91579M

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SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tuanaki, Kaupae 3, 2018

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

9.30 i te ata Rātū 13 Whiringa-ā-rangi 2018
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–27 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int\left(6x-\frac{8}{x^3}\right)\mathrm{d}x$

(b) Whakaotihia te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{2x}+\frac{1}{x}$, ina ko $x = 1$, kāti ko $y = 2$.

(c) Whiriwhiria $\int_6^8\frac{2x-7}{x-5}\mathrm{d}x$

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Find $\int \left(6x - \frac{8}{x^3}\right) \mathrm{d}x$

(b) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x} + \frac{1}{x}$, given that when $x = 1$, $y = 2$.

(c) Find $\int_6^8 \frac{2x-7}{x-5} \mathrm{d}x$

You must use calculus and show the results of any integration needed to solve the problem.

MĀ TE KAIMĀKA ANAKE

(d) Whakaotihia te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 2x}{\mathrm{e}^{y}}$ ina ko $y = 0$ mēnā $x = \frac{\pi}{4}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

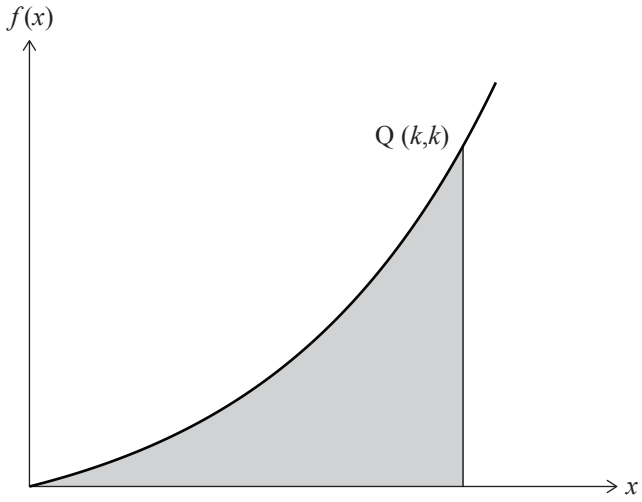
ASSESSOR'S USE ONLY

(d) Solve the differential equation $\frac{dy}{dx} = \frac{\cos 2x}{e^y}$ given that $y = 0$ when $x = \frac{\pi}{4}$.

You must use calculus and show the results of any integration needed to solve the problem.

(e) E whakaatu ana te hoahoa o raro nei i te kauwhata o te pānga $f(x) = \frac{1}{2}\left(e^x - 1\right)$.

MĀ TE KAIMĀKA ANAKE



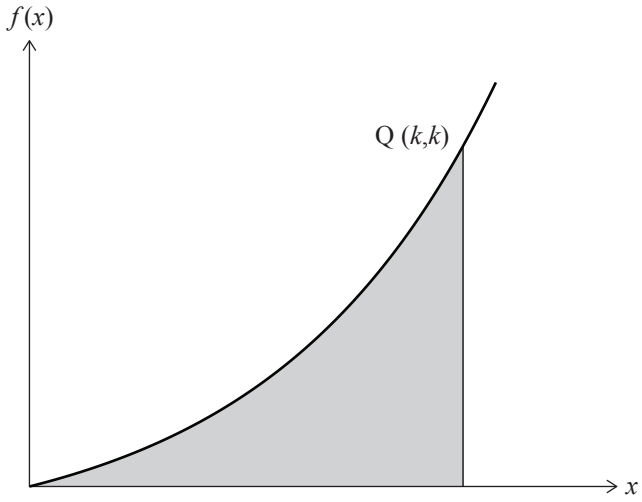
E takoto ana te pūwāhi Q(k,k) ki te ānau.
Kei te rohea te wāhanga kauruku i te hoahoa nei e te ānau, e te tuaka-x me te rārangi $x = k$.

Me whakaatu ko te horahanga o te rohe kua kaurukutia he $\frac{1}{2}k$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(e) The diagram below shows the graph of the function $f(x) = \frac{1}{2}\left(e^x - 1\right)$.



The point Q (k,k) lies on the curve.
The shaded region in the diagram is bounded by the curve, the x -axis and the line $x = k$.

Show that the shaded region has an area of $\frac{1}{2}k$.

