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91262

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NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
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SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2015

91262 Apply calculus methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Low Achievement

TOTAL 9

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR USE ONLY

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

Find the gradient of the graph of the function at the point where $x = -1$.

$f'(x) = 4x^3 + 4x$

$f'(-1) = 4(-1)^3 + 4(-1)$

$= -4 + -4 = -8$

1

(b) $f(x) = 8 + 3x + x^2 - \frac{x^3}{3}$

For what values of x is f a decreasing function?

Justify your answer.

You must show the use of calculus.

$f'(x) = 3 + 2x - 3x^2$

$0 < 3 + 2x - 3x^2$

$-3 < 2x - 3x^2$

$-3 < x(2 - 3x)$

$\frac{-3}{(2-3x)} < x$

$2 - x < x$

$2x > 2$

$x > 1$ when x is greater than 1

2

(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.

(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, a m s^{-2}, t seconds after it passes the signal, can be modelled by the function

$$a(t) = (16 - 2t)$$

(i) What is the greatest speed attained by the train after it passes the signal?

$a(t) = v'(t)$

$v(t) = 16t - \frac{2t^2}{2} + c$

$40 = 16t - t^2 + c$

(ii) How far past the signal does the train travel before it stops?

ASSESSOR'S USE ONLY

N2

QUESTION TWO

ASSESSOR USE ONLY

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

$f(x) = \frac{4x^2}{2} - 3x + c$ $\quad$ $f(x) = 2x^2 - 3x - 14$

$f(x) = 2x^2 - 3x + c$

$6 = 2(4)^2 - 3(4) + c$

$6 = 32 - 12 + c$

$6 = 20 + c$

$c = -14$

4

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

$g'(x) = 2x - 3$ $\quad$ $y = x^2 - 3x + 18$

$0 = 2x - 3$ $\quad$ $= 1.5^2 - 3(1.5) + 18$ $\quad$ ~~$y - y_1 = m(x - x_1)$~~

$3 = 2x$ $\quad$ $= 15.75$

$x = 1.5$

5

(ii) In relation to the graph, fully describe the point where this tangent meets the function.

(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

ASSESSOR'S USE ONLY

The mound can be modelled by

$$h = -0.5x^2 + 3x - 1.5$$

(i) What is the maximum height of the mound?

$\frac{dh}{dx} = -x + 3$

$-x + 3 = 0$

$x = 3m$

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6

(ii) A ramp up the side of the mound is a tangent to the mound.

The ramp can be modelled by the function

$$h = 0.5x - c$$

Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

$\frac{dh}{dx} = 0.5$

(iii) The height h metres of a skateboard path at a horizontal distance r metres from another point Q, can be modelled by the function

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.

$$\frac{dh}{dr} = r^2 - 4r + 3$$

ASSESSOR'S USE ONLY

7

N

A4

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

$V'(t) = a(t)$

$a(t) = 12t^2 - 2t + 2$

8

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

$f'(x) = 3x^2 - 4x + 1$ $\quad$ ~~$y - y_1 = m(x - x_1)$~~ $y =$

$f'(2) = 3(2)^2 - 4(2) + 1$

$= 12 - 8 + 1$

$= 5$

$f''(x) = 6x - 4$

9

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

Use calculus methods to justify your answer.

