2

91262M

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Te Pāngarau me te Tauanga, Kaupae 2, 2014

91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

2.00 i te ahiahi Rāapa 19 Whiringa-ā-rangi 2014
Whiwhinga: Rima

Paetae	Paetae Kaiaka	Paetae Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mehemea e ōrite ana te Tau Ākonga ā-Motu (NSN) kei tō pepa whakauru ki te tau kei runga ake nei.

Me whakautu e koe ngā pātai KATOA kei roto i te pukapuka nei.

Tirohia mēnā kei a koe te Rau Rauemi L2–MATHF.

Whakaaturia ngā mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te (ngā) whārangi kei muri i te pukapuka nei, ka āta tohu ai i ngā tau pātai.

Tirohia mehemea kei roto nei ngā whārangi 2–33 e raupapa tika ana, ā, kāore hoki he whārangi wātea.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

MĀ TE KAIMĀKA ANAKE

PĀTAI TUATAHI

(a) Ka tohua he pānga g mā te $g(x) = x^3 - 4x + 5$.

Tātaihia te rōnaki o te kauwhata o g i te pūwāhi $x = 2$.

(b) Mō te pānga f

$$f'(x) = 3x^2 + 4x - 1$$

Ka whakawhiti te kauwhata o f mā te pūwāhi (2,5).

Tātaihia te whārite mō te pānga f.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function g is given by $g(x) = x^3 - 4x + 5$.

Find the gradient of the graph of g at the point where $x = 2$.

(b) For a function f

$$f'(x) = 3x^2 + 4x - 1$$

The graph of f passes through the point (2,5).

Find the equation of the function f.

MĀ TE KAIMĀKA ANAKE

(c) Ka ruku tētahi kairuku ki roto i tētahi hōpua kaukau. Ko te hōhonu d ā-mita ka tae ia i ngā t hēkona i muri i tōna pānga ki te wai ka tohua mā te

$$d(t) = 1.25t^2 - 4t$$

Tātaihia te hōhonutanga i taea rawatia e ia.

(d) He pūwāhi huringa i (3,4) tō te kauwhata $f(x) = -x^2 + \mathrm{k}x - 5$.

Tātaihia te rōnaki o te pānga kei te pūwāhi $x = 4$.

ASSESSOR'S USE ONLY

(c) A diver dives into a pool. The depth d metres that she reaches t seconds after she hits the water is given by

$$d(t) = 1.25t^2 - 4t$$

Find the greatest depth that she reaches.

(d) The graph of $f(x) = -x^2 + kx - 5$ has a turning point at (3,4).

Find the gradient of the function at the point where $x = 4$.

(e) $g(x) = \frac{2x^3}{3} + \frac{3x^2}{2} - 20x + 4$

He aha ngā uara o x e noho ai a g hei pānga heke?

Me mātua whakamahi e koe te tuanaki i roto i tō otinga.

MĀ TE KAIMĀKA ANAKE

(e) $g(x) = \frac{2x^3}{3} + \frac{3x^2}{2} - 20x + 4$

For what values of x is g a decreasing function?

You must use calculus in finding your solution.

ASSESSOR'S USE ONLY

(f) I te wā o tētahi hāereere eke pahikara mahi moni, ko te tawhiti s ā-kiromata o tētahi kaieke pahikara mai i tētahi pūwāhi pūmau o tana haere ka whakatauiratia e te pānga

$$s(t) = 0.1875t^3 - 2.25t^2 + 21t + 0.5$$

ina ko t te wā ā-hāora mai i te hipatanga o te kaieke pahikara i te pūwāhi pūmau.

E hia te tawhiti o te kaieke pahikara mai i te pūwāhi pūmau ina eke i a ia tana tere mōkito?

MĀ TE KAIMĀKA ANAKE

(f) During a fund-raising cycle ride, the distance s kilometres of a cyclist from a fixed point on his ride is modelled by the function

$$s(t) = 0.1875t^3 - 2.25t^2 + 21t + 0.5$$

where t is the time in hours since the cyclist passed the fixed point.

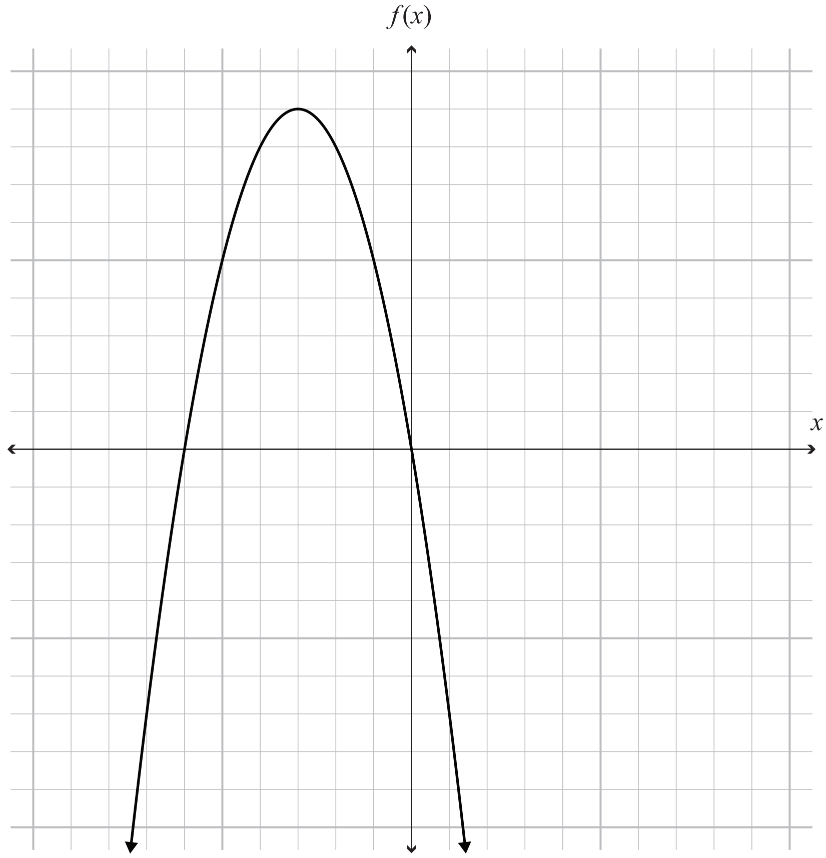
How far will the cyclist be from the fixed point when he reaches his minimum speed?

