See back cover for an English translation of this cover

**2**

91262M

912625

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

SUPERVISOR'S USE ONLY

# Te Pāngarau me te Tauanga, Kaupae 2, 2013

## 91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

2.00 i te ahiahi Rāhina 18 Whiringa-ā-rangi 2013
Whiwhinga: Rima

| Paetae | Paetae Kaiaka | Paetae Kairangi |
|---|---|---|
| Te whakahāngai tikanga tuanaki hei whakaoti rapanga. | Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga. | Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga. |

Tirohia mehemea e ōrite ana te Tau Ākonga ā-Motu (NSN) kei tō pepa whakauru ki te tau kei runga ake nei.

**Me whakautu e koe ngā pātai KATOA kei roto i te pukapuka nei.**

Whakaaturia ngā mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te (ngā) whārangi kei muri i te pukapuka nei, ka āta tohu ai i ngā tau pātai.

Tirohia mehemea kei roto nei ngā whārangi 2–29 e raupapa tika ana, ā, kāore hoki he whārangi wātea.

**HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.**

**TAPEKE**

MĀ TE KAIMĀKA ANAKE

Kia 60 meneti hei whakautu i ngā pātai o tēnei pukapuka.

MĀ TE KAIMĀKA ANAKE

## PĀTAI TUATAHI

(a) Ka tohua he pānga $f$ mā te $f(x) = 4x^2 - 5x + 2$.

Tātaihia te rōnaki o te kauwhata o $f$ i te pūwāhi $x = 3$.

(b) Mō tētahi pānga $g$,

$$g'(x) = 6x^2 - 5.$$

Ka whakawhiti te kauwhata o $g$ mā te pūwāhi (1,4).

Kimihia te pānga $g(x)$.

You are advised to spend 60 minutes answering the questions in this booklet.

ASSESSOR'S USE ONLY

## QUESTION ONE

(a) A function $f$ is given by $f(x) = 4x^2 - 5x + 2$.

Find the gradient of the graph of $f$ at the point where $x = 3$.

(b) For a function $g$,

$$g'(x) = 6x^2 - 5.$$

The graph of $g$ passes through the point $(1,4)$.

Find the function $g(x)$.

MĀ TE KAIMĀKA ANAKE

(c) Ka tukuna tētahi mura ahotea[1] mai i tētahi poti.

Ko tōna teitei, he $h$ mita i runga ake i te mata o te wai, ka tohua mā te

$$h = 90t - 5t^2 + 2$$

ko $t$ te wā ā-hēkona mai i te tukutanga o te mura.

He aha te teitei mōrahi rawa i taea e te mura?

(d) Ko te tawhiti āwhio (te paenga porowhita) o tētahi rākau he $g$ mita, he $t$ tau te roa o te wā mai i tōna whakatōnga, ka whakatauiratia mā te pānga

$$g = -0.005t^2 + 0.15t + 0.3 \qquad 0 \leq t \leq 15.$$

Āhea ka eke te pāpātanga o te tipu o te paenga porowhita o te rākau ki te 0.04 mita i te tau?

[1] ohotata

ASSESSOR'S USE ONLY

(c) An emergency flare is fired from a boat.

Its height, $h$ metres above the surface of the water, is given by

$$h = 90t - 5t^2 + 2$$

where $t$ is the time in seconds since the flare was fired.

What is the maximum height reached by the flare?

(d) The distance around a tree (its girth) $g$ metres, at a time $t$ years after it is planted, is modelled by the function

$$g = -0.005t^2 + 0.15t + 0.3 \qquad 0 \leq t \leq 15.$$

When will the rate of growth of the tree's girth be 0.04 metres per year?

MĀ TE KAIMĀKA ANAKE

(e) $g(x) = -x^3 + 3x + 2$

Mō ēhea uara o $x$ ko $g$ te pānga heke haere?

Me mātua **whakaatu e koe ngā whakamahinga tuanaki** i roto i ō mahinga.

(f) Ko te pānga rōnaki o tētahi kōpiko ko te $f'(x) = mx + 2$. Ka pā te kōpiko ki ngā pūwāhi (2,10) and (–1,–8).

Kimihia a $f(x)$, te whārite o te kōpiko.

ASSESSOR'S USE ONLY

(e) $g(x) = -x^3 + 3x + 2$

For what values of $x$ is $g$ a decreasing function?

You must **show the use of calculus** in your working.

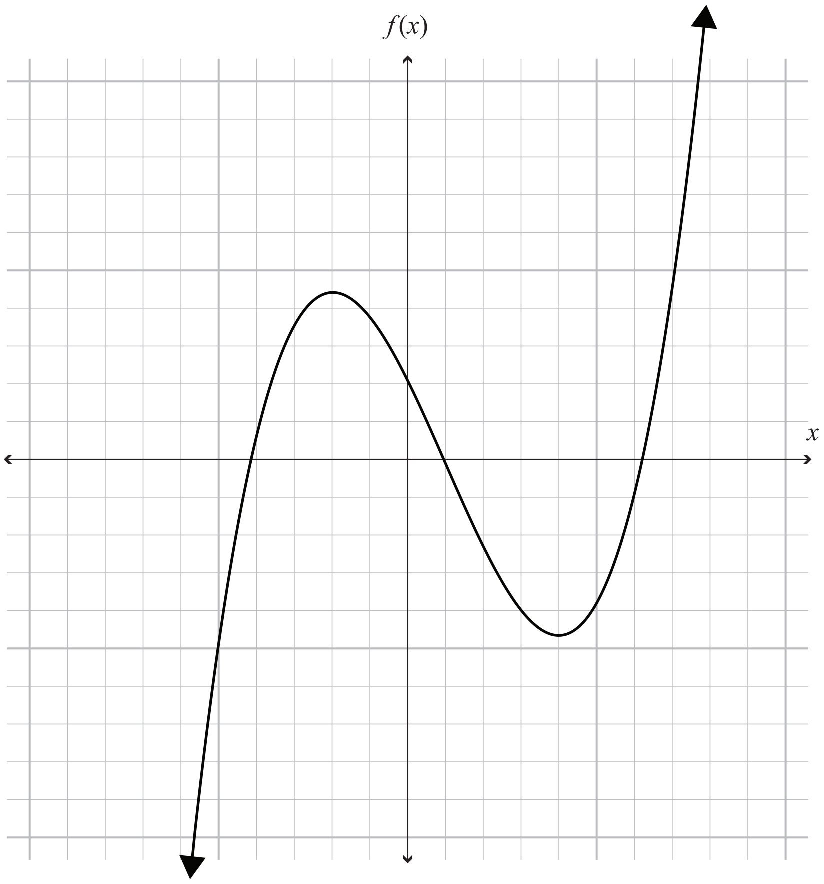
(f) A curve has gradient function $f'(x) = mx + 2$. The curve passes through the points (2,10) and (–1,–8).

