See back cover for an English translation of this cover

2

91262M

912625

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Te Pāngarau me te Tauanga, Kaupae 2, 2012

91262M Te whakahāngai i ngā tikanga tuanaki hei whakaoti rapanga

2.00 i te ahiahi Rāhina 19 Whiringa-ā-rangi 2012
Whiwhinga: Rima

Paetae	Paetae Kaiaka	Paetae Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mehemea e ōrite ana te Tau Ākonga ā-Motu (NSN) kei tō pepa whakauru ki te tau kei runga ake nei.

Me whakautu e koe ngā pātai KATOA kei roto i te pukapuka nei.

Whakaaturia tō mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te (ngā) whārangi kei muri i te pukapuka nei, ka āta tohu ai i ngā tau pātai.

Tirohia mehemea kei roto nei ngā whārangi 2–27 e raupapa tika ana, ā, kāore hoki he whārangi wātea.

HOATU TE PUKAPUKA NEI KI TE KAIWHAKAHAERE HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

Kia 60 meneti hei whakautu i ngā pātai o tēnei pukapuka.

MĀ TE KAIMĀKA ANAKE

PĀTAI TUATAHI

(a) Ka tukuna he pānga f mā te $f(x) = 2x^3 - 10x + 5$.

Tātaihia te rōnaki o te kauwhata o f i te pūwāhi $x = 2$.

(b) Mō tētahi pānga p,

$p'(x) = 4 - 6x^2$

Ka whakawhiti te kauwhata o p mā te pūwāhi (2,5).

Kimihia te pānga $p(x)$.

You are advised to spend 60 minutes answering the questions in this booklet.

ASSESSOR'S USE ONLY

QUESTION ONE

(a) A function f is given by $f(x) = 2x^3 - 10x + 5$.

Find the gradient of the graph of f at the point where $x = 2$.

(b) For a function p,

$$p'(x) = 4 - 6x^2$$

The graph of p passes through the point (2,5).

Find the function $p(x)$.

MĀ TE KAIMĀKA ANAKE

(c) Ko te mamao ā-mita, s, o tētahi tarakihana mai i te pou kēti ka whakatauirahia e

$$s = 0.1\,t^2 + 5t,$$

ina ko t te roa o te wā ā-hēkona mai i te hipanga o te tarakihana i te pou kēti me te $0 \le t \le 20$.

Homai te whārite mō te tere o te tarakihana, ā, ka whakamahi i tēnei hei kimi i te wā ka eke te tere o te tarakihana ki te 8 m s^{-1}.

(d) E whai pānga ana te whakapeto kōhinu[1], f, o tētahi motukā (i te 100 km ki te rita) ki te tere, v, (i te km h^{-1}), e ai ki tēnei tātai

$$f = 16 - 0.2v + \frac{v^2}{250}$$

Whakaatuhia ko te whakapeto kōhinu he **mōkito** ina ko $v = 25$ km h^{-1}.

[1] penehīni

ASSESSOR'S USE ONLY

(c) The distance, s, of a tractor from a gate-post, in metres, is modelled by

$$s = 0.1\,t^2 + 5t,$$

where t is the time in seconds since the tractor passed the gate-post and $0 \leq t \leq 20$.

Give the equation for the velocity of the tractor, and use this to find when the velocity of the tractor is 8 m s^{-1}.

(d) The fuel consumption, f, of a car, in litres per 100 km, is related to the velocity, v, in km h^{-1}, by the formula

$$f = 16 - 0.2v + \frac{v^2}{250}$$

Show that the fuel consumption is a **minimum** when $v = 25$ km h^{-1}.

(e) Ka tīmata te tereina mai i te tūnga okioki.
Ko tōna whakaterenga t hēkona whai muri i tōna tīmatanga he

$$a = \frac{1}{4}\left(20 - t\right) \text{ m s}^{-2}$$

He aha te mamao ka oti i te tereina i roto i te 30 hēkona tuatahi?

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) A train starts from rest.
Its acceleration t seconds after it starts is given by

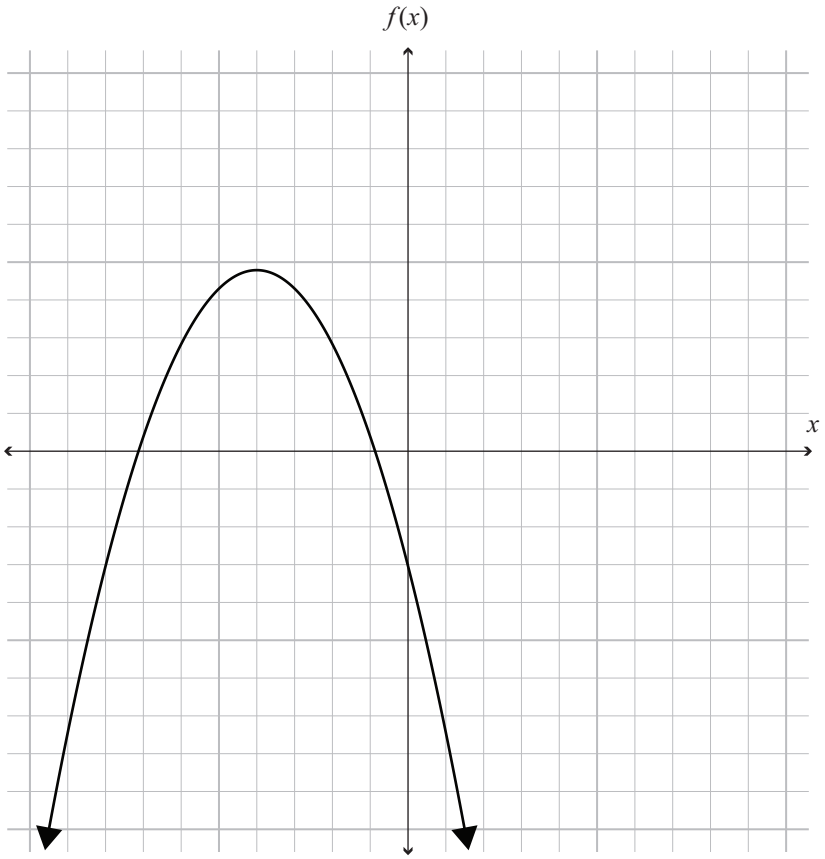
$$a = \frac{1}{4}\left(20 - t\right) \text{ m s}^{-2}$$