$\frac{dh}{dt} = 20 - 10t$

$50 = 20 - 10t$

$30 = -10t$

$-30 = 10t$

$t = \frac{-30}{10}$

$t = -3$

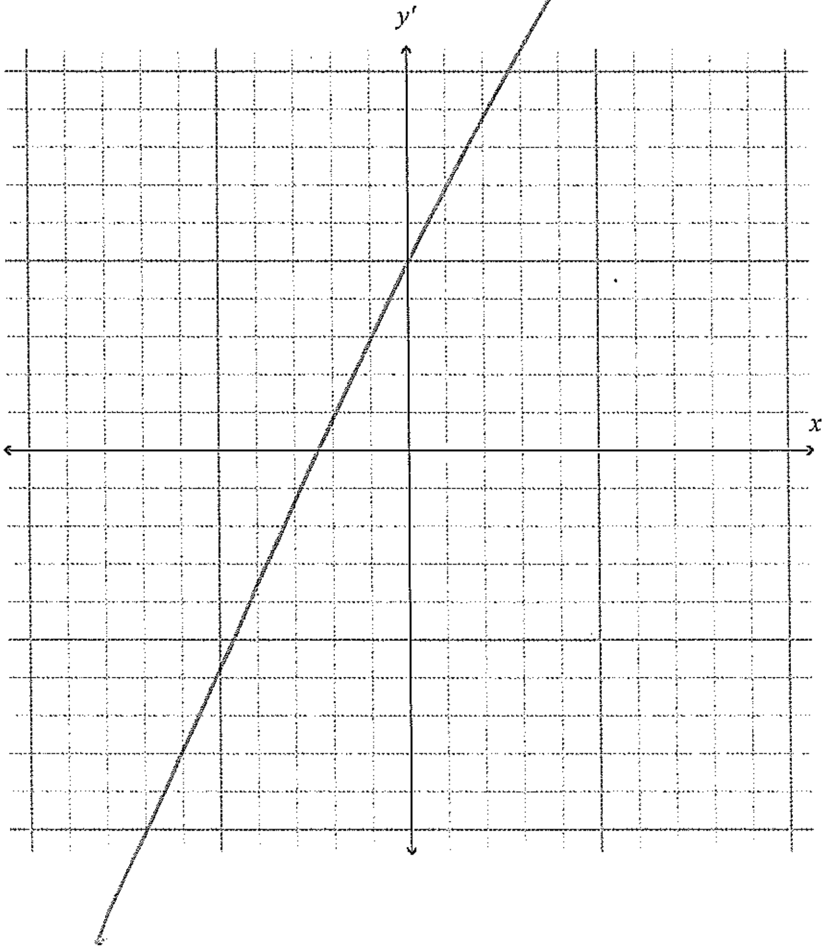
10

(d) For a function $y = -ax^2 + bx + c$,

a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.



If you need to redraw this graph, use the grid on page 10.

ASSESSOR'S USE ONLY

(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

Find the maximum and minimum values of the product of x^2y.

Justify your answer.

ASSESSOR'S USE ONLY

A3

91262 2015

Apply Calculus methods in solving problems

Descriptions for the exemplar numbers:

NOTE: Top down marking was used in this standard.

GRADE = LOW ACHIEVED

1. Correct derivative found and x=-1 correctly substituted into the derivative to calculate the gradient.
2. Incorrect derivative leading to only one x value given instead of two. Also incorrect value of x, missing the negative sign.
3. Correct integration of the acceleration formula, not sufficient as the +C was not evaluated to 40.
4. Function integrated correctly including the use of +C. The point (4,6) correctly substituted into the equation to find the constant of integration.
5. Correct derivative found, equated to zero and the x value of 1.5 found. Equation for the tangent not shown, only the corresponding y value (which 15.75).
6. Correct derivative found and equated to zero. This gives an x value of 3. No working to demonstrate the substitution back into the height formula to gain the height =3m.
7. Correct derivative found but not further working towards the answer given.
8. Correct equation for the acceleration of the object given.
9. Correct derivative found and the gradient evaluated when x=2. No equation for the tangent given.
10. Correct derivative found, however it was then equated to 50 instead of zero, resulting in an incorrect t value.

