91579M

915795

3

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tuanaki, Kaupae 3, 2019

91579M Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga

9.30 i te ata Rātū 26 Whiringa-ā-rangi 2019
Whiwhinga: Ono

Paetae	Kaiaka	Kairangi
Te whakahāngai i ngā tikanga pāwhaitua hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai i ngā tikanga pāwhaitua mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tuhia ō mahinga KATOA.

Tirohia mēnā kei a koe te Pukapuka Tikanga Tātai me ngā Tūtohi L3–CALCMF.

Mēnā ka hiahia whārangi atu anō koe mō ō tuhinga, whakamahia te (ngā) whārangi wātea kei muri o tēnei pukapuka, ka āta tohu ai i te tau tūmahi.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–24 kei roto i tēnei pukapuka, ā, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

© Mana Tohu Mātauranga o Aotearoa, 2019. Pūmau te mana.
Kia kaua rawa he wāhi o tēnei tuhinga e whakahuatia ki te kore te whakaaetanga tuatahi a te Mana Tohu Mātauranga o Aotearoa.

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int\left(2+\frac{2}{\sqrt{x}}\right)\mathrm{d}x$.

(b) Whakamahia ngā uara i raro ki te kimi i tētahi āwhiwhitanga ki $\int_2^5 f\left(x\right)\mathrm{d}x$, mā te whakamahi i te Ture Taparara.

x	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.6	1.1	1.7	2.6	3.2	3.4	2.6

(c) Whiriwhiria $\int_0^{\frac{\pi}{12}}\cos 4x.\cos 2x\,\mathrm{d}x$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Find $\int\left(2+\frac{2}{\sqrt{x}}\right)\mathrm{d}x$.

(b) Use the values given in the table below to find an approximation to $\int_2^5 f\left(x\right)\mathrm{d}x$, using the Trapezium Rule.

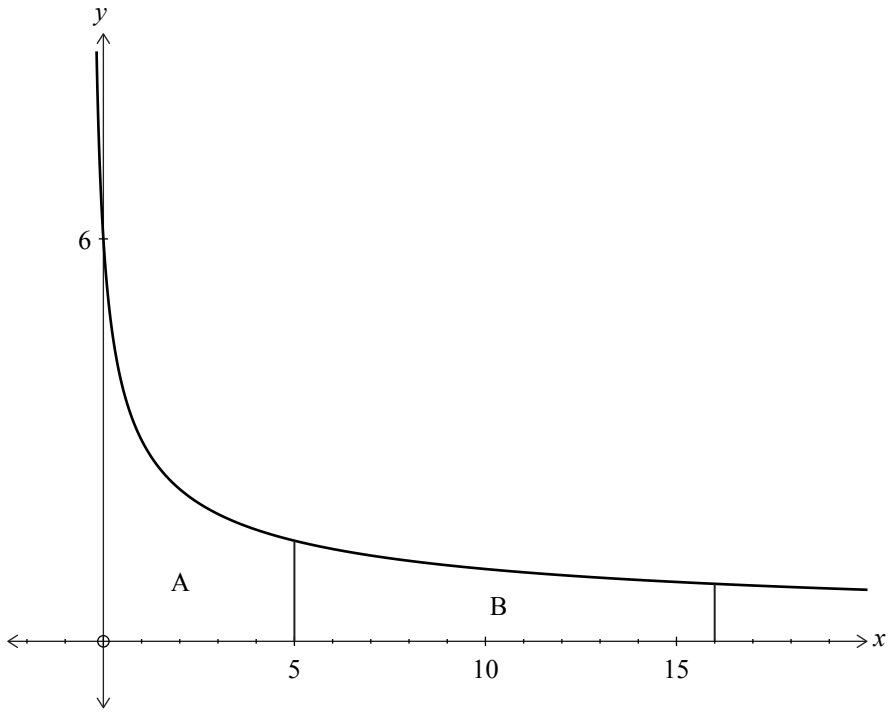
x	2	2.5	3	3.5	4	4.5	5
$f(x)$	0.6	1.1	1.7	2.6	3.2	3.4	2.6

(c) Find $\int_0^{\frac{\pi}{12}}\cos 4x.\cos 2x\,\mathrm{d}x$.

You must use calculus and show the results of any integration needed to solve the problem.

(d) E whakaaturia ana te kauwhata i raro i te ānau $y = \dfrac{6}{\sqrt{3x+1}}$.