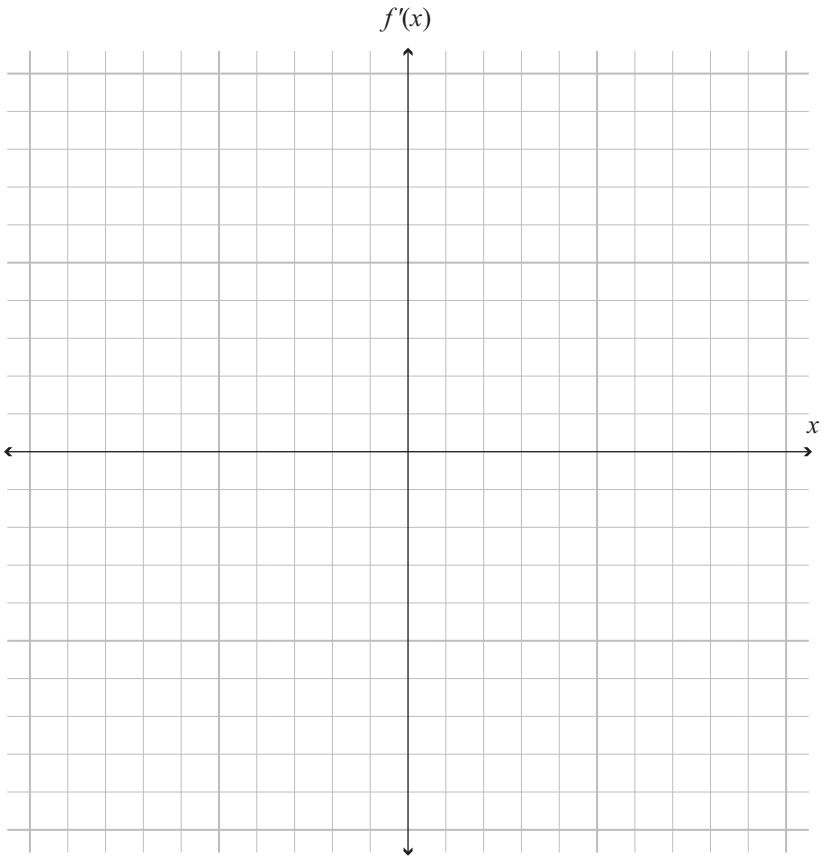
What distance does the train cover in the first 30 seconds?

PĀTAI TUARUA

MĀ TE KAIMĀKA ANAKE

(a) Tuhia te pānga rōnaki $f'(x)$ mō te pānga $f(x)$ kei raro:



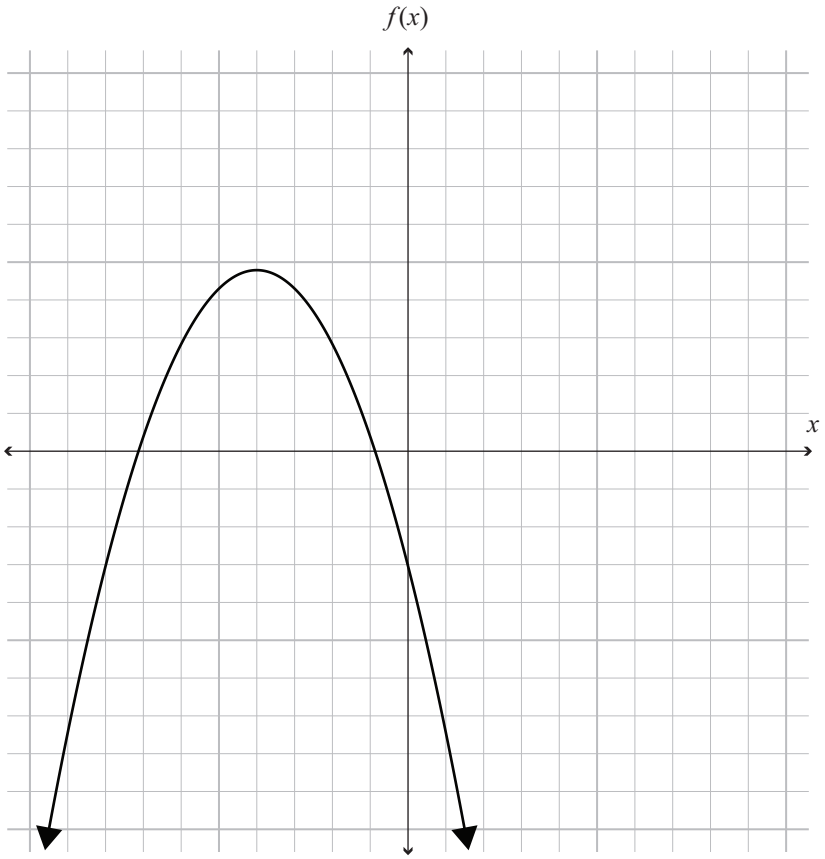


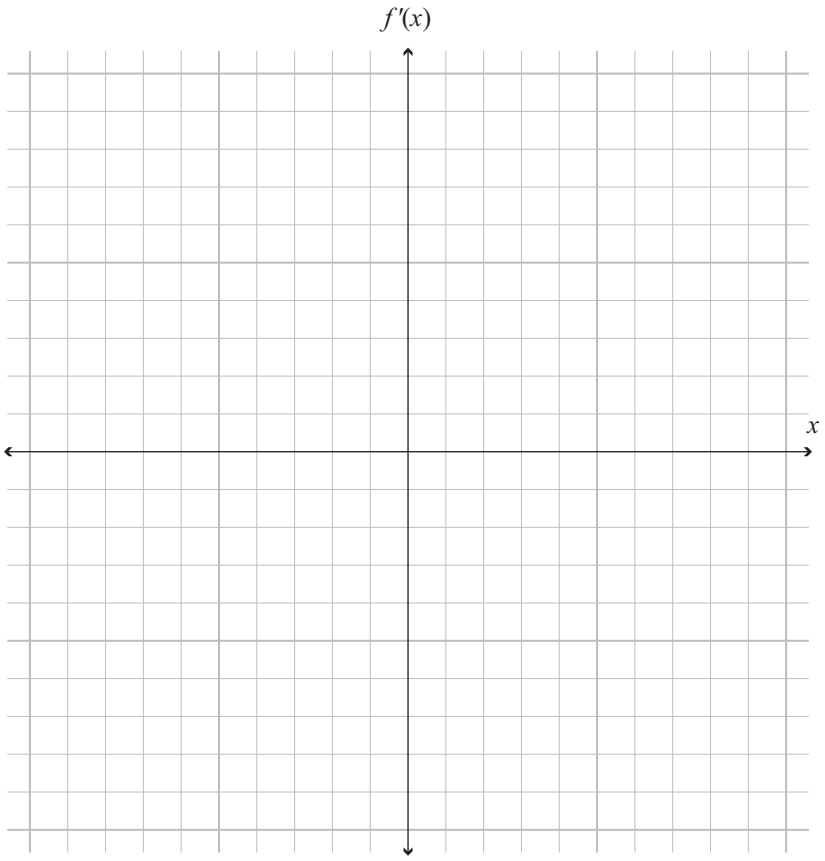
Ki te hiahia koe ki te tā anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 22.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Sketch the gradient function $f'(x)$ for the function $f(x)$ below:





If you need to redraw this graph, use the grid on page 23.

MĀ TE KAIMĀKA ANAKE

(b) (i) Whiriwhiria te taunga-x o te pūwāhi ki te kauwhata o $h(x) = x^2 - 12x$ ina e ōrite ana te rōnaki ki te 4.

(ii) Whiriwhiria te whārite o te pātapa ki te ānau o te $h(x) = x^2 - 12x$ i te pūwāhi (1,–11).

(c) $g(x) = x^3 - 9x^2 + 24x - 8$

He aha ngā uara o x he pānga paheke a g?

Me parahau koe i tō whakautu mā ngā tikanga tuanaki.

ASSESSOR'S USE ONLY

(b) (i) Find the x-coordinate of the point on the graph of $h(x) = x^2 - 12x$ where the gradient is equal to 4.

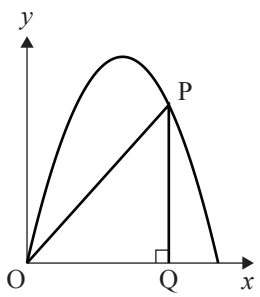
(ii) Find the equation of the tangent to the curve of $h(x) = x^2 - 12x$ at the point $(1,-11)$.

(c) $g(x) = x^3 - 9x^2 + 24x - 8$

For what values of x is g a decreasing function?

You must justify your answer, using calculus.

(d) Ka tuhia tētahi tapatoru hāngai OPQ pēnei e whakaatu nei, ā, ko te pūwāhi o O ko (0,0).



Ko P he pūwāhi ki te unahi $y = ax - x^2$

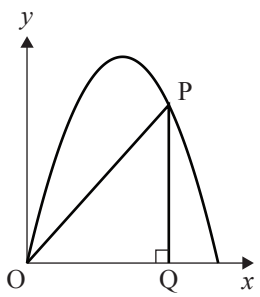
ā, kei runga a Q i te tuaka-x.

Whakaatuhia ko te horahanga mōrahi rawa o te tapatoru OPQ

ko te $\frac{2a^3}{27}$

MĀ TE KAIMĀKA ANAKE

(d) A right angled triangle OPQ is drawn as shown where O is at (0,0).
P is a point on the parabola $y = ax - x^2$
and Q is on the x-axis.



Show that the maximum possible area for the triangle OPQ is $\frac{2a^3}{27}$

ASSESSOR'S USE ONLY

PĀTAI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Ka whakawhiti te ānau $y = f(x)$ mā te pūtake (0,0), ā, ko tana pānga rōnaki ko

$$\frac{dy}{dx} = x^2 - 2x$$

Whiriwhiria ngā taunga o te pūwāhi o te ānau ina ko $x = 6$.

(b) Ka panoni haere te rōrahi, V, o tētahi mataono rite ina ka panoni te roa x o ngā taha.

$$V = x^3$$

Whiriwhiria te pāpātanga panoni o te rōrahi, e ai ki te roa o te taha, ina he 5 cm te taha.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) A curve $y = f(x)$ passes through the origin (0,0) and has a gradient function

$$\frac{dy}{dx} = x^2 - 2x$$

Find the co-ordinates of the point on the curve where $x = 6$.

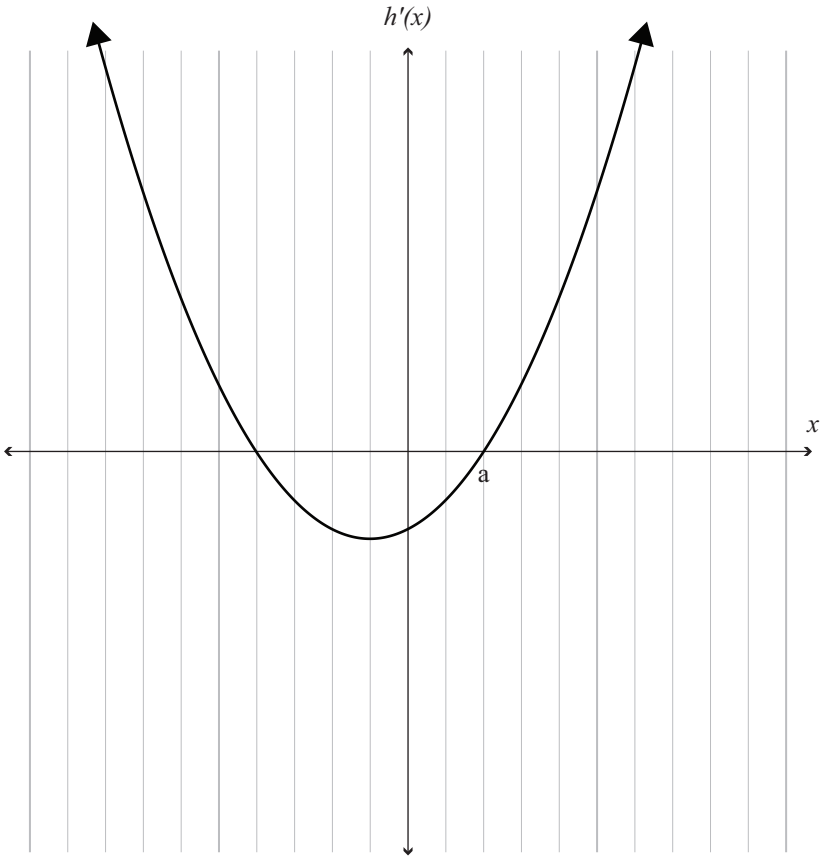
(b) The volume, V, of a cube changes as the length x of the sides changes.

