2

SUPERVISOR'S USE ONLY

91261

912610

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91261 Apply algebraic methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.

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Merit

TOTAL 17

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Annotated Exemplar Template

Merit exemplar for 91261 2015		Total score	17
Q	**Grade score**	**Annotation**	
1	M6	The two quadratic solutions in 1aiii provided r evidence, consideration of discriminant was irrelevant. Logs incorrectly applied in 1cii so only u evidence for setting up correct equations.	
2	E7	Question 2dii provided the only t evidence so overall E7.	
3	A4	There was only partial evidence in the first three questions. Question 3ai was not fully simplified, 3aii a correct answer was undone by incorrect simplification and in 3b a correct common denominator was formed on LHS. In 3d no inequality was given for the discriminant, the discriminant was not consistently calculated.	

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QUESTION ONE

(a) (i) Find the value of $\log_2 1024$.

$1024 = 2^x$

$\log 1024 = x \log 2$

$x = \log 1024 / \log 2$

$x = 10$ //

u

(ii) Solve the equation $\log_4(3w + 1) = 2$.

$3w+1 = 4^2$

$3w+1 = 16$

$3w = 15$

$w = 5$ //

u

(iii) Luka says that the equation $\log_x(4x + 12) = 2$ has only one solution.

Is he correct?

Find the solution(s), justifying your answer.

$4x+12 = x^2$

$-x^2 + 4x + 12 = 0$

$b^2 - 4ac$

$4^2 - 4 \times -1 \times 12 = 64$

$b^2 - 4ac > 0$ so 2 solutions //

$x = 6$ and $x = -2$

(He's) wrong

r

(b) Make x the subject of the equation $a^{2x} = b^{x+1}$.

$2x \log a = (x+1) \log b$

$\log a / \log b \; (-1) = \frac{x}{2x}$

$x = 2\left(\frac{\log a}{\log b} - 1\right)$ //

u

ASSESSOR'S USE ONLY

(c) The market value of Sue's house has been increasing at a constant exponential rate of 3% per annum since she bought it sixteen years ago at the start of 1999. At the start of 2015 it was worth \$350 000.

(i) Assuming the exponential growth is of the form $y = A\,r^t$, what was the value of the house at the start of 1999 when she bought it?

y = market value   A = 1999 value   r = 3% increase

$350000 = A \times 1.03^{16}$

$A = \$218108$

(ii) A friend also bought a house at the start of 1999 that cost \$200 000.

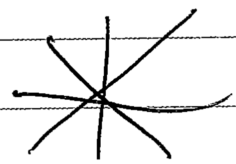
Its market value also has been steadily increasing, but at a slightly higher exponential rate of 3.5%.

Its value, \$$y$, t years after the start of 1999, is given by the function

$$y = 200\,000 \times (1.035)^t$$

If the houses continue to keep increasing in value at the original rates, in which year will the two houses be worth the same amount?

$218108 \times 1.03^{t} = 200000 \times 1.035^{t}$



$\frac{218108}{200000} \times 1.03^{t} = 1.035^{t}$

$1.1x + \log 1.03 = t \log 1.035$

$1.1t = \frac{\log 1.035}{\log 1.03}$

$1.1t = 1.16$

r

4

M6

ASSESSOR'S USE ONLY

QUESTION TWO

(a) Simplify $\frac{2x^2+7x-4}{2x^2-32}$

$\frac{\cancel{(2x-1)(x+4)}}{\cancel{(2x+8)(x+4)}}$ $\frac{(2x-1)\cancel{(x+4)}}{(2x-8)\cancel{(x+4)}}$

$\frac{2x+1}{2x-8}$ //

T

r

(b) If $a = y^{\frac{3}{4}}$, find an expression for a^7 in terms of y.

