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91028



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NEW ZEALAND QUALIFICATIONS AUTHORITY
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SUPERVISOR'S USE ONLY

Level 1 Mathematics and Statistics, 2015

91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Monday 9 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL

21

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QUESTION ONE

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A plant is growing on the surface of a pond. Hank noticed the plant on Day 1. Two days later Hank was worried about the plant and started measuring the area that the plant covered.

- (a) Each day (at 5 pm) Hank measures the area of water (in square metres) covered by the plant. He records his measurements in the table below.

Day, d	Area covered by plant, A
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128

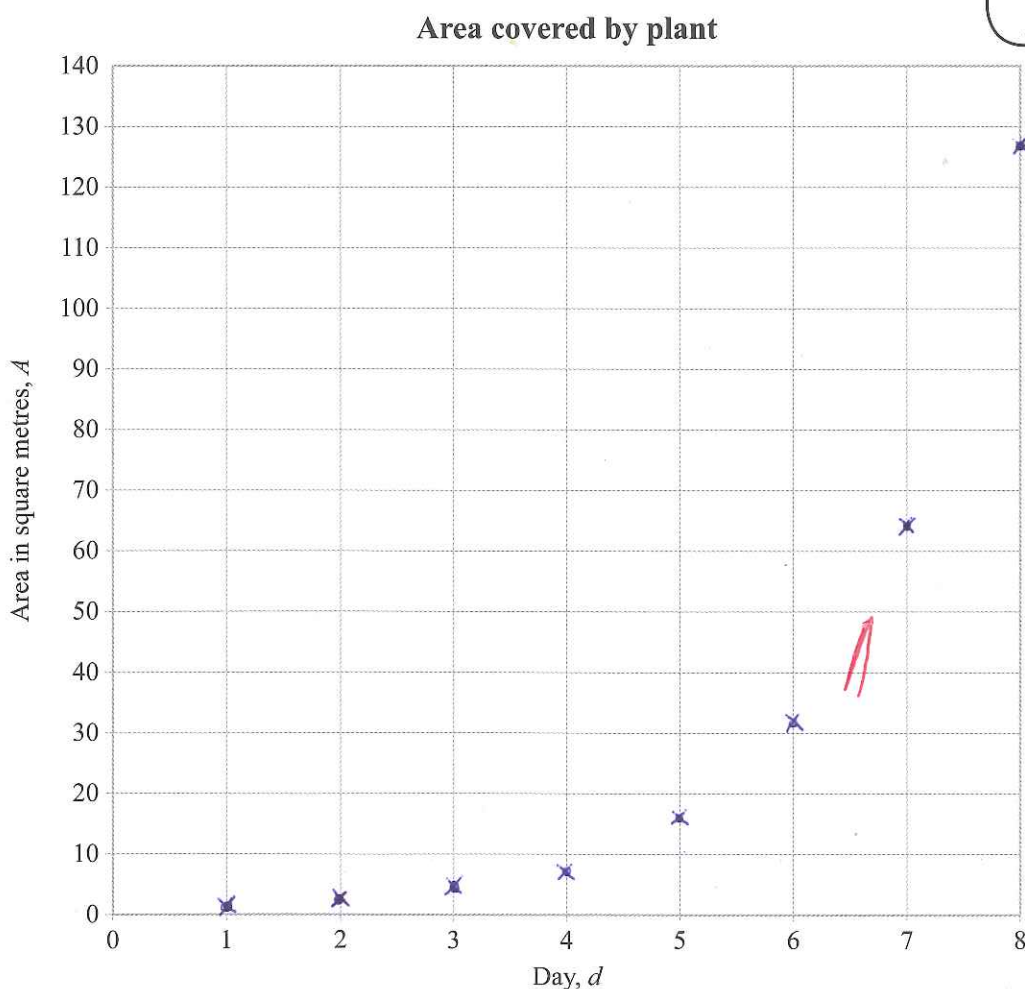
Handwritten notes next to the table showing the doubling of the area each day:

- 1 → 2 $\div 2$
- 2 → 4 $\div 2$
- 4 → 8 $\div 2$
- 8 → 16 $\div 2$
- 16 → 32 $\div 2$
- 32 → 64 $\div 2$
- 64 → 128 $\div 2$

d is the number of days since Hank first noticed the plant.