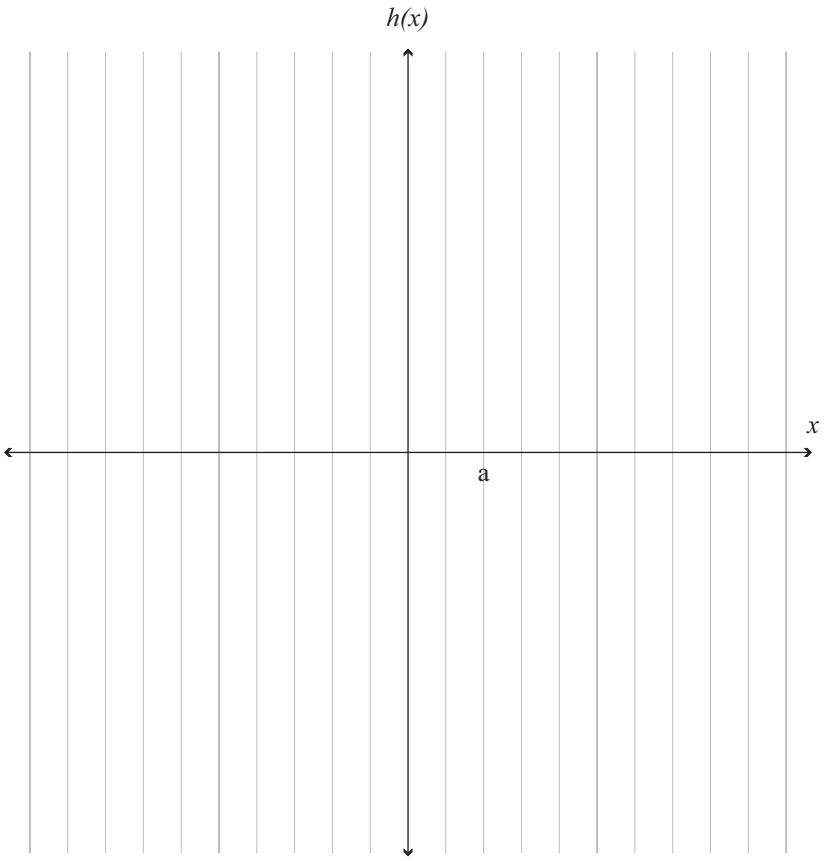
$$V = x^3$$

Find the rate of change of the volume, with respect to the length of the side, when the side is 5 cm.

(c) Tuhia te pānga $h(x)$ mō te pānga rōnaki $h'(x)$ ki raro, i runga i te mōhio ko $h(a) = 0$.

