2

SUPERVISOR'S USE ONLY

91262

912620

NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91262 Apply calculus methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Low Merit

TOTAL 16

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

Find the gradient of the graph of the function at the point where $x = -1$.

$f'(x) = 4x^3 + 4x$

$f'(-1) = 4(-1)^3 + 4(-1)$

$m = -8$

1

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(b) $f(x) = 8 + 3x + x^2 - \frac{x^3}{3}$

For what values of x is f a decreasing function?

*

Justify your answer.

You must show the use of calculus.

$f'(x) = 3 + 2x - x^2$

$x = 3$, $x = -1$

2

$f(3) = 8 + 3(3) + (3)^2 - (3)^3/3$

$= 23$

$f(-1) = 8 + 3(-1) + (-1)^2 - (-1)^3/3$

$= 6.33$

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(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.

$V = 5 \times 5 \times 5$

$V = 5^3$

$V = L^3$

$V' = 3L^3$

$V'(5) = 3(5)^3$

$= 375$

3

~~When the volume is changing~~

When Lengths are 5cm, volume changes at rate of ~~...~~ 375 cm^3

n

ASSESSOR'S USE ONLY

(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, a m s^{-2}, t seconds after it passes the signal, can be modelled by the function

$a(t) = (16 - 2t)$

(i) What is the greatest speed attained by the train after it passes the signal?

4

$\cancel{v(t) = 16t - t^2 + c}$ $\qquad$ $16 - 2t = 0$

$40 = 16t - t^2 + c$ $\qquad$ $16 = 2t$

$\cancel{16t - t^2 + c - 40 = 0}$ $\qquad$ $16/2 = t$

$a(8) = (16 - 2(8))$ $\qquad$ $8 = t$

$a(8) = 0$

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(ii) How far past the signal does the train travel before it stops?

5

$v(t) = 16t - t^2 + c$

$v(8) = 16(8) - (8)^2 + c$

$0 = 64 \cancel{+c}$

$c = +64$

$d(t) = 8t^2 - \frac{1}{2}t^3 + 64t$

$d(8) = 8(8)^2 - \frac{1}{2}(8)^3 + 64(8)$

$d = 256$ m

n

A4

QUESTION TWO

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

$f(x) = 2x^2 - 3x + C$

$f(4) = 2 \times (4)^2 - 3(4) + C$

$f(4) = 20 + C$

$C = -20$

$f(x) = 2x^2 - 3x - 20$

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

$g'(x) = 2x - 3$

$0 = 2x - 3$

$2x = 3$

$x = 3/2$

$g(3/2) = (3/2)^2 - 3(3/2) + 18$

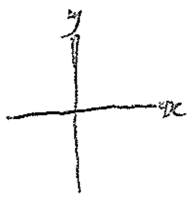
$y = 15.75$

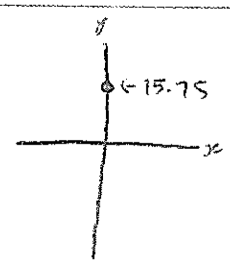
$y - 15.75 = 0(x - 3/2)$

$y = 15.75$

(ii) In relation to the graph, fully describe the point where this tangent meets the function.

The point stays on 0 on the x axis and is placed 15.75 up on the y axis





(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

The mound can be modelled by

$$h = -0.5x^2 + 3x - 1.5$$

(i) What is the maximum height of the mound?

$\frac{dh}{dx} = -1x + 3$

$0 = -1x + 3$

$+x = 3$

$h(-3) = -0.5(+3)^2 + 3(+3) - 1.5$

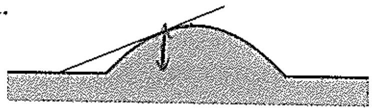
$h = 3$ metres

6

(ii) A ramp up the side of the mound is a tangent to the mound.