$\cancel{a^7 = y^{21/4}}$ $a^7 = y^{21/4}$ //

u

(c) Solve the equation $2u^{\frac{2}{3}} + 7u^{\frac{1}{3}} = 4$

$2u^2 + 7u = 64$

$2u^2 + 7u - 64 = 0$

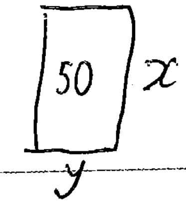
$u = 4.17$ $u = -7.67$ //

n

ASSESSOR'S USE ONLY

(d) Talia used timber to form the exterior sides of her rectangular garden. The length of the garden is x metres, and its area is 50 m^2.

(i) Show that the perimeter of the garden is given by $2x+\frac{100}{x}$



$\cancel{xy=50}$ $yx=50$

$2x+2y=2x+\frac{100}{x}$

$2y=\frac{100}{x}$

$2yx=100$

$yx=50$

A

n

(ii) If she uses 33 m of timber to build the sides, find the dimensions of the garden.

$2y+2x=33$

$y+x=16.5$

$yx=50$

$y=50/x$

$50/x+x=16.5$

$50+x^2=16.5x$

$x^2-16.5x+50=0$

$x=12.5 \quad x=4$

$12.5y=50$

$y=4$

$4y=50 \quad y=12.5$

$12.5\times4=50m^2$

$(2\times12.5)+(2\times4)=33m$

The garden is ~~12.5 × 4m~~

12.5 metres by 4 metres

t

(e) David and Sione are competing in a cycle race of 150 km.

Sione cycles on average 4 km per hour faster than David, and finishes half an hour earlier than David.

Find David's average speed.

You MUST use algebra to solve this problem. (Hint: average speed = $\frac{distance}{time}$*)*

~~v(t) = d+4~~ ~~v(t)~~ $v(t) = St^2 - 4t +$

$v(t) = xt^2 - 4t + 30$ AS

$150 = xt^2 - 4t - 30$

N

/

E7

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Simplify, giving your answer with positive exponents:

(i) $\left(\frac{a^{10}}{4a^5}\right)^{-2}$

$\frac{16a^{10}}{a^{20}}$

u

(ii) $\sqrt[5]{\left(\frac{32}{x^5}\right)^3}$

$\sqrt[5]{\frac{32^3}{x^{15}}}$

$\frac{32^{3/5}}{x^3}$

$8/x^3 = 2/x$ #

u

(b) Solve the following equation for t:

$$\frac{1}{t(t-1)} - \frac{1}{t} = \frac{3}{t-1}$$

$\frac{1}{t^2-t} - \frac{1}{t} = \frac{3}{t-1}$

$\frac{t}{t^3-t^2} - \frac{t^2-t}{t^3-t^2} = \frac{3}{t-1}$

$t^3 = \frac{3}{t-1}$

$t^3(t-1) = 3$

$t^4 - t^3 = 3$

$t^4 - t^3 - 3 = 0$

$t = 1.66$

$t = -1.12$

u

Question Three continues on the following page.

ASSESSOR'S USE ONLY

(c) For what value(s) of k does the graph of the quadratic function

$$y = x^2 + (3k - 1)x + (2k + 10)$$

never touch the x-axis?

R'S
.Y

ASSESSOR'S
USE ONLY

(d) The quadratic equation

$$mx^2 - (m+2)x + 2 = 0$$

has two positive real roots.

Find the possible value(s) of m, and the roots of the equation.

$\Delta = b^2 - 4ac$

$(m+2)^2 - (4 \times m \times 2)$

$m^2 + 4m + 4 - \cancel{4}\ 8m = 0$ $m^2 + 12m + 4 = 0$

$\cancel{m^2}\ m^2 \cancel{+} 12m + 4 = 0$

n

A4

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High Merit

TOTAL 19

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Annotated Exemplar Template

High Merit exemplar for 91261 2015		Total score	19
Q	**Grade score**	**Annotation**	
1	M5	M5 was awarded because 1aiii gave both quadratic solutions which provided r but gave wrong justification as it is the base which can't be negative. Question 1ci only contributed u evidence as the equation was correct but the answer was not presented in a meaningful context.	
2	E7	Question 2dii provided t evidence to give candidate E7, no correct final answers were given in 2c or 2e to provide further t evidence.	
3	E7	Candidate solution to 3d omitted the square in the m^2 term, consequently subsequent working was incorrect, E7 was awarded as only 3c provided t evidence.	