2

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High Achievement

TOTAL 13

ASSESSOR'S USE ONLY

QUESTION ONE

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

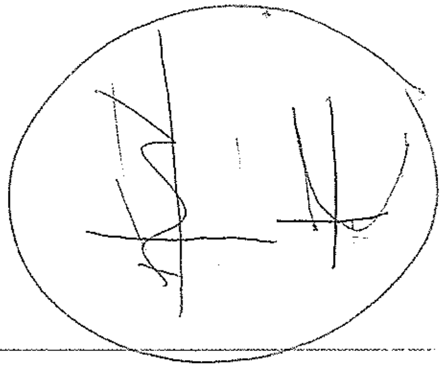
Find the gradient of the graph of the function at the point where $x = -1$.

$f'(x) = 4x^3 + 4x$

$f'(1) = 4 \times 1^3 + 4 \times 1$

$= 18$ $\qquad f'(1) = 18$

(b) $f(x) = 8 + 3x + x^2 - \dfrac{x^3}{3}$



For what values of x is f a decreasing function?

Justify your answer.

You must show the use of calculus.

$f'(x) = \cancel{8}\ 3 + 2x - \dfrac{3x^2}{3}$

$f'(x) = 3 + 2x - x^2$

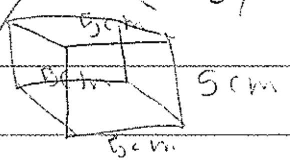
When $f'(x) < 0$ f is decreasing

$3 + 2x - x^2 < 0$

$\cancel{x <}\ (x-3)(x+1) < 0$

when x is between 3 and -1. f is a decreasing function.

(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.



ASSESSOR'S USE ONLY

(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, a m s^{-2}, t seconds after it passes the signal, can be modelled by the function

$a(t) = (16 - 2t)$

(i) What is the greatest speed attained by the train after it passes the signal?

4

$\cancel{v = \int 16 + 2t \, .dx}$ $\cancel{a(t) =}$ $a(0) = 16 - 2 \times 0$

$a(0) = 16 ms^{-2}$

$v = 16t + \frac{2t^2}{2} + c$ $16 = 16 - 2t$ $\frac{16}{2} = 16 + t$

$v = 16t + t^2 + c$ $v = 40$ $t = 0$ $t = 8$ secs

$40 = 16 \times 0 + 0^2 + c$ $c = 40$ $v = 16t + t^2 + 40$

$v = 16 \times 8 + 8^2 + 40$ $v = 232 ms^{-1}$

u

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(ii) How far past the signal does the train travel before it stops?

$d = \int 16t + t^2 + 40 \, .dx$

$d = \frac{16t^2}{2} + \frac{t^3}{3} + 40t + c$ when $v = 0$

$d = 8t^2 + \frac{t^3}{3} + 40t + c$

5

$v = 0$ $t = 2$

$1t^2 + 16t + 40 = 0$

$t = -3.1$ & -12.9

n

A3

ASSESSOR'S USE ONLY

QUESTION TWO

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

$f(x) = \frac{4x^2}{2} + 3x + C \quad (4,6)$

$f(4) = 6 \quad 6 = 2\times4^2 + 3\times4 + C$ ~~$2\times4^2 + 3x + 6$~~

~~6 =~~ $6 = 44 + C \quad C = -38$

$f(x) = 2x^2 + 3x - 38.$

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

$g'(x) = 2x - 3$ ~~$y = 2\times1.5 - 3$~~ ~~$y = 0$~~

$2x - 3 = 0$

$2x = 3 \quad g'(1.5) = 0$

$x = 1.5$

~~$g'(1.5) = 2$ when $g'(x) = 0$~~

6

crossed out

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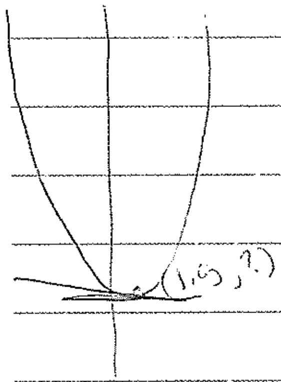
(ii) In relation to the graph, fully describe the point where this tangent meets the function.