ASSESSOR'S USE ONLY

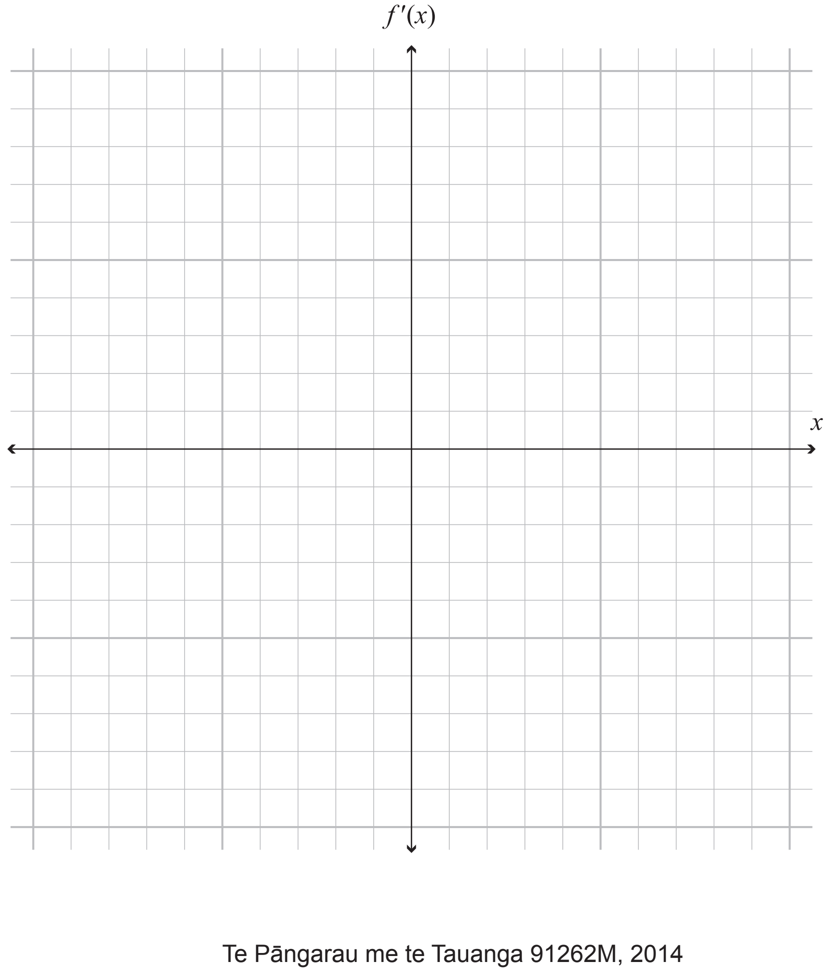
PĀTAI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.



Ki ngā tuaka o raro, tuhia te pānga rōnaki $y = f'(x)$.

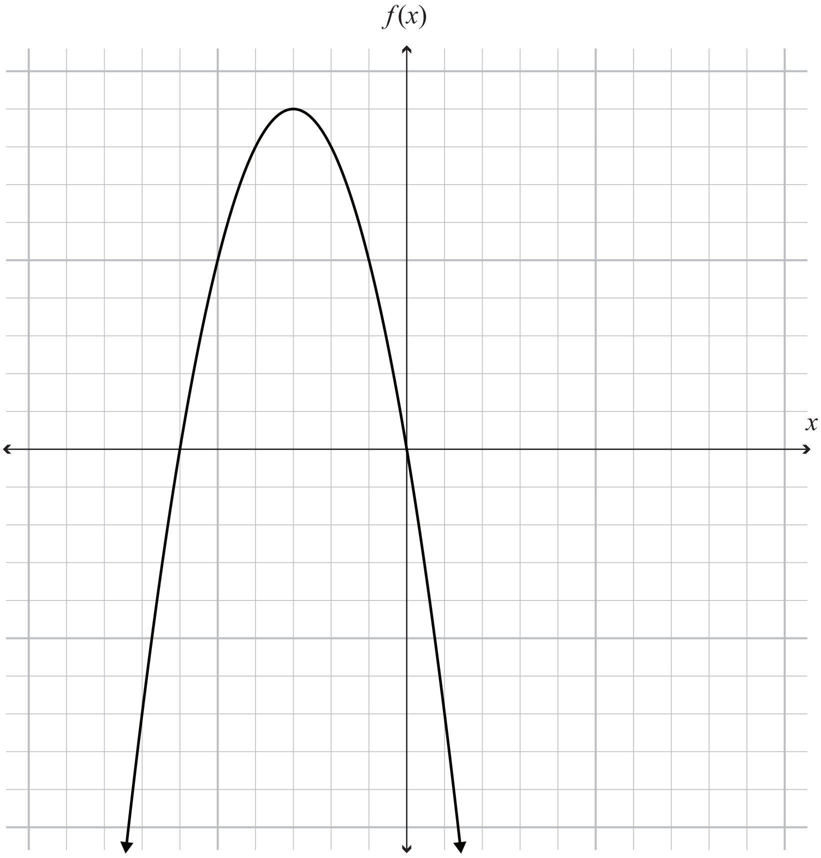


Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 28

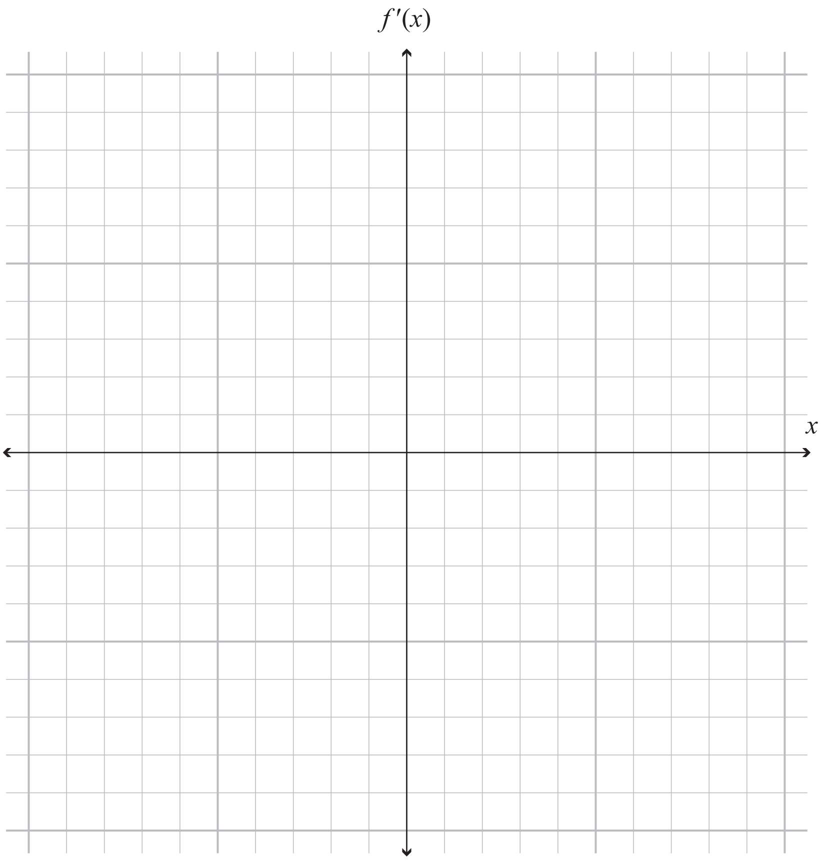
ASSESSOR'S USE ONLY

QUESTION TWO

(a) The graph of the function $y = f(x)$ is shown on the axes below.