- (i) Show how the area of the pond covered by the plant changes with time.

If you need to redraw this graph, use the grid on page 14.



- (ii) The plant followed the same pattern of growth from the time when it was first noticed.

What area of the pond was covered by the plant when it was first noticed?

Explain your answer.

1m² on the first day because the area covered by plant doubles each day. On the 3rd day it is 4m², on the 2nd day it is 2m² (4÷2=2), therefore on the first day it is 1m² (2÷2=1). //

- (iii) Give the equation that describes the area of the plant covering the pond after d days.

$$A = 2^d \div 2$$

$$A = \frac{2^d}{2} //$$

- (iv) If no intervention takes place, on which day will Hank first measure the area of the plant to be more than 500 square metres?

$$500 = 2^d \div 2$$

$$1000 = 2^d$$

~~10~~

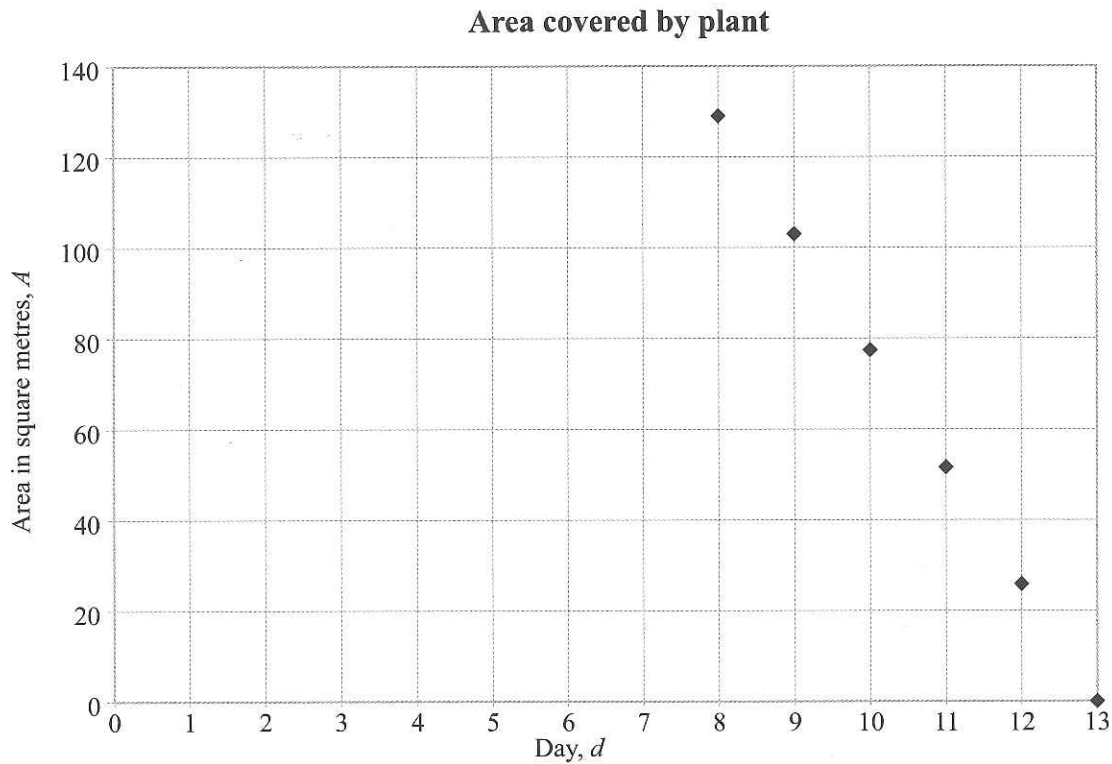
$$2^{10} = 1024$$

$$d = 10$$

On the 10th day, the plant will be measured to be 512m² //

- (b) Hank and some friends start removing the plant on Day 9.

The graph of the area covered by the plant from **Day 8** (when it covers 128 square metres), below, shows what Hank hopes will happen to the area of pond covered by the plant.



