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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2016

91579 Apply integration methods in solving problems

9.30 a.m. Wednesday 23 November 2016
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL **13**

ASSESSOR'S USE ONLY

QUESTION ONE

For parts (a) and (b) find each integral.

Remember the constant of integration.

(a) $\displaystyle\int \frac{2x^4 - x^2}{x^3}\,dx$

$\int ((2x^4 - x^2)(x^{-3})\,dx$

$= \frac{4}{5}x^5 - \frac{1}{3}x^3$

(b) $\displaystyle\int \sec(3x)\tan(3x)\,dx$

$= \dfrac{\sec(3x)}{3} + c$

(c) If $\dfrac{dy}{dx} = \dfrac{\cos x}{3y}$ and $y = 1$ when $x = \dfrac{\pi}{6}$, find the value of y when $x = \dfrac{7\pi}{6}$.
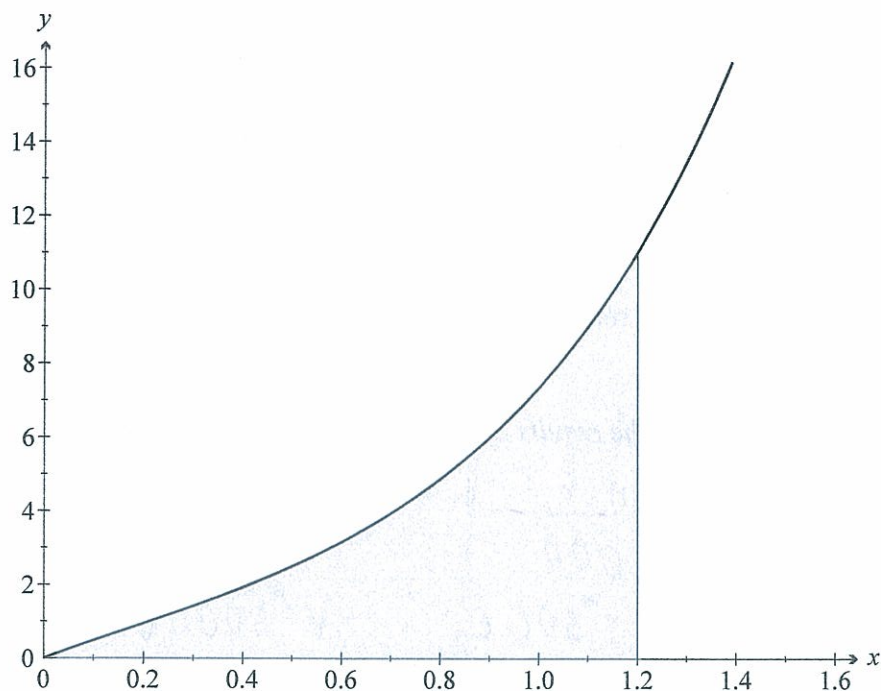
You must use calculus and give the results of any integration needed to solve this problem.

$\int \dfrac{\cos x}{3y}$ $y = \dfrac{\sin x \cdot \ln 3y}{3}$

$\dfrac{\sin 30 \cdot \ln 3}{3} = 1$

(d) Use integration to find the area enclosed between the curve $y = e^{2x} - \dfrac{1}{e^{3x}}$

and the lines $y = 0$, $x = 0$, and $x = 1.2$ (the area shaded in the diagram below).

You must use calculus and give the results of any integration needed to solve this problem.

$$\int_{0}^{1.2} e^{2x} - e^{-3x} \, dx$$

$$= \left[\frac{e^{2x}}{2} + \frac{1}{3} e^{-3x} \right]_{0}^{1.2}$$

$$= \left(\frac{e^{2.4}}{2} + \frac{1}{3} e^{-3.6} \right) - \left(\frac{e^{0}}{2} + \frac{1}{3} e^{0} \right)$$

$$= \left(5.5115 8819 + 9.107901 \times 10^{-3} \right) - \frac{5}{6}$$

$$= 4.687 \text{ units}^2$$

(e) Mr Newton has a container of oil and places it in the garage. Unfortunately, he puts the container on top of a sharp nail and it begins to leak.