MĀ TE KAIMĀKA ANAKE

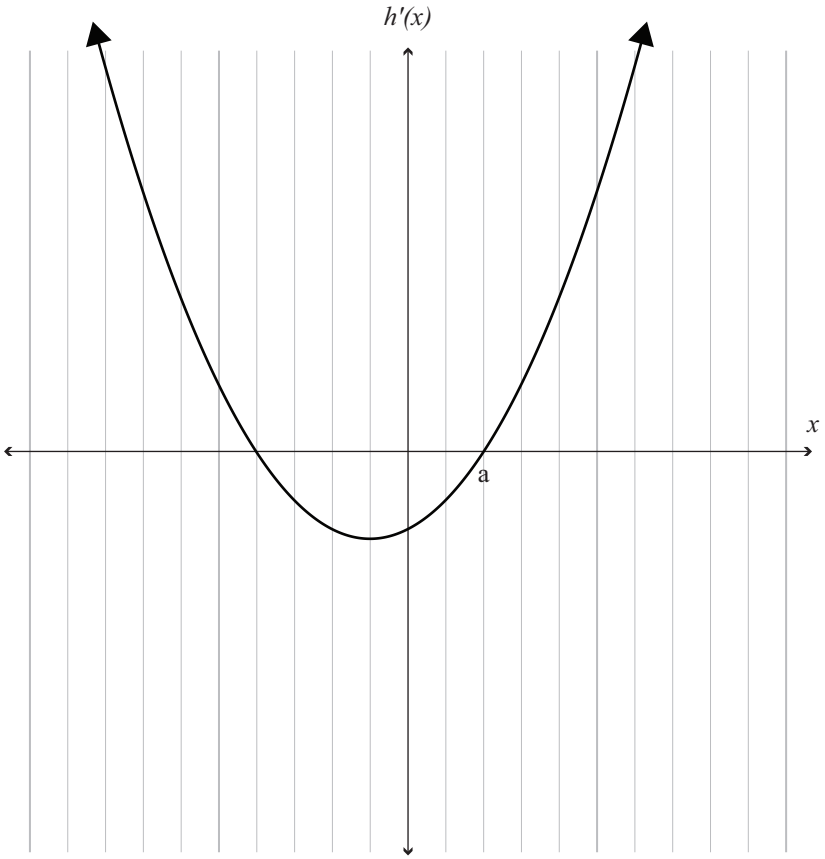


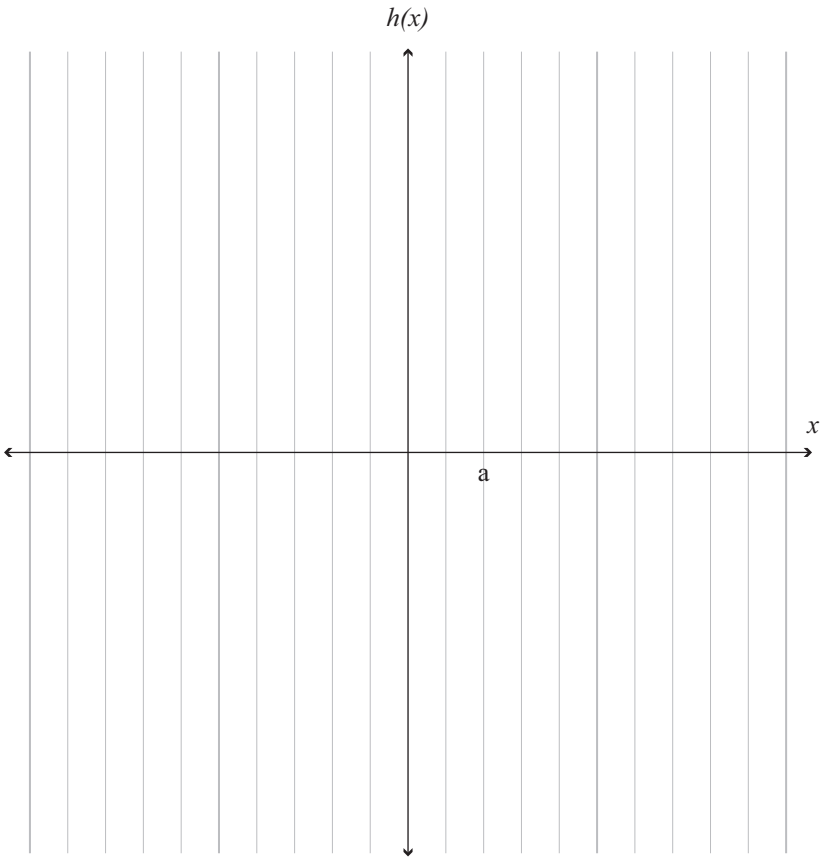


Ki te hiahia koe ki te tā anō i tēnei kauwhata, whakamahia te tukutuku i te whārangi 24.

ASSESSOR'S USE ONLY

(c) Sketch the function $h(x)$ for the gradient function $h'(x)$ below, given that $h(a) = 0$.





If you need to redraw this graph, use the grid on page 25.

(d) He pātapa tō te ānau o $f(x) = 2x^3 + Ax + B$ me te rōnaki 10 i te pūwāhi (–2,33).

Whiriwhiria ngā taunga o te pūwāhi o te ānau ina ko $x = 4$.

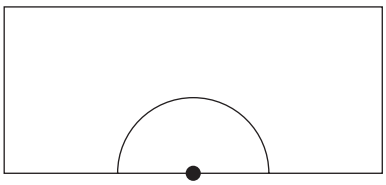
MĀ TE KAIMĀKA ANAKE

(d) The curve of $f(x) = 2x^3 + Ax + B$ has a tangent with gradient 10 at the point (–2,33).

Find the coordinates of the point on the curve where $x = 4$.

ASSESSOR'S USE ONLY

(e) Ka makeretia tētahi matū ki roto i te wai o tētahi hōpua wai tapawhā hāngai ki te pūwāhi o waenganui o te roanga.



Whai muri i te 0.1 meneti, ka horapa te matū ki te āhua o te porowhita haurua me te pūtoro r mita, ina

ko $r = 1 + 2t$

ā, ko t te wā ā-meneti atu i te tāpiritanga o te matū ki roto i te wai.

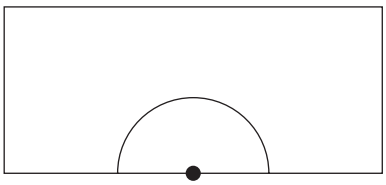
Ina tae tuatahi atu te matū ki te taha tawhiti o te hōpua wai, e kake ana te horahanga o te porowhita haurua mā te pāpātanga 60 m^2 min^{-1}.

Tātaitia te whānui o te hōpua wai. (Ko te horahanga o tētahi porowhita = πr^2)

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) A chemical is dropped into the water in a rectangular swimming pool at a point half-way along its length.



After 0.1 minutes, the chemical spreads in a semi-circular shape with radius r metres, where

$$r = 1 + 2t$$

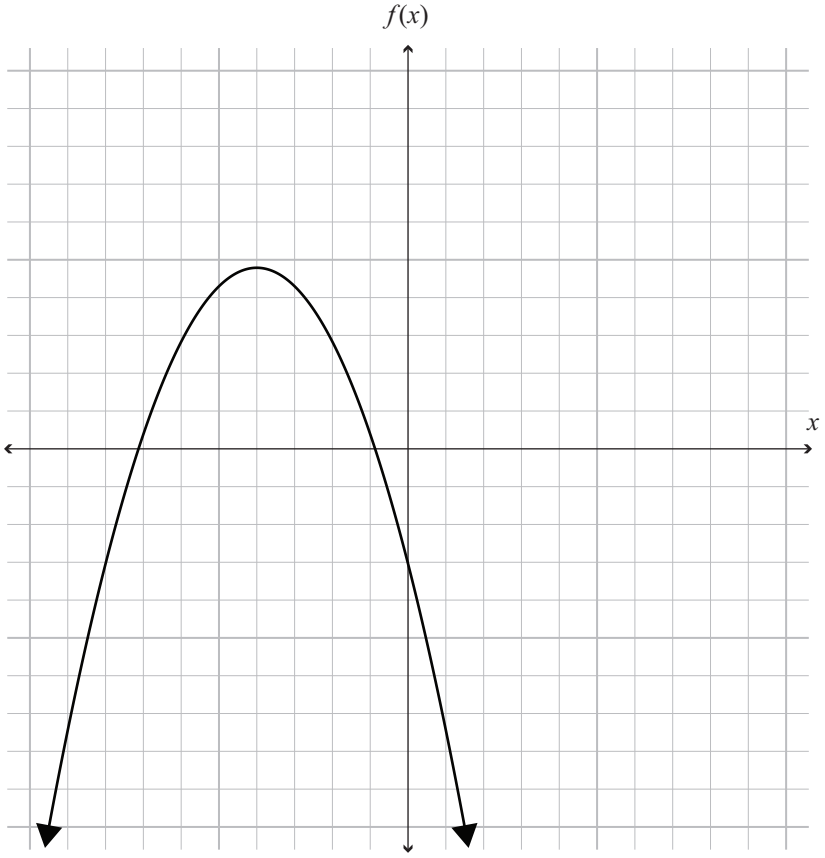
and t is the time in minutes since the chemical was added to the water.

