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MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
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Level 2 Mathematics and Statistics, 2015

91262 Apply calculus methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Excellence

TOTAL 22

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QUESTION ONE

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

Find the gradient of the graph of the function at the point where $x = -1$.

$f'(x) = 4x^3 + 4x$

$= 4 + 4$

$= 8$

(b) $f(x) = 8 + 3x + x^2 - \dfrac{x^3}{3}$

For what values of x is f a decreasing function?

Justify your answer.

You must show the use of calculus.

$f'(x) = 3 + 2x - x^2$

$0 = -x^2 + 2x + 3$

$x = 3$ and -1

$x = -2$ $f'(x) = -ve$

$x = 0$ $f'(x) = +ve$ $\therefore -1$ min

$x = 4$ $f'(x) = -ve$ $\therefore 3$ max

$x = 2$ $f'(x) = +ve$

x decreasing: $-1 > x > 3$

1

(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.

$V = L^3$

$V'(~~L~~) = 3L^2$

$V'(5) = 3 \times 25$

$= 75$ ~~cm³~~

(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, a m s^{-2}, t seconds after it passes the signal, can be modelled by the function

$t=0$ $v=40$

$$a(t) = (16 - 2t)$$

(i) What is the greatest speed attained by the train after it passes the signal?

$0 = 16 - 2t$ $\quad v = 16t - t^2 + c$ $\quad t=0$ $v=40 \therefore c=40$

$\frac{-16}{-2} = t$ $\quad v = 16t - \cancel{2}t^2 + 40$

$8 = t$ $\quad v = 16 \times 8 - 64 + 40$

$= 104 ms^{-1}$

(ii) How far past the signal does the train travel before it stops?

$0 = -t^2 + 16t + 40$

$t = 18.2$ or $\cancel{-2.2}$ invalid no -ve time

$d = \frac{-t^3}{3} + 8t^2 + 40t$ $\quad c=0$ as (d,t) $(0,0)$

$d = -\frac{(18.2)^3}{3} + 8 \times 18.2^2 + 40 \times 18.2$

$= 1368m$

$= 1400m.$

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t

E7

QUESTION TWO

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

$f(x) = 2x^2 - 3x + C$

$6 = 2 \times 16 - 12 + C$

$-14 = C$

$f(x) = 2x^2 - 3x - 14$

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

$g'(x) = 2x - 3$ $\quad$ $g(x) = 1.5^2 - 3 \times 1.5 + 18$

$0 = 2x - 3$ $\quad$ $y = 15.75$

$1.5 = x$

$y - y_1 = m(x - x_1)$

$y - 15.75 = 0(x - 1.5)$

$y = 15.75$

(ii) In relation to the graph, fully describe the point where this tangent meets the function.

When gradient function = 0. is the turning point of the graph. The tangent is ∴ a horizontal straight line which cuts the minimum of the graph ~~at the~~, point (1.5, 15.75) and cuts the y axis at 15.75

(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

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The mound can be modelled by

$$h = -0.5x^2 + 3x - 1.5$$

(i) What is the maximum height of the mound?

$h'(x) = -x + 3$

$0 = -x + 3$

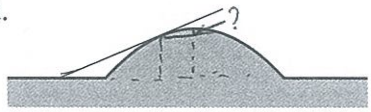
$-3 = -x$

$3 = x$

$h = -0.5 \times 9 + 9 - 1.5$

$y = \underline{3m}$

(ii) A ramp up the side of the mound is a tangent to the mound.



The ramp can be modelled by the function

$$h = 0.5x - c$$

← ~~find c~~ find y co-ordinate

$3 - y$ = answer

Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

$h'(x) = 0.5 \therefore m = 0.5$

$0.5 = -x + 3$

$2.5 = x$ of point on curve where tangent occurs.

~~$h = -0.5 \times 2.5^2 + 3 \times 2.5 - 1.5$~~

~~$= 2.575$~~

~~2.575~~

$h = -0.5 \times 2.5^2 + 3 \times 2.5 - 1.5$

$= 2.875$

$\therefore 3 - 2.875$

$= 0.125$

$(2.5, 2.875)$

ramp hit 0.125 m below the apex of the mound

(iii) The height h metres of a skateboard path at a horizontal distance r metres from another point Q, can be modelled by the function

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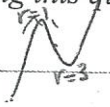
$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.

$h'(r) = r^2 - 4r + 3$



~~r = 3 and 1~~

$0 = r^2 - 4r + 3$

$r = 3$ and 1

$r = 2 \quad h'(r) = -ve \quad \therefore 1$ max 3 min.

$r = 0 \quad h'(r) = +ve$

$r = 4 \quad h'(r) = +ve$

$h = \frac{3^3}{3} - 2 \times 9 + 9 = 0 \quad$ min at $(3, 0)$

$h = \frac{1^3}{3} - 2 \times 1 + 3 = 1.33 \quad$ max at $(1, 1.33)$

$\cancel{h = \frac{3.4^3}{3} - 2 \times 3.4^2 + 3 \times 3.4 = 0.16}$

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~~due to the restrictions any vert~~

The maximum vertical height of skateboard is ~~at~~ 1.33m (When board is 1 m from Q) $\therefore$ does not break height regulation //

t

e8

QUESTION THREE

ASSESSOR'S USE ONLY

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

$a(t) = 12t^2 - 2t + 2$

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

$f'(x) = 3x^2 - 4x + 1$

$f'(2) = 3 \times 4 - 8 + 1$

$m = 5$

$y - 2 = 5(x - 2)$

$y = 5x - 8$

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

Use calculus methods to justify your answer.