- (i) What is the equation for the area covered by the plant as shown in this graph?

$$A = -25.6d + 332.8$$

(ii) What is unrealistic about this graph?

Write at least TWO comments with justification.

If you were removing the plant, the area of the pond covered by the plant wouldn't reduce exactly 25.6 m^2 each day which is shown as the gradient of the graph. The graph also uses discrete points, this means that Hank thinks the plant will suddenly be smaller in area at the same time each day. ^{which is unlikely.} Moreover, the equation $A = -25.6d + 332.8$ is only true from the 8th day, until the 13th day. If you used this equation to find the area covered by the plant on the 1st day, it would be $(A = -25.6 \times 1 + 332.8)$ 358.4 m^2 which is not true and not possible.

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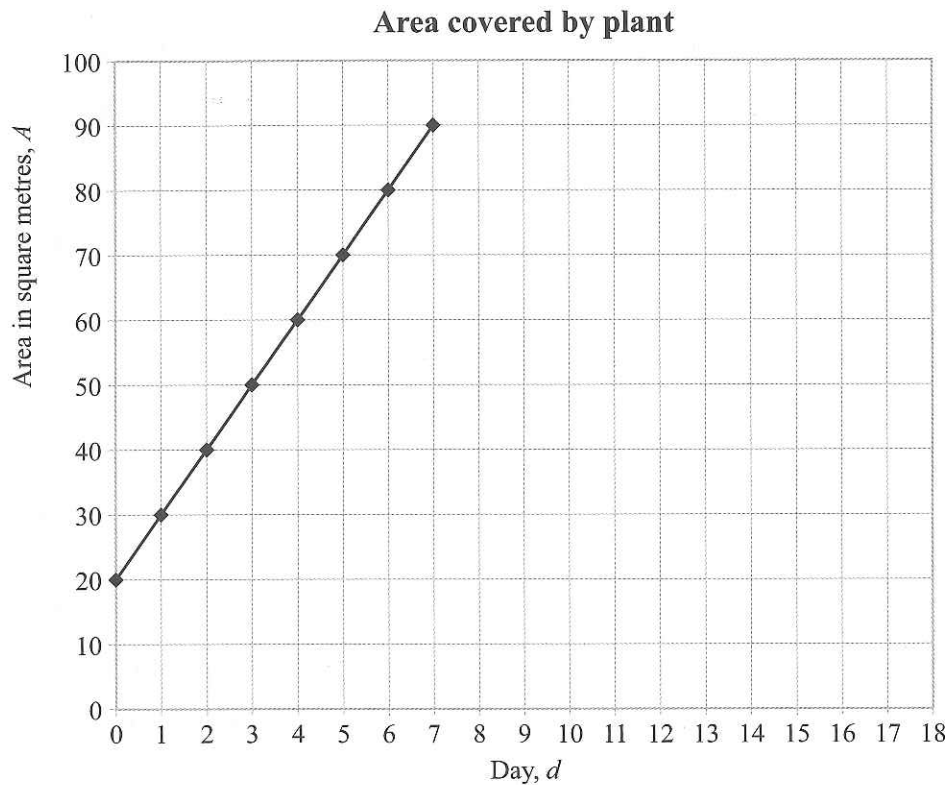
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QUESTION TWO

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USE ONLY

The next year, when the plant begins to grow back, Hank tries to stop it from spreading across the pond so quickly. As soon as he notices the plant, he begins removing it. The graph of the area of pond covered by the plant in this year is shown below:



- (a) How much more area is the plant covering each day?

10m^2

- (b) What day will it be when the plant covers 200 square metres if the conditions remain the same?