QUESTION ONE

ASSESSOR'S USE ONLY

(a) (i) Find the value of $\log_2 1024$.

$2^x = 1024$

$x = 10$ //

~~Value =~~

u

(ii) Solve the equation $\log_4(3w + 1) = 2$.

$4^2 = 3w + 1$

$16 = 3w + 1 \quad \therefore \quad 3w = 15$

$w = \frac{15}{3}$

$w = 5$ //

u

(iii) Luka says that the equation $\log_x(4x + 12) = 2$ has only one solution.

Is he correct?

Find the solution(s), justifying your answer.

$x^2 = 4x + 12$

$x^2 - 4x - 12 = 0$

$\Rightarrow x = 6, -2$ Sub $x = 6$ into $4x + 12$

$\log_6(24 + 12) = 2 \quad \therefore \log_6(36) = 2$

AC.

Sub $x = -2$ $\quad \log_{-2}(-8 + 12) = 2$ Only one answer as cannot log a negative number

r

(b) Make x the subject of the equation $a^{2x} = b^{x+1}$.

$(2x)\log a = (x + 1)\log b$

$\frac{2x \log a}{x + 1} = \log b$

$\frac{2x}{x + 1} = \frac{\log b}{\log a}$ //

u

ASSESSOR'S USE ONLY

(c) The market value of Sue's house has been increasing at a constant exponential rate of 3% per annum since she bought it sixteen years ago at the start of 1999. At the start of 2015 it was worth \$350 000.

(i) Assuming the exponential growth is of the form $y = A\,r^t$, what was the value of the house at the start of 1999 when she bought it?

~~y = 350,000~~ $350,000 = A \cdot (1.03)^{16}$

~~350,000 = A ×~~ $\frac{350,000}{(1.03)^{16}} = A$

$A = 218,108.4287$

u

(ii) A friend also bought a house at the start of 1999 that cost \$200 000.

Its market value also has been steadily increasing, but at a slightly higher exponential rate of 3.5%.

Its value, \$$y$, t years after the start of 1999, is given by the function

$y = 200000 \times (1.035)^t$

If the houses continue to keep increasing in value at the original rates, in which year will the two houses be worth the same amount?

$218,108.43 (1.03)^t = 200,000 \times (1.035)^t$

u

M5

$2x^2 + 7x - 4$

$\frac{(2x + 8)(2x - 1)}{2 \quad\quad 1}$

$(x+4)(2x-1)$

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Simplify $\dfrac{2x^2+7x-4}{2x^2-32}$

$\cancel{2x^2 + 7x - 4 \ -8}$

$\cancel{2x^2 \ 64}$

$= \dfrac{(2x-1)(x+4)}{\cancel{\frac{(2x+8)(2x-8)}{2 \quad\quad 2}}}$

$= \dfrac{(2x-1)\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}$

$= \dfrac{2x-1}{x-4}$ //

a

(b) If $a = y^{\frac{3}{4}}$, find an expression for a^7 in terms of y.

$a = \sqrt[4]{y^3}$, $a^7 = \left(y^{\frac{3}{4}}\right)^7$

$a^7 = y^{5.25}$

$a^7 = \underline{y^{\frac{21}{4}}}$ //

u

(c) Solve the equation $2u^{\frac{2}{3}} + 7u^{\frac{1}{3}} = 4$

$2 \times \sqrt[3]{u^2} + 7\sqrt[3]{u} = 4$

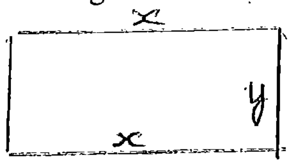
$2^3 \times u^2 + 7^3 u = 4^3$

$8u^2 + 343u = 64$ //

n

ASSESSOR'S USE ONLY

(d) Talia used timber to form the exterior sides of her rectangular garden. The length of the garden is x metres, and its area is 50 m^2.