$h'(t) = 20 - 10t$

$0 = 20 - 10t$

$\frac{-20}{-10} = t$

$2 = t$

$h = 20 \times 2 - 5 \times 4$

$= 20\text{m}$

will not break limit.

(d) For a function $y = -ax^2 + bx + c$,

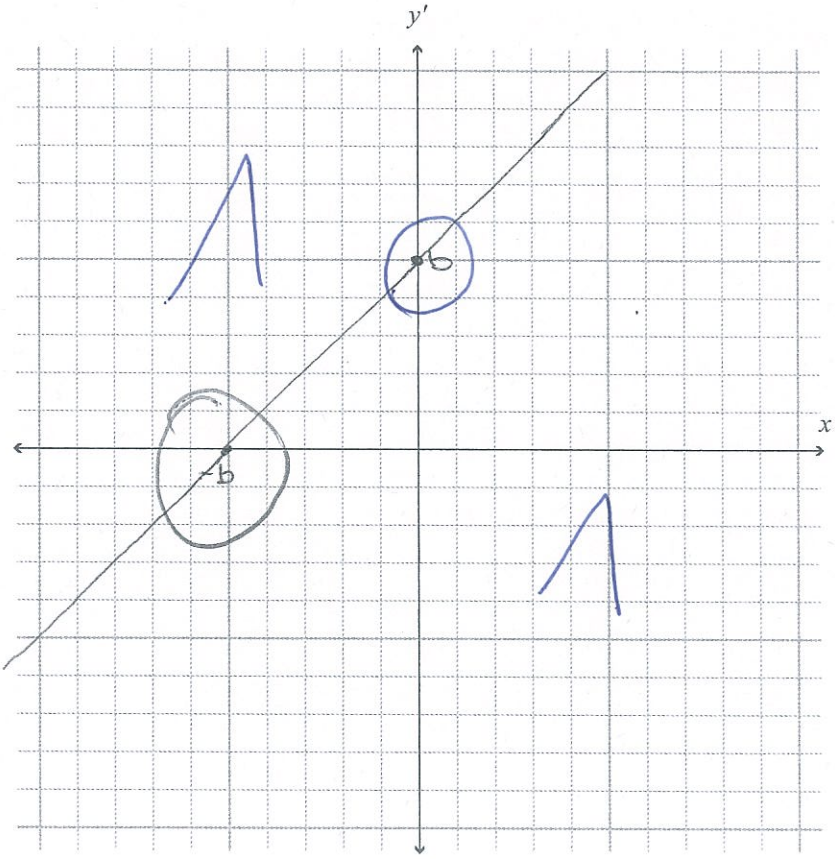
a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.

$\frac{dy}{dx} = -2ax + b$ $\quad -2cx = -b$

$\frac{dy}{dx} = -b + b$



If you need to redraw this graph, use the grid on page 10.

$y = 2(x - 3)$ $y = 2x - 6$

(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

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Find the maximum and minimum values of the product of x^2y

Justify your answer.

$y = 2x - 6$

$A = x^2(2x - 6)$

$A = 2x^3 - 6x^2$

$A'(x) = 6x^2 - 12x$

$0 = 6x^2 - 12x$

$x = 0$ and 2

$x = -0.5$ $A'(x) = +ve$ $0 = max$ max at $(0, 6)$

$x = 1$ $A'(x) = -ve$ $2 = min$ min at $(2, -2)$

$x = 3$ $A'(x) = +ve$

$x = 2$ $P = -8$ and $x = 0$ $P = 0$

6

t

E7

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Apply Calculus methods in solving problems

Descriptions for the exemplar numbers:

NOTE: Top down marking was used in this standard.

GRADE = EXCELLENCE

1. Correct derivative given and the two values for x found. Correct substitution into values on either side of the turning points to decide which turning point was a minimum and which was a maximum. Incorrect region of where the function was decreasing, -1>x>3.
2. Identified that v=0 and both of the t values found, ignoring the negative solution as invalid. Correct integration to find the distance formula, including finding c=0. Correct distance evaluated.
3. Correct derivative recognised. Equated the gradient function to 0.5 and found the correct x value of 2.5. Substituted this into the height formula to find the height of 2.875m. Complete solution given by subtracting the height from 3m.
4. Correct derivative found and both of the r values correctly evaluated. Both height values found for both of the r values. Small graph drawn with r values stated and an answer to the statement about compliance of the skate park.
5. X missing from the derivative and graph with a positive gradient instead of a negative gradient.
6. Relationship formed and correctly differentiated. Both x values correctly evaluated. Correct evaluation of the gradient on either side of the turning points to decide which turning point was a minimum and which was a maximum. Product at the minimum and maximum found.