MĀ TE KAIMĀKA ANAKE

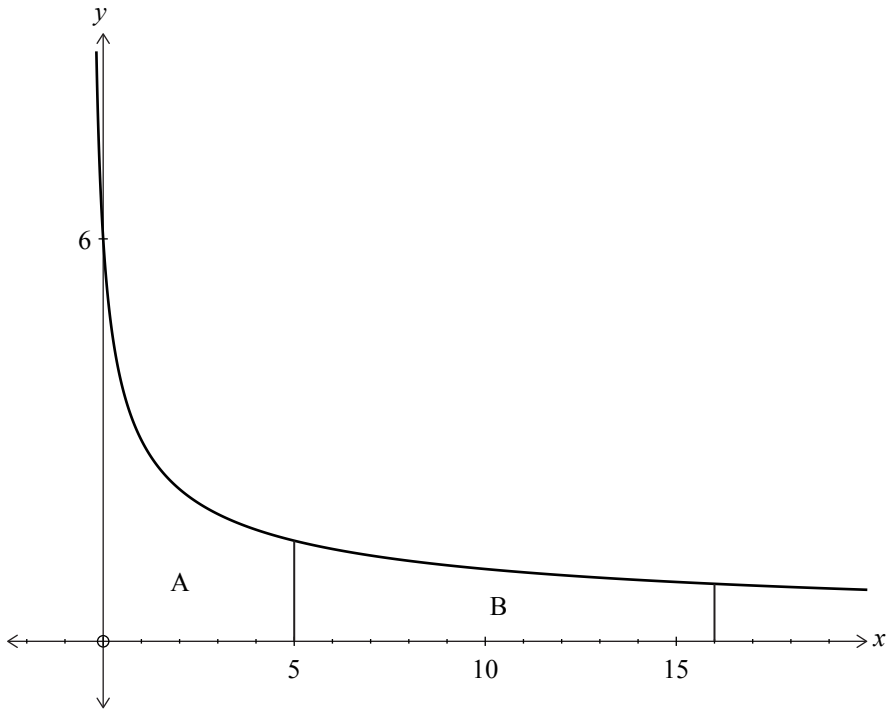


Me whakaatu he ōrite ngā horahanga o ngā rohe A me B.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(d) The graph below shows the curve $y = \dfrac{6}{\sqrt{3x+1}}$.



Show that the areas of regions A and B are equal.

You must use calculus and show the results of any integration needed to solve the problem.

(e) Ko te pāpātanga panoni o te rahinga N i tētahi wā tonu ka tukuna mā te whārite pārōnaki.

MĀ TE KAIMĀKA ANAKE

$$\frac{\mathrm{d}N}{\mathrm{d}t} = kN$$

Mēnā ko N, he whai uara tōrunga N_1 i te wā t_1 me N_2 i te wā $2t_1$, me hāpono ko

$$k = \frac{1}{t_1}\ln\left(\frac{N_2}{N_1}\right)$$

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(e) The rate of change of quantity N at any instant is given by the differential equation:

$$\frac{dN}{dt} = kN$$

If N has positive values N_1 and N_2 at times t_1 and $2t_1$ respectively, prove that