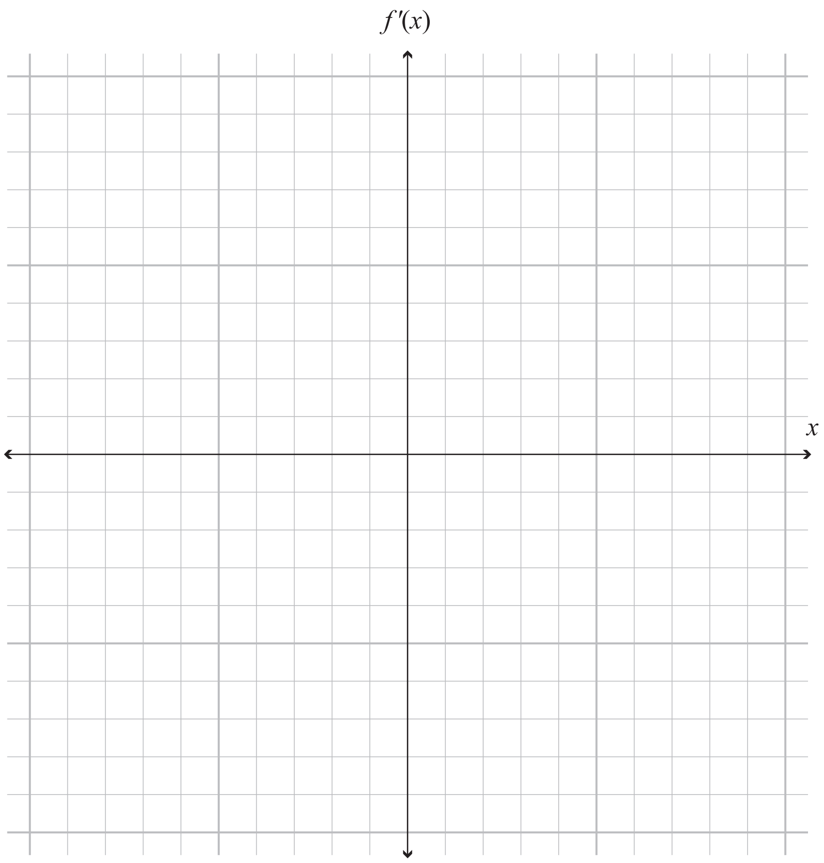
Find $f(x)$, the equation of the curve.

MĀ TE KAIMĀKA ANAKE

## PĀTAI TUARUA

(a) Tuhia te pānga rōnaki $f'(x)$ mō te pānga $f(x)$ i raro:



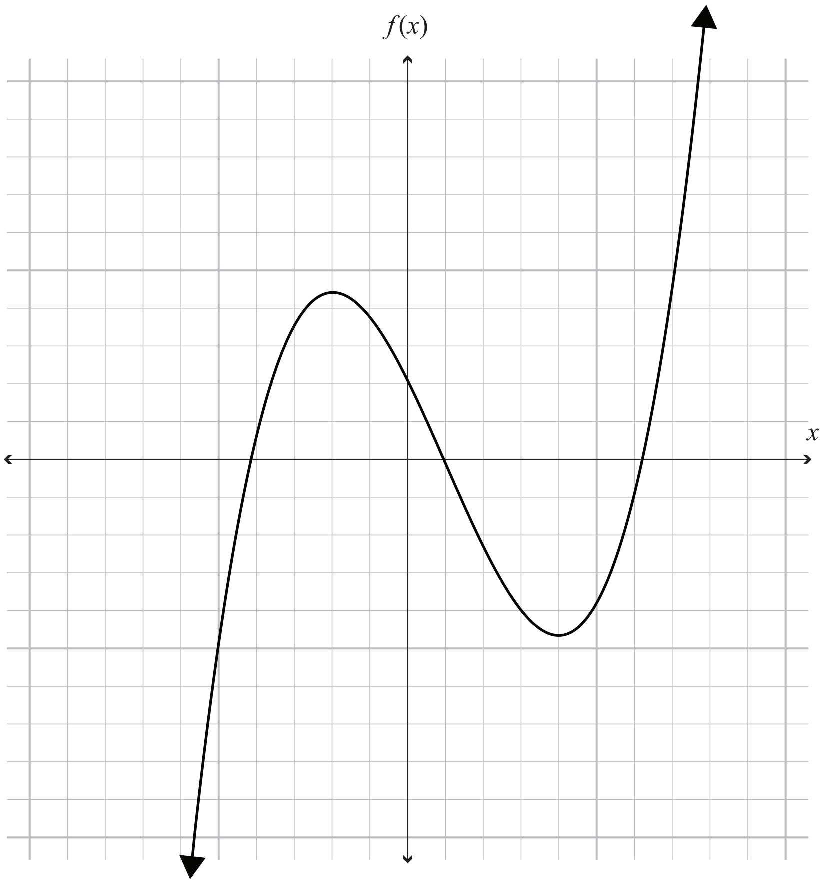



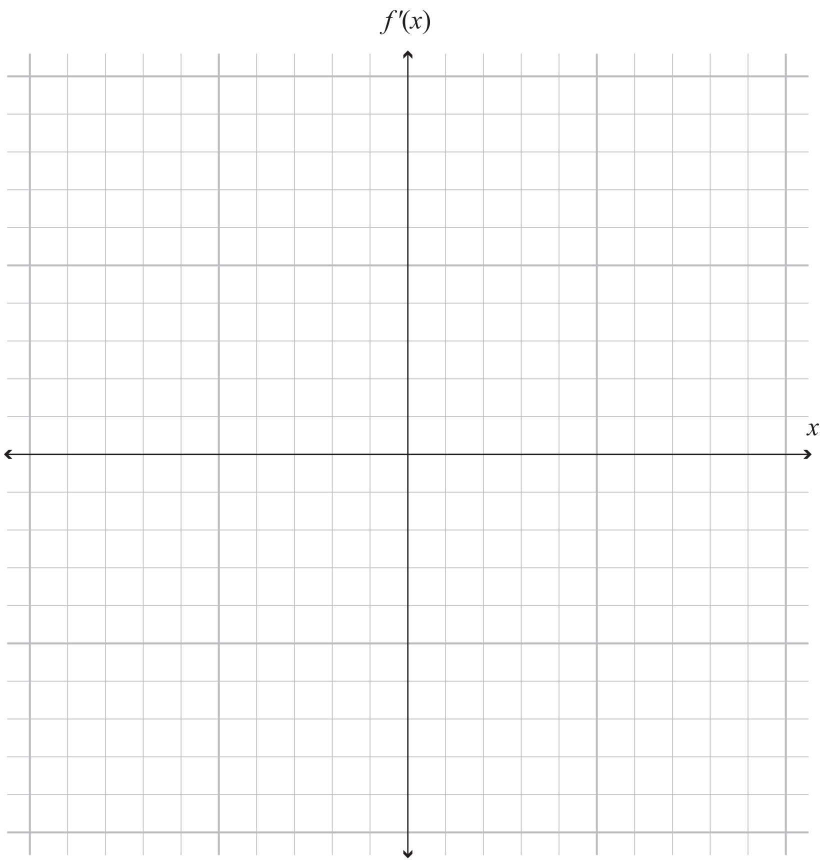

Ki te hiahia koe ki te tā anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 26.

## QUESTION TWO

ASSESSOR'S USE ONLY

(a) Sketch the gradient function $f'(x)$ for the function $f(x)$ below:




If you need to redraw this graph, use the grid on page 27

(b) Kei te whakakīa he taika[2] ki te miraka. Ko te hōhonu o te miraka he $d$ cm, i te $t$ meneti i muri i te rututanga ka tohua mā te

$$\mathrm{d}(t) = \frac{t^2}{4} + t$$

Kimihia te pāpātanga e rerekē ai te hōhonu o te miraka i te 5 meneti i muri i te tīmatanga o te rututanga.

(c) Ka whakatakahia he kōhatu ki roto i tētahi hōpua wai.

Ka whakaputa i ngā pōkare porohitahita i te mata o te wai.

Ko te horahanga $A$ o tētahi pōkare porohitahita, ā-mita pūrua, ka tohua mā te

$$A = \pi r^2$$

ina ko te pūtoro he $r$ mita.

Kimihia te pāpātanga o te whiti o te horahanga o te pōkare, e pā ana ki te pūtoro, ina ko te horahanga he $49\pi$ m$^2$.

[2] kura

ASSESSOR'S USE ONLY

(b) A tank is being filled with milk. The depth of the milk *d* cm, at a time *t* minutes after pouring started is given by

$$d(t) = \frac{t^2}{4} + t$$

Find the rate at which the depth of the milk is changing 5 minutes after the pouring started.

(c) A stone is dropped into a pool.

This makes circular ripples on the surface of the water.

