No part of the candidate evidence in this exemplar material may be presented in an external assessment for the purpose of gaining credits towards an NCEA qualification.

1

SUPERVISOR'S USE ONLY

91028

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NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
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QUALIFY FOR THE FUTURE WORLD
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Level 1 Mathematics and Statistics, 2018

91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Tuesday 20 November 2018
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Grids are provided on some pages. This is working space for the drawing of a graph or a diagram, constructing a table, writing an equation, or writing your answer.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

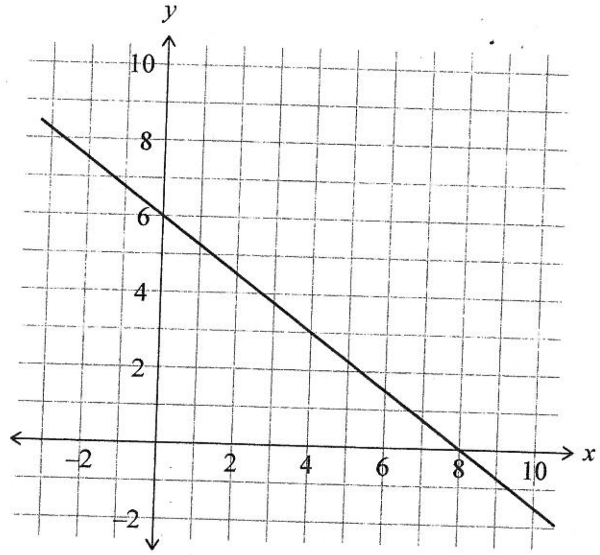
Excellence

TOTAL 24

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Give the equation of the graph shown below.



Equation: $y = -0.75x + 6$

(b) James takes 40 minutes to jog the 5 km from his home to school.

(i) What is James's average speed when he is jogging from his home to school?

~~[illegible] km per hour~~ 2.08 meters per second

~~7.5 km per hour~~

7.5 km per hour or

or

0.125 km per minute

(ii) Emma lives further away from the school than James.

They leave their homes at the same time.

Emma rides her bike to school, and James jogs to school.

They meet 20 minutes after they leave their homes.

After they meet, both James and Emma change their travelling speeds so they are the same.

Faster

Slower

James begins running and Emma rides her bike at ¾ of the speed she had been travelling before they met.

They arrive at school 30 minutes after they left their homes.

(30, 9600)

(20, 6600)

Represent Emma and James's journeys from their homes on a graph.