On the axes below sketch the gradient function $y = f'(x)$.



If you need to redraw this graph, use the grid on page 29

(b) Ka whakawhiti te kauwhata o te pānga $y = g(x)$ mā (0,0), ā, ko tana pānga rōnaki he $g'(x) = 2x - 5$.

Tātaihia te taunga-y o te pūwāhi o te ānau ina ko $x = 3$.

(c) E kerēme ana a Sione ko te uara hoko atu anō o tētahi waka, ngā tau t mai i tōna hokotanga atu, ka taea te whakatauira mā te pānga

$R = 150t^2 - 2250t + 38\,000$, ina ko R te uara ā-tāra hoko atu anō.

E hia te roa mai i tōna hokotanga atu ka huri te uara o te waka ki te pāpātanga o te -\$150 i te tau?

ASSESSOR'S USE ONLY

(b) The graph of a function $y = g(x)$ passes through (0,0), and its gradient function is $g'(x) = 2x - 5$.

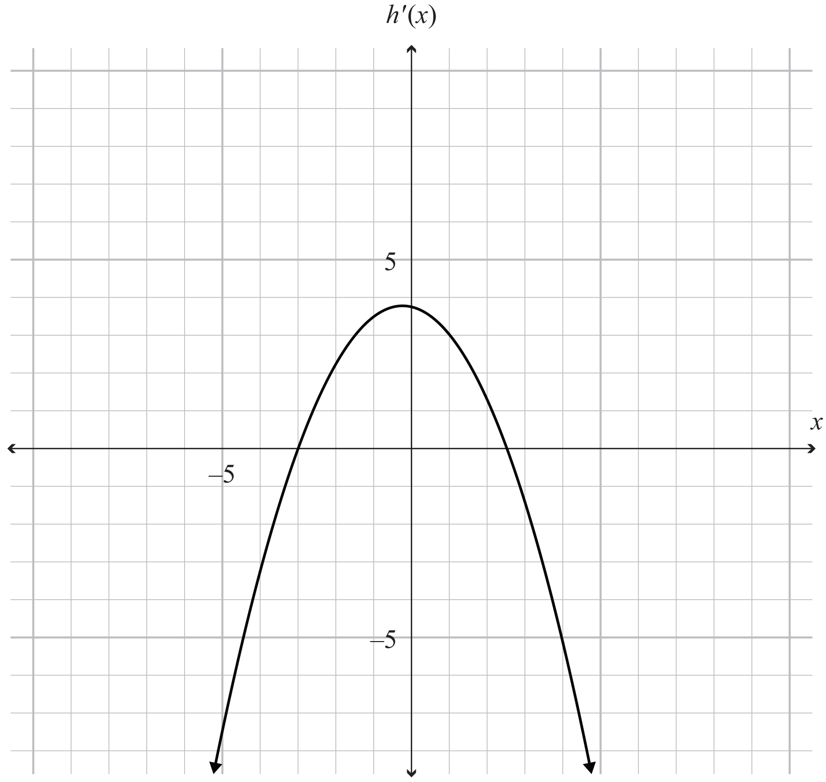
Find the y-coordinate of the point on the curve where $x = 3$.

(c) Sione claims that the resale value of a car, t years after it is sold, can be modelled by the function

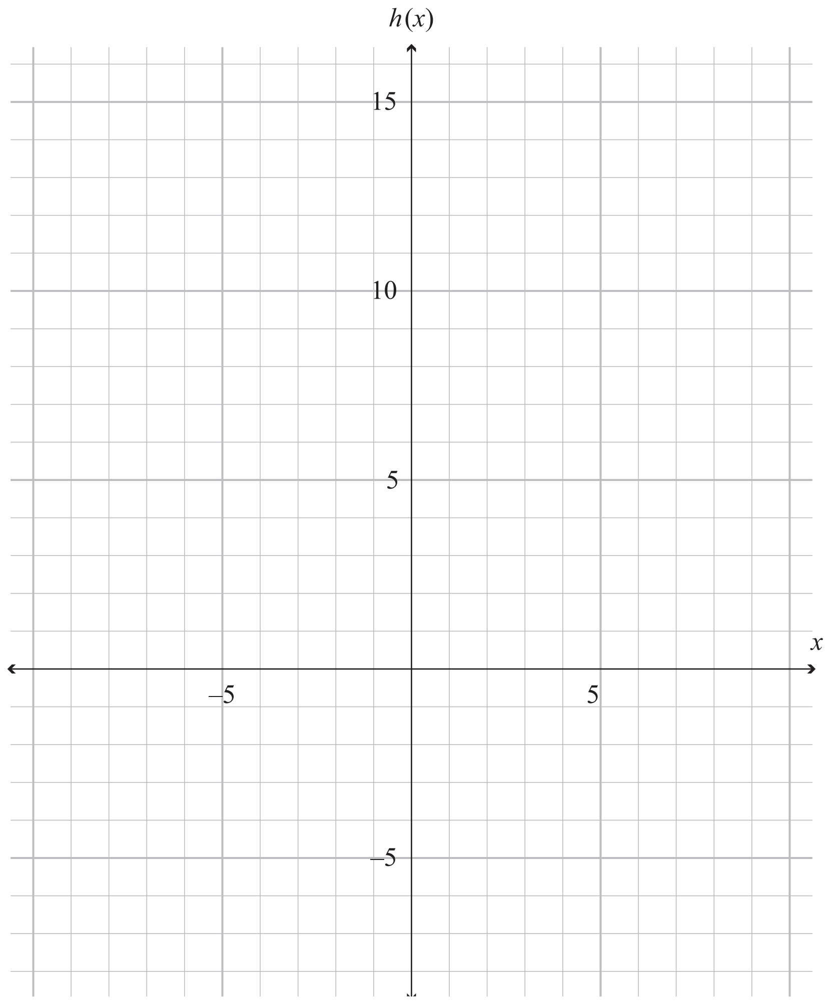
$R = 150t^2 - 2250t + 38\,000$, where R is the resale value in dollars.

How long after it is sold will the car's value be changing at a rate of –\$150 per year?

(d) E whakaaturia ana te pānga rōnaki $h'(x)$ ki ngā tuaka i raro.



Tātuhia te pānga $h(x)$ mēnā ka hipa a $h(x)$ mā te pūwāhi (–3,2).