The rate of decrease of the volume of oil in the container is given by the differential equation

$$\frac{dV}{dt} = -kVt$$

where V is the volume of oil remaining in the container t hours after the container was put in the garage.

The volume of oil in the container when it was placed in the garage was 3000 mL.

After 20 hours, the volume of oil in the container was 2400 mL.

How much, if any, of the oil will remain in the container 96 hours after it was placed in the garage?

You must use calculus and give the results of any integration needed to solve this problem.

$$dv \frac{1}{v} = -kdt \qquad \ln v = -kt \qquad V = Ae^{-kt} \qquad V = -Ae^{-kt}$$

when $t = 0$, $V = 3000$

$3000 = Ae^0 \qquad A = 3000 \qquad V = 3000e^{-kt}$

when $t = 20$, $V = 2400$

$2400 = 3000e^{-20k} \qquad e^{-20k} = 0.8$

$\ln 0.8 = -20k \qquad k = 0.011157\ldots$

$V = ?$ when $t = 96$

$V = 3000e^{-0.011157 \times 96}$

$V = 1027.905$

$= 1027.9 \text{ mL}$

M5

QUESTION TWO
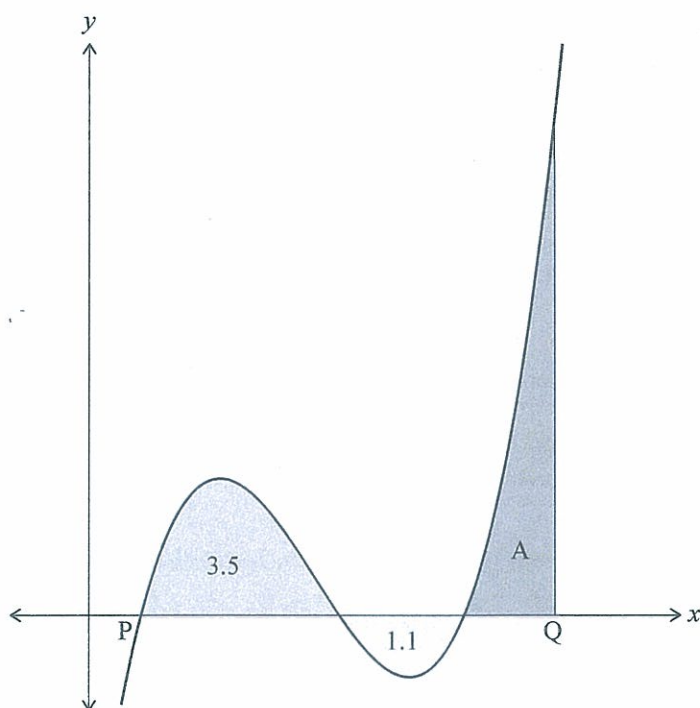
(a) Find $\int (5x^2 - 1)^2 \, dx.$

y12

(b) The graph of a function $y = f(x)$ is shown below.

The areas of two of the shaded regions are given.

If $\int_{P}^{Q} f(x)\,dx = 9.4$, what is the area of shaded region A?

A = 4.8

(c)   The acceleration of an object is given by $a(t) = 0.2t + 0.3\sqrt{t}$ for $0 \leq t \leq 10$.

where a is the acceleration of the object in m s^{-2}
and t is the time in seconds from when the object started to move.

The object was moving with a velocity of 5 m s^{-1} when $t = 4$.

How far was the object from its starting point after 9 seconds?

You must use calculus and give the results of any integration needed to solve this problem.