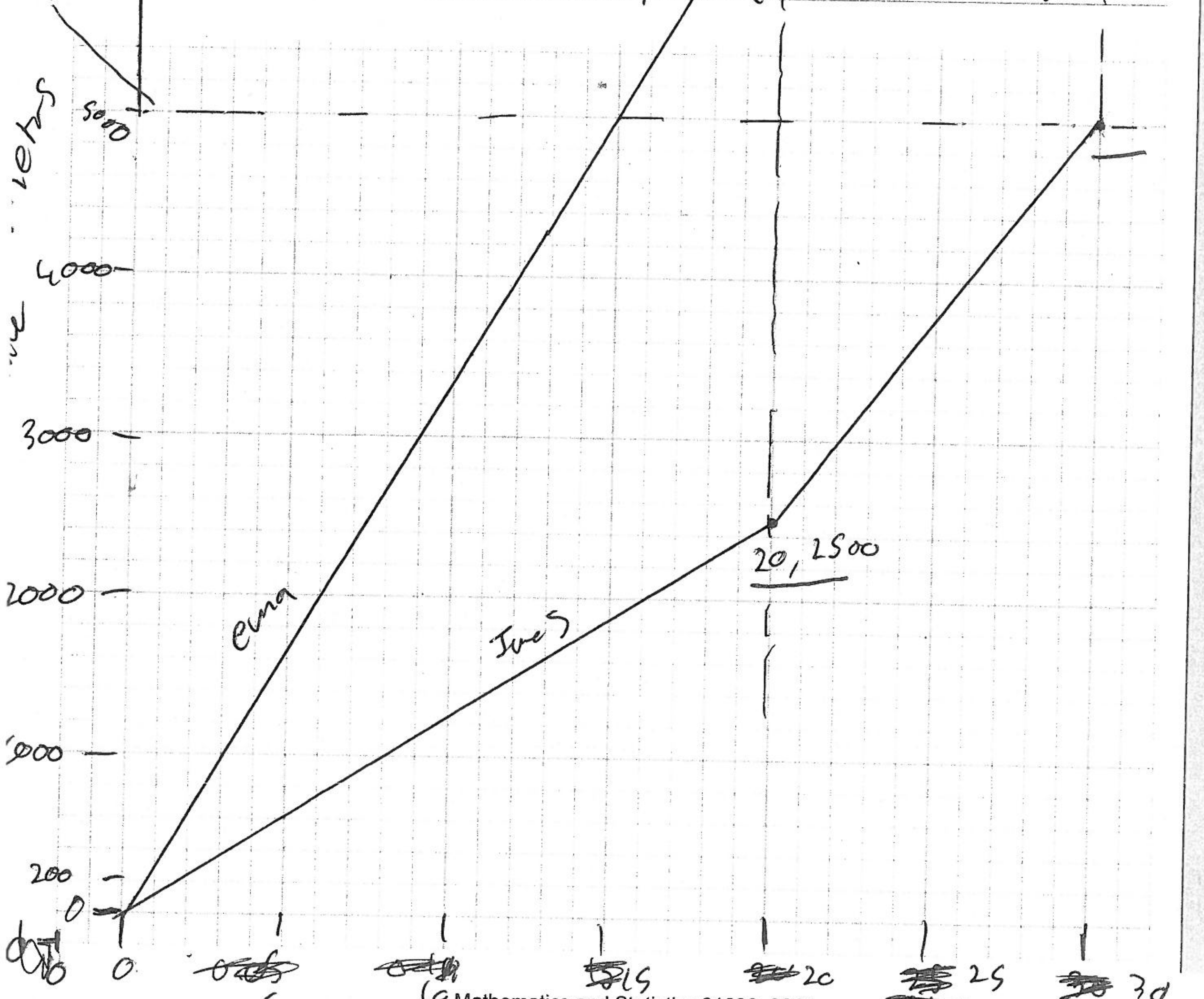
James $d = 0.125X$ (0.125 km per minute)

James new $d = 0.250X$ (0.250 km per minute)

Emma new $d = 0.250X$ (3/4 of prev speed)

Emma old ~~#~~ $d = 0.333X$ (0.333 km per minute)

(James to school = 5000m, Emma to school = 9100 m



ASSESSOR'S USE ONLY

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Let d = distance travelled t = time in min

(iii) Give the equations that represent Emma's and James's journeys.

James 0 – 20 minuts : ~~d = 0.125~~ $d = 0.125t$

James 20 – 30 minuts : $d = 0.250t$

Emma 0 – 20 minuts : $d = 0.333t$

Emma 20-30 minuts : $d = 0.250t$

(iv) Describe Emma's and James's journeys to school, including their speeds and how far Emma's home is from the school.