Show your working.

$$A = 10d + 20$$

$$200 = 10d + 20$$

$$180 = 10d$$

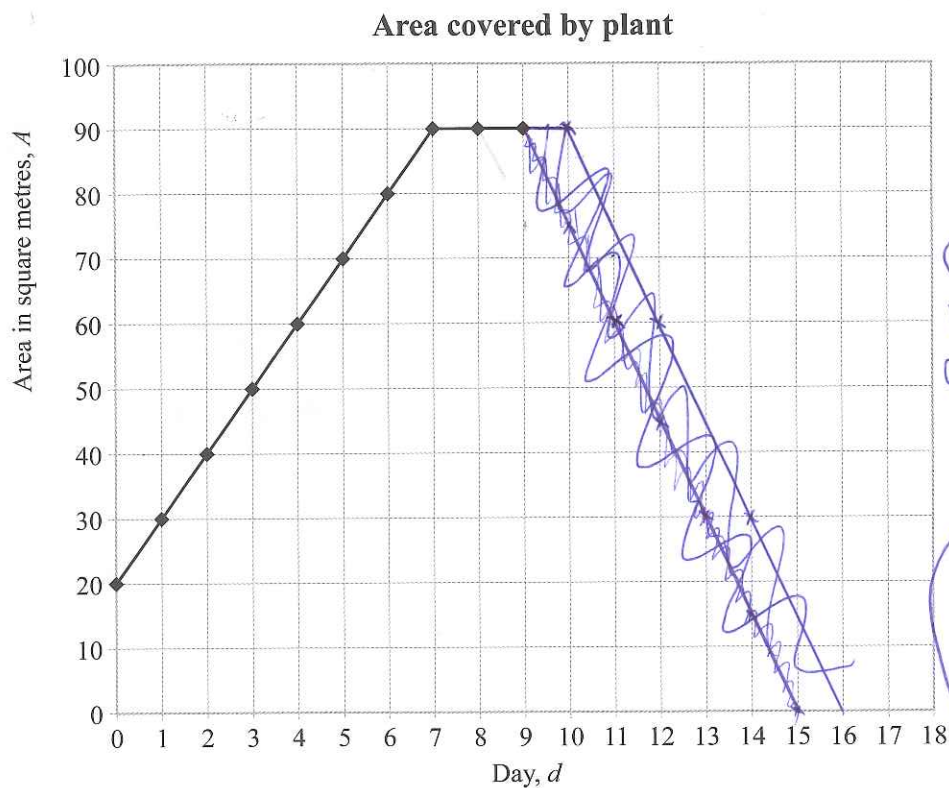
$$d = 18$$

The plant will cover 200 square meters on the 18th day.

After 7 days removing some of the plant by himself, Hank decides to get help.

- (c) One friend helps on Day 8 and Day 9.

The area covered by the plant stays the same for Day 8 and Day 9.



Graph
redrawn
on pg 14.

If you
need to
redraw this
graph, use
the grid on
page 14.

- (i) What is the equation of this new section of the graph on Day 8 and Day 9?

$$A = 90$$

- (ii) What does this section of the graph mean?

The plant is staying at 90 square meters because each day the plant grows, Hank and his friend helps remove 10m^2 each day so it becomes 90m^2 again; the plant doesn't increase in size.

- (d) Two more friends come to help. Now the area covered by the plant decreases by 15 square metres each day until the plant is completely removed.

- (i) Draw a graph on the grid above to show the area of pond covered by the plant from Day 10.

- (ii) On what day will there be no plant left?

~~On the 16th day.~~ On day 15. ~~On 2 days to.~~

- (e) The equation of the line for Day 9 onwards is $A = 225 - 15d$.