The ramp can be modelled by the function

$$h = 0.5x - c$$



Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

ASSESSOR'S USE ONLY

(iii) The height h metres of a skateboard path at a horizontal distance r metres from another point Q, can be modelled by the function

r = metres

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.

~~$h' = 3r^2 - 4r + 3$~~

~~$0 = 3r^2 - 4r + 3$~~

~~$3 = \frac{r^3}{3} - 2r^2 + 3r$~~

~~$\frac{r^3}{3} - 2r^2 + 3r - 3 = 0$~~

Xed out

$h' = r^2 - 4r + 3$

$0 = r^2 - 4r + 3$

$r = 3$

$r = 1$

~~using $r=3$ ... $(3)^2 - 4(3) + 3$~~

turning points are $(1, 4/3)$ and $(3, 0)$.

using $r = 3$

$h = \frac{(3)^3}{3} - 2(3)^2 + 3(3)$

$= 0$ m

Using $r = 1$

$h = \frac{(1)^3}{3} - 2(1)^2 + 3(1)$

$= 4/3$ m

The curve does comply with the height regulations.

7

ASSESSOR'S USE ONLY

QUESTION THREE

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

$a(t) = 12t^2 - 2t + 2$

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

$f'(x) = 3x^2 - 4x + 1$

$f'(2) = 3(2)^2 - 4(2) + 1$

$m = 5$

$y - 2 = 5(x - 2)$

$y - 2 = 5x - 10$

$y = 5x - 8$

8

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

Use calculus methods to justify your answer.

$h' = 20 - 10t$

$0 = 20 - 10t$

$20 = 10t$

$20/10 = t$

$t = 2$

$h = 20(2) - 5(2)^2$

$h = 20$

The firework doesn't break the limit

9

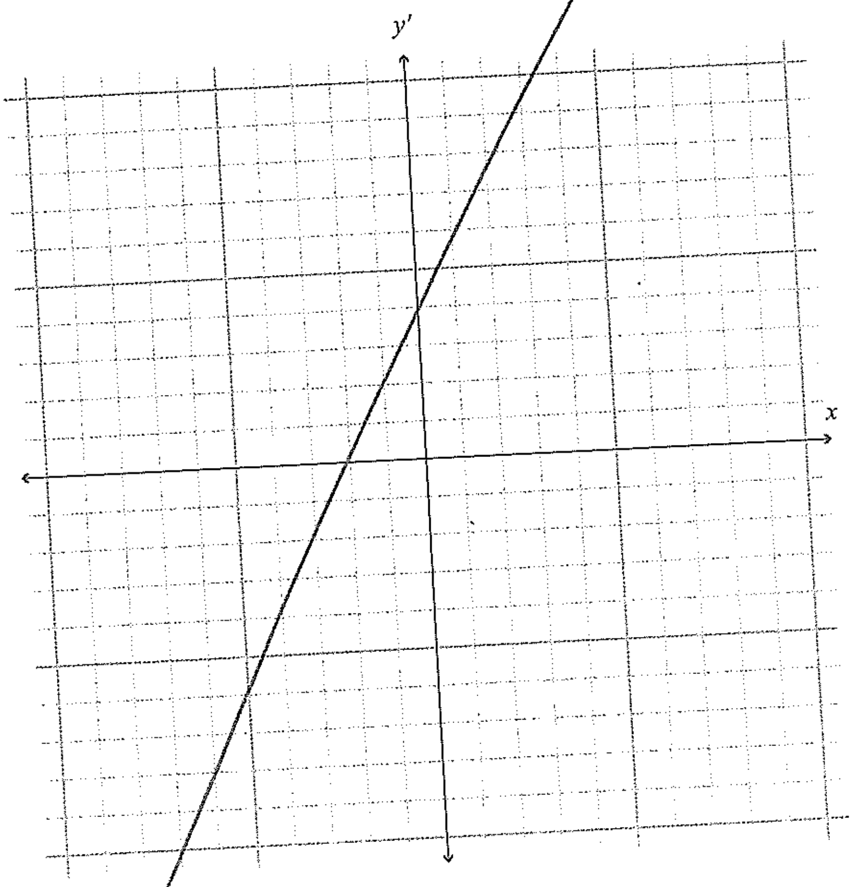
ASSESSOR'S USE ONLY

(d) For a function $y = -ax^2 + bx + c$,

a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.



