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Level 1 Mathematics and Statistics, 2016

91031 Apply geometric reasoning in solving problems

9.30 a.m. Thursday 17 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 21

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THE SKY TOWER

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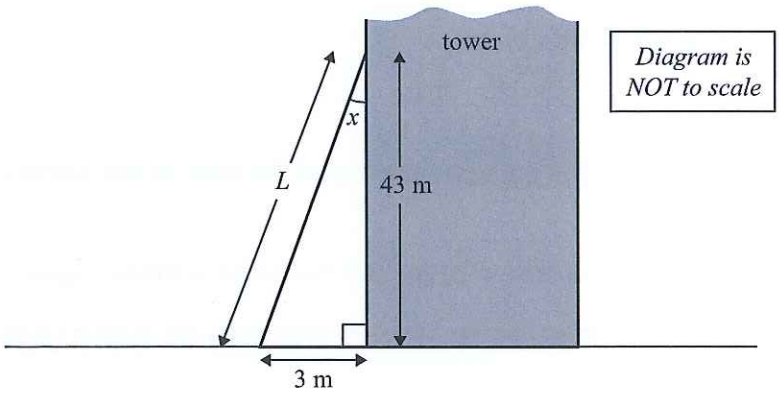
Auckland's Sky Tower is the tallest man-made structure in the Southern Hemisphere.

QUESTION ONE

(a) The base of the tower is supported by 8 legs.
These legs are L metres long and are 3 metres away from the tower at ground level.
The legs join the tower 43 m above ground level.



(i) Calculate the length, L, of the leg from the ground to the tower.

$c = \sqrt{a^2+b^2} = \sqrt{3^2+43^2} = 43.1\text{ m}$

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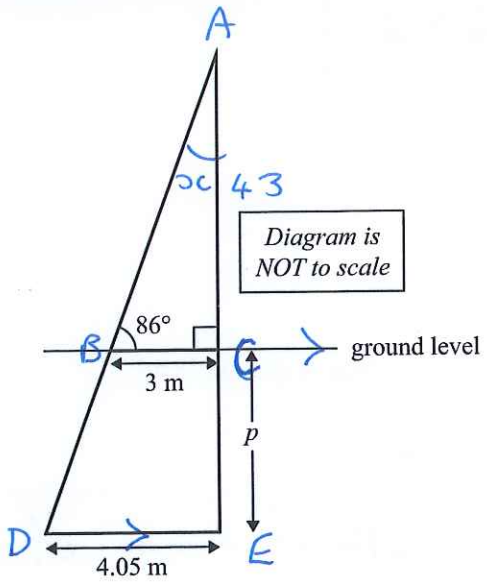
(ii) Use trigonometry to calculate the size of angle x, where the leg joins the tower.

$x = \tan^{-1}\left(\frac{3}{43}\right) = 3.99°$

(iii) The legs of the tower go below ground level.

The horizontal distance from the tower to the bottom of the leg under the ground is 4.05 metres.



Calculate p, the vertical distance that the legs are built into the ground.

Show your working clearly.

△ at ground level & △ below ground level are similar.

AE = 4.05 ÷ tan 3.99 = 58.06 m

CE = AE − 43 = 58.06 − 43 = 15.06 m

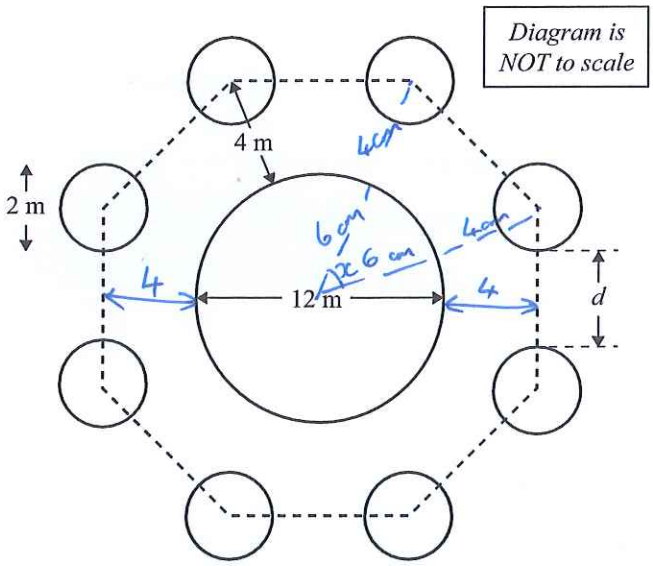
p = 15.06 m

M

(b) The centres of the 8 circular legs form a regular octagonal shape.

The tower has a diameter of 12 metres and each leg has a diameter of 2 metres.

The distance from the outside edge of the tower to the centre of the legs at the ground is 4 metres.