$g(1.5) = 1.5^2 - 1.5\times3 + 18$

$g(1.5) = 2.25 - 4.5 + 18$

$g(1.5) = 15.74$

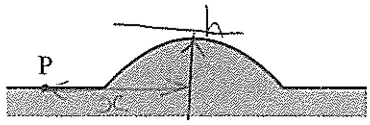
$pt = (1.5, 15.74)$

(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

ASSESSOR'S USE ONLY

The mound can be modelled by

$$h = -0.5x^2 + 3x - 1.5$$



(i) What is the maximum height of the mound?

$h' = -x + 3$ $\qquad h = -0{,}5 \times 3^2 + 3 \times 3 - 1{,}5$

$h' = 0$ $\qquad h = 3\text{ m}$

$-x + 3 = 0$

$-x = -3$

$x = 3$

(ii) A ramp up the side of the mound is a tangent to the mound.

The ramp can be modelled by the function

$$h = 0.5x - c$$

Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

$0{,}5x - c < 0$

$x = 3$

$0{,}5 \times 3 - c < 0$

$1{,}5 - c < 0$

$c > 1{,}5$

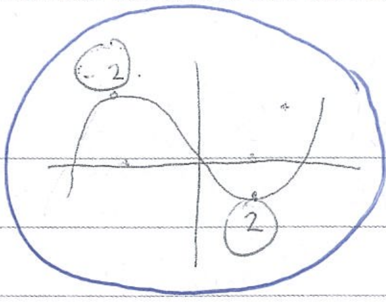
(iii) The height h metres of a skateboard path at a horizontal distance r metres from another point Q, can be modelled by the function

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.



$h' = \frac{3r^2}{3} - 4r + 3$

$h' = r^2 - 4r + 3$

$r = 3$ & 1

$h'' = 2r - 4$ $\qquad$ $h'' = 2r - 4$

$h'' = 2 \times 3 - 4$ $\qquad$ $h'' = 2 \times 1 - 4$

$h'' = 6 - 4$ $\qquad$ $h'' = -2$ max turning pt

$h'' = 2$ min turning pt

the skate path complies with the hiegnt regulagion

9

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

$a = v'(t)$

$a = 12t^2 - 2t + 2$

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

$f'(x) = 3x^2 - 4x + 1$

$f'(2) = 3 \times 2^2 - 4 \times 2 + 1$ $(2,2)$

$f'(2) = 5$

f

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

Use calculus methods to justify your answer.

~~$20t - 5t^2 = 50$~~

~~$20t - 5t^2 - 50 = 0$~~

$h' = 20 - 10t$

$20 - 10t = 0$

$-10t = -20$

$t = \frac{-20}{-10}$

$t = 2$ s

$h = 20 \times 2 - 5 \times 2^2$

$h = 20$ m

no the firework will not break the 50 m limit

(d) For a function $y = -ax^2 + bx + c$,

a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.