If you need to redraw this graph, use the grid on page 10.

(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

Find the maximum and minimum values of the product of x^2y.

Justify your answer.

$y = 2(x-3)$

$y = 2x-6$

10

$x^2(2x-6)$ = product

$2x^3 - 6x^2 \rightarrow$ product' $= 6x^2 - 12x$

$x = 0$ and $x = -2$

using $x=0$ $y = 2(0)-6$

$y = -6$

$\rightarrow 2(0)^3 x - 6$

$= 0$

using $x = -2$ $y = 2(-2) - 6$

$y = -10$

$\rightarrow (-2)^2 \times (-10)$

x^2 y

-40

2x²(

ELSD

maximum value of 0

minimum value of -40

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M6

91262 2015

Apply Calculus methods in solving problems

Descriptions for the exemplar numbers:

NOTE: Top down marking was used in this standard.

GRADE = LOW MERIT

1. Correct derivative found and x=-1 correctly substituted into the derivative to calculate the gradient.
2. Correct derivative given and the two values for x found. No region identified and no justification for when the function was decreasing.
3. Equation incorrectly set up leading to an incorrect solution.
4. Acceleration equation correctly equated to zero and t=8 second found. No velocity value given.
5. No relevant working.
6. Correct derivative found and equated to zero. This gives an x value of 3. This is then substituted into the height formula to get the correct height of 3m.
7. Correct derivative found and the two correct r values stated. Both the corresponding y values correctly evaluated. No justification for the nature of the turning points and the curve not described.
8. Correct derivative found and x=2 substituted accurately into the derivative to find m=5. The gradient correctly substituted into the point gradient formula to find the correct equation of the tangent.
9. Correct derivative found, equated to zero and the x value of 2 seconds found. Correctly substituted into the height formula to find h=20m. Then a correct and consistent answer to the question "will the firework break the 50m limit"?
10. Correct relationship formed and differentiated accurately. Incorrect x value.

2

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High Merit

TOTAL 19

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = x^4 + 2x^2 - 5$

Find the gradient of the graph of the function at the point where $x = -1$.

$f'(x) = 4x^3 + 4x$

$f'(-1) = -8$

(b) $f(x) = 8 + 3x + x^2 - \frac{x^3}{3}$

For what values of x is f a decreasing function?

Justify your answer.

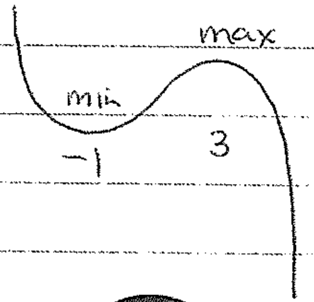
You must show the use of calculus.

$f'(x) = 3 + 2x - x^2$

(negative cubic)

$0 = 3 + 2x - x^2$

$x = 3, -1$



Decreasing $x > 3$

Decreasing $x < -1$

(c) Calculate the rate at which the volume of a cube is changing with respect to its length, at the instant when the length of each edge of the cube is 5 cm.

$V = L^3$

$V' = 3L^2 \Rightarrow 75$

$V'(5) = 3 \times 5^2$

(d) A train passes a signal at a velocity of 40 m s^{-1}.
The train's acceleration, a m s^{-2}, t seconds after it passes the signal, can be modelled by the function

$$a(t) = (16 - 2t)$$

(i) What is the greatest speed attained by the train after it passes the signal?

$a = 16 - 2t$ $\qquad$ $V = 16t - t^2 + 40$

$0 = 16 - 2t$ $\qquad$ $V = 16(8) - 8^2 + 40$

$\therefore t = 8$ seconds $\qquad$ $V = 104$ m/s

3

(ii) How far past the signal does the train travel before it stops?

