2

SUPERVISOR'S USE ONLY

91262

912620

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91262 Apply calculus methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Not Achieved

TOTAL 7

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

Find the gradient of the graph of the function at the point where $x = -1$.

$f'(x) = 4x^3 + 4x$ | $f'(-1) = 4(-1) + 4(-1)$

$= -4 - 4$

$f'(-1) = 4(-1)^3 + 4(-1)$ | $= -8$

1

(b) $f(x) = 8 + 3x + x^2 - \frac{x^3}{3}$

For what values of x is f a decreasing function?

Justify your answer.

You must show the use of calculus.

(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.

ASSESSOR'S USE ONLY

(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, a m s^{-2}, t seconds after it passes the signal, can be modelled by the function

$$a(t) = (16 - 2t)$$

(i) What is the greatest speed attained by the train after it passes the signal?

(ii) How far past the signal does the train travel before it stops?

N2

QUESTION TWO

ASSESSOR'S USE ONLY

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

$f(x) = \frac{4x^2}{2} - 3x + c$

$= 2x^2 - 3x + c$

$f(x=4) = 6$

$6 = 2(4)^2 - 3(4) + c$

$6 = 2(16) - 12 + c$

$6 = 32 - 12 + c$

$6 = 20 + c$

$c = 6 - 20$

$c = -14$

Therefore

$f(x) = 2x^2 - 3x - 14$

2

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

$g'(x) = 2x - 3$

$g'(x) = 0$

$2x - 3 = 0$

$2x = 3$

$x = 3/2$

$g(x) = \frac{2x^2}{2} - 3x + c$

$= x^2 - 3x + c$

$g(3/2) = (3/2)^2 - 3(3/2) + c$

$= 2.25 - 4.5 + c$

$c = 4.5 - 2.25$

$= 2.25$

$g(x) = x^2 - 3x + 2.25$

3

(ii) In relation to the graph, fully describe the point where this tangent meets the function.

The tangent is horizontal as its gradient is 0. It will meet with the highest point of the arch

4

ASSESSOR'S USE ONLY

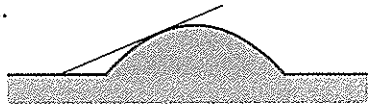
(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

The mound can be modelled by

$$h = -0.5x^2 + 3x - 1.5$$

(i) What is the maximum height of the mound?

(ii) A ramp up the side of the mound is a tangent to the mound.



The ramp can be modelled by the function

$$h = 0.5x - c$$

Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

(iii) The height h metres of a skateboard path at a horizontal distance r metres from another point Q, can be modelled by the function

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.

ASSESSOR USE ONLY

A3

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

$a(t) = 12t^2 - 2t + 2$ //

5

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

$f'(x) = 3x^2 - 4x + c$

$f'(2) = 2$

$f'(2) = 3(2)^2 - 4(2) + c$

$3(4) - 8 + c = 2$

$12 - 8 + c = 2$

$4 + c = 2$

$c = -2$

$f(x) = x^3 - 2x^2 - 2$

6

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

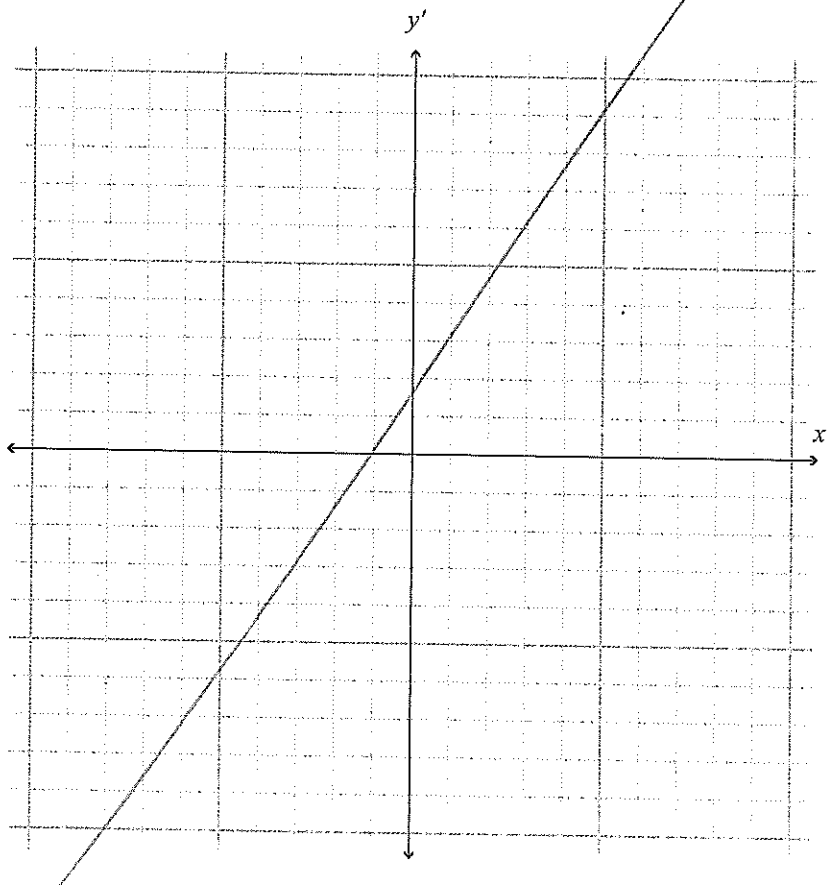
Use calculus methods to justify your answer.

(d) For a function $y = -ax^2 + bx + c$,

a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.



If you need to redraw this graph, use the grid on page 10.

(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

ASSESSOR'S USE ONLY

Find the maximum and minimum values of the product of x^2y.

Justify your answer.

N2

91262 2015

Apply Calculus methods in solving problems

Descriptions for the exemplar numbers: NOTE: Top down marking was used in this standard.

GRADE = NOT ACHIEVED

1. Correct derivative found and x=-1 correctly substituted into the derivative to calculate the gradient.
2. Function integrated correctly including the use of +C. The point (4,6) correctly substituted into the equation to find the constant of integration.
3. Correct derivative found, equated to zero and the x value of 3/2 found. Incorrect equation for the tangent.
4. To gain the "u" grade the coordinates of the turning point must be given.
5. Correct equation for the acceleration of the object given.
6. Incorrect use of the +C in a differentiation question that leads to an incorrect equation for the tangent.