The area $A$ of a circular ripple, in square metres, is given by

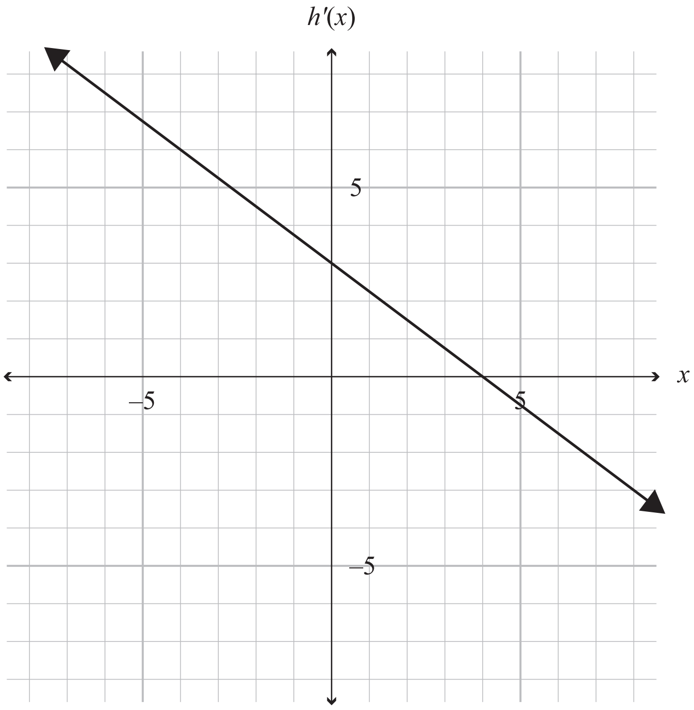
$$A = \pi r^2$$

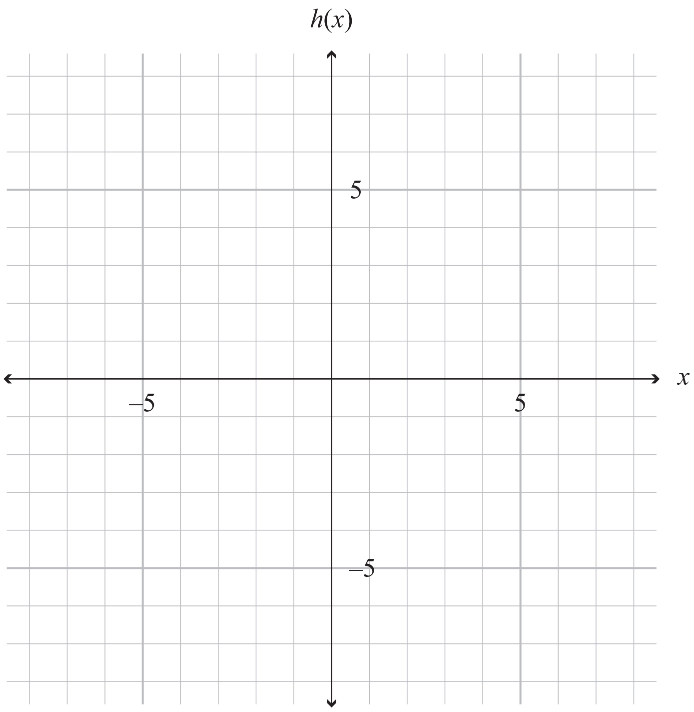
where the radius is $r$ metres.

Find the rate of change of the area of the ripple, with respect to the radius, when the area is $49\pi$ m$^2$.

(d) Tuhia te pānga $h(x)$ mō te pānga rōnaki $h'(x)$ i raro, ina ko te uara mōrahi o $h$ he 5.

Āta whakaaturia te akitu.



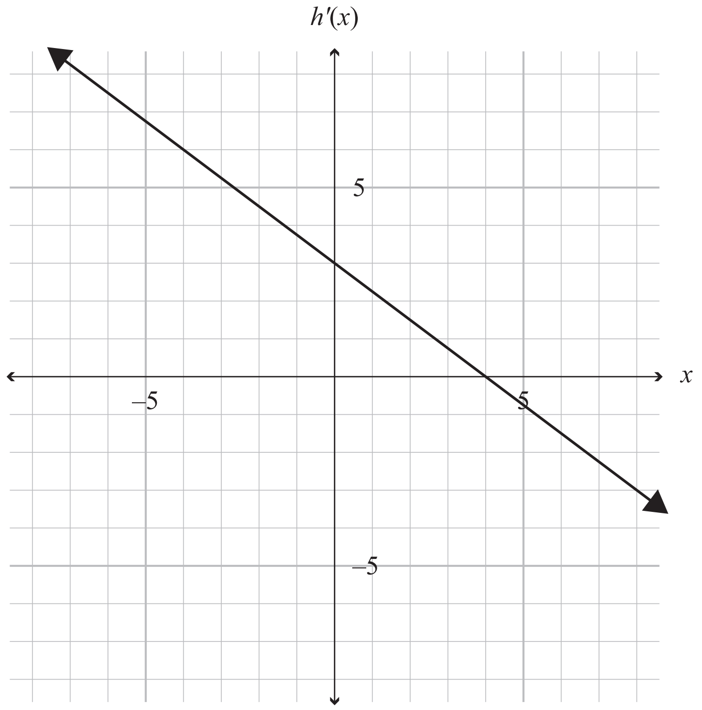



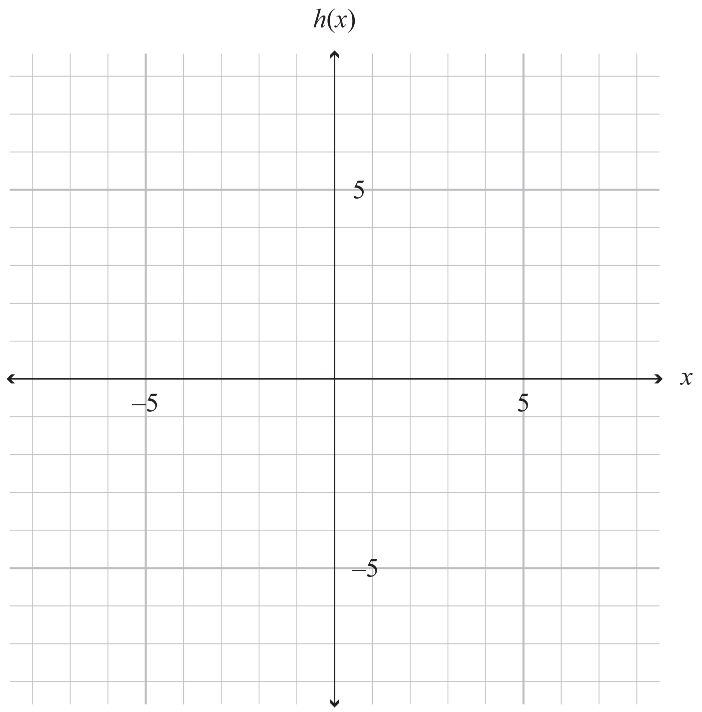

*Ki te hiahia koe ki te tā anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 26.*

MĀ TE KAIMĀKA ANAKE

(d) Sketch the function $h(x)$ for the gradient function $h'(x)$ below, given that the maximum value of $h$ is 5.

Show the vertex clearly.




If you need to redraw this graph, use the grid on page 27

MĀ TE KAIMĀKA ANAKE

(e) Ka tohua te rōnaki o tētahi kōpiko ko $\frac{dy}{dx} = 6x^2 - 12x$.

Ko te taunga-$y$ o te pūwāhi huringa mōkito o te kōpiko he 10.

Kimihia te whārite o te kōpiko.

ASSESSOR'S USE ONLY

(e) The gradient of a curve is given by $\frac{dy}{dx} = 6x^2 - 12x$.

The $y$-coordinate of the minimum turning point of the curve is 10.

Find the equation of the curve.

(f) E haere ana tētahi motokā ki tētahi tere aumou kia whakamahia rā anō ngā pereki o te motokā.

Ka huri te tere o te motokā ki te pāpātanga o te $-0.08t$ mita hēkona$^{-2}$ i muri i te whakamahinga o ngā pereki, ina ko $t$ hēkona te wā mai i te whakamahinga o ngā pereki.