$y = -ax^2 + 2ax + c$

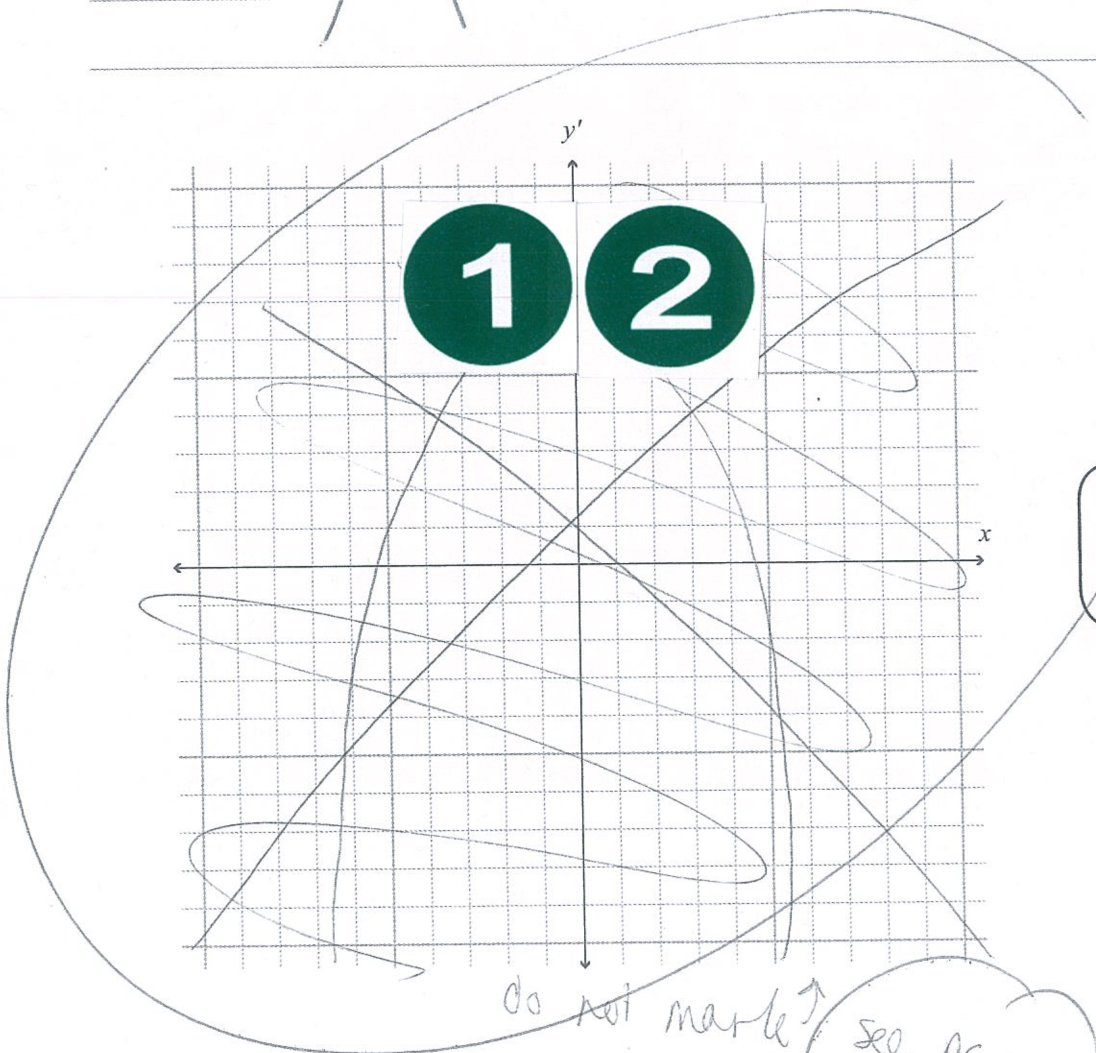
$y' = -2ax + 2a$.

~~y~~ ~~$-2ax$~~ $-2ax + 2a = 0$

$-2ax = -2a$

$x = \frac{-2a}{-2a}$

$(-a$



do not mark↑ see pg 10

If you need to redraw this graph, use the grid on page 10.

ASSESSOR'S USE ONLY

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(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

Find the maximum and minimum values of the product of x^2y.

Justify your answer.

