91261

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912610

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91261 Apply algebraic methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 23

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Annotated Exemplar Template

Excellence exemplar for 91261 2015		Total score	23
Q	**Grade score**	**Annotation**	
1	E7	This is E7 because candidate successfully made x the subject in 1b, they failed to gain E8 because 1aiii justification was insufficient and answer to 1cii was not in the context of the question.	
2	E8	Candidate gained E8 because all three answers which contributed t evidence were fully correct.	
3	E8	Solution to 3d omitted the constraint 0 < m < 2 for positive real roots but identification of constraint m>2 and both roots determined was sufficient abstract thinking for t to be awarded.	

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QUESTION ONE

(a) (i) Find the value of $\log_2 1024$.

$2^x = 1024$

$x = 10$ //

u

(ii) Solve the equation $\log_4(3w+1) = 2$.

$4^2 = 3w+1$

$16 = 3w+1$

$w = 5$ //

u

(iii) Luka says that the equation $\log_x(4x+12) = 2$ has only one solution.

Is he correct?

Find the solution(s), justifying your answer.

$x^2 = 4x+12$

$x^2 - 4x - 12 = 0$

$x = -2$ or $x = 6$

x cannot be -2 because it makes the equation untrue.

A

r

(b) Make x the subject of the equation $a^{2x} = b^{x+1}$.

$2x \log a \quad (x+1)\log b$

$2x\log a = x\log b + \log b$

$2x\log a - x\log b = \log b$

$x(2\log a - \log b) = \log b$

$x = \frac{\log b}{(2\log a - \log b)}$ //

t

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(c) The market value of Sue's house has been increasing at a constant exponential rate of 3% per annum since she bought it sixteen years ago at the start of 1999. At the start of 2015 it was worth \$350 000.

(i) Assuming the exponential growth is of the form $y = A\,r^t$, what was the value of the house at the start of 1999 when she bought it?

$350000 = A \times (1.03)^{16}$

$350000 = A \times 1.03^{16}$

$A = \$218108.43$

(ii) A friend also bought a house at the start of 1999 that cost \$200 000.

Its market value also has been steadily increasing, but at a slightly higher exponential rate of 3.5%.

Its value, \$$y$, t years after the start of 1999, is given by the function

$y = 200\,000 \times (1.035)^t$

If the houses continue to keep increasing in value at the original rates, in which year will the two houses be worth the same amount?

$200000 \times (1.035)^t = 218108.43 \times (1.03)^t$

$\frac{(1.035)^t}{(1.03)^t} = \frac{218108.43}{200000}$

$\left(\frac{1.035}{1.03}\right)^t = 1.09 \quad (2dp)$

$t \log\left(\frac{1.035}{1.03}\right) = \log 1.09$

$t = \frac{\log 1.09}{\log\left(\frac{1.035}{1.03}\right)}$

$t = 17.8$

in 17.8 years.

r

r

E7

QUESTION TWO

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(a) Simplify $\frac{2x^2+7x-4}{2x^2-32}$

$= \frac{(2x-1)(\cancel{x+4})}{2(x-4)(\cancel{x+4})}$

$= \frac{2x-1}{2x-8}$ //

r

(b) If $a = y^{\frac{3}{4}}$, find an expression for a^7 in terms of y.

$a^7 = (y^{\frac{3}{4}})^7$

$a^7 = y^{\frac{21}{4}}$

$a^7 = y^{5\frac{1}{4}}$ //

u

(c) Solve the equation $2u^{\frac{2}{3}} + 7u^{\frac{1}{3}} = 4$

let x be $u^{\frac{1}{3}}$

$2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$

$x = 0.5$ or $x = -4$

$u^{\frac{1}{3}} = 0.5$ or $u^{\frac{1}{3}} = -4$

$u = 0.125$ or $u = -64$ //

t

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(d) Talia used timber to form the exterior sides of her rectangular garden. The length of the garden is x metres, and its area is 50 m^2.