Calculate the shortest distance, d, between adjacent legs at ground level.

Show your working clearly.

$x = \frac{1}{8} \times 360 = 45^\circ$

cosine rule finds length of 1 side of the octagon

$a^2 = -2bc\cos A + b^2 + c^2$

$a^2 = -2(10)(10)(\cos 45) + 10^2 + 10^2$

$a^2 = 58.58$

$a = 7.65$

$d = a - 2$ radii

$d = 7.65 - 2(1) = 5.65$ m.

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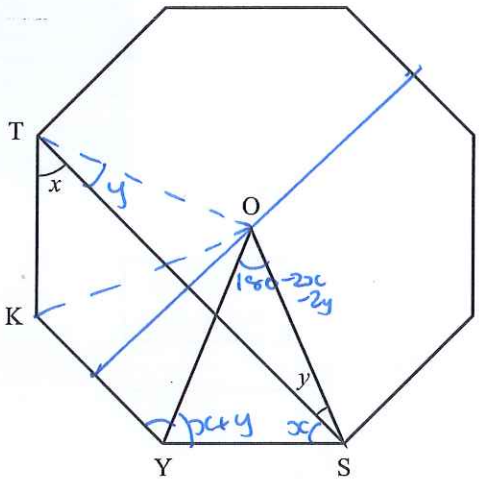
(c) A simplified diagram of the position of the legs is shown below as a regular octagon. Point O is at the centre of the octagon.



Show that angle y is half the size of angle x.

Justify your answer with clear geometric reasoning.

YOS = 45° because angles at a point sum to 360° and a regular octagon cuts this point evenly into 8. 360 ÷ 8 = 45.

YST = x because a line drawn in a regular shape creates each angle in pairs because it is symmetrical, shown by line of symmetry drawn

OYS = x + y because base angles of an isosceles triangle are equal

45 + x + y + x + y = 180 //

E7

QUESTION TWO

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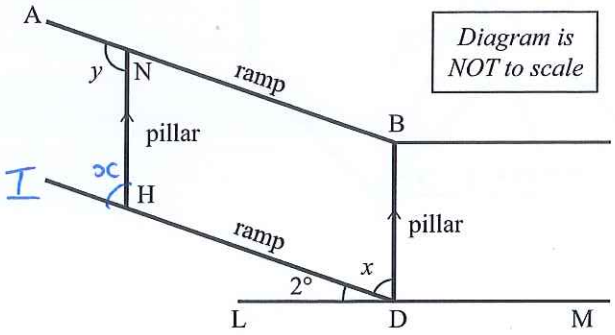
Below the Sky Tower is a car park made of ramps.

The ramps are at a 2° angle.

There are vertical pillars regularly placed along the ramps for strength.

(a) All pillars are parallel to each other. LM is horizontal.



(i) Calculate the size of angle x in the diagram above.

Justify your answer with clear geometric reasoning.

BD is perpendicular to LD

$90 - 2 = 88°$

$x = 88°$

(ii) Calculate the size of angle y in the diagram above.

Justify your answer with clear geometric reasoning.

corresponding Ls are equal $\therefore$ HIA = 88

cointerior Ls are supplementary $\therefore y = 180 - x$

$= 180 - 88$

$= 92°$

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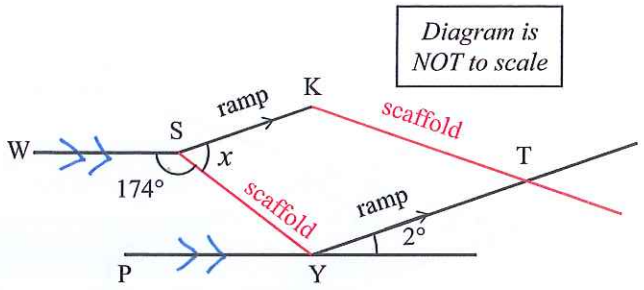
(iii) Part of the ramp had extra scaffolding added for support, as shown in the diagram below. The lines SK and YT are parallel.

Angle WSY is 174°.

The lines WS and PY are both horizontal.



Calculate the size of angle x in the diagram above.

Justify your answer with clear geometric reasoning.

∠PYS = 6 because cointerior angles are supplementary

∠SYT = 172 because angles in a line are supplementary

x = 8° because cointerior angles are supplementary.