You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int \left(\sec^2 x + \sec 2x \tan 2x \right) \mathrm{d}x$

(b) Whiriwhiria te uara o k, ina ko $\int_1^k \sqrt{x}\, \mathrm{d}x = \frac{52}{3}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION TWO

ASSESSOR'S USE ONLY

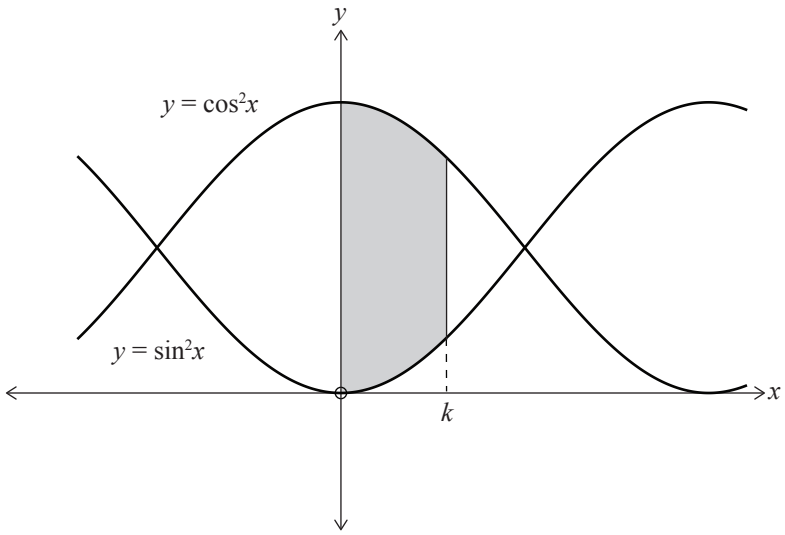
(a) Find $\int \left(\sec^2 x + \sec 2x \tan 2x \right) \mathrm{d}x$

(b) Find the value of k, given that $\int_1^k \sqrt{x}\, \mathrm{d}x = \frac{52}{3}$.

You must use calculus and show the results of any integration needed to solve the problem.

(c) E whakaatu ana te hoahoa i raro i ngā kauwhata o ngā pānga $y = \cos^2 x$ me $y = \sin^2 x$.

MĀ TE KAIMĀKA ANAKE

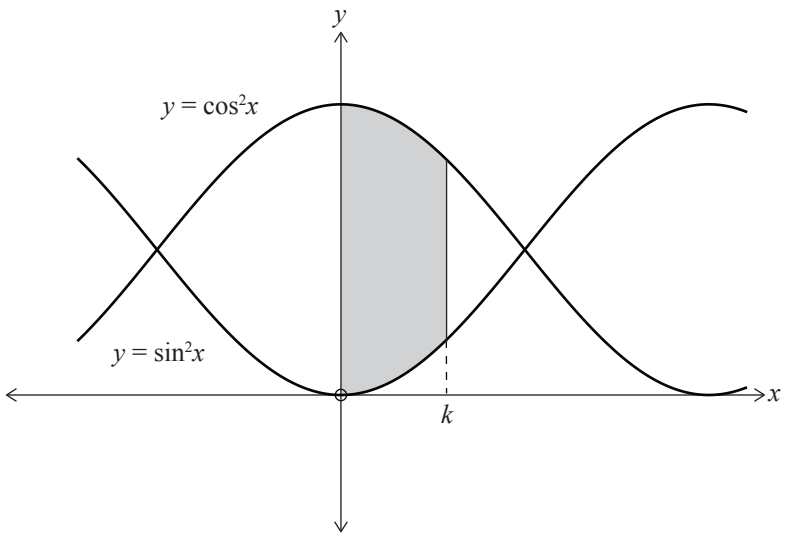


Whiriwhiria te uara o k kia $\int_0^k \left(\cos^2 x - \sin^2 x\right) \mathrm{d}x = \frac{1}{2}$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) The diagram below shows the graphs of the functions $y = \cos^2 x$ and $y = \sin^2 x$.



Find the value of k such that $\int_0^k \left(\cos^2 x - \sin^2 x\right) \mathrm{d}x = \frac{1}{2}$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) Ka taea te whakaterenga o tētahi ahanoa te whakatauira mā te whārite $a(t) = \frac{2}{\sqrt{t+1}}$, ina ko $t \geq 0$.

ina ko a te whakaterenga o te ahanoa i te m s^{-2}
ko t te wā ā-hēkona mai i te tīmatanga o te ine wā.

He 9 m s^{-1} te terenga o te ahanoa ina ko $t = 3$.

E hia te tawhiti i haere ai te ahanoa i ngā hēkona e 8 tuatahi i te wā e inea ana te nekehanga?

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(d) An object's acceleration can be modelled by the equation $a(t) = \dfrac{2}{\sqrt{t+1}}$, where $t \geq 0$.

where a is the acceleration of the object in m s^{-2}
and t is the time in seconds from the start of timing.

The object has a velocity of 9 m s^{-1} when $t = 3$.

How far did the object travel in the first 8 seconds of its timed motion?

You must use calculus and show the results of any integration needed to solve the problem.