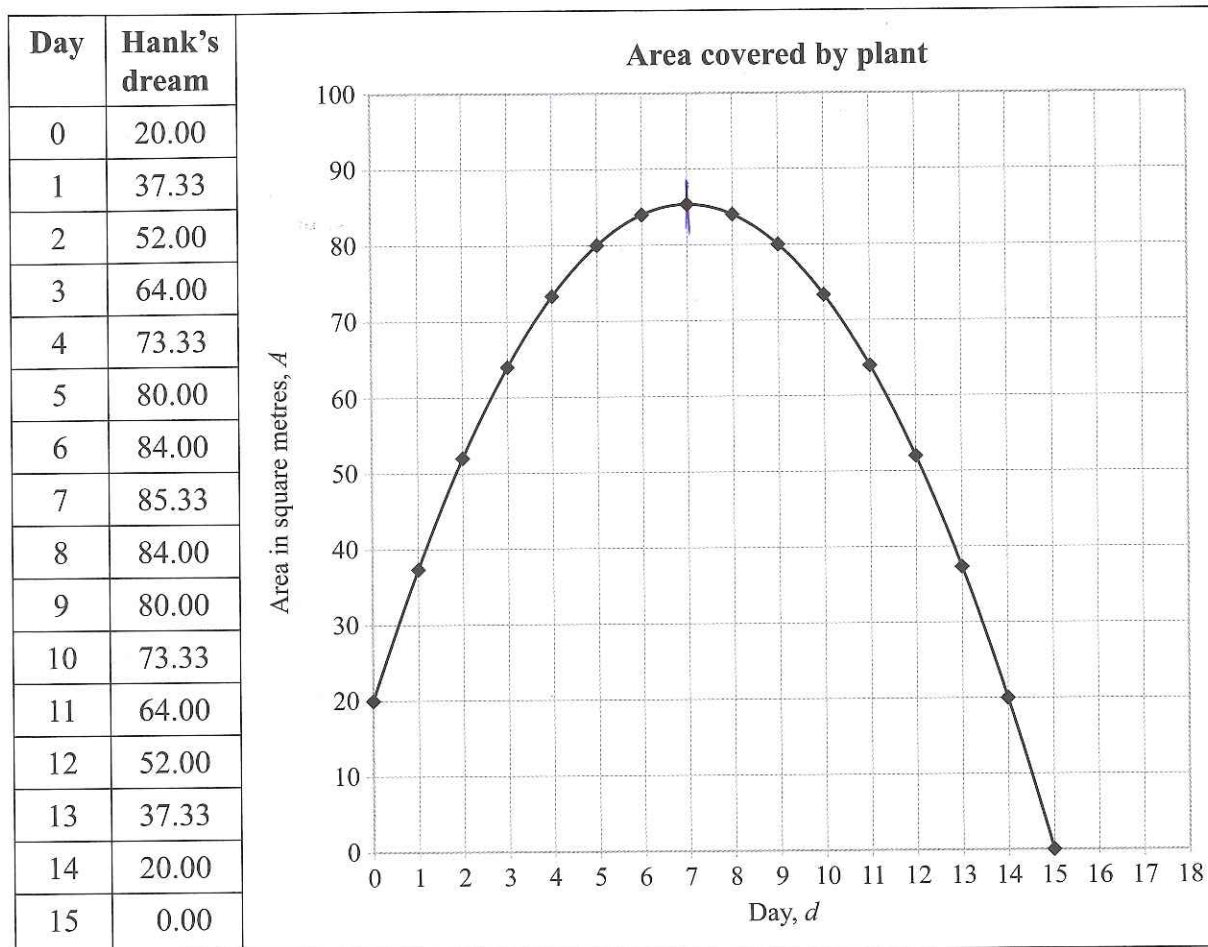
If Hank's 2 friends had come on Day 8, what would the equation of this line have been?

Explain your reasoning.

$$A = 210 - 15d$$

The gradient $(-15d)$ stays the same because they will still do the same amount of work. However 15 is subtracted from the y intercept as his friends come one day earlier so they have done one day's more work than if they came on ~~the~~ day 9

- (f) Hank had a dream that he and his friends made the area of the pond covered by the plant follow the parabola given below:

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USE ONLY

What is the equation of this graph?

vertex $(7, 85.33)$

x intercepts: $(-1, 0)$ $(15, 0)$

$$A = (d+1)(d-15)$$

$$A = k(d+1)(d-15)$$

$$85.33 = k(7+1)(7-15)$$

$$85.33 = -64k$$

$$k = -1.333$$

$$A = -1.333(d+1)(d-15)$$

E7

QUESTION THREE

ASSESSOR'S
USE ONLY

- (a) Jodie sets her friends a mathematical problem. She says:

I think of an integer

When I add 1 to my number, I get A

But if I take 4 off my number, I get B

When A is multiplied by B, I get an answer of 6.