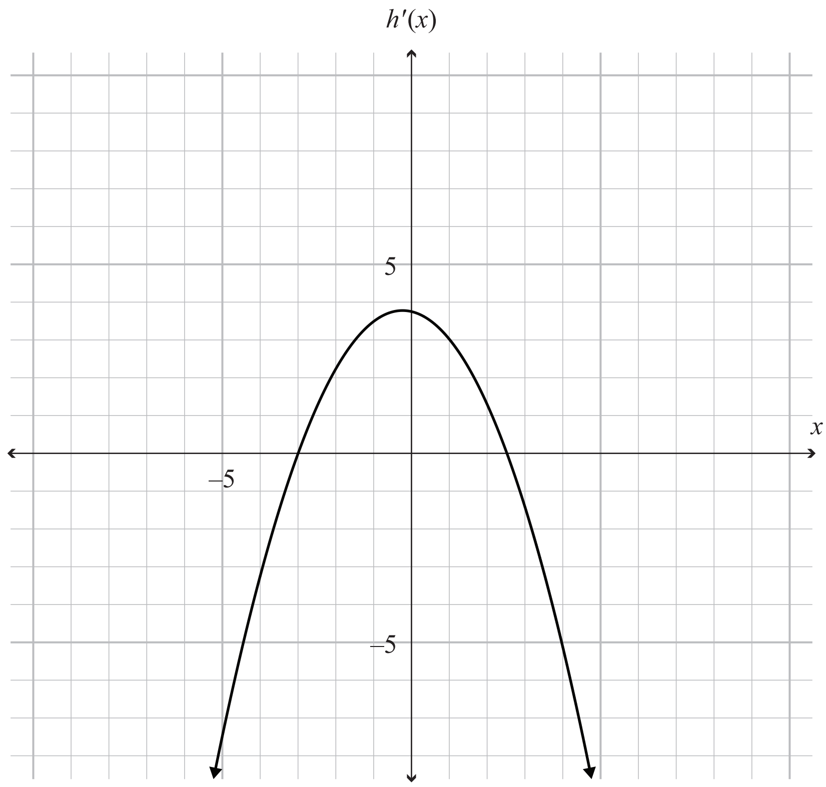


Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 30

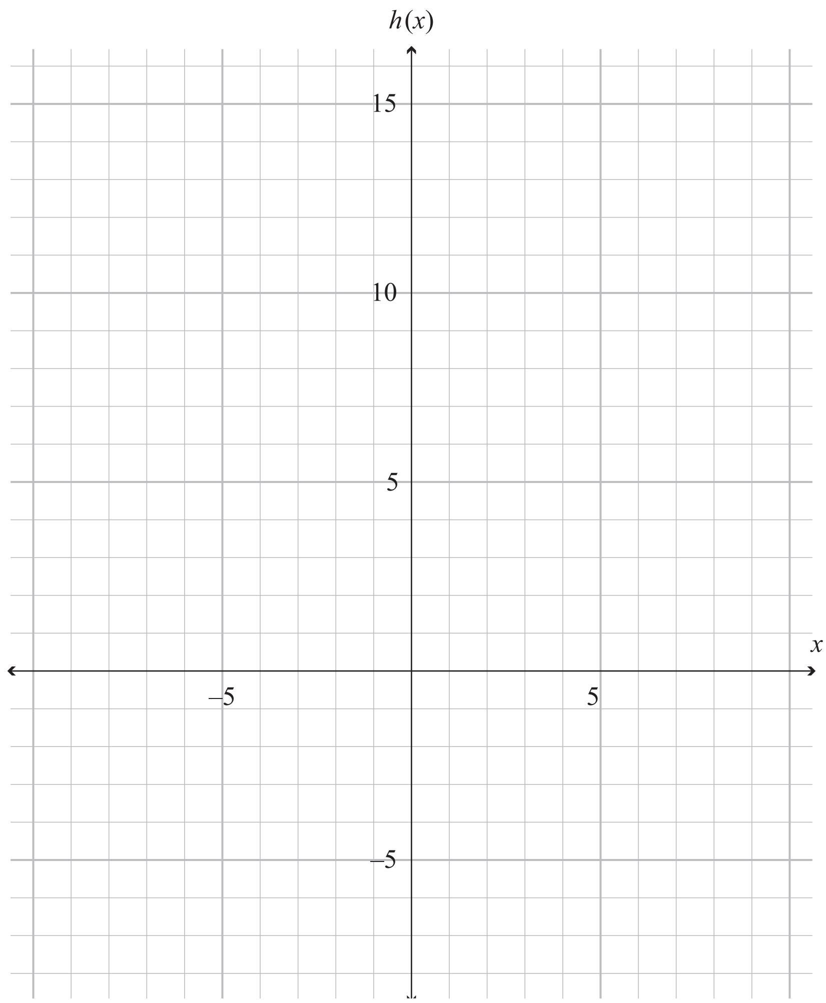
MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(d) A gradient function $h'(x)$ is shown on the axes below.



Sketch the function $h(x)$ if $h(x)$ passes through the point (–3,2).



If you need to redraw this graph, use the grid on page 31

(e) Ka tohua te rōnaki o te kauwhata o te pānga mā te $\frac{dy}{dx} = 9x - 3x^2$.

I te pūwāhi huringa mōrahi o te kauwhata o te pānga, $y = -4$.

Tātaihia te whārite o te kauwhata.

Kia mōhio mai: Ko te pūwāhi huringa ko $y = -4$ me mātua whakaatu hei mōrahi.

MĀ TE KAIMĀKA ANAKE

(e) The gradient of the graph of a function is given by $\frac{dy}{dx} = 9x - 3x^2$.

At the maximum turning point of the graph of the function, $y = -4$.

Find the equation of the graph.

Note: The turning point where $y = -4$ must be shown to be a maximum.

ASSESSOR'S USE ONLY

(f) Ko te pātapa ki te kauwhata o te pānga $y = px^2 + 4x - 5$ i te pūwāhi ko $x = 3$ ka hipa mā te pūwāhi (0,4).

Tātaihia te uara o p kia puta ko te whārite o te pātapa.

MĀ TE KAIMĀKA ANAKE

(f) The tangent to the graph of the function $y = px^2 + 4x - 5$ at the point where $x = 3$ passes through the point (0,4).

Find the value of p, and hence the equation of the tangent.

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

PĀTAI TUATORU

(a) Kimihia te taunga-x o te pūwāhi o te kauwhata o te pānga $f(x) = 4x - x^2$ ina ōrite te rōnaki ki te 10.

(b) Kei te whakakīa tētahi hōpua kaukau ki te wai.

I te wāhi hōhonu rawa, ko te hōhonu o te wāi i tētahi wā he h m.

E tohua ana te rōrahi V m^3 o te wai i roto i te hōpua kaukau ko te

$$V(h) = 20h^2 + 40h$$

Tātaihia te pāpātanga e huri ai te rōrahi o te wai e ai ki te hōhonu ina eke te hōhonu o te wai i roto i te hōpua kaukau ki te 0.75 m.

(c) Kei te whakamakohatia tētahi poihau ki te haumāmā.

E tohua ana te rōrahi V cm^3 o te haumāmā i roto i te poihau ko te $V = \frac{4}{3}\pi r^3$, ina ko r cm te pūtoro o te poihau.

Tātaihia te pāpātanga e huri ai te rōrahi o te hau i roto i te poihau e ai ki te pūtoro ina ko te rōrahi he 288π cm^3.

ASSESSOR'S USE ONLY

QUESTION THREE

(a) Find the x-coordinate of the point on the graph of the function $f(x) = 4x - x^2$ where the gradient is equal to 10.

(b) A swimming pool is being filled.