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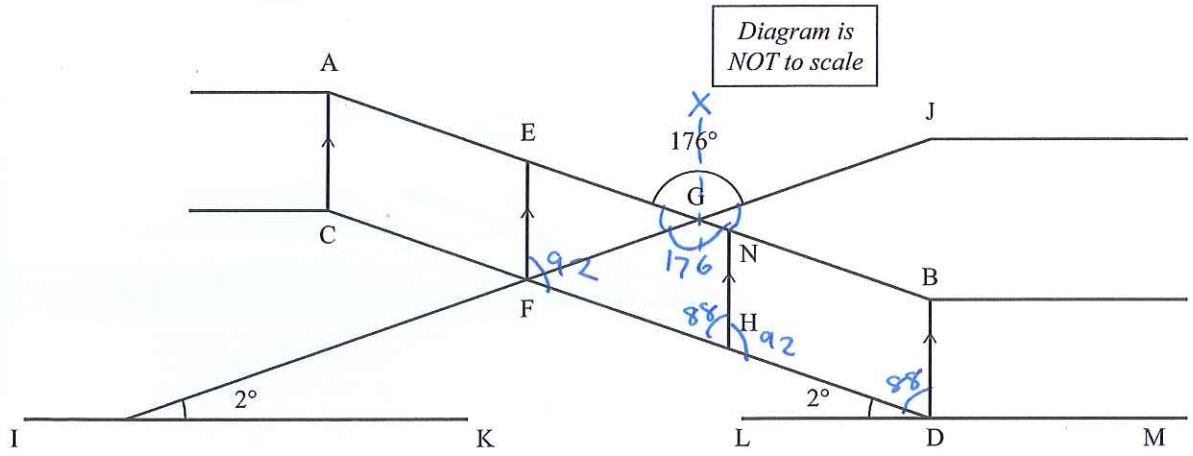
(iv) From the side, the carpark looks like the diagram below.

Angle EGJ is 176°.

IK and LM are horizontal.



Prove that the lines AB and CD are parallel.

Justify your answer with clear geometric reasoning.

FGB = 176 because vertically opposite angles are equal.

JGB = 4° because angles at a point equal 360 and vertically

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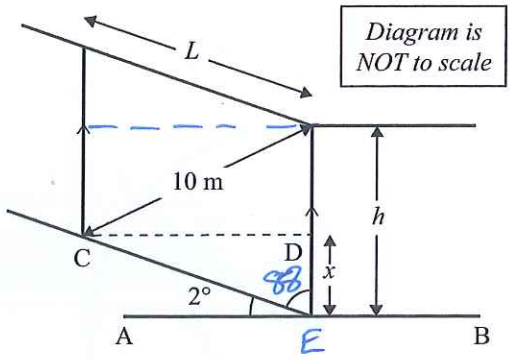
(b) The length along the slope between two pillars is L metres.

The diagonal distance between the top of one pillar and the base of the next higher pillar is 10 m.

AB and CD are horizontal.



(i) Find the height, x, in terms of the length L.

Show your working clearly.

CE = L

DEC = 88 degrees because DEC and AEC are complementary.

$x = L \times \cos 88^\circ$

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(ii) Calculate h, the height in metres of a pillar, in terms of L.

Show your working clearly.

M

M6

QUESTION THREE

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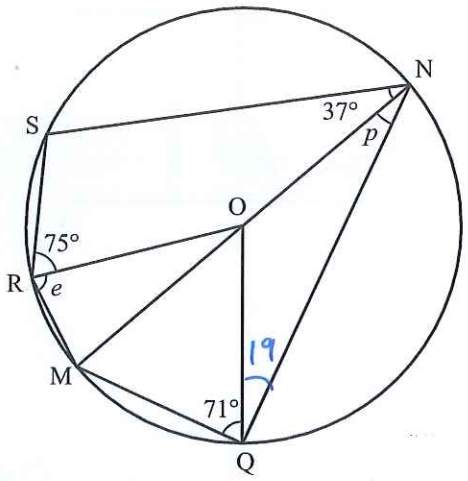
(a) In the diagram below, the line MN passes through the centre of the circle, O.
Angle MQO is 71°, angle SNO is 37° and angle SRO is 75°.

Diagram is NOT to scale



(i) Find the size of angle p.

Justify your answer with clear geometric reasoning.

Angle of Δ when hypotenuse is a diameter is 90
OQN = 90 − 71 = 19
Base angles of isosceles Δ are equal
p = OQN = 19°

(ii) Find the size of angle e.

Justify your answer with clear geometric reasoning.

Opposite angles in a cyclic quadrilateral are equal
e = 180 − 37 − 75 = 68°

(iii) In the diagram below, angle SRO is 75°.