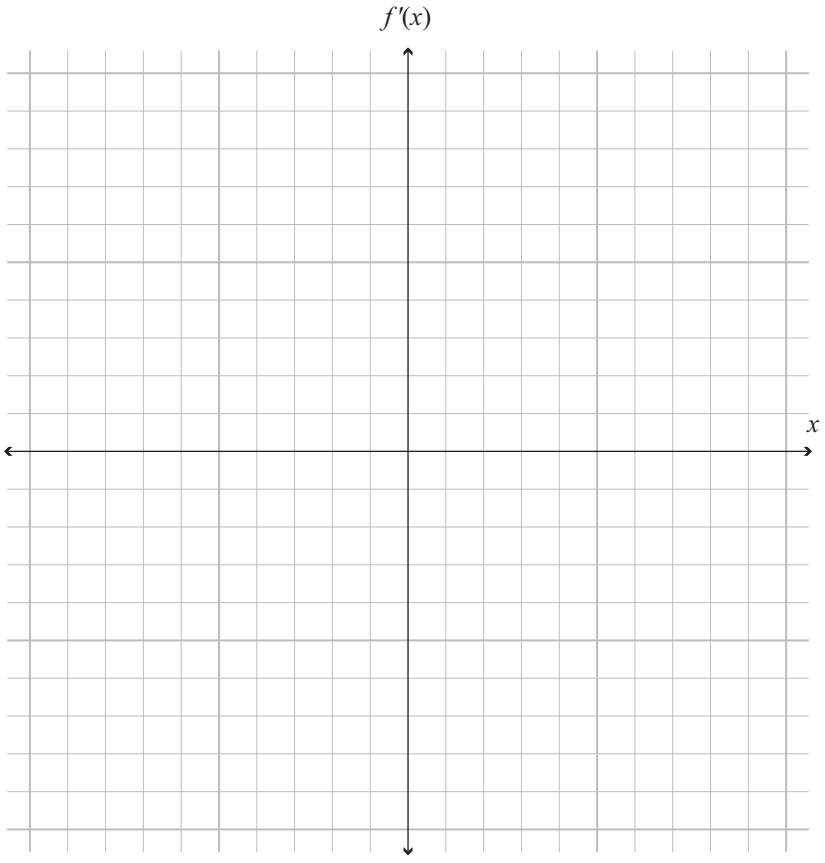
When the chemical first reaches the far side of the pool, the area of the semi-circle is increasing at the rate of 60 m^2 min^{-1}.

Find the width of the pool. (Area of a circle = πr^2)

Ki te hiahia koe ki te tuhi anō i te kauwhata mō te Pātai Tuarua (a), tuhia ki te tukutuku i raro.
Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