At the deepest point, the depth of the water in the pool at any instant is h m.

The volume of water in the pool V m^3 is given by

$$V(h) = 20h^2 + 40h$$

Find the rate at which the volume of water is changing with respect to the depth when the water in the pool is 0.75 m deep.

(c) A balloon is being inflated with helium.

The volume V cm^3 of helium in the balloon is given by $V = \frac{4}{3}\pi r^3$, where r cm is the radius of the balloon.

Find the rate at which the volume of gas in the balloon is changing with respect to the radius when the volume is 288π cm^3.

(d) He toru whakareatanga ake te roa o tētahi poro-tapawhā hāngai i tōna whānuitanga.

Ko te tapeke o te teitei, whānui, me te roa he 150 cm.

Ka kīia te rōrahi V cm^3 ko te

$V = 450x^2 - 12x^3$ ina ko x cm te whānui.

Tātaihia te teitei o te poro-tapawhā hāngai ki tōna rōrahi mōrahi.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(d) The length of a cuboid is three times its width.

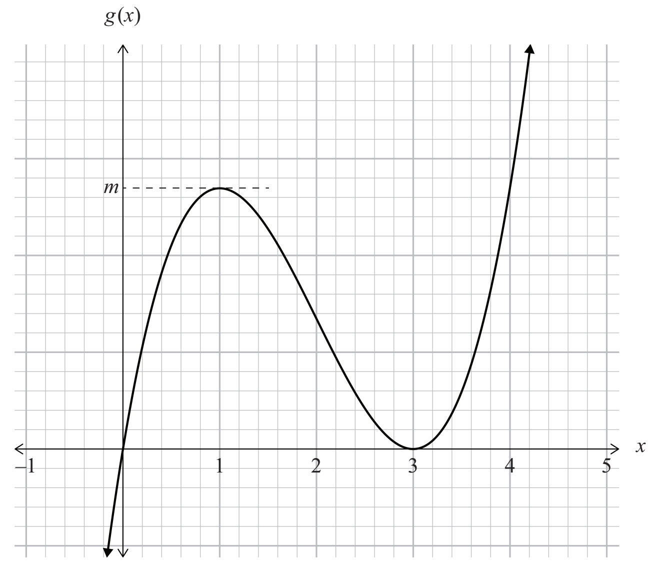
The sum of the height, width, and length is 150 cm.

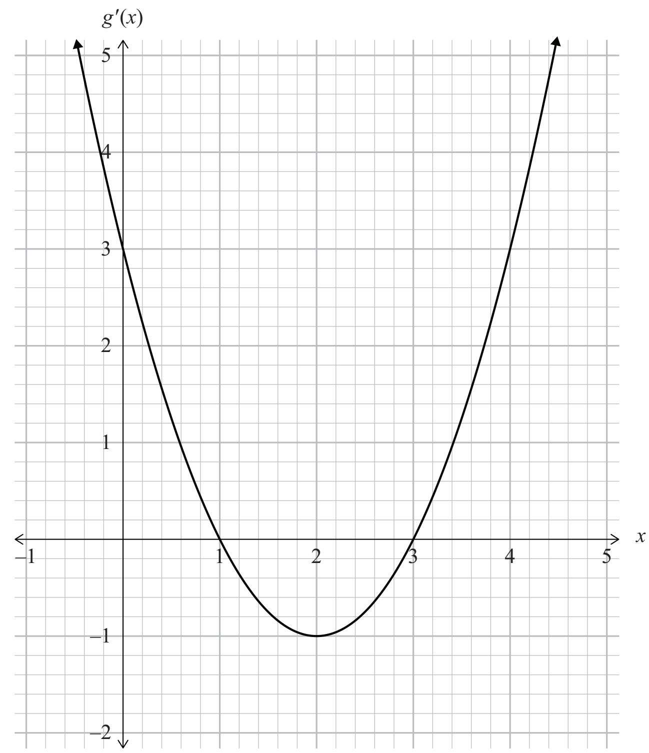
The volume V cm^3 can be expressed as

$V = 450x^2 - 12x^3$ where the width is x cm.

Find the height of the cuboid for which the volume is maximum.

(e) E whakaaturia ana i raro ko te kauwhata o te pānga $g(x)$ me te kauwhata o tōna pānga rōnaki.





Tātaihia te uara m, te uara-y mō te pūwāhi huringa mōrahi o te pānga $g(x)$.

MĀ TE KAIMĀKA ANAKE

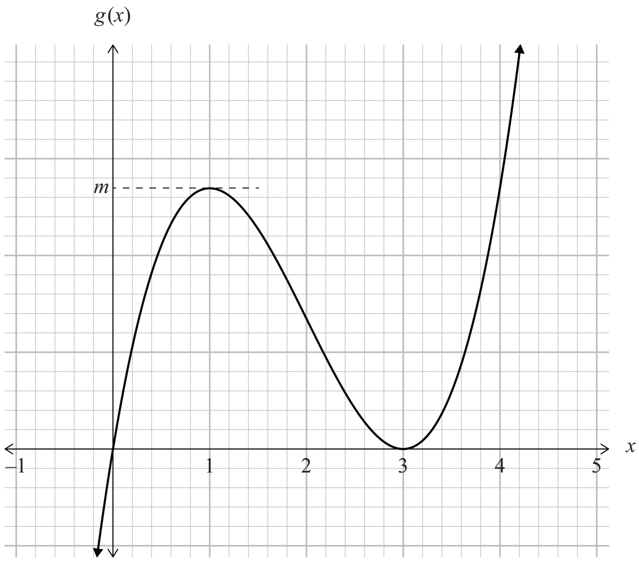
(f) He 70 m s^{-1} te tere o te haere o tētahi waka rererangi i tōna taunga.

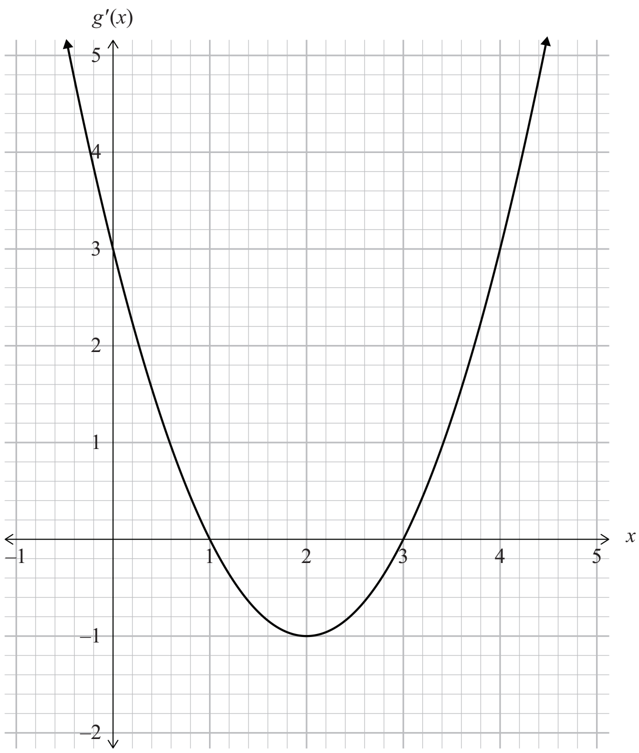
Ka huri tōna tere ki te rerenga aumou o te –3.3 m s^{-2}.