$v = \int 0.2t + 0.3 t^{1/2} \, dt$

$\quad = 0.1t^2 + 0.2t^{3/2} + C$

when $t = 4$ $v = 5$

$(0.1 \times 16) + 0.2(4^{3/2}) + C = 5$ $C = 1.8$

$V = 0.1t^2 + 0.2t^{3/2} + 1.8$

$\int_0^9 0.1t^2 + 0.2t^{3/2} + 1.8 \, dt$

$\left[\dfrac{t^3}{30} + 0.08t^{5/2} + 1.8t \right]_0^9$ when $t = 9$

$= 24.3 + 19.44 + 16.2$

$= 59.94 \, m.$

(d)   Find the value of the constant m such that $\displaystyle\int_{m}^{2m} (2x - m)^2 \, dx = 117$.

You must use calculus and give the results of any integration needed to solve this problem.
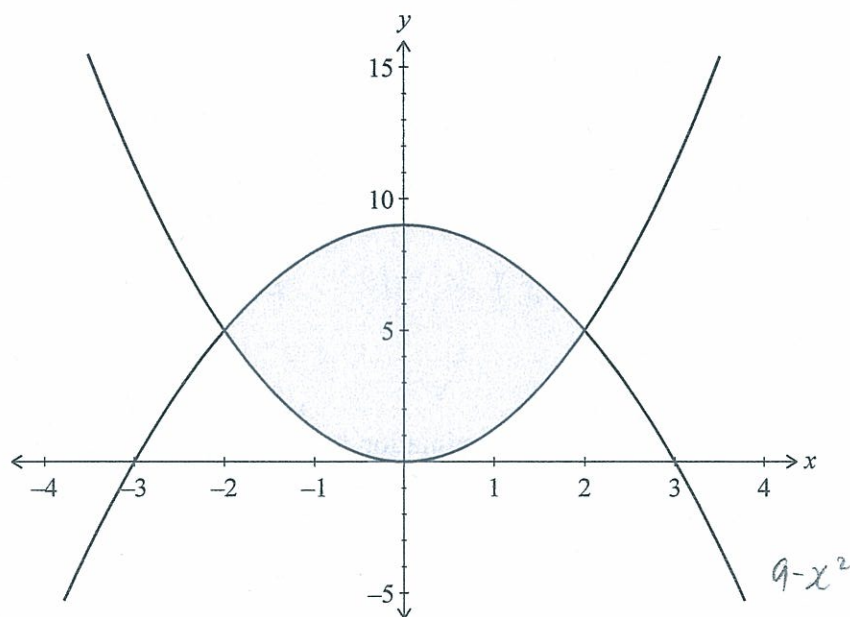
$= \left[\dfrac{1}{6} (2x - m)^3 \right]_{m}^{2m}$

$= \left(\dfrac{(4m - m)^3}{6} \right) - \left(\dfrac{(2m - m)^3}{6} \right)$

$= \dfrac{9m^3}{6} - \dfrac{m^3}{6} = 117$ $8m^3 = 702$

$m = 4.443$

(e)   The graphs of $y = (k-1)x^2$, $k > 1$ and $y = 9 - x^2$ are shown in the diagram below.

The shaded region has an area of 24.

Find the value of k.

You must use calculus and give the results of any integration needed to solve this problem.

$$\left(\left(\int_{-2}^{2} (k-1)x^2 \, dx\right) - \left(\int_{-2}^{2} 9 - x^2 \, dx\right)\right) = 24$$

$$= \int_{-2}^{2} kx^2 - x^2 \, dx$$

$$= \left[\frac{kx^3}{3} - \frac{x^3}{3}\right]_{-2}^{2} - \left[9x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \left(\frac{8k-8}{3} - \left(\frac{-8k+8}{3}\right)\right) - \left(\frac{46}{3} - \left(-18 + \frac{46}{3}\right)\right)$$

$$= \frac{16k-16}{3} - \frac{146}{3}$$

$$= \frac{16k - 162}{3} = 24 \qquad 16k = 72 + 162$$

$$k = 14.625$$

QUESTION THREE

(a) Find the value of k if $\displaystyle\int_1^4 \left(4 + \frac{k}{x^2}\right) dx = 0$.