$$k = \frac{1}{t_1}\ln\left(\frac{N_2}{N_1}\right)$$

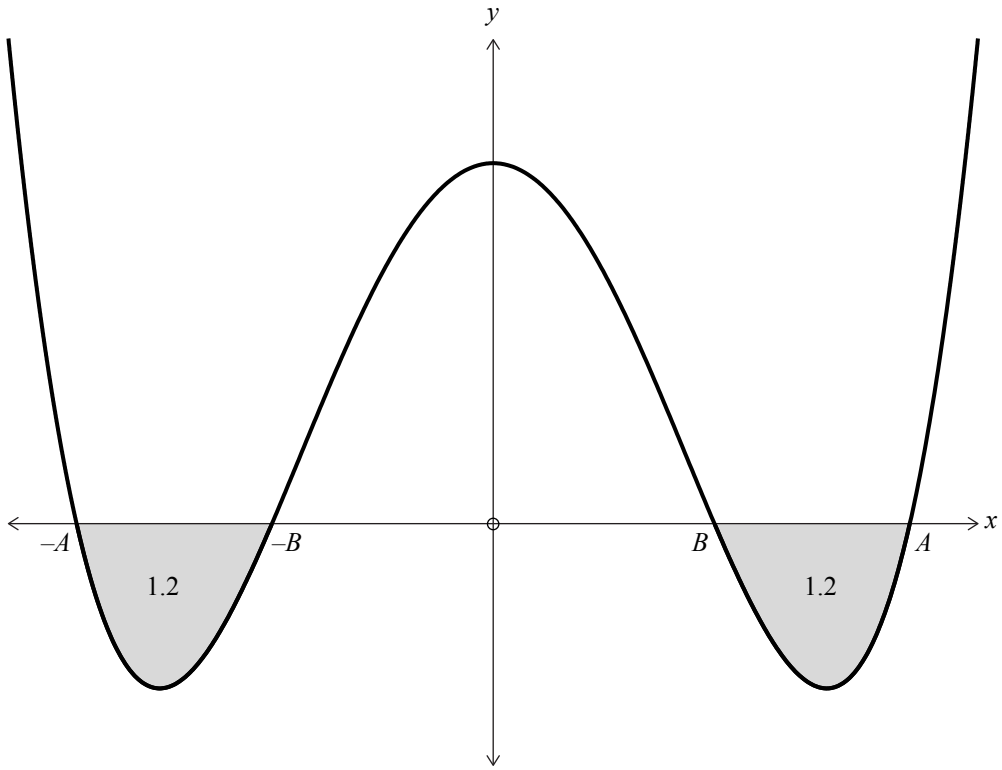
You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int\left(1+2\mathrm{e}^{4x}\right)\mathrm{d}x$.

(b) Kei te kauwhata o $y = f(x)$ e whakaaturia ana i raro nei, te tuaka-y hei rārangi hangarite. Kua tukuna ngā horahanga o ngā wāhi kauruku.



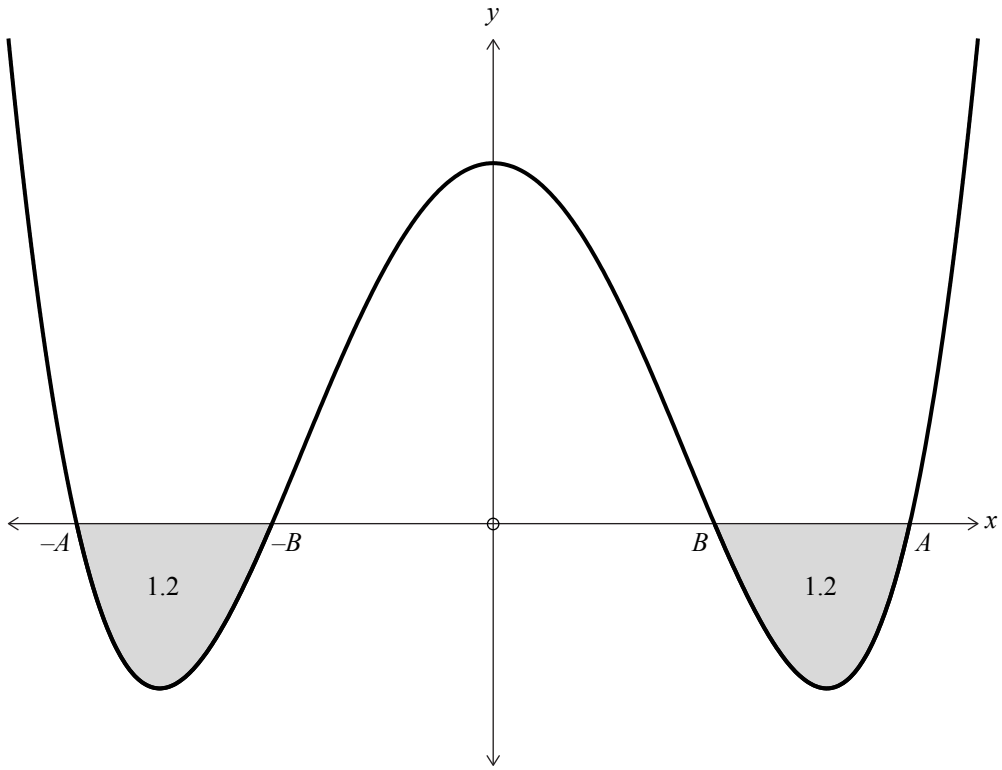
Mēnā $\int_{-A}^{A} f(x)\mathrm{d}x = 5.8$, he aha te uara o $\int_{-B}^{B} f(x)\mathrm{d}x$?