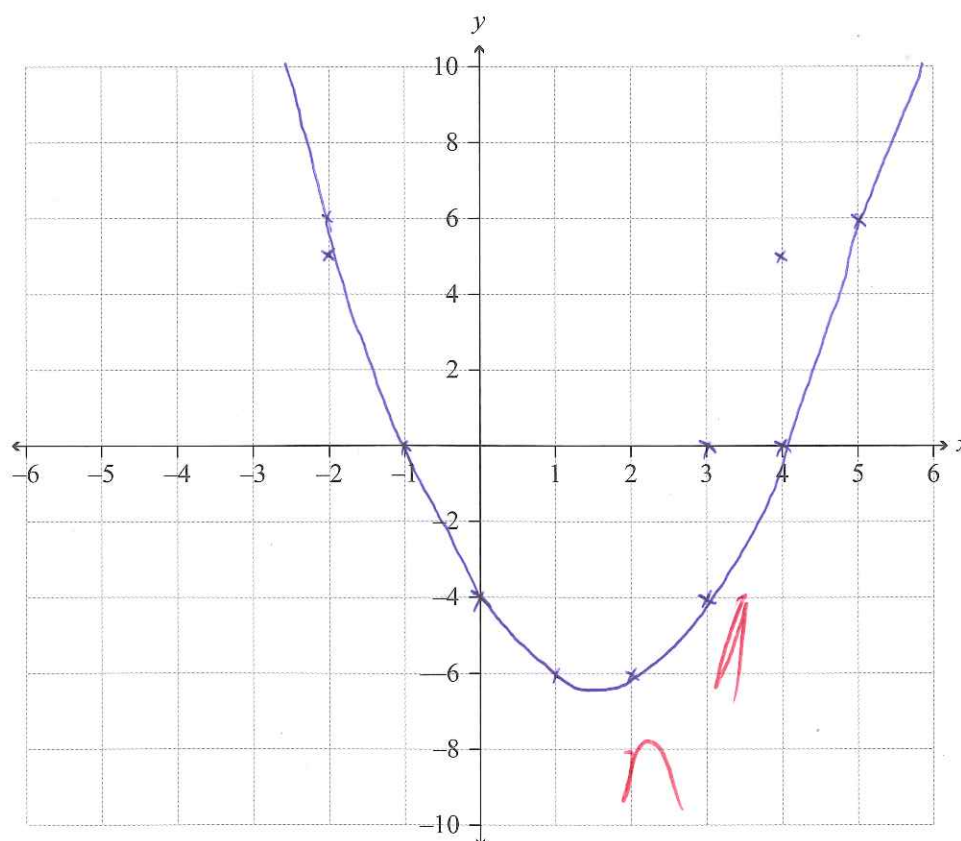
What's my number?

Her friends start by writing a table:

x : Jodie's number	$A = x + 1$	$B = x - 4$	$y = AB$
0	1	-4	-4
1	2	-3	-6
2	3	-2	-6
3	4	-1	-4
4	5	0	0
5	6	1	6
6	7	2	14

- (i) Draw the graph of y against x .

Use the set of axes below.



If you
need to
redraw this
graph, use
the grid on
page 15

- (ii) What is the equation of the graph that matches the table above, in terms of x ?

$$y = (x+1)(x-4)$$

$$y = (x+1)(x-4)$$

- (iii) Explain how Jodie's number can be found from the graph if the answer is 6.

When $y=6$, there is an exact point (5,6). This means $x=5$, and her starting number is 5. Or you can find her number using the graph's equation:

$$6 = (x+1)(x-4) \quad 6 = (x+1)(x-4)$$

The parabola shows that y also equals 6 when x is -2. This means Jodie's number can be -2 or 5.

$$6 = x^2 - 3x - 4$$

$$0 = x^2 - 3x - 10$$

$$0 = (x+2)(x-5)$$

$$x = -2 \text{ or } x = 5$$

- (iv) Suppose Jodie had said "A multiplied by B gives me -10".

What does your graph tell you about the solutions to this new problem?

There is no point on the graph where $y=-10$, the parabola's lowest point is (1.5, -6.25). Jodie must have made a mistake as it is impossible to get an answer of -10 using the equation $y=(x+1)(x-4)$.

- (b) Tom thinks of a puzzle to challenge Jodie.