ASSESSOR'S USE ONLY

E7

ASSESSOR' USE ONLY

QUESTION TWO

(a) The gradient of function f is given by $f'(x) = 4x - 3$

The point (4,6) lies on the graph of the function.

Find the equation of the function f.

$f(x) = 2x^2 - 3x + c$

$6 = 2(4)^2 - 3(4) + c \quad c = 14$

$f(x) = 2x^2 - 3x - 14$

(b) A function g is given by $g(x) = x^2 - 3x + 18$.

(i) Find the equation of the tangent at the point on the graph of g where the gradient is 0.

$g'(x) = 2x - 3$

$0 = 2x - 3$

$x = 3/2$

$y - 15.75 = 0(x - 3/2)$

$y - 15.75 = 0$

$y = 15.75$

(ii) In relation to the graph, fully describe the point where this tangent meets the function.

It a turning point because the gradient is always zero at a turning point. The coordinates are $(3/2, 15.75)$.

ASSESSOR'S USE ONLY

(c) A skateboard park has a mound that is h metres high at the point where the horizontal distance, from a fixed point P, is x metres.

The mound can be modelled by

$$h = -0.5x^2 + 3x - 1.5$$

(i) What is the maximum height of the mound?

$h' = -1x + 3$ $\quad\quad h = -0.5(3)^2 + 3(3) - 1.5$

$0 = -1x + 3$

$-3 = -1x$ $\quad\quad h = 3$ meters

$3 = x$

(ii) A ramp up the side of the mound is a tangent to the mound.

The ramp can be modelled by the function

$$h = 0.5x - c$$

Use calculus to find the vertical distance below the top of the mound where the ramp will meet the mound.

Ignore the thickness of the ramp.

$h'(x) = 0.5$ ∴ gradient $= 0.5$

$0.5 = -1x + 3$

$-2.5 = -1x$

$x = 2.5$

height $= -0.5(2.5)^2 + 3 \times (2.5) - 1.5$

$= 2.875$

r

∴ height is 2.875 m

ns

(iii) The height h metres of a skateboard path at a horizontal distance r metres from another point Q, can be modelled by the function

$$h = \frac{r^3}{3} - 2r^2 + 3r \qquad (0.15 < r < 3.5)$$

There is a height regulation that requires no part of the skateboard path to be more than 3 m above the ground.

Fully describe this curve including its turning points, and state whether or not the skateboard path complies with the height regulation.

You must show calculus in answering this question.

ASSESSOR'S USE ONLY

$h' = r^2 - 4r + 3$

$0 = r^2 - 4r + 3$

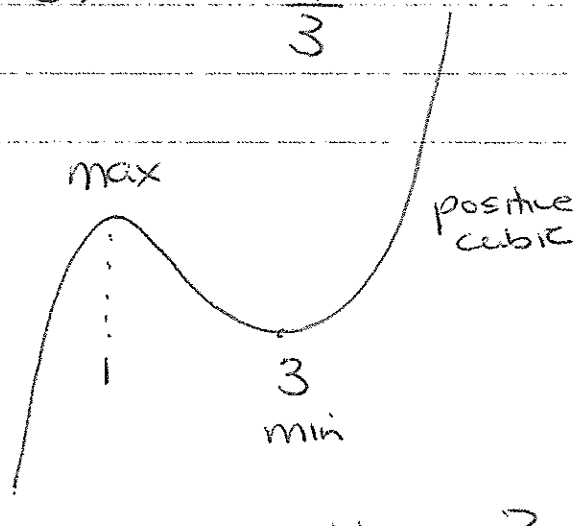
$r = 3, 1$

$h = \frac{3^3}{3} - 2 \times 3^2 + 3 \times 3 = 0$

$(3, 0)$

$h = \frac{1^3}{3} - 2 \times