$[4x \cancel{+ k \log x}$ $\displaystyle\int_1^4 (4 + kx^{-2})\,dx$

$[4x - kx^{-1}]_1^4$

$\left[4x - \dfrac{k}{x}\right]_1^4$ $\qquad \left(16 - \dfrac{k}{4}\right) - \left(4 - k\right) = 0$

$\qquad\qquad\qquad\qquad 16 - \frac{1}{4}k - 4 + k = 0$

$\qquad\qquad\qquad\qquad \frac{3}{4}k = -12 \qquad k = -16$

(b) Use the values given below to find an approximation to $\displaystyle\int_1^4 f(x)\,dx$, using Simpson's rule.

x	1	1.5	2	2.5	3	3.5	4
$f(x)$	1.4	2	3	3.8	2.8	2.2	1.8

(c) Use integration to find the area enclosed between the graphs of the functions $y = 2 - x^2$ and $y = -x$.

You must use calculus and give the results of any integration needed to solve this problem.

interception between graphs

$2 - x^2 = -x$ $-x^2 + x + 2 = 0$ $x = 2$ or $x = -1$ lower limit

$2 - x^2 = 0$ $x = \sqrt{2}$ or $-\sqrt{2}$ upper limit

$\text{area} = \left[\left(\int_{-1}^{\sqrt{2}} 2 - x^2 \, dx \right) - \left(\int_{-1}^{\sqrt{2}} -x \, dx \right) \right]$

$= \left[2x - \frac{x^3}{3} \right]_{-1}^{\sqrt{2}} - \left[-\frac{x^2}{2} \right]_{-1}^{\sqrt{2}}$

$= \left(1.8856 - \frac{5}{3} \right) - \left(-1 + \frac{1}{2} \right)$

$= 1.661 \text{ units}^2.$

(d) Find $\int \left(\dfrac{e^{3x} - x^2}{e^{3x} - x^3} \right) dx.$

$= \int (e^{3x} - x^2)(e^{3x} - x^3)^{-1} \, dx$

$=$

(e)   If $\sec x \cdot \dfrac{dy}{dx} = e^{y+\sin x}$ and $y = -1$ when $x = 0$, find the value of y when $x = \dfrac{\pi}{2}$.

QUESTION
NUMBER

91579

Achieved Exemplar: **Total Score = 13**

Question 1 = This question provides evidence for M5 because the candidate has gained 1 r grade for their efforts in part d

    a)   They have integrated the equation correctly

    b)   They have correctly substituted the limits of 0 and 1.2 into the integrand to gain an answer of 4.687

Question 2= = This question provides evidence for M5 because the candidate has gained 1 r grade for their efforts in part c

    a)   They have correctly recognised that you need to integrate the acceleration function to get the velocity function. They have then used the given variable to correctly find c = 1.8

    b)   They have correctly recognised that you need to integrate the velocity function to get the distance function.

    c)   They have then recognised the limits are 0 and 9 and have substituted these in correctly to get an answer of 59.94

They did not get a grade of r for part c because they incorrectly calculated that 3 to the power of 3 = 9

They did not get a grade of t for part e because they did not calculate what the limits for the question were

Question 3 = = This question provides evidence for A3 because the candidate has gained 2 u grades for their efforts in parts a and c

In a

    a)   They have integrated correctly and then substituted the limits of 1 and 4 in to get an answer of -16

in c)

    a)   They have set up a correct integral involving the equation of the top curve minus the equation of the bottom curve

    b)   They have integrated this integral correctly

    c)   However, they have incorrectly found the x values of the points of intersection of the two curves meaning they cannot go on to get a grade of r