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91031



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SUPERVISOR'S USE ONLY

Level 1 Mathematics and Statistics, 2016

91031 Apply geometric reasoning in solving problems

9.30 a.m. Thursday 17 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL

11

ASSESSOR'S USE ONLY

THE SKY TOWER

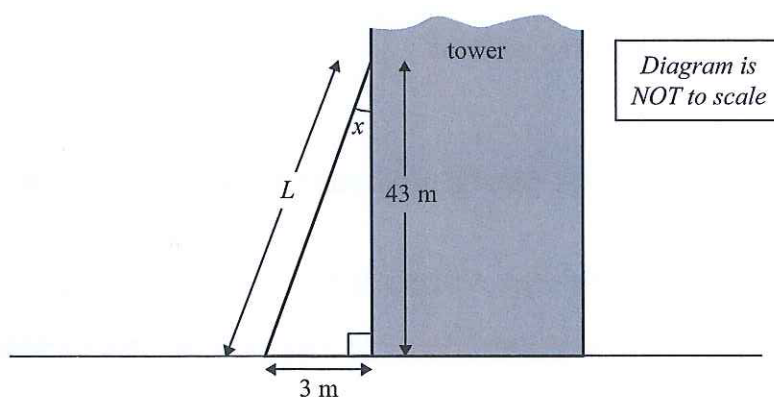
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Auckland's Sky Tower is the tallest man-made structure in the Southern Hemisphere.

QUESTION ONE

- (a) The base of the tower is supported by 8 legs.
These legs are L metres long and are 3 metres away from the tower at ground level.
The legs join the tower 43 m above ground level.



- (i) Calculate the length, L , of the leg from the ground to the tower.

~~$3^2 + 43^2 = L^2$~~

$$L = \sqrt{1858}$$

$$a^2 + b^2 = c^2$$

$$L = 43.10452412m$$

$$3^2 + 43^2 = L^2$$

$$L = 43.1m$$

$$L^2 = 1858$$

- (ii) Use trigonometry to calculate the size of angle x , where the leg joins the tower.

$$\tan(x) = \frac{3}{43}$$

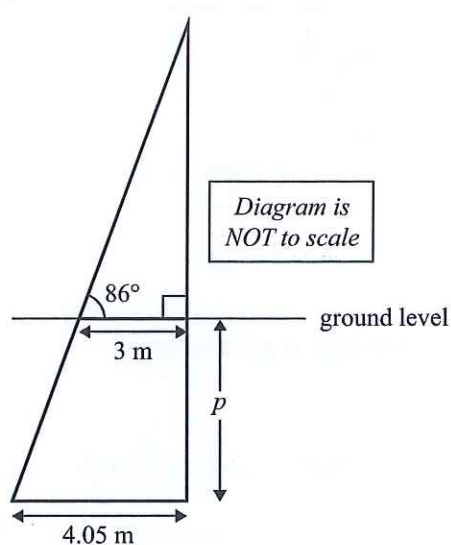
$$\therefore x = 3.99^\circ$$

$$x = \tan^{-1}\left(\frac{3}{43}\right)$$

$$x = 3.9909130198^\circ$$

- (iii) The legs of the tower go below ground level.

The horizontal distance from the tower to the bottom of the leg under the ground is 4.05 metres.



Calculate p , the vertical distance that the legs are built into the ground.