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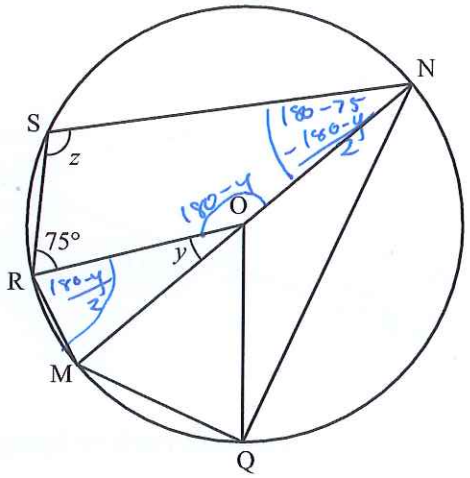
Diagram is NOT to scale



Find an expression for z in terms of y.

Justify your answer with clear geometric reasoning.

RON = 180−y because angles on a line are supplementary

MRO = $\frac{180-y}{2}$ because angles in a triangle sum to 180 and base angles of a isoseles triangle are equal.

MNS = 180−75 − $\frac{180-y}{2}$ because opposite angles of a cyclic quadrilateral are equal.

MNS = 105 − $\frac{180-y}{2}$

$z = 360 - 75 - (180-y) - (105 - \frac{180-y}{2})$

because angles in a quadrilateral sum to 360

$z = 285 - 180 + y - 105 + \frac{180-y}{2}$

$z = y + \frac{180-y}{2}$

$2z = 2y + 180 - y$

$2z = y + 180$

$z = \frac{y+180}{2}$

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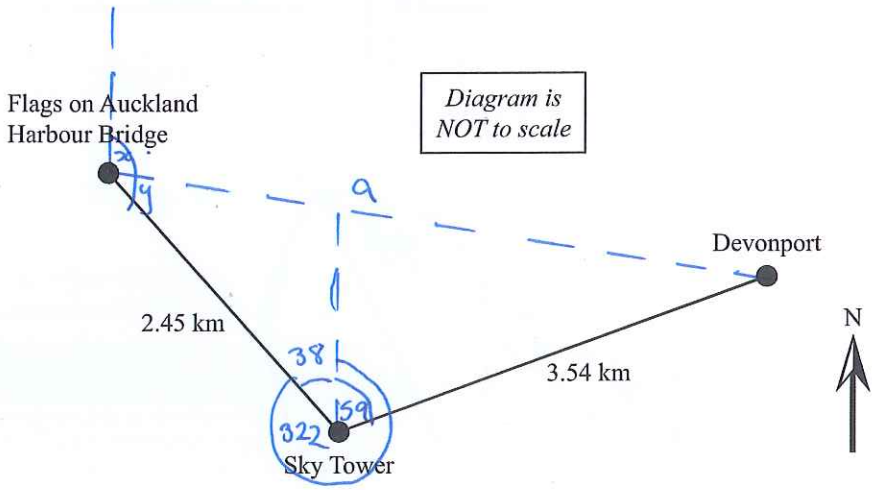
(b) Devonport is at a bearing of 059° and 3.54 km from the Sky Tower.

The flags on the Auckland Harbour Bridge are at a bearing of 322° and 2.45 km from the Sky Tower.



Calculate the bearing from the flags on the Auckland Harbour Bridge to Devonport.

Show your working clearly.

$a^2 = -2(2.45)(3.54)(\cos 97) + 2.45^2 + 3.54^2$

$a^2 = 20.64$

$a = 4.54$

$\frac{\sin y}{3.54} = \frac{\sin 97}{4.54}$

$\sin y = 0.77$

$y = 50.65°$

$x + y = 180 - 38$ because cointerior angles sum to 180°

$x + 50.65 = 180 - 38$

$x = 91.35$

Bearing = 091°

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E8

Annotated Exemplar Excellence

Excellence exemplar 2016

Subject:	Mathematics	Standard:	91031	Total score:	21

Q	Grade score	Annotation
1	E7	Candidate has used a range of methods to solve problems. Pythagoras and Trigonometry have been correctly used to find a side and an angle. Trigonometry and similar triangles has been used as a strategy to solve (a) (iii) correctly. In 1(b), the candidate has developed a logical sequence of steps to solve the problem. To gain an E8, the candidate would need to find both angle x and angle y and correctly reason each step towards solution.
2	M6	Candidate has used a range of methods in parallel line geometry and has reasoned correctly to find the angles in 2 (a) (i)–(iii). Each step towards the solution is correctly reasoned. To gain a grade of E7, the candidate would need to develop a chain of logical reasoning to prove that the lines AB and CD are parallel or trigonometry in the abstract to find the height in 2(b)(ii).
3	E8	Candidate has used a range of methods in circle geometry to correctly calculate angle p and angle e in 3(a)(i) and (ii). Each step towards the solution is correctly reasoned. In 3a) iii the candidate has used circle geometry rules to find the correct expression for z in terms of y. A chain of reasoning was developed and communicated well. In 3(b), the candidate used trigonometry and parallel line geometry to correctly solve the problem.