(i) Show that the perimeter of the garden is given by $2x+\dfrac{100}{x}$

$x \times y = 50 \quad \therefore \; x = \dfrac{50}{y}, \; y = \dfrac{50}{x}$

$\therefore \; \dfrac{50}{x} + \dfrac{50}{x} + 2x = \text{perimeter}$

$\therefore \; \dfrac{100}{x} + 2x = \text{perimeter}$ //

u.

(ii) If she uses 33 m of timber to build the sides, find the dimensions of the garden.

$\dfrac{100}{x} + 2x = 33$

$100 + 2x^2 = 33x$

$2x^2 - 33x + 100 = 0$

$(x - 12.5)(x - 4) = 0$

$x = 12.5, \text{ or } x = 4$

Sub $x = 12.5$ into $y = \dfrac{50}{x} \; \therefore \; y = 4$

Sub $x = 4$ into $y = \dfrac{50}{x} \; \therefore \; y = 12.5$

t.

$\therefore$ Dimensions of garden are

4m by 12.5m

(e) David and Sione are competing in a cycle race of 150 km.

Sione cycles on average 4 km per hour faster than David, and finishes half an hour earlier than David.

Find David's average speed.

You MUST ~~use algebra to~~ solve this problem. (Hint: average speed = $\frac{distance}{time}$*)*

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~~Speed~~ = 150 //

N

0

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Simplify, giving your answer with positive exponents:

(i) $\left(\frac{a^{10}}{4a^5}\right)^{-2}$ $= \left(\frac{4a^5}{a^{10}}\right)^2 = \frac{16a^{10}}{a^{20}} = \frac{16}{a^{10}}$

Ans $= \frac{16}{a^{10}}$ //

r

(ii) $\sqrt[5]{\left(\frac{32}{x^5}\right)^3}$ $= \left(\frac{32}{x^5}\right)^{\frac{3}{5}} = \frac{8}{x^3}$ /

r

(b) Solve the following equation for t:

$$\frac{1}{t(t-1)} - \frac{1}{t} = \frac{3}{t-1}$$

$\frac{1}{t^2 - t} - \frac{1}{t} = \frac{3}{t-1}$ //

n.

Question Three continues on the following page.

(c) For what value(s) of k does the graph of the quadratic function

$$y = x^2 + (3k - 1)x + (2k + 10)$$

never touch the x-axis?

ASSESSOR'S USE ONLY

Graph never touch x axis when $b^2 - 4ac < 0$

$b = 3k - 1$, $a = 1$, $c = 2k + 10$

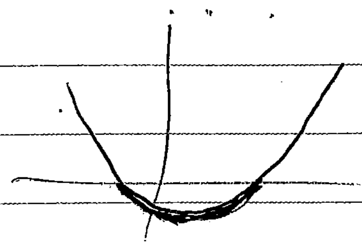
$\therefore \quad (3k-1)^2 - 4(1)(2k+10) < 0$

$9k^2 - 6k + 1 - 4(2k + 10) < 0$

$9k^2 - 6k + 1 - 8k - 40 < 0$

$9k^2 - 14k - 39 < 0$

$k < 3$ and $k > -1.4$



$-1.4 < k < 3$ //

t

ASSESSOR'S USE ONLY

(d) The quadratic equation

$$mx^2 - (m+2)x + 2 = 0$$

has two positive real roots.

Find the possible value(s) of m, and the roots of the equation.

Two positive real roots when $b^2 - 4ac > 0$

$a = m,\ b = -(m+2),\ c = 2$

$a = m,\ b = -m-2,\ c = 2$

$(-m-2)^2 - 4(m)2 > 0$

$\{-(m+2)\}^2$

$m^2 + 4m + 4 - 8m > 0$

$5m + 4 - 8m > 0$

$4 - 3m > 0$

$-3m > -4$

$m < \frac{-4}{-3}$

$m < \frac{4}{3}$

u

E7