Whakamahia te tuanaki ki te kimi e hia te tawhiti o te haere o te waka rererangi mai i tōna taunga atu ki te wāhi ko tōna tere he 4 m s^{-1}.

(e) The graph of a function $g(x)$ and the graph of its gradient function, are shown below.

ASSESSOR'S USE ONLY





Find the value m, the y-value for the maximum turning point of the function $g(x)$.

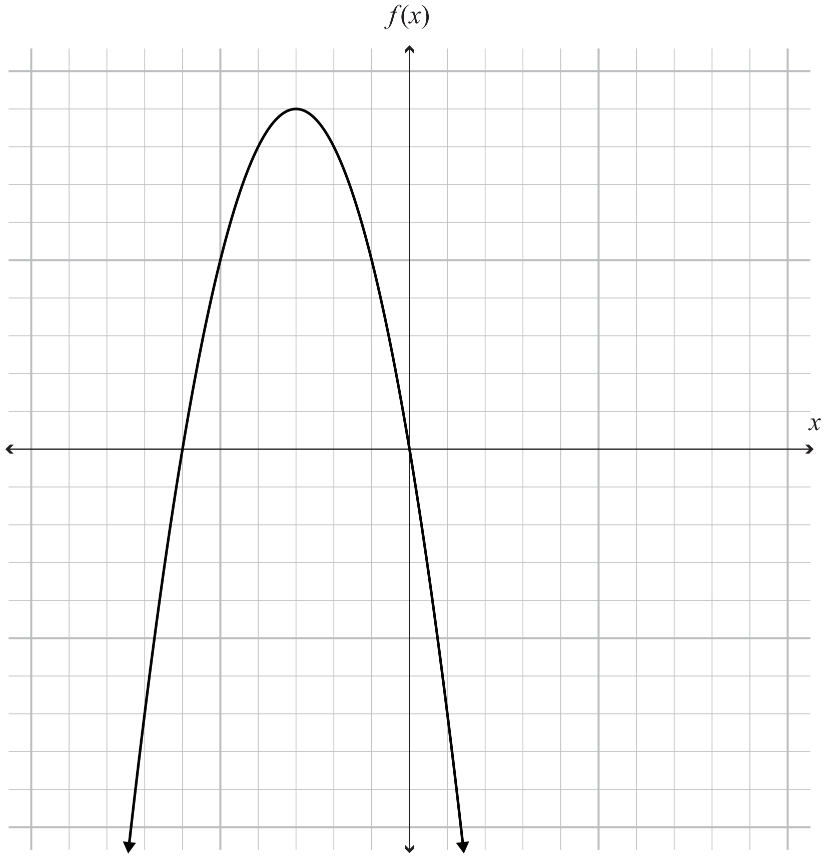
ASSESSOR'S USE ONLY

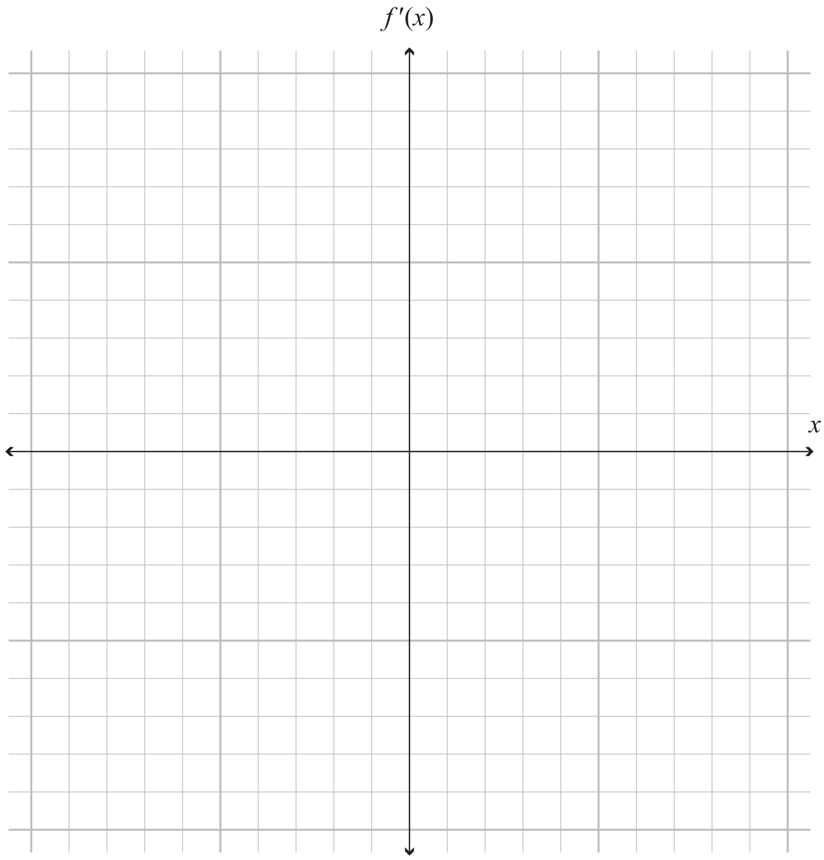
(f) An aircraft is travelling at 70 m s^{-1} when it lands.

Its speed changes at a constant rate of –3.3 m s^{-2}.

Use calculus to find how far the aircraft will travel from where it lands to where it has a speed of 4 m s^{-1}.

Ki te hiahia koe ki te tuhi anō i te kauwhata mō te Pātai Tuarua (a), tuhia ki te tukutuku i raro. Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