He starts by saying:

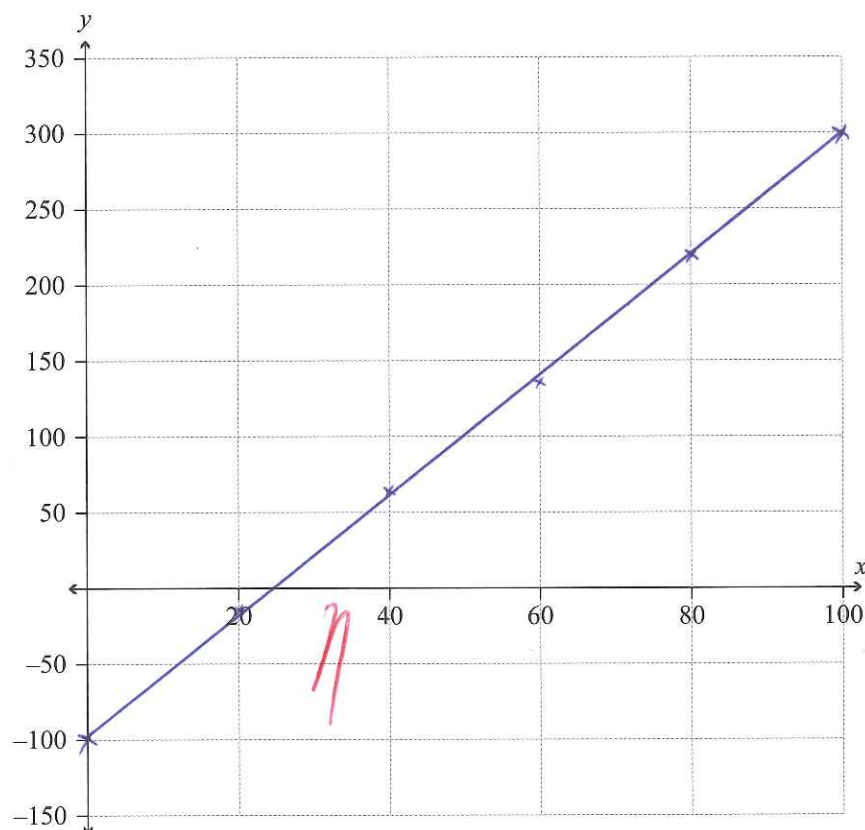
I think of a two-digit number.

I multiply it by 4 and take away 100 ...

- (i) What equation would you use to describe this relationship?

$$y = 4x - 100$$

- (ii) Draw the graph of this relationship on the axes below.



If you
need to
redraw this
graph, use
the grid on
page 15

(iii) Tom's whole puzzle is:

Guess my 2-digit number:

If I multiply it by 4 and take away 100 ...

I get the same as when I add 47 to it and then multiply the result by 1.12

Explain how the solution to Tom's question can be found, and give the solution as accurately as possible.

$$y = 4x - 100$$

$$y = 1.12(x + 47)$$

$$1.12(x + 47) = 4x - 100$$

$$1.12x + 52.64 = 4x - 100$$

$$152.64 = 2.88x$$

$$x = 53$$

The solution to Tom's question can be found using simultaneous equations, to find out that his two digit number is 53.

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Annotated Exemplar Template

Excellence exemplar for 91028 2015			Total score	21
Q	Grade score	Annotation		
1	E7	a(i). Points correctly plotted. a(ii). Correct answer and explanation. a(iii). Correct equation. a(iv). Correct answer. b(i). Correct equation. b(ii).Candidate has been unable to make connection between graph and context.		
.2	E7	a) Correct answer. b) Correct answer and explanation. c) Equation correct and explanation recognises that removal and growth are cancelling each other out. d) Both parts correct. (Graph redrawn page 14.) e) Equation correct for u. Candidate has recognised that coming earlier will change the graph but only did one day instead of two. f) Correct.		
3	E7	a(i). Graph OK but should be discrete points for r. a(ii). Correct a(iii). Candidate has found both solutions but not referred to graph. a(iv). Candidate has recognised minimum point of parabola and therefore impossibility of situation for r. (Very few candidates were able to do this.) b(i). Correct. b(ii).Correct. b(iii).Correct equation formed and solved.		