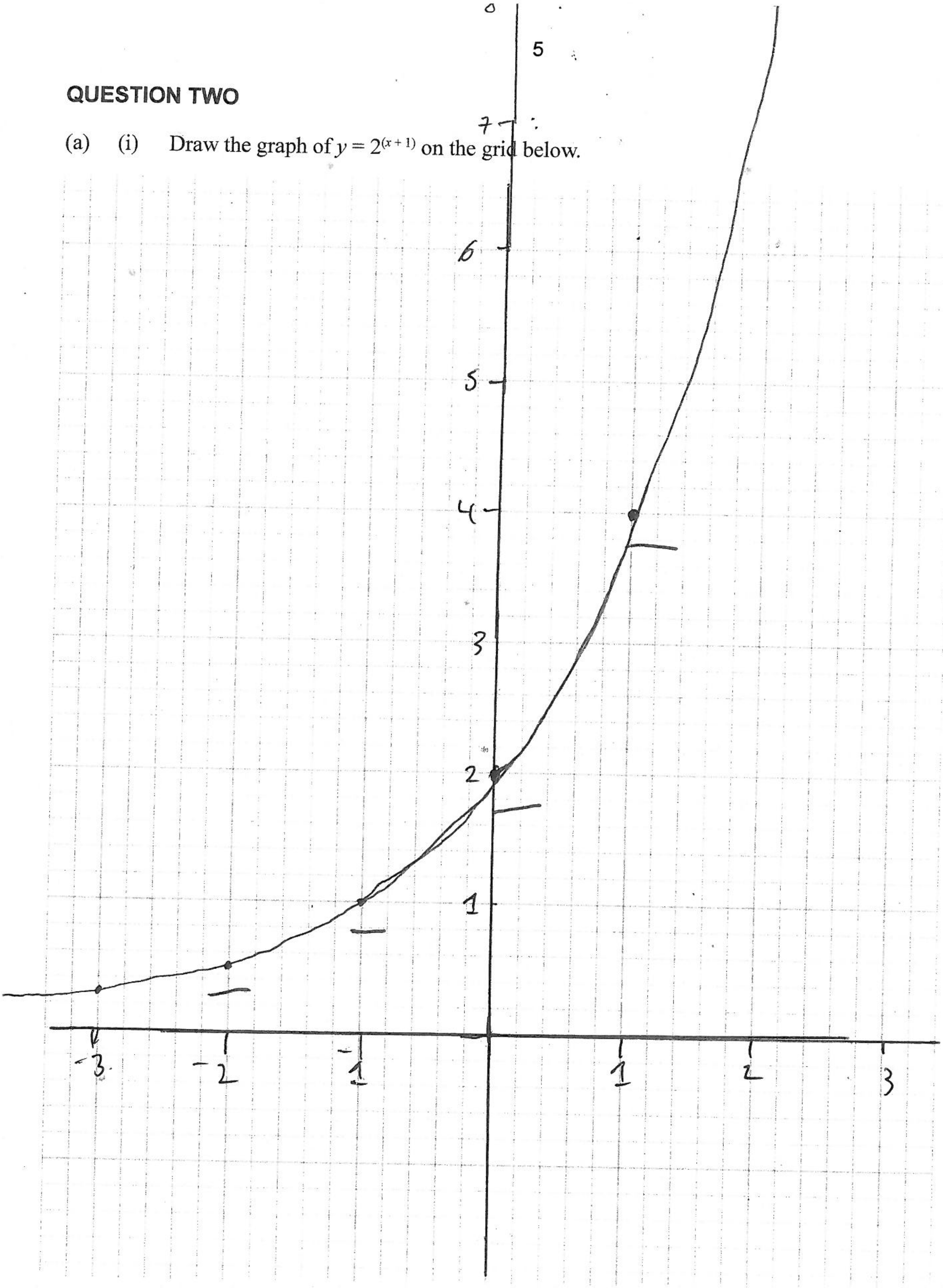
James lives 5km from school, he jogs ~~at 0.125 km~~ ~~[illegible]~~ slowly taking 20 minuts to reach half way (2.5 km). He jogs at 125 m a minute, ~~[illegible]~~ emma however lives father away from school and bikes quikly for 20 minuts, she travels at 330 meters a minute. At 20 minuts they meet and match speeds, this their gradients become prallel, both travel at 250 meters a minute. Emma lives 9100 meters or 9.1 km from school

E8

ASSESSOR'S USE ONLY

QUESTION TWO

(a) (i) Draw the graph of $y = 2^{(x+1)}$ on the grid below.



(ii) If this graph was moved 3 units to the right and 4 units up, give the equation of the translated graph.

$y = 2^{(x-2)} + 4$

(b) A stomach bug spreads through a large school.

The **total** number of different students who go to the nurse at least once because of the stomach bug is recorded. Each student's name is recorded only once.

The **total** number of students whose name has been recorded can be modelled by:

$$y = 2^n + 3$$

where n is the number of days since the first students visit the school nurse with the stomach bug.

(i) How many **more** students visited the nurse for the first time on the fourth day than on the third day?

Show your working.

$2^4+3 = 19$ students (day 4), $2^3+3 = 11$ (day 3)

$19-11 = 8$, $\therefore$ 8 more students visited the nurse

ASSESSOR'S USE ONLY

u

(ii) Give the equation that best represents the **number** of students who were recorded as going to the nurse **on any day** n, when $n \geq 1$.

Give your equation in the simplest form.