Show your working clearly.

$$\frac{p+43}{4.05} = \frac{43}{3}$$

$$p+43 = \frac{43 \times 4.05}{3}$$

$$p = 58.05 - 43$$

$$p = 15.05 \text{ m}$$

$$p+43 = \frac{4.05}{\tan(3.99^\circ)}$$

$$p = 43 = 58.05$$

$$p = 58.05 - 43$$

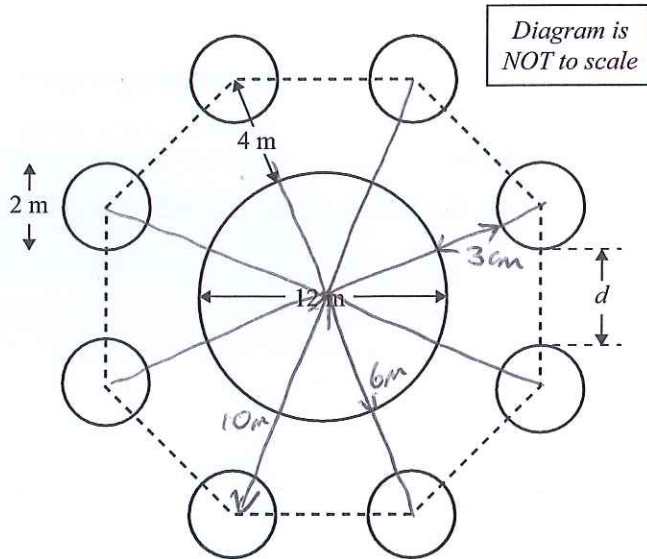
$$p = 15.05$$

$$\therefore p = 15.05 \text{ m}$$

- (b) The centres of the 8 circular legs form a regular octagonal shape.

The tower has a diameter of 12 metres and each leg has a diameter of 2 metres.

The distance from the outside edge of the tower to the centre of the legs at the ground is 4 metres.



Calculate the shortest distance, d , between adjacent legs at ground level.

Show your working clearly.

The radius of the tower is 6m

$$a = \sqrt{c^2 - b^2 + 2bc}$$

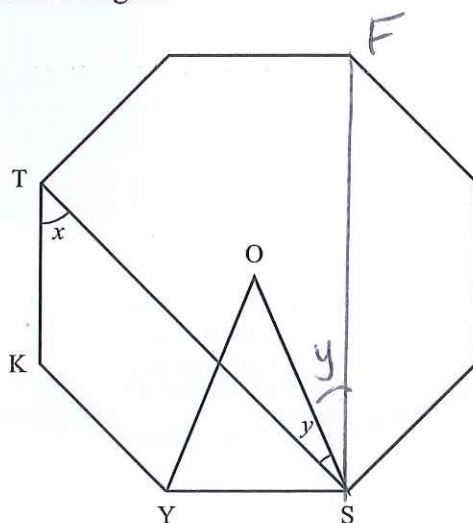
$$a = \sqrt{10^2 - 6^2 + (2 \times 10 \times 6)}$$

$$a = 14.14m$$

$$d = 14.14 - 1 - 1$$

$$d = 12.14m$$

- (c) A simplified diagram of the position of the legs is shown below as a regular octagon. Point O is at the centre of the octagon.



Show that angle y is half the size of angle x .

Justify your answer with clear geometric reasoning.

The lines TK and FS are parallel.
 This means that angle KTS and FST
 are equal. So $x = 2y$. Because adjacent
 angles on parallel lines are equal.

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M5

QUESTION TWO

Below the Sky Tower is a car park made of ramps.

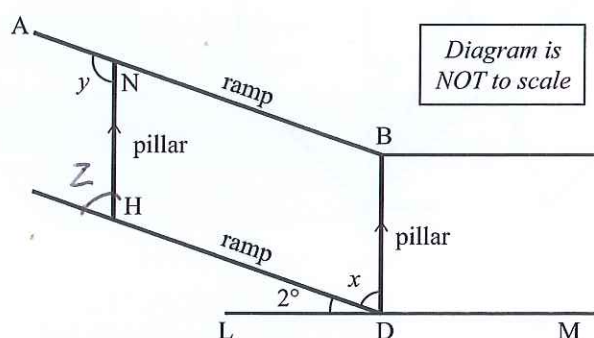
The ramps are at a 2° angle.

There are vertical pillars regularly placed along the ramps for strength.



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- (a) All pillars are parallel to each other. LM is horizontal.