E 3 hēkona i muri i te whakamahinga o ngā pereki, ko te tere o te motokā he 5 mita hēkona$^{-1}$.

E hia te tawhiti o te haere o te motokā i mua i te tūnga ina whakamahia ngā pereki?

MĀ TE KAIMĀKA ANAKE

(f) A car is travelling at a constant speed until the car's brakes are applied.

The car's speed changes at a rate given by $-0.08t$ metres $sec^{-2}$ after the brakes are applied, where $t$ sec is the time since the brakes were applied.

3 seconds after the brakes are applied, the speed of the car is 5 metres $sec^{-1}$.

How far will the car travel with the brakes applied before it stops?

ASSESSOR'S USE ONLY

MĀ TE KAIMĀKA ANAKE

**PĀTAI TUATORU**

(a) Ko tētahi kōpiko $y = f(x)$ ka whiti mā te (0,0), ā, ko tana pānga rōnaki he

$f'(x) = 4x + 3$.

Kimihia ngā taunga o te pūwāhi o te kōpiko ko $x = -3$.

(b) (i) Kimihia te taunga-$x$ o te pūwāhi o te kauwhata o $g(x) = 0.5x^2 - 5x$ ko te rōnaki e rite ana ki te 2.

(ii) Kimihia te whārite o te pātapa ki te kōpiko $g(x) = 0.5x^2 - 5x$ i te pūwāhi (8,–8).

ASSESSOR'S USE ONLY

## QUESTION THREE

(a) A curve $y = f(x)$ passes through (0,0) and has gradient function

$$f'(x) = 4x + 3.$$

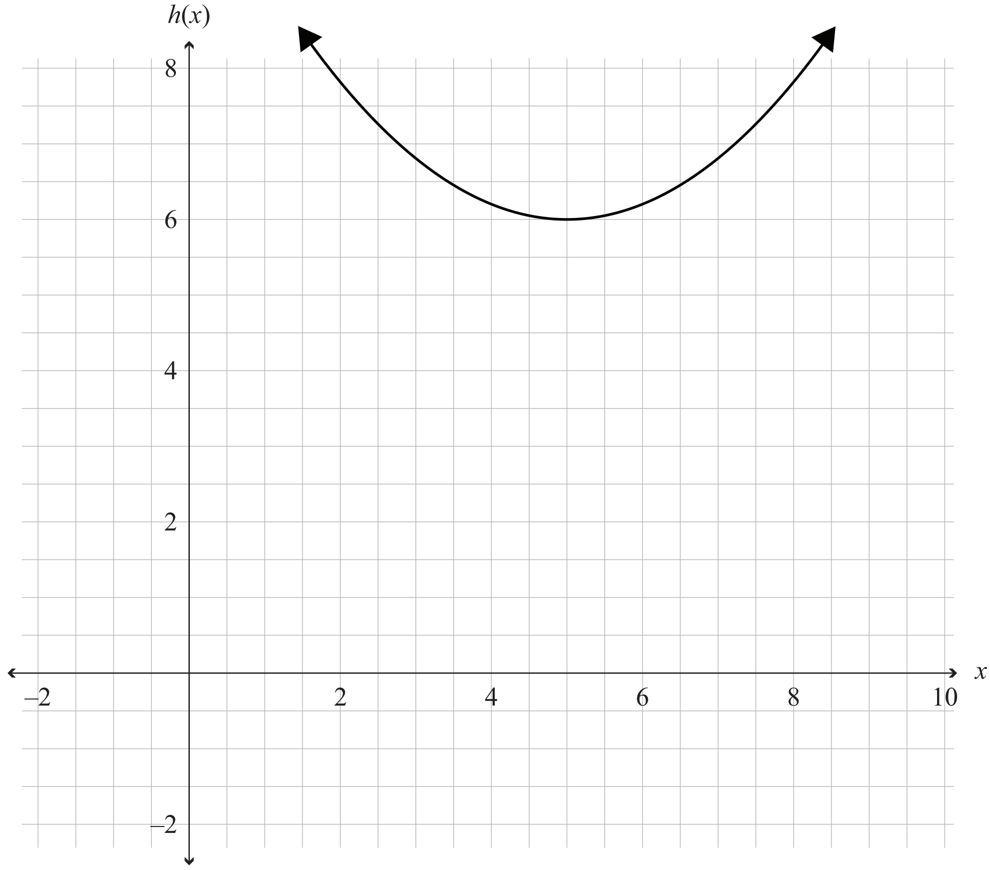
Find the coordinates of the point on the curve where $x = -3$.

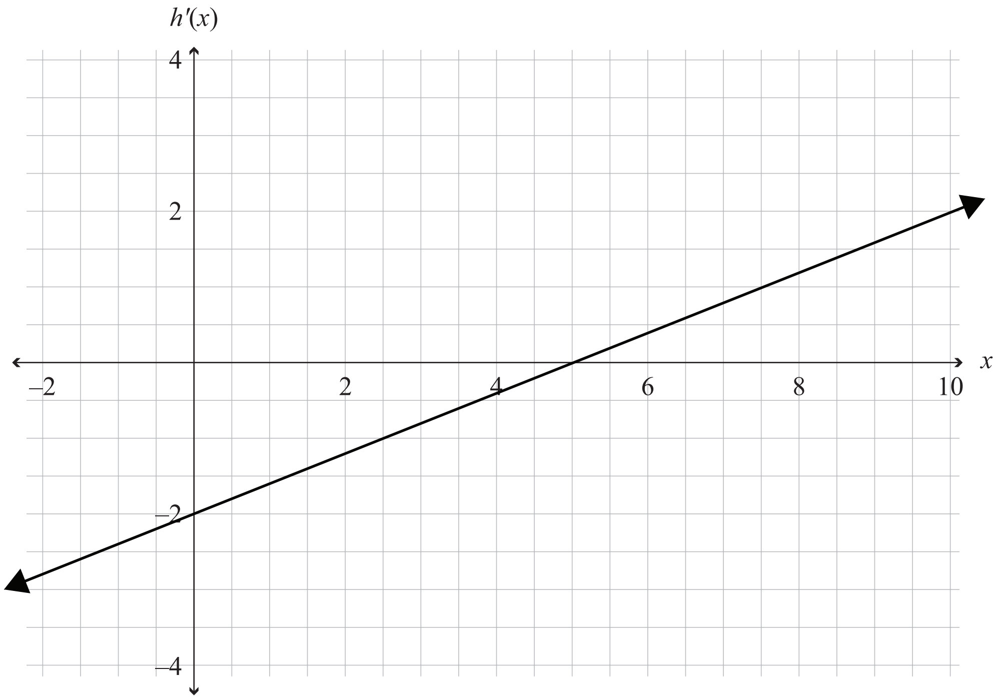
(b) (i) Find the $x$-coordinate of the point on the graph of $g(x) = 0.5x^2 - 5x$ where the gradient is equal to 2.