Equation: # of students visit on day $= 2^{n-1}$

day	visits
2	2
3	4
4	8
5	16

2, 4, 8; 2, 2

$a = 1$
$c = na$ $(n > 1)$

$n^2 - 1$

total

day	total
1	5
2	7
3	11
4	19
5	35

+2 ← students visit
+4 4 more students
+8 ← 8 more students
+16 16 more students

E

	1	5	+2	+2
2	2	7	+4	+4
4	3	11	+8	+8
8	4	19	+16	+16
16	5	35	+32	
13	6	67		

2	2	2	2
3	4	4	2
4	8	8	2
5	16	16	
6	32		

2^{x-1}

$2^n + 3$

(iii) After the total number of different students who have visited the nurse reaches 67, the daily number of students who visit begins to decrease.

2^{n-1}

The number of different students going to the nurse can now be modelled by:

$$M = -(x-5)(x+3) + 9$$

where x is the number of days after the daily number of students visiting the nurse starts to decrease.

days after 6 ($x = 1 \rightarrow$ day 7)

How many days after the first students went to the school nurse with the stomach bug would there be no students going to the school nurse with the same stomach bug?

Number of days: 12 days after first students

days	total students
1	5
2	7
3	11
4	19
5	35
6	67
7	
8	
9	

decrease starts

days	daily
~~1~~	
2	2
3	4
4	8
5	16
6	32
(1) 7	25
(2) 8	24
(3) 9	21
(4) 10	16
(5) 11	9
(6) 12	0

2^{n-1}

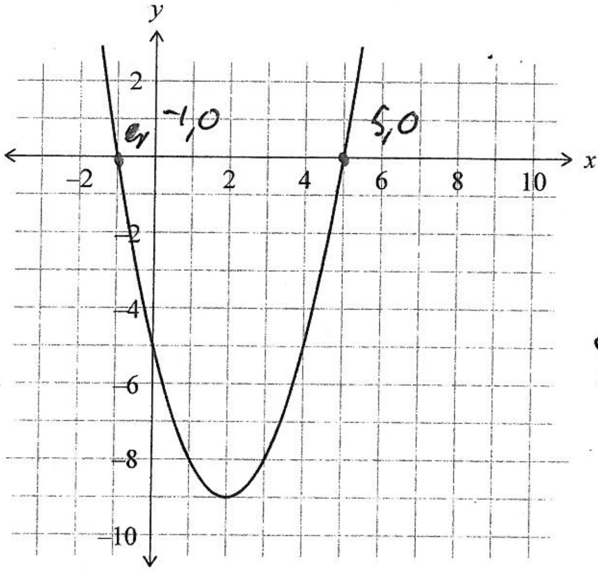
$m = -(x-5)(x+3)+9$

t

E8

QUESTION THREE

(a) Give the equation of the graph shown below.



$y = a(x+1)(x-5)-9$

$y = x^2 - 4x - 5 - 9$

$y = x^2 - 4x - 14$

$y = (x-2)^2 - 9$

Equation: ~~y = (x+1)(x-5)~~ $y = x^2 - 4x - 14$ or $y = (x-2)^2 - 9$

u

(b) Pippa is designing a new label for a drink bottle.

The design is made up of two circles placed one on top of the other as shown in the diagram.

The maximum height of the two circles is to be 10 cm.

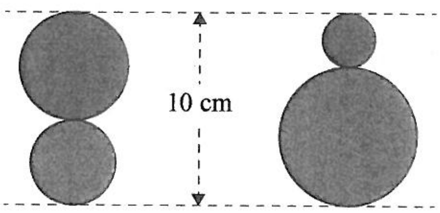
The minimum diameter of either circle is 2 cm (radius is at least 1 cm).

The bottom circle is coloured red and the top one blue.

She wants to know the approximate area of each circle.

Remember $A = \pi r^2$.

Pippa uses π as 3.