(e) Ko te papatipu, m karamu, o tētahi kānara e mura ana i te t haora i muri i te whakakātanga tuatahi ka taea te whakatauira mā te whārite pārōnaki

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -k(m-10) \text{ ina ko } k > 0 \text{ me } m \geq 10.$$

Ko te papatipu tīmata o te kānara he 140 karamu.

I ngā haora e 3 i muri mai kua haurua kē te papatipu o te kānara.

Whiriwhiria te roa o te wā e heke ai te papatipu o te kānara ki te 50 karamu.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) The mass, m grams, of a burning candle t hours after it was first lit can be modelled by the differential equation

$$\frac{dm}{dt} = -k(m-10) \text{ where } k > 0 \text{ and } m \geq 10.$$

The initial mass of the candle was 140 grams.

3 hours later the mass of the candle had halved.

Find the length of time it will take for the mass of the candle to reduce to 50 grams.

You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int\left(\left(4x\right)^2+4x+\frac{4}{x}\right)\mathrm{d}x$

(b) Whakamahia ngā uara i raro hei whiriwhiri i tētahi āwhiwhitanga ki $\int_0^3 f(x)\mathrm{d}x$, mā te whakamahi i te Ture a Simpson.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0.3	0.75	1.1	1.35	1.6	1.15	0.5

(c) Whiriwhiria te uara o k ina ko $\int_0^k 3\mathrm{e}^{0.5x}\,\mathrm{d}x = 75$

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Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int\left(\left(4x\right)^2+4x+\frac{4}{x}\right)\mathrm{d}x$

(b) Use the values given in the table below to find an approximation to $\int_0^3 f(x)\mathrm{d}x$, using Simpson's Rule.

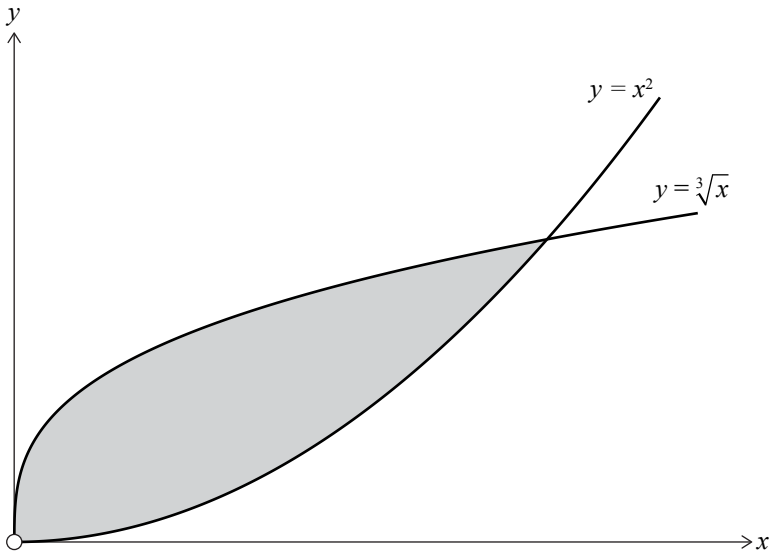
x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0.3	0.75	1.1	1.35	1.6	1.15	0.5

(c) Find the value of k given that $\int_0^k 3\mathrm{e}^{0.5x}\,\mathrm{d}x = 75$.

Question Three continues on page 19.

MĀ TE KAIMĀKA ANAKE

(d) E whakaatu ana te hoahoa o raro i ngā kauwhata o ngā pānga $y = x^2$ me te $y = \sqrt[3]{x}$.

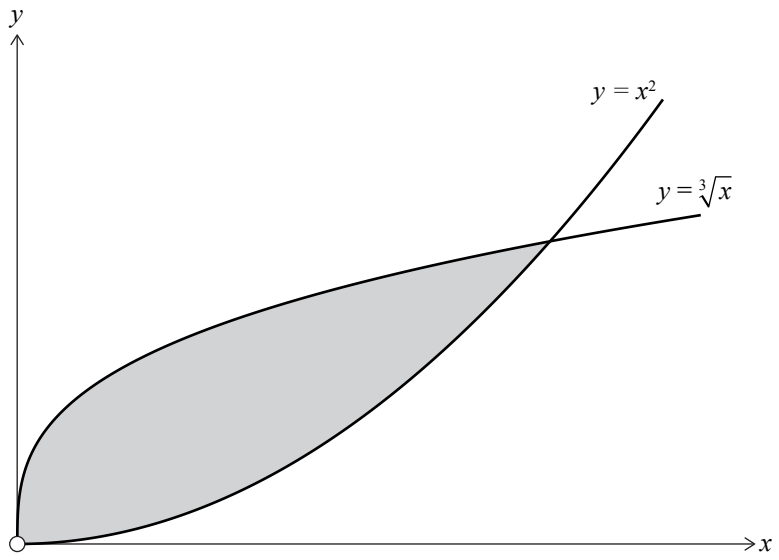


Whiriwhiria te horahanga o te rohe i waenga i ngā kauwhata (kua kaurukutia i roto i te hoahoa).