ASSESSOR'S USE ONLY

$y = 2(x - 3)$

$y = 2x - 6$ $\qquad$ ~~x~~ $-0.5 < x < 3$

$x^2 y = x^2(2x - 6)$

$x \ 2x^3 - 6x^2$

$x = 3 \ \& \ 0$

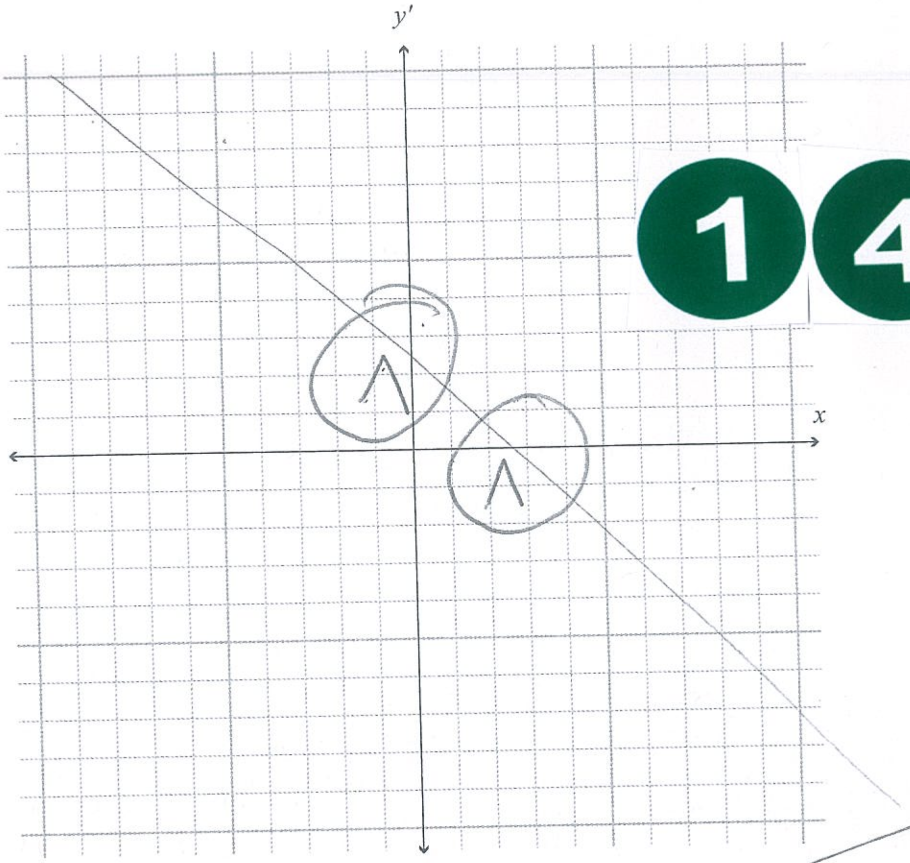
x is between 3 and 0

13

Mathematics and Statistics 91262, 2015

If you need to redraw your graph from Question Three (d), draw it on the grid below. Make sure it is clear which answer you want marked.

ASSESSOR'S USE ONLY



91262 2015

Apply Calculus methods in solving problems

Descriptions for the exemplar numbers:

NOTE: Top down marking was used in this standard.

GRADE = HIGH ACHIEVED

1. Correct derivative found. Incorrect value substituted into the derivative. No minor errors (MEI) were awarded on achievement only questions.
2. Correct derivative given and the two values for x found. No region identified and no justification for when the function was decreasing.
3. No relevant working.
4. Acceleration equation correctly equated to zero and t=8 second found. Incorrect velocity.
5. Correct integration of the velocity formula, not sufficient as the +C was not evaluated to 40.
6. Correct derivative found, equated to zero and the x value of 1.5 found. Equation for the tangent not evaluated.
7. Correct derivative found and equated to zero. This gives an x value of 3. This is then substituted into the height formula to get the correct height of 3m.
8. No relevant working.
9. Correct derivative found and the two correct r values stated. No region for where the function is decreasing is given, however the candidate has used the double derivative to state which TP is a max and which is a min. The y values on the graph are incorrect as the candidate has used the double derivative values rather than evaluating the y coordinates.
10. Correct derivative found and x=2 substituted accurately into the derivative to find m=5. No equation of the tangent given.
11. Correct derivative found, equated to zero and the x value of 2 seconds found. Correctly substituted into the height formula to find h=20m. Then a correct and consistent answer to the question “will the firework break the 50m limit”?
12. Correct derivative found. No further working towards the solution given.
13. Equation incorrectly set up.
14. Second graph has the correct linear function but neither intercepts are labelled.