One possible logo design | Another possible logo design

ASSESSOR'S USE ONLY

(i) Use a table or graph to investigate the relationship between the area of the red circle and its radius as the radius increases.

let Radius of red = R

Radius us must add to 5 cm

(diameter must add to 10)

$r_b + R_R = 5$ cm

$r = 5 - R$

area of blue =

$A = \pi r^2$

area of red =

$A = \pi R^2$

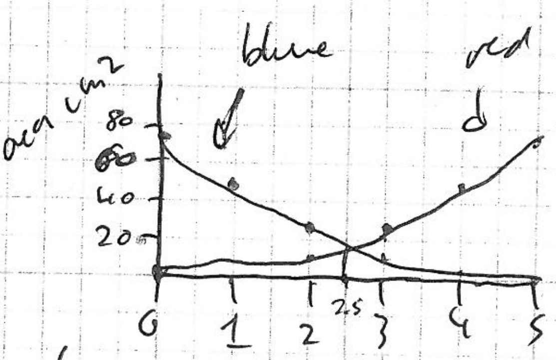
as $r = 5 - R$ ∴

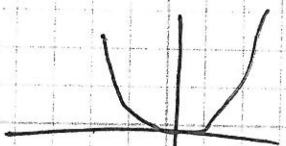
area of ~~blue~~ = blue now

$A = \pi \cdot (5-R)^2$

max red radius = 5

radius	area R	area b
0	0	75
1	3	48
2	12	27
3	27	12
4	48	3
5	75	0





1 2

0 0
1 3
2 12
3 27
4 48
5 75

3 9 15 21

6 6 6

6/2

3

a = 3

c = 0

$= 3x^2 + 6x$

$y = 3(x-5)^2 - 5$

$y = 3(x-5)^2 \rightarrow \cancel{y = 3(x-5)(x-5) \rightarrow y = 3x^2 - 10x + 25}$

(ii) Describe the major features of the graph that represents the relationship between the area of the red circle and its radius as the radius increases.

The graph is a quadratic, so the amount ~~area increases~~ that the area increases by each time the radius gets bigger increases linearly. As the max radius is 5 the graph stops at 5 cm additionally the radius is 0 the area is 0 as the blue circle takes up all the space. The radius ~~cannot be negative~~.

(iii) State the equation of the graph that represents the **total** of the areas of the red and the blue circles as the radii change.

$6R^2 - 30R + 75$ = total area

$3r^2 + 3R^2 = A_{total}$

as $R + r = 5$ ∴

$5 - R = R$ ∴

$3(5-R)^2 + 3R^2 = A_{total}$

$3(5-R)(5-R)$

$\cancel{3(5-R)(5-R)} + 3R^2 = A_{tot}$

$-R+5 \; (-R+5)$

$3(R^2 - 10R + 25) + 3R^2 = A_{tot}$

$\cancel{6R^2 - 10R + 25 = A_{total}}$

$3R^2 - 30R + 75 + 3R^2 = A_{tot}$

$6R^2 - 30R + 75 = A_{tot}$

~~Radius Red | Area Red | Area~~

R Red	A Red	A blue	tot
0	0	75	75
1	3	48	51
2	12	27	39
3	27	12	39
4	48	3	51
5	75	0	75

$R + r = n$

$n - R = r$

$(n-R)(n-R)$

$n^2 - 2Rn + R^2$

$3(n-R)^2 + 3R^2 = tot$

$3(n^2 + R^2 - 2Rn) + 3R^2 = tot$

$3n^2 + 3R^2 - 6Rn + 3R^2 = tot$

$\cancel{6R^2 + 3n^2 - 6Rn = tot}$

(iv) Give the general equation of the graph which represents the **total** of the areas of the red and blue circles, where the sum of their radii is n cm.

$6R^2 + 3n^2 - 6Rn$ = total area where sum of radii is n cm

Excellence Exemplar 2018

Subject	Level 1 Mathematics and Statistics	Standard	91028	Total score	24

Q	Grade score	Annotation
1	E8	The candidate successfully found the equation of the line and the speed for James. Candidate was able to graph the journey. The candidate also gave a detailed description of the journey with correct speed and distances and mostly correct equations.
2	E8	The candidate successfully drew and translated an exponential. They used a table to find the correct simplified equation in b) ii. The candidate also used both an exponential and quadratic equation to solve a problem with several steps in b) iii. The candidate communicated their solutions clearly and with detailed evidence to support their answers.
3	E8	The candidate successfully found the equation of the parabola from the graph. The candidate found and commented on the relationship between the radius and the area. The candidate successfully found the quadratic relationship for the total area and generalised the relationship between the total area where the sum of the radii is n.