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(d) The diagram below shows the graphs of the functions $y = x^2$ and $y = \sqrt[3]{x}$.

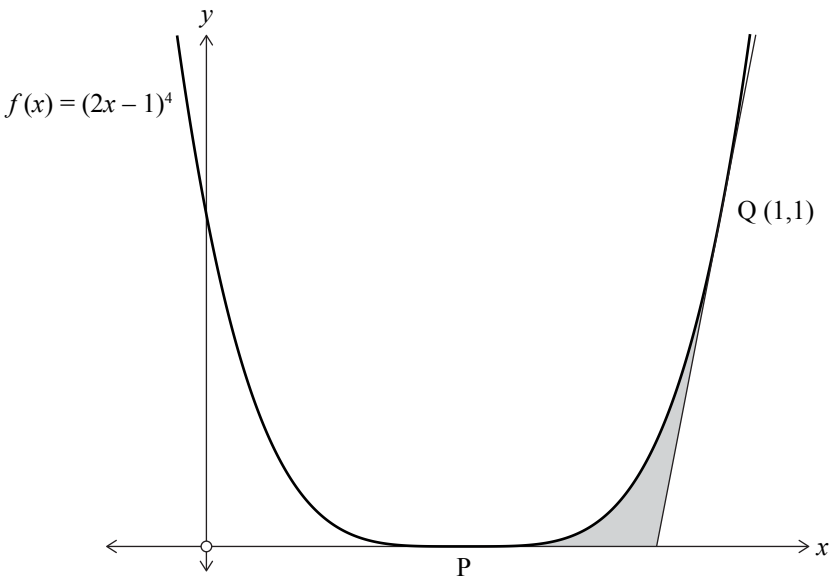


Find the area of the region between the graphs (shown shaded in the diagram).

You must use calculus and show the results of any integration needed to solve the problem.

(e) E whakaatu ana te hoahoa o raro nei i te kauwhata o te pānga $f(x) = (2x - 1)^4$.

MĀ TE KAIMĀKA ANAKE



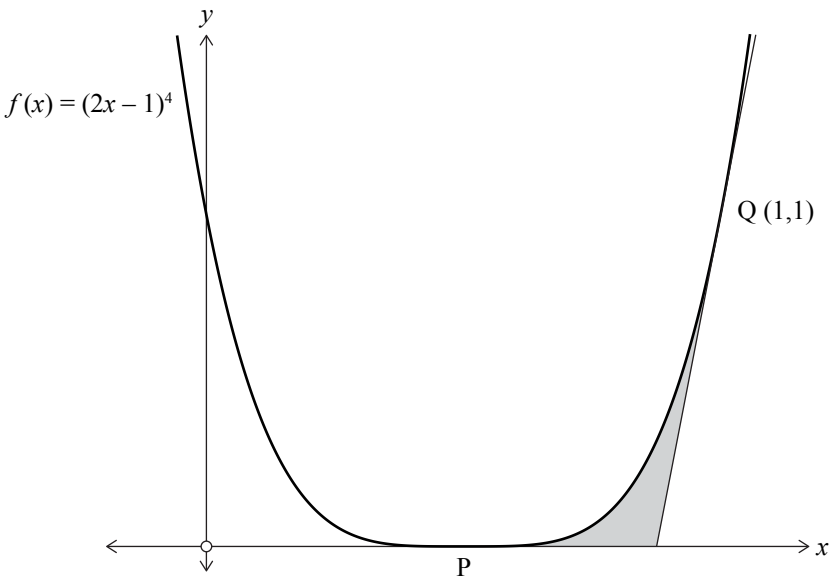
Ka tūtaki te ānau i te tuaka-*x* i P me te rārangi o te kauwhata he pātapa ki te ānau i te pūwāhi Q (1,1).

Whiriwhiria te horahanga kua rohea e te ānau, te tuaka-*x*, me te pātapa ki te ānau i Q (kua kaurukutia i roto i te hoahoa).

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(e) The diagram below shows the graph of the function $f(x) = (2x - 1)^4$.



The curve meets the x-axis at P and the line on the graph is a tangent to the curve at the point Q (1,1).

Find the area of the region bounded by the curve, the x-axis, and the tangent to the curve at Q (shown shaded in the diagram).

You must use calculus and show the results of any integration needed to solve the problem.

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2018

91579 Apply integration methods in solving problems

9.30 a.m. Tuesday 13 November 2018
Credits: Six

91579M

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.