(i) Show that the perimeter of the garden is given by $2x+\frac{100}{x}$

The width is $\frac{50}{x}$

$P = 2(x + \frac{50}{x})$

$P = 2x + \frac{100}{x}$ //

u.

(ii) If she uses 33 m of timber to build the sides, find the dimensions of the garden.

$2x + \frac{100}{x} = 33$

$2x^2 + 100 = 33x$

$2x^2 - 33x + 100 = 0$

$x = 12.5$ or $x = 4$

~~$\frac{33}{2} - 12.5 = 4$ (ignore)~~

the garden is 4m × 12.5m

t

(e) David and Sione are competing in a cycle race of 150 km.

Sione cycles on average 4 km per hour faster than David, and finishes half an hour earlier than David.

Find David's average speed.

You MUST use algebra to solve this problem. (Hint: average speed = $\frac{distance}{time}$*)*

~~Sione's time: $4 = \frac{150}{t}$~~

~~$t = 37.5$ hours.~~

~~Sione took 37.5 hours.~~

~~This means~~

~~David took 38 hours~~

Let David's time be x

Sione's time is $x+4$

$t(x+4) = 150$

$(t+0.5)x = 150$

$tx + 0.5x = tx + 4t$

$0.5x = 4t$

$x = 8t$

$t(8t+4) = 150$

$8t^2 + 4t - 150 = 0$

$t = 4.09$ or $t = -4.59$ (ignore)

it took Sione 4.09 hours.

it took David 4.59 hours.

$\frac{150}{4.59} = 32.68$

David travelled at an average speed of 32.68 $kmhr^{-1}$

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t

E8

QUESTION THREE

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(a) Simplify, giving your answer with positive exponents:

(i) $\left(\frac{a^{10}}{4a^5}\right)^{-2}$

$= \left(\frac{4a^5}{a^{10}}\right)^2$

$= \frac{16a^{10}}{a^{20}}$

$= 16a^{-10}$

$\cancel{= 16 \cdot a} = \frac{16}{a^{10}}$ //

(ii) $\sqrt[5]{\left(\frac{32}{x^5}\right)^3}$

$= \left(\left(\frac{32}{x^5}\right)^3\right)^{\frac{1}{5}}$

$= \left(\frac{32}{x^5}\right)^{\frac{3}{5}}$

$= \frac{8}{x^3}$ //

r

r

(b) Solve the following equation for t:

$$\frac{1}{t(t-1)} - \frac{1}{t} = \frac{3}{t-1}$$

$1 - 1(t-1) = 3t$

$-4t = -2$

$t = \frac{1}{2}$ //

r

Question Three continues on the following page.

(c) For what value(s) of k does the graph of the quadratic function

$$y = x^2 + (3k - 1)x + (2k + 10)$$

never touch the x-axis?

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$\Delta = b^2 - 4ac$

$\Delta < 0$ because it doesn't touch the y-axis.

$(3k-1)^2 - 4(2k+10) < 0$

$9k^2 - 6k + 1 - 8k - 40 < 0$

$9k^2 - 14k - 39 < 0$

$(k-3)(13k+9) < 0$

$-\frac{13}{9} < k < 3$ //

t

(d) The quadratic equation

$$mx^2 - (m+2)x + 2 = 0$$

has two positive real roots.

Find the possible value(s) of m, and the roots of the equation.

$\cancel{m}\ 2 \quad (-m-2)(-m-2)$

$(-m-2)^2 - 8m > 0$

$m^2 + 4m + 4 - 8m > 0$

$m^2 - 4m + 4 > 0$

$(m-2)^2 > 0$

$m - 2 > 0$

$m > 2$

roots are: $x = \frac{(m+2) + \sqrt{m^2-4m+4}}{2m}$

$x = \frac{m+2+m-2}{2m}$

$x = 1$

and

$x = \frac{(m+2) - \sqrt{m^2-4m+4}}{2m}$

$x = \frac{m+2-m+2}{2m}$

$x = \frac{2}{m}$

$x = 1$ and $x = \frac{2}{m}$

ASSESSOR'S USE ONLY

t

E8