- (i) Calculate the size of angle x in the diagram above.

Justify your answer with clear geometric reasoning.

$$x = 90 - z$$

$$\text{L's on a line} = 180^\circ$$

$$x = 88$$

- (ii) Calculate the size of angle y in the diagram above.

Justify your answer with clear geometric reasoning.

$$x = z$$

$$y + z = 180$$

$$y + 88 = 180$$

$$y = 92^\circ$$

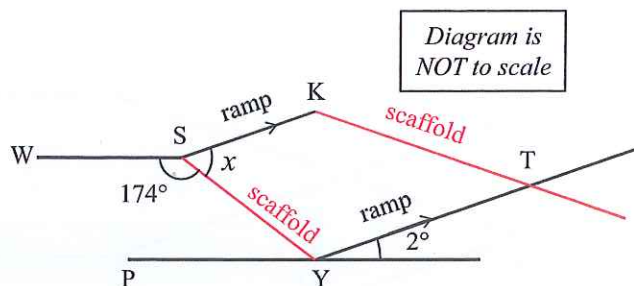
$$\text{adj L's of } \parallel \text{ lines} =$$

$$\text{int L's of } \parallel \text{ lines} = 180^\circ$$

- (iii) Part of the ramp had extra scaffolding added for support, as shown in the diagram below. The lines SK and YT are parallel.

Angle WSY is 174° .

The lines WS and PY are both horizontal.



Calculate the size of angle x in the diagram above.

Justify your answer with clear geometric reasoning.

$$174 + x = 180 + 2$$

$$x = 182 - 174$$

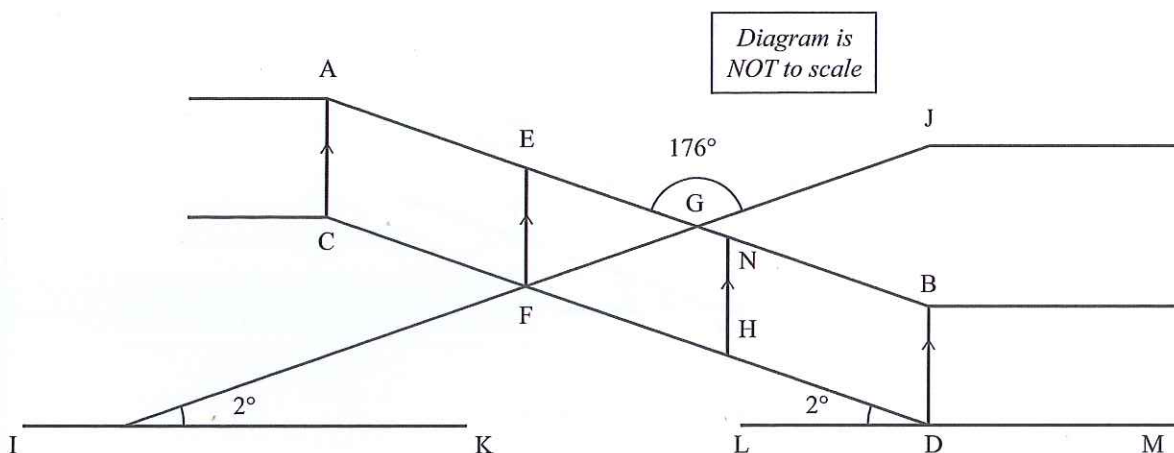
$$x = 8^\circ$$

This is because the lines SK and YT have the same elevation.

- (iv) From the side, the carpark looks like the diagram below.

Angle EGJ is 176° .

IK and LM are horizontal.



Prove that the lines AB and CD are parallel.