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Find $\int \left(1+2e^{4x}\right) dx$.

(b) The graph of $y = f(x)$ shown below has the y-axis as a line of symmetry.
The areas of the shaded regions are shown.



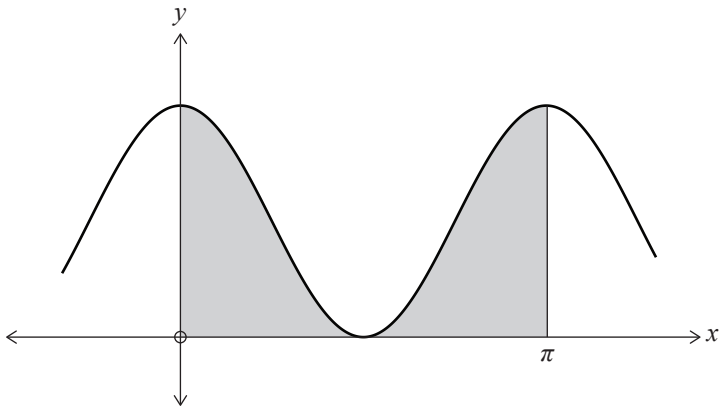
If $\int_{-A}^{A} f(x)\,dx = 5.8$, what is the value of $\int_{-B}^{B} f(x)\,dx$?

MĀ TE KAIMĀKA ANAKE

(c) Whiriwhiria te uara o k kia $\int_3^k \frac{8}{2x-5}\,\mathrm{d}x = 10$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

(d) E whakaatu ana te hoahoa o raro nei i te kauwhata o te pānga $y = \cos^2 x$.



Whiriwhiria te horahanga o te wāhi kauruku.

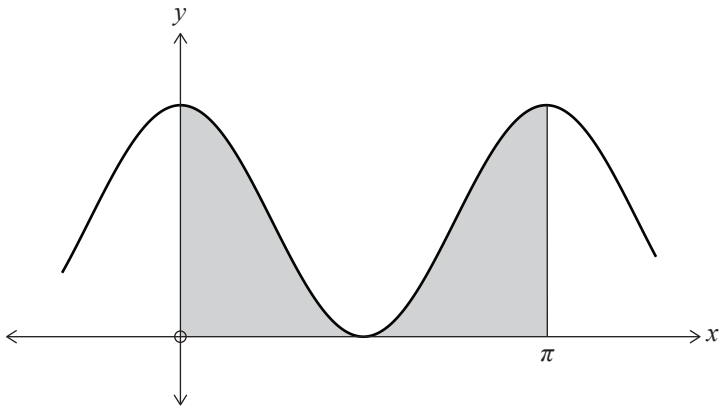
Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

ASSESSOR'S USE ONLY

(c) Find k such that $\int_3^k \frac{8}{2x-5}\,dx = 10$.

You must use calculus and show the results of any integration needed to solve the problem.

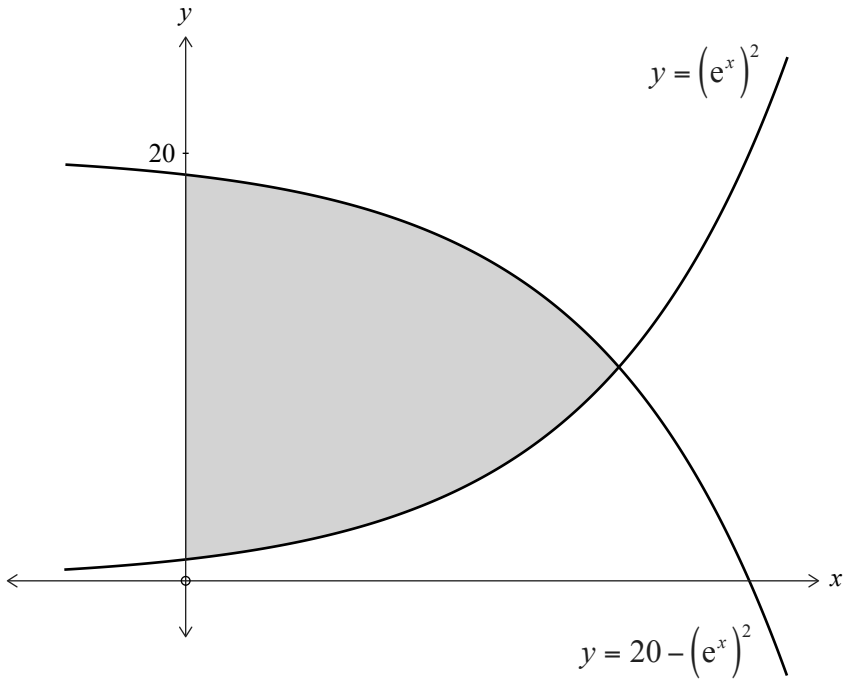
(d) The diagram below shows the graph of the function $y = \cos^2 x$.