(ii) Find the equation of the tangent to the curve $g(x) = 0.5x^2 - 5x$ at the point (8,–8).

(c) E tohua tahitia ana te kauwhata o te pānga $h(x)$ me tana pānga rōnaki i raro.

MĀ TE KAIMĀKA ANAKE



MĀ TE KAIMĀKA ANAKE

Kimihia te whārite o $h(x)$.

Me mātua **whakamahi koe i ngā tikanga tuanaki** kia whiwhi ai i tō whakautu.

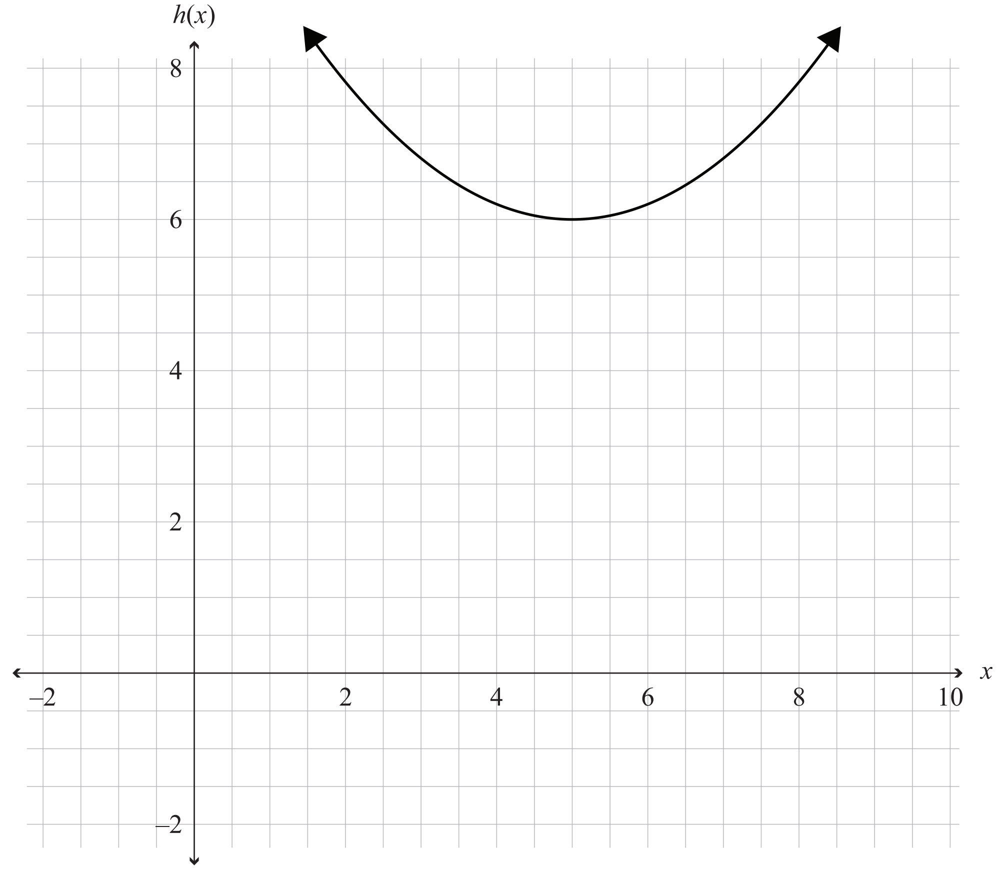
(d) He pūwāhi huringa tō te kōpiko o $f(x) = \mathrm{P}x^2 + \mathrm{Q}x + 2$ ina ko $x = \frac{2}{3}$.

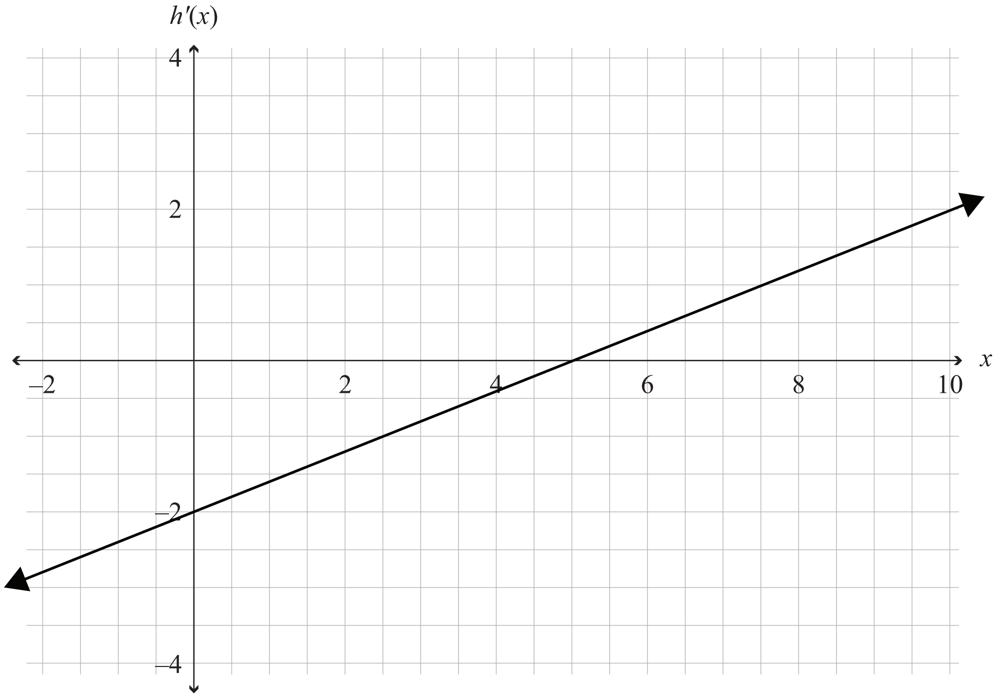
Ka whiti te kōpiko mā te pūwāhi (1,9).