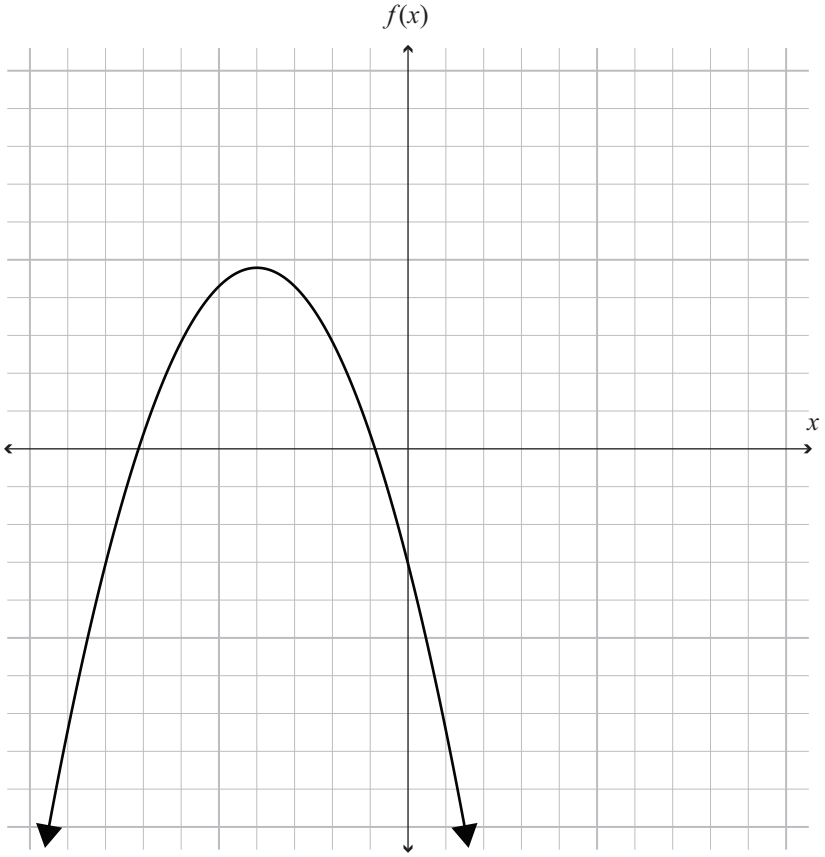
MĀ TE KAIMĀKA ANAKE

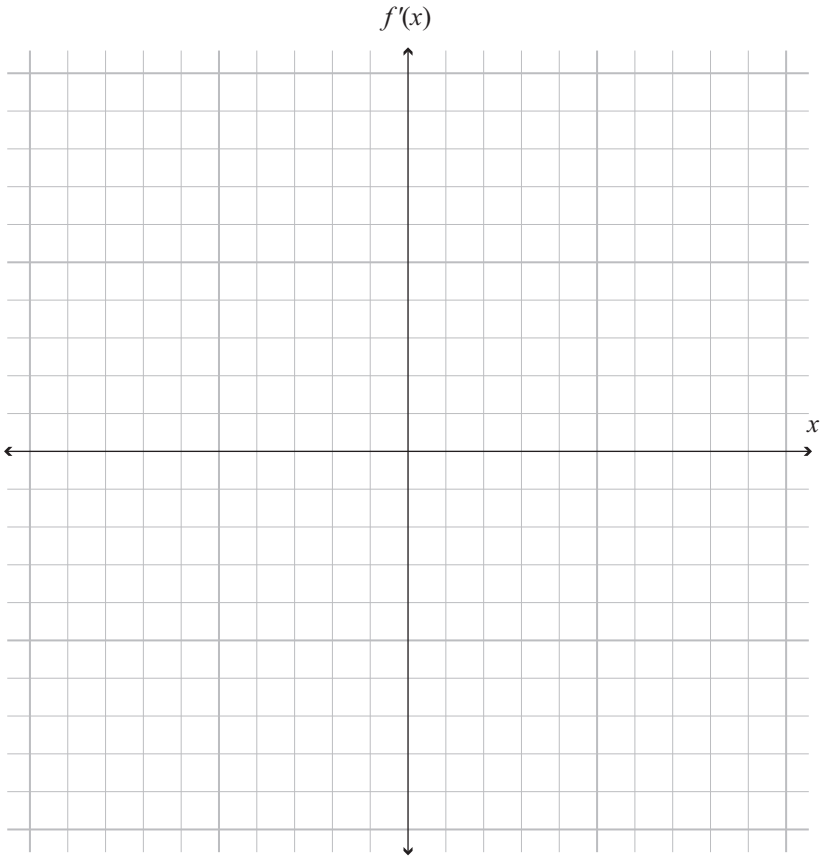




If you need to redraw your graph from Question Two (a), draw it on the grid below. Make sure it is clear which graph you want marked.

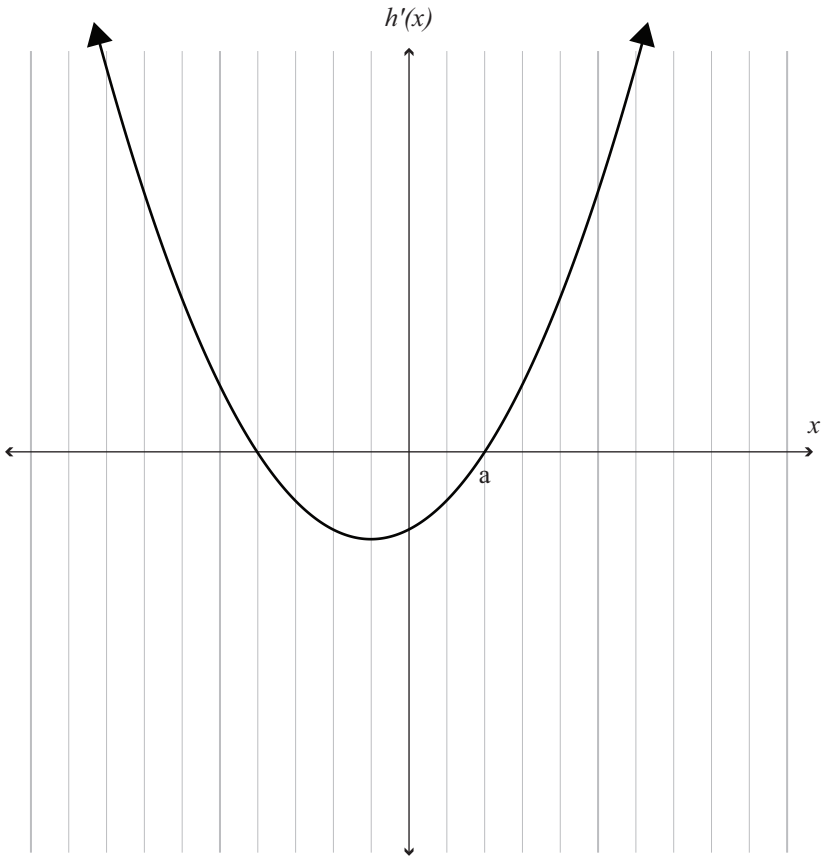
ASSESSOR'S USE ONLY

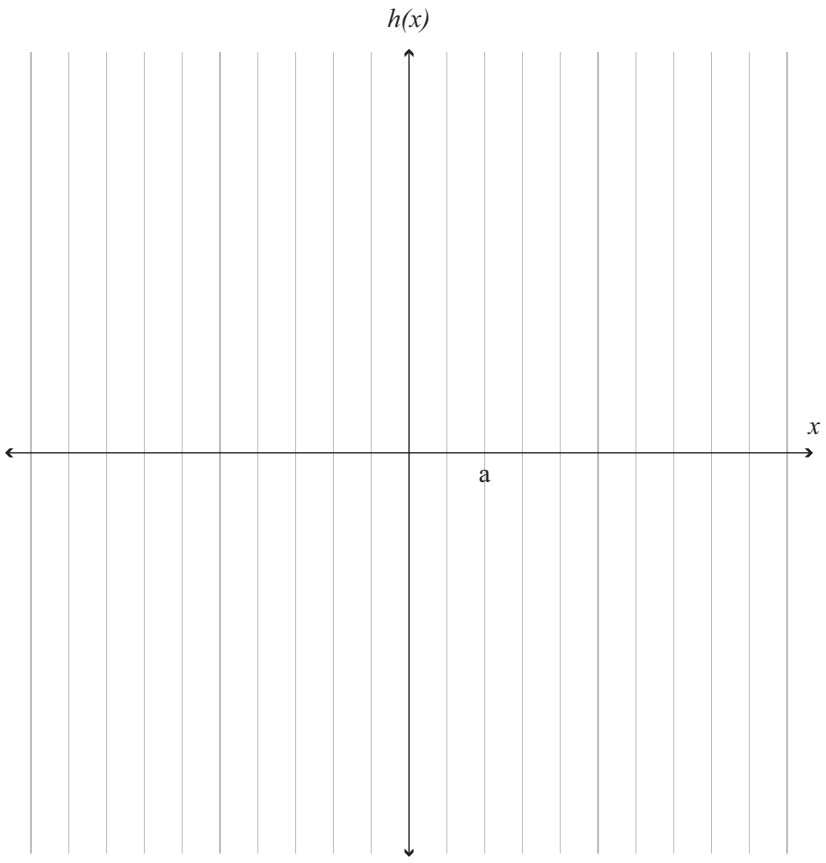




Ki te hiahia koe ki te tuhi anō i te kauwhata mō te Pātai Tuatoru (c), tuhia ki te tukutuku i raro.
Kia mārama te tohu ko tēhea te kauwhata ka hiahia koe kia mākahia.