Find the area of the shaded region.

You must use calculus and show the results of any integration needed to solve the problem.

(e) E whakaatu ana te hoahoa i raro i ngā kauwhata o ngā pānga $y = \left(e^x\right)^2$ me $y = 20 - \left(e^x\right)^2$.



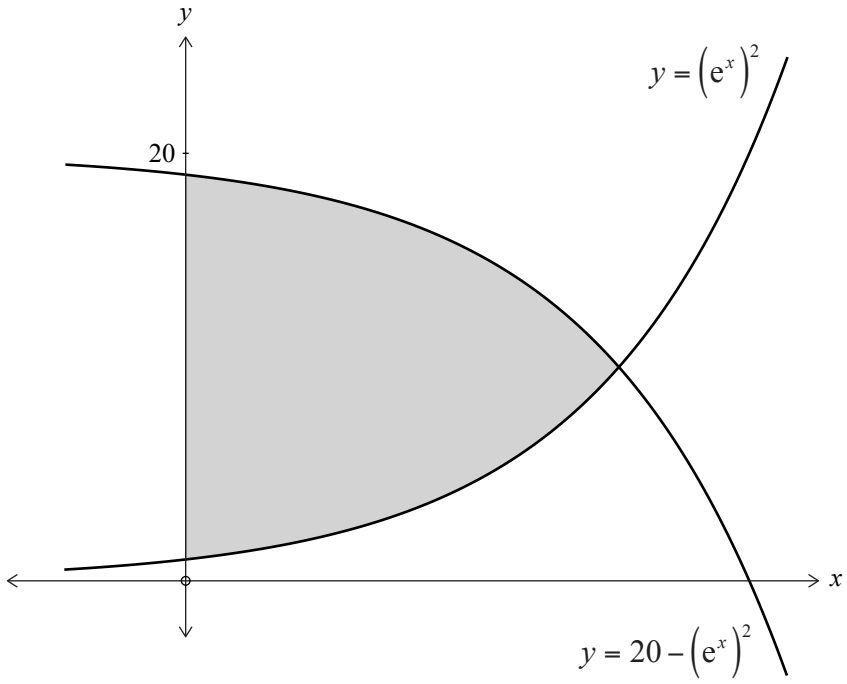
Whiriwhiria te horahanga o te wāhi kauruku i roto i te hoahoa.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) The diagram below shows the graphs of the functions $y = \left(e^x\right)^2$ and $y = 20 - \left(e^x\right)^2$.



Find the area of the region shaded in the diagram.

You must use calculus and show the results of any integration needed to solve the problem.

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Whiriwhiria $\int 24(2x-1)^3 \, \mathrm{d}x$.

(b) Whakaotihia te whārite pārōnaki $\frac{\mathrm{d}y}{\mathrm{d}x} = 4\sec^2 2x$, mēnā ko $y = 5$ ina $x = \frac{\pi}{8}$.

(c) Whiriwhiria $\int_1^4 x + 1 + \frac{x}{x+1} \, \mathrm{d}x$.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find $\int 24(2x-1)^3 \, dx$.

(b) Solve the differential equation $\frac{dy}{dx} = 4\sec^2 2x$, given that when $x = \frac{\pi}{8}$, $y = 5$.

(c) Find $\int_1^4 x + 1 + \frac{x}{x+1} \, dx$.

You must use calculus and show the results of any integration needed to solve the problem.

MĀ TE KAIMĀKA ANAKE

(d) Mēnā ko $\frac{dy}{dx} = \frac{4x}{4x^2 - 3} + \sqrt{x}$ ā, ko $y(1) = 2$, whiriwhiria $y(4)$.

Ka haere tonu te Tūmahi Tuatoru i te whārangi 18.