Kimihia ngā taunga o te pūwāhi o te kōpiko ina ko $x = 3$.

(c) The graph of the function $h(x)$ together with that of its gradient function are given below.

ASSESSOR'S USE ONLY



Find the equation of $h(x)$.

You must **use calculus methods** to obtain your answer.

ASSESSOR'S USE ONLY

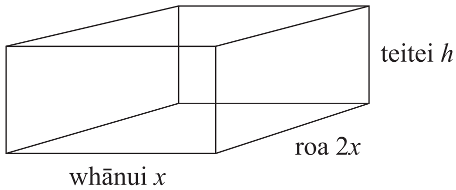
(d) The curve of $f(x) = \text{P}x^2 + \text{Q}x + 2$ has a turning point when $x = \frac{2}{3}$.

The curve passes through the point (1,9).

Find the coordinates of the point on the curve where $x = 3$.

(e) E 12 ngā toko maitai o te anga o tētahi kereti, ā, ko te roa tapeke o ngā toko he $L$ cm.

Ko te roa o te kereti e rua whakareanga ake i te whānui.

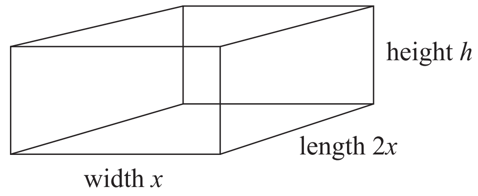



Whakaaturia ko te roa o te kereti he $\frac{L}{9}$ cm ina ko te rōrahi he mōrahi.

MĀ TE KAIMĀKA ANAKE

(e) The frame of a crate is made up of 12 steel rods that have a total length of $L$ cm.

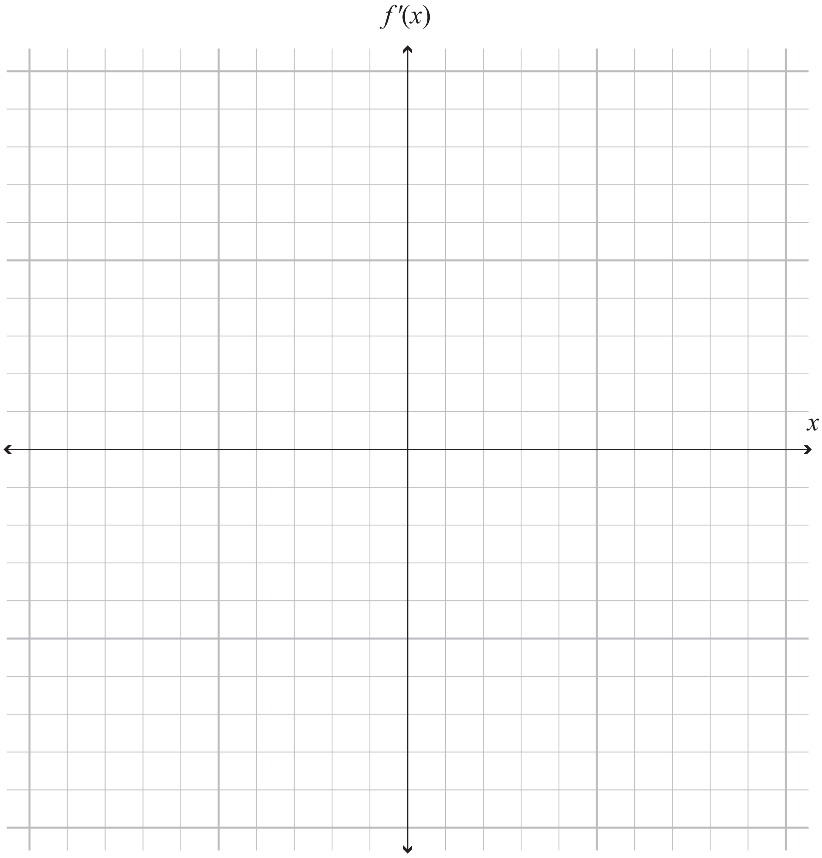
The length of the crate is twice the width.



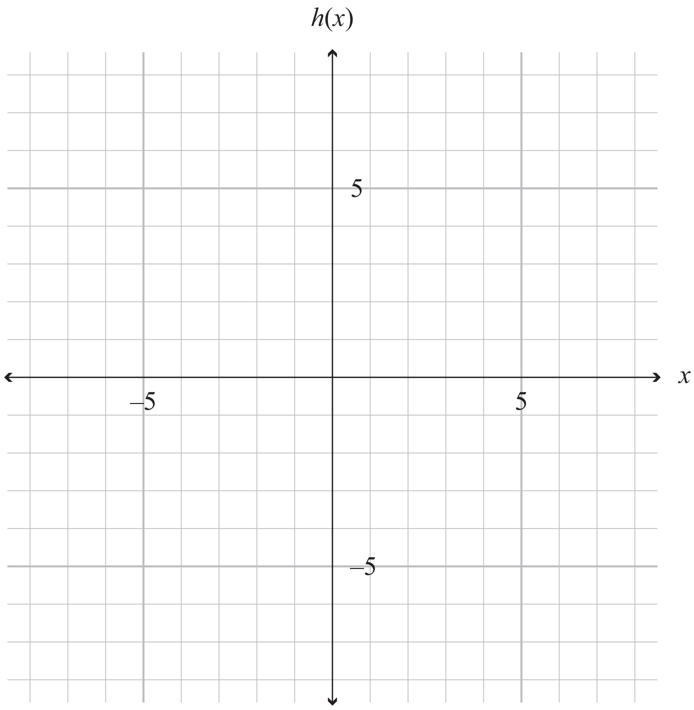

Show that the length of the crate will be $\dfrac{L}{9}$ cm when the volume is a maximum.