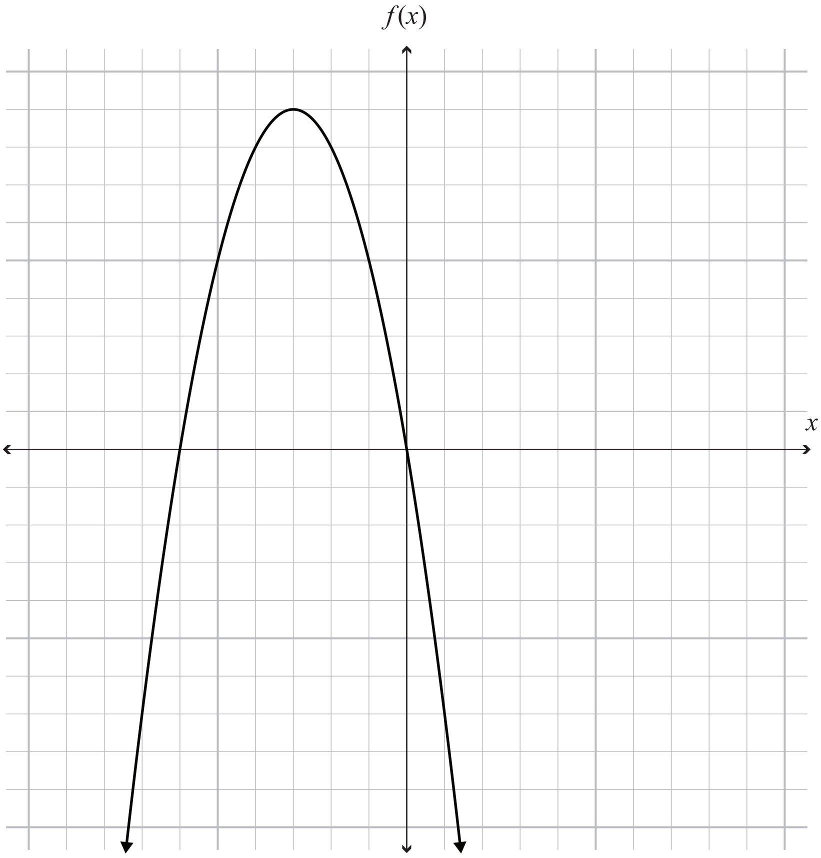


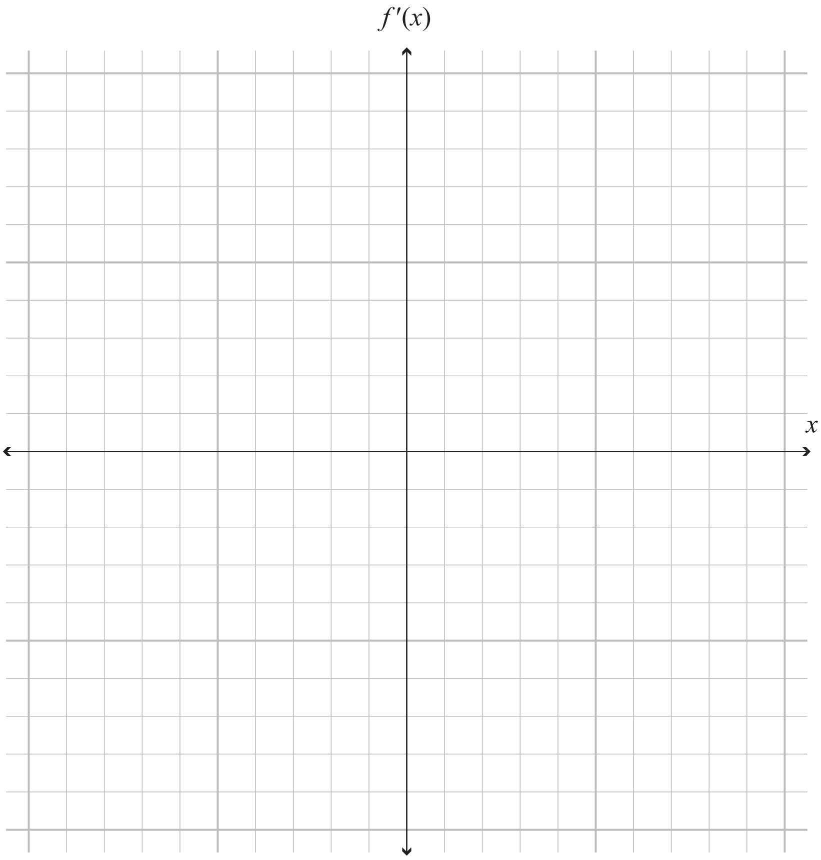


MĀ TE KAIMĀKA ANAKE

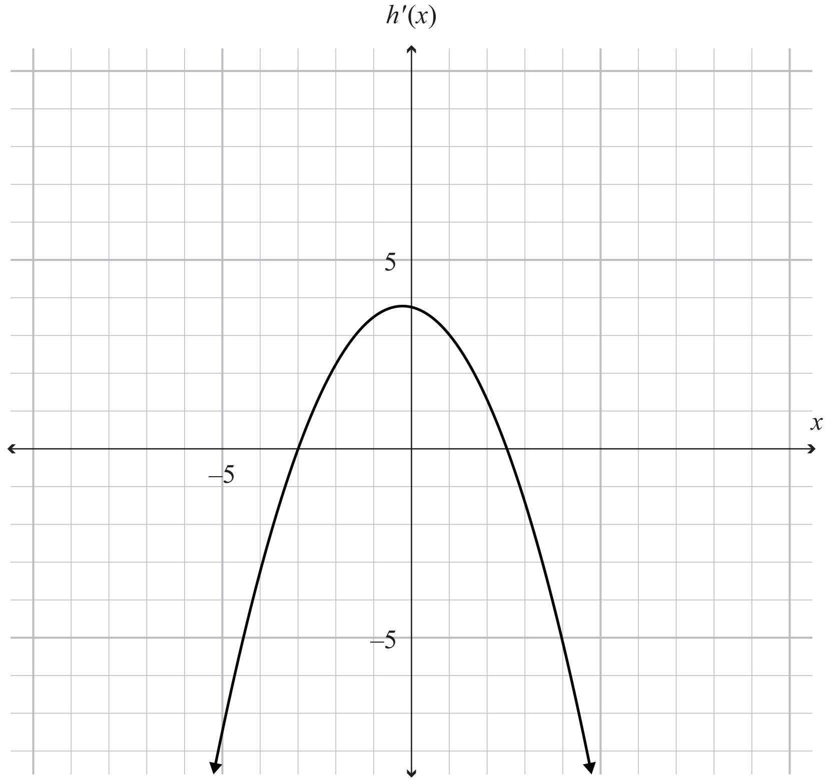
ASSESSOR'S USE ONLY

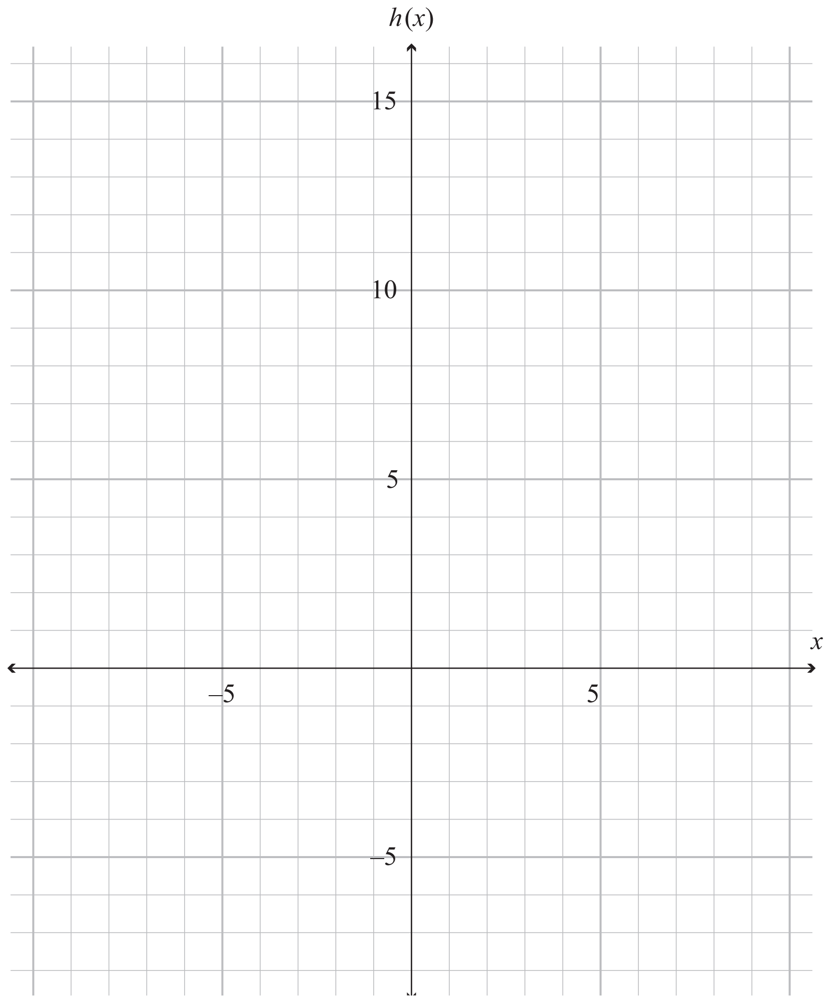
If you need to redraw your graph from Question Two (a), draw it on the lower grid. Make sure it is clear which answer you want marked.





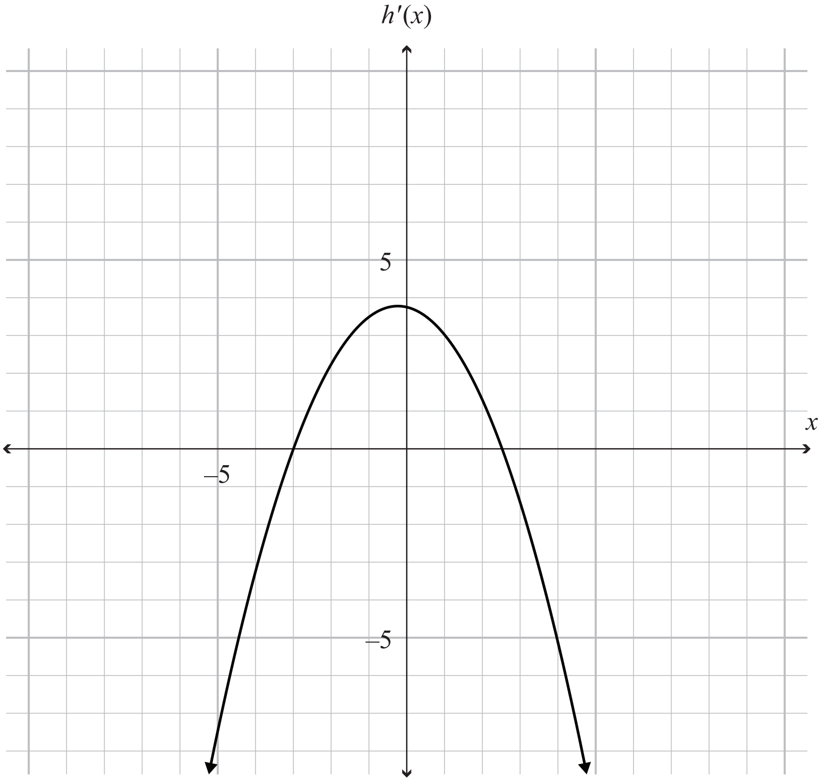
Ki te hiahia koe ki te tuhi anō i te kauwhata mō te Pātai Tuarua (d), tuhia ki te tukutuku i raro. Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

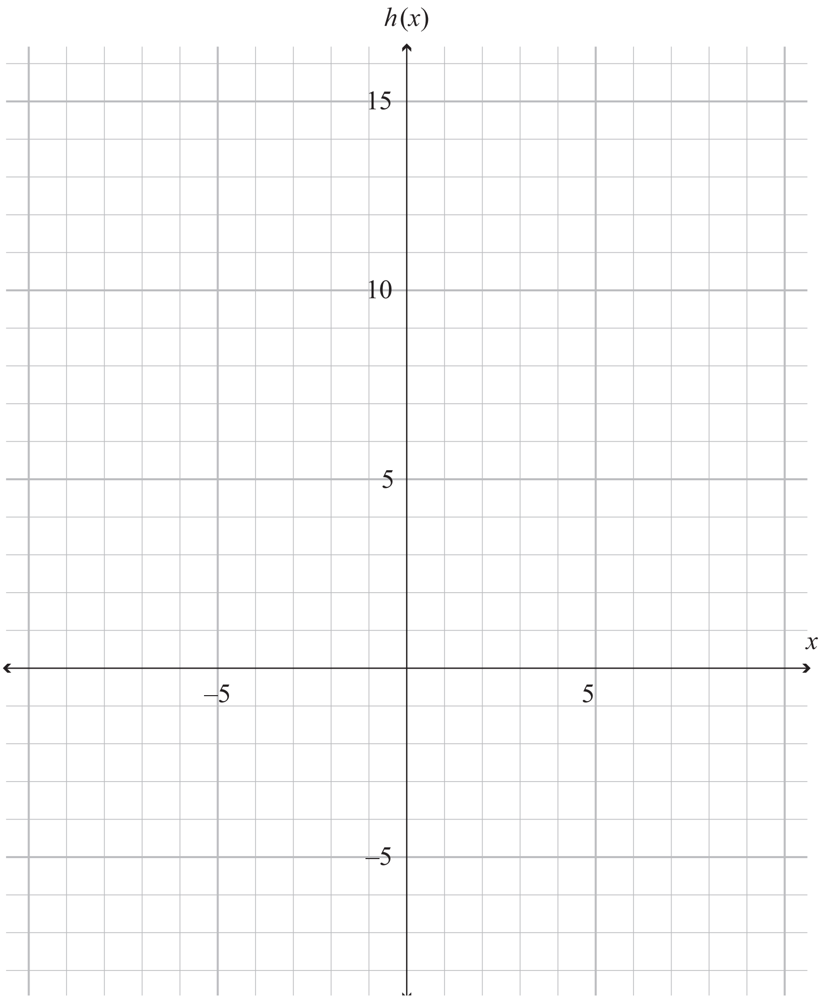




MĀ TE KAIMĀKA ANAKE

If you need to redraw your graph from Question Two (d), draw it on the lower grid. Make sure it is clear which answer you want marked.





ASSESSOR'S USE ONLY

He puka anō mēnā ka hiahiatia.
Tuhia te (ngā) tāu pātai mēnā e hāngai ana.

TAU PĀTAI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 2 Mathematics and Statistics, 2014

91262 Apply calculus methods in solving problems

2.00 pm Wednesday 19 November 2014
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

91262M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–33 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.