object originally moving at a constant velocity suddenly starts to accelerate. From the start of the object's acceleration the motion of the object can be modelled by the differential equation

$$\frac{dv}{dt} = t + e^{0.2t} \quad \text{for } 0 \le t \le 15$$

where v is the velocity of the object in m s^{-1}
and t is the time in seconds after the object starts to accelerate.

When $t = 0$, the velocity of the object was 8 m s^{-1}.

Find the velocity of the object when $t = 10$.

You must use calculus and give the results of any integration needed to solve this problem.

(d) I te horo iraruke, ko te pāpātanga e popo ai te matū iraruke ahakoa te wā he ōwehenga ki te maha o ngā ngota iraruke kei reira i tētahi wā.

MĀ TE KAIMĀKA ANAKE

Ka taea tēnei te whakatauira mā te whārite pārōnaki

$$\frac{\mathrm{d}N}{\mathrm{d}t} = kN$$

ina ko N te maha o ngā ngota iraruke, ā, ko t te wā ki ngā rā.

Ka whakaputaina he rahinga konupango-52.

He kanoirite iraruke te konupango-52 nō te konupango.

He 5.6 rā te houanga memeha o te konupango-52 (arā, whai muri i te 5.6 rā, ka popo te haurua o ngā ngota o te konupango-52).

E hia te roa e popo ai te 95% o te konupango-52?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 20.

ASSESSOR'S USE ONLY

(d) In radioactive decay, the rate at which the radioactive substance decays at any instant is proportional to the number of radioactive atoms present at that instant.

This can be modelled by the differential equation

$$\frac{dN}{dt} = kN$$

where N is the number of radioactive atoms present and t is the time in days.

A quantity of manganese-52 is produced.

Manganese-52 is a radioactive isotope of manganese.

Manganese-52 has a half-life of 5.6 days (i.e. after 5.6 days, half of any atoms of manganese-52 would have decayed).

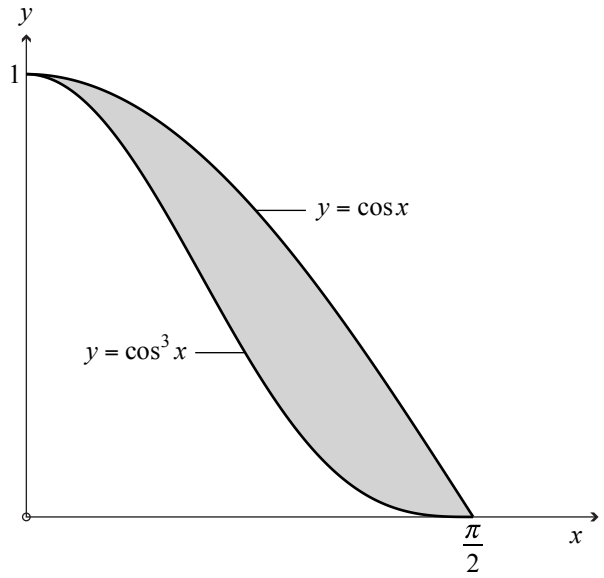
How long would it take for 95% of the manganese-52 to decay?

You must use calculus and give the results of any integration needed to solve this problem.

Question Three continues on page 21.

(e) E whakaatu ana te kauwhata i raro i ngā kōpiko $y = \cos x$ me te $y = \cos^3 x$ mō $0 \le x \le \frac{\pi}{2}$.

MĀ TE KAIMĀKA ANAKE

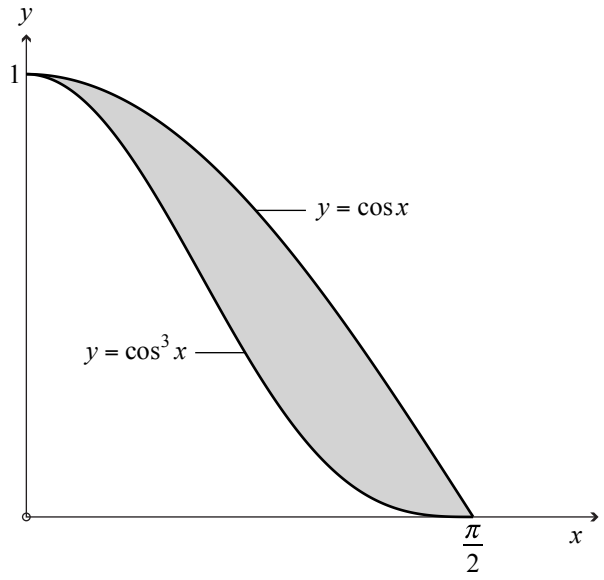


Tātaihia te horahanga o te wāhi kauruku.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(e) The graph below shows the curves $y = \cos x$ and $y = \cos^3 x$ for $0 \le x \le \frac{\pi}{2}$.

ASSESSOR'S USE ONLY



Find the shaded area.

You must use calculus and give the results of any integration needed to solve this problem.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus 2020

91579 Apply integration methods in solving problems

9.30 a.m. Monday 23 November 2020
Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.