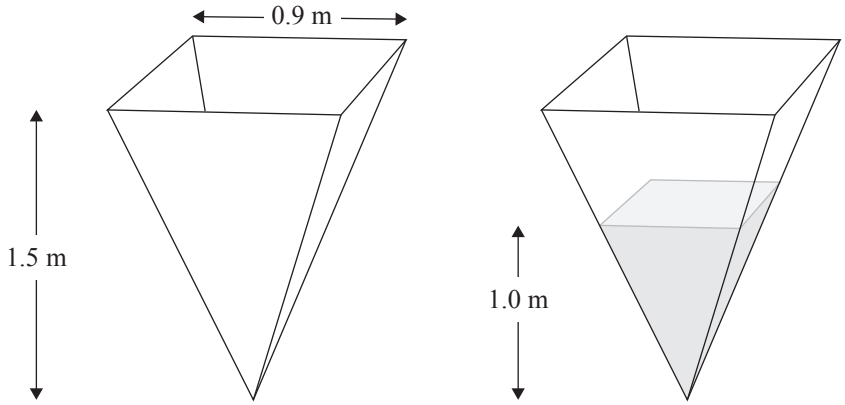
ASSESSOR'S USE ONLY

(d) If $\frac{dy}{dx} = \frac{4x}{4x^2 - 3} + \sqrt{x}$ and $y(1) = 2$, find $y(4)$.

Question Three continues on page 19.

(e) He pūtake tapawhā rite tō tētahi koeko hāngai kōaro me te teitei o te 1.5 m.

I whakakīa te koeko ki te wai i te tuatahi ki te 1 m te hōhonu, e ai ki raro.



Ko te pūngao e hiahiatia ana hei papu i te wai mai i tētahi taika he H te teitei, ka tukuna mā te:

$$E = 9800 \int_{H-d}^{H} (H-h)A(h)\,\mathrm{d}h$$

ina ko E te pūngao ā-wae pūngoi
d te hōhonu tīmata o te wai i rō taika
h te hōhonu o te wai i rō taika i tētahi wā tonu
$A(h)$ te horahanga o te mata o te wai i tēnei wā tonu.

Whiriwhiria te pūngao e hiahiatia ana hei papu i te wai mai i te taika, e ai ki te whakaaturanga.

Me whakamahi rawa i te tuanaki ka whakaatu i ngā otinga o te mahi pāwhaitua ka hiahiatia hei whakaoti i te rapanga.

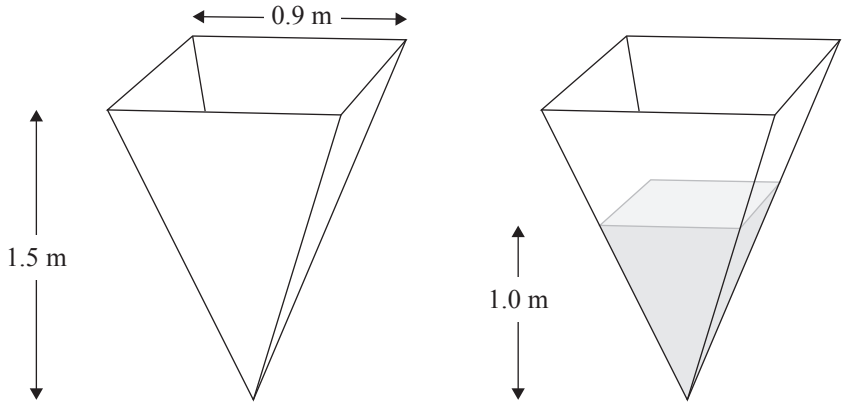
MĀ TE KAIMĀKA ANAKE

He wāhi anō mō tō tuhinga mō tēnei tūmahi kei te whārangi 20.

ASSESSOR'S USE ONLY

(e) An inverted right pyramid has a square base and a height of 1.5 m.

The pyramid is initially filled with water to a depth of 1 m, as shown.



The energy required to pump water out of a tank of height H is given by:

$$E = 9800 \int_{H-d}^{H} (H-h)A(h)\,\mathrm{d}h$$

where E is the energy in joules
d is the initial depth of the water in the tank
h is the depth of the water in the tank at any instant
$A(h)$ is the area of the surface of the water at this instant.

Find the energy required to pump the water out of the tank shown.

You must use calculus and show the results of any integration needed to solve the problem.

There is more space for your answer to this question on page 21.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 3 Calculus, 2019

91579 Apply integration methods in solving problems

9.30 a.m. Tuesday 26 November 2019
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

91579M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCMF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–23 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.