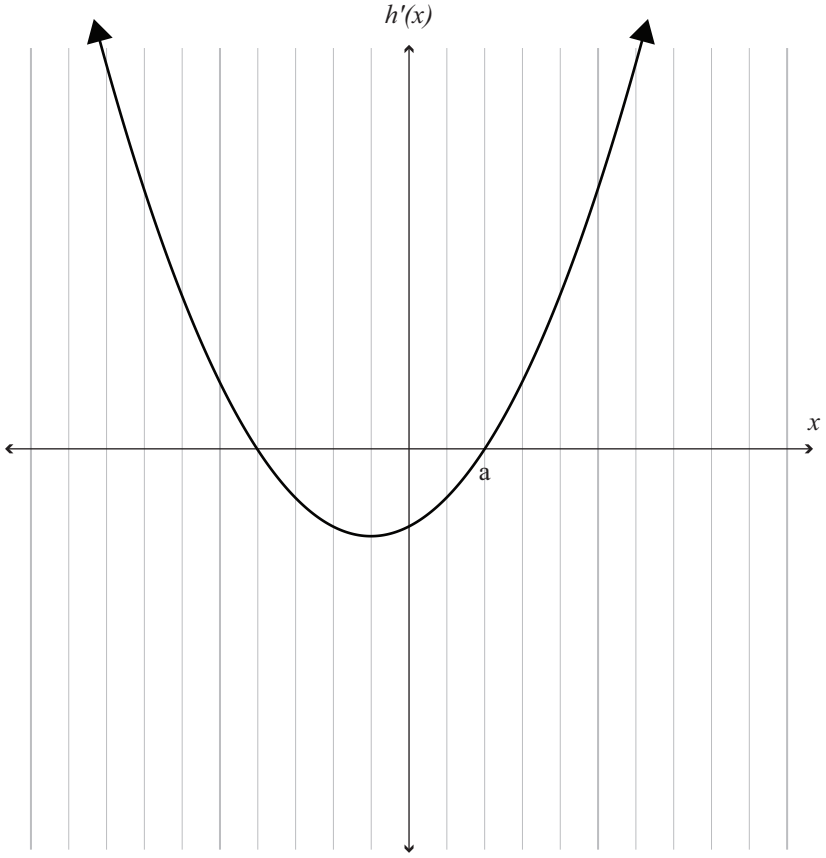
MĀ TE KAIMĀKA ANAKE

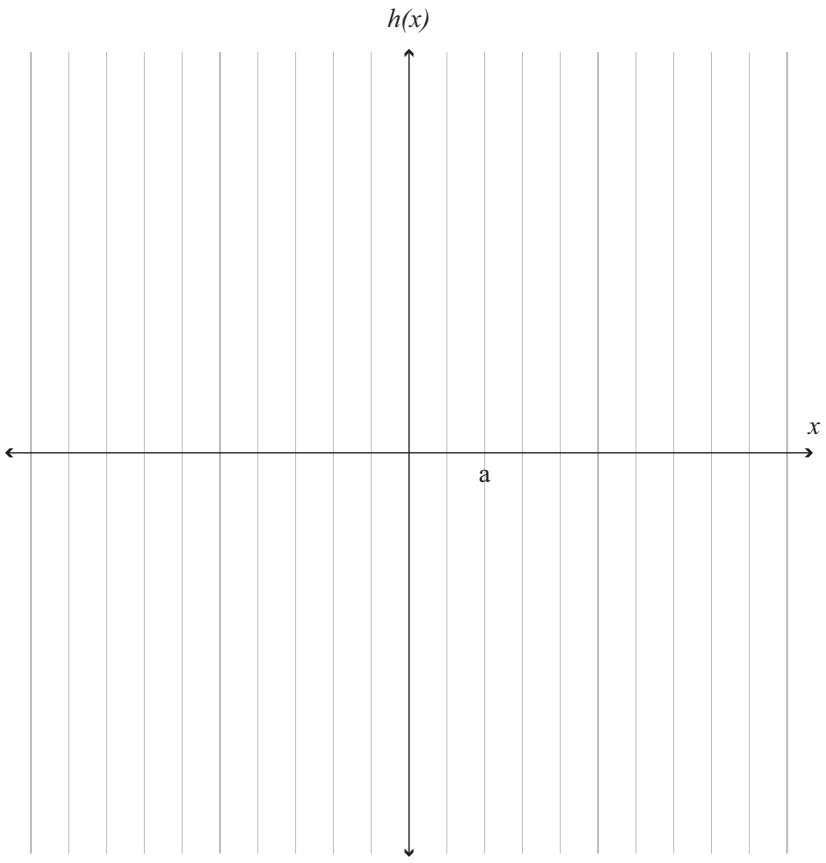




If you need to redraw your graph from Question Three (c), draw it on the grid below. Make sure it is clear which graph you want marked.

ASSESSOR'S USE ONLY





He puka anō mēnā ka hiahiatia.
Tuhia te (ngā) tau pātai mēnā e hāngai ana.

TAU PĀTAI

MĀ TE KAIMĀKA ANAKE

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 2 Mathematics and Statistics, 2012

91262 Apply calculus methods in solving problems

2.00 pm Monday 19 November 2012
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

9 1 2 6 2 M

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.