ASSESSOR'S USE ONLY

Ki te hiahia koe ki te tuhi anō i te kauwhata mō te Pātai Tuarua (a), tuhia ki te tukutuku i raro. Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

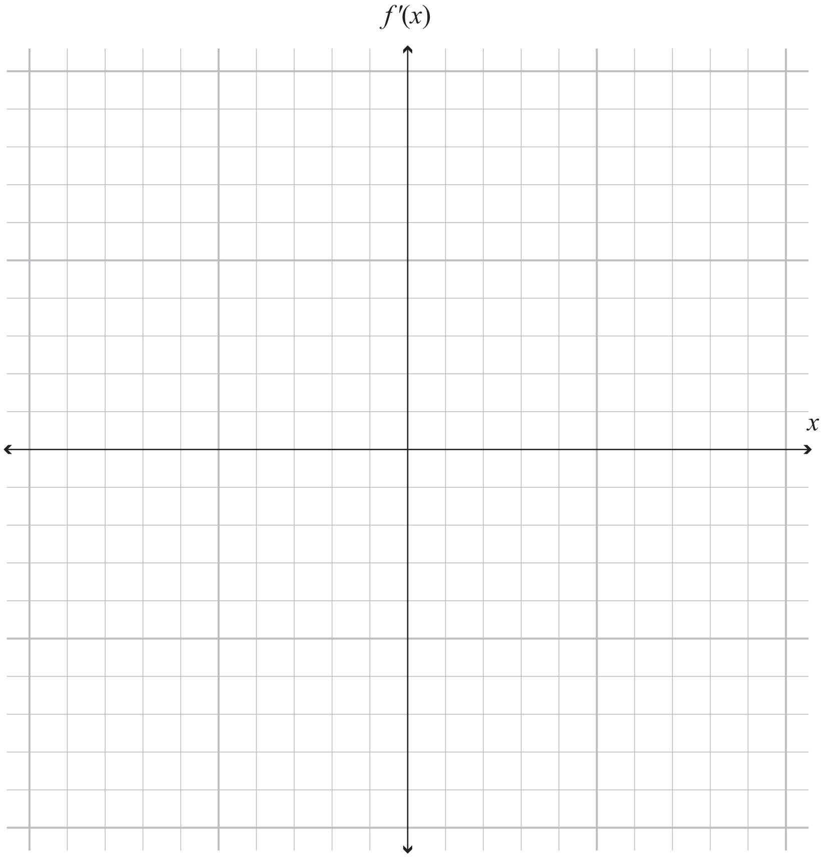



Ki te hiahia koe ki te tuhi anō i te kauwhata mō te Pātai Tuarua (d), tuhia ki te tukutuku i raro. Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

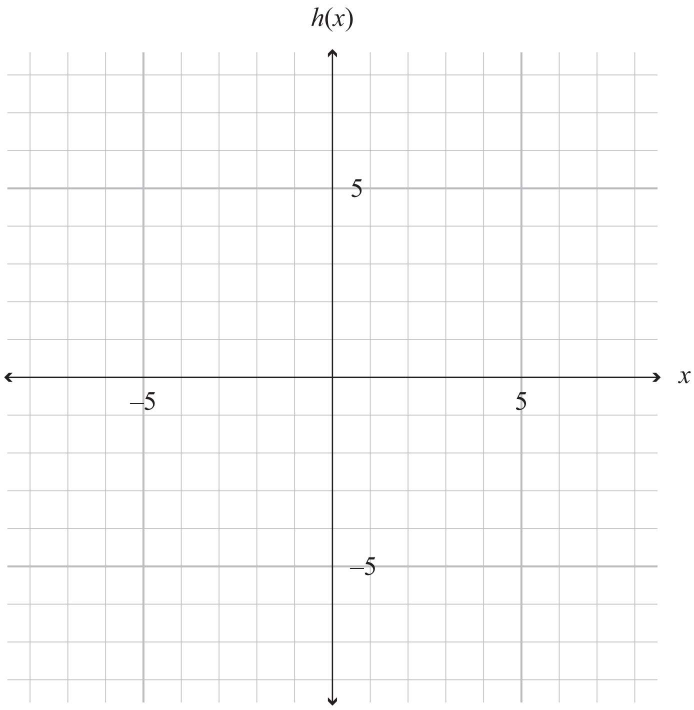


ASSESSOR'S USE ONLY

If you need to redraw your graph from Question Two (a), draw it on the grid below. Make sure it is clear which graph you want marked.




If you need to redraw your graph from Question Two (d), draw it on the grid below. Make sure it is clear which graph you want marked.



**He puka anō mēnā ka hiahiatia.**
**Tuhia te (ngā) tau pātai mēnā e hāngai ana.**

TAU PĀTAI

MĀ TE KAIMĀKA ANAKE

**Extra paper if required.**
**Write the question number(s) if applicable.**

QUESTION NUMBER

ASSESSOR'S USE ONLY

*English translation of the wording on the front cover*

# Level 2 Mathematics and Statistics, 2013

## 91262 Apply calculus methods in solving problems

2.00 pm Monday 18 November 2013
Credits: Five

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| Apply calculus methods in solving problems. | Apply calculus methods, using relational thinking, in solving problems. | Apply calculus methods, using extended abstract thinking, in solving problems. |

91262M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–29 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**