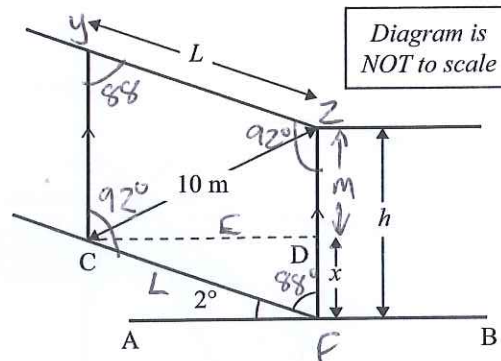
Justify your answer with clear geometric reasoning.

All the supports AC , EF , NH , BD are all the same length and they are parallel. This means that the ramp and the roof are parallel because all the supports are the same length // \wedge

- (b) The length along the slope between two pillars is L metres.

The diagonal distance between the top of one pillar and the base of the next higher pillar is 10 m.

AB and CD are horizontal.



- (i) Find the height, x , in terms of the length L .

Show your working clearly.

$$x = \sqrt{L^2 - E^2}$$

CF and YZ are equal
so that means that

$$E = CD$$

$$x = \sqrt{L^2 + E^2}$$

- (ii) Calculate h , the height in metres of a pillar, in terms of L .

Show your working clearly.

$$E = \sin 46 \times 10$$

$$E = 7.19 \text{ m}$$

$$M + x = h$$

$$h = 6.95 + 0.25$$

$$h = 7.2 \text{ m}$$

$$M = \cos 46 \times 10$$

$$M = 6.95 \text{ m}$$

$$x = \frac{7.19}{\tan 88}$$

$$x = 0.25 \text{ m}$$

7

7

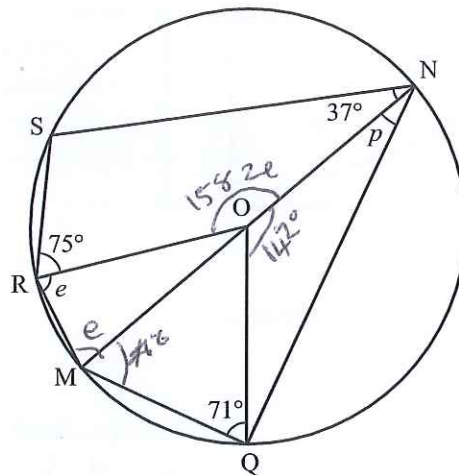
A3

QUESTION THREE

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- (a) In the diagram below, the line MN passes through the centre of the circle, O.
Angle MQO is 71° , angle SNO is 37° and angle SRO is 75° .

Diagram is
NOT to scale



- (i) Find the size of angle p .

Justify your answer with clear geometric reasoning.

$$90 - 71 = 19 \quad \text{base } \angle\text{'s of isos } \Delta =$$

$$p = 19^\circ$$

- (ii) Find the size of angle e .

Justify your answer with clear geometric reasoning.

$$180 - 142 = 38$$

$$180 - 38 - 71 = 71^\circ$$

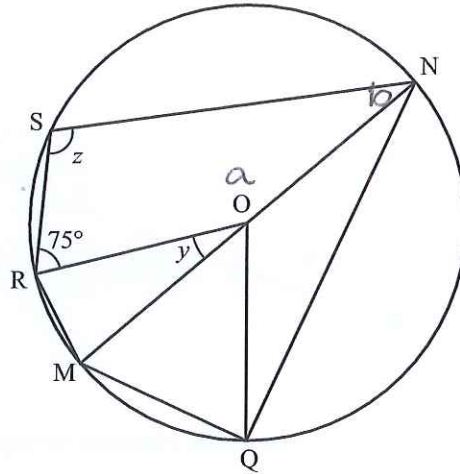
$$360 - 75 - 90 - 37 = 158$$

$$\frac{158}{2} = 79^\circ$$

$$e = 79^\circ$$

- (iii) In the diagram below, angle SRO is 75° .

Diagram is
NOT to scale



Find an expression for z in terms of y .

Justify your answer with clear geometric reasoning.

$$y = 180 - a$$

$$z = 360 - 75 - b - a$$

$$z = 360 - 75 - b - 180 - y$$

$$z = 105 - b - y$$

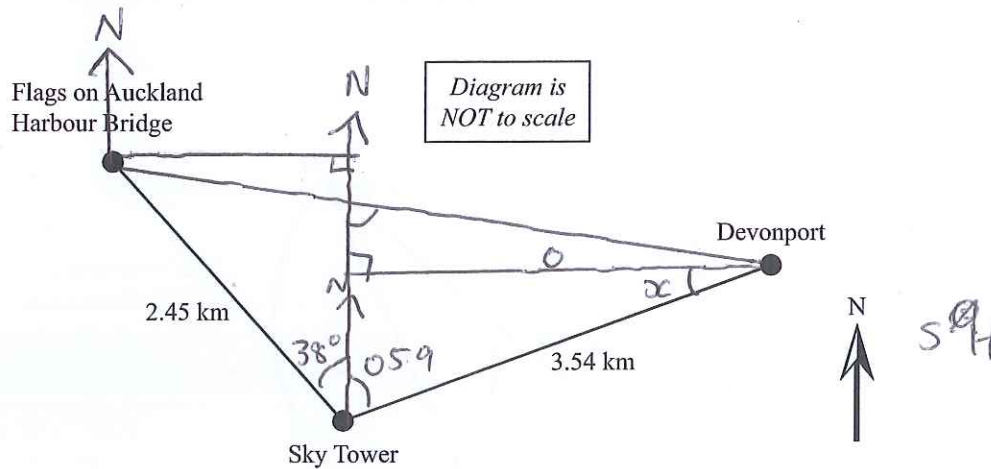
$$z = 105 - b - y$$

$$\angle \text{sum of } \Delta = 180^\circ$$

$$\angle \text{'s on a line} = 180^\circ //$$

- (b) Devonport is at a bearing of 059° and 3.54 km from the Sky Tower.

The flags on the Auckland Harbour Bridge are at a bearing of 322° and 2.45 km from the Sky Tower.



Calculate the bearing from the flags on the Auckland Harbour Bridge to Devonport.

Show your working clearly.

$$0 = \sin 59 \times 3.54$$

$$0 = 3.03 \text{ km}$$

$$x = \cos^{-1} \left(\frac{3.03}{3.54} \right)$$

$$x = 31^\circ$$

The bearing from Auckland Harbour Bridge to Devonport is $90^\circ + 31^\circ$
the bearing is 121

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A3

Annotated Exemplar Achievement

Achievement exemplar 2016

Subject:	Mathematics	Standard:	91031	Total score:	11
Q	Grade score	Annotation			
1	M5	<p>Candidate has used a range of methods to solve problems. Pythagoras and Trigonometry have been correctly used to find a side and an angle. Similar triangles has been used as a strategy to solve (a) (iii) correctly.</p> <p>Candidate has been able to relate and communicate the solutions in context.</p> <p>To gain M6, the candidate would have needed to recognise the correct right angle triangle to use in 1(b) or make progress in a chain of reasoning in 1(c).</p>			
2	A3	<p>The candidate has used parallel line geometry in 2(a) to find angles. The candidate has not completely reasoned their answers and has not connected the different concepts adequately. Interior angles is not a sufficient reason.</p> <p>To gain a M5 candidate would have needed to fully reason each angle found in 2a) i and ii with correct mathematical statements.</p>			
3	A3	<p>The candidate has used circle geometry to find angle p in 3(a) (i). The candidate has not completely reasoned their answers and has not connected the different concepts adequately. Evidence for Achievement is also found in the diagram with the correct angles found.</p> <p>The candidate has also started 3) (b) with a correct step towards solution</p> <p>To gain an M5, the candidate would have needed to fully reason each angle found in 3(a) (i) with correct mathematical statements or make further progress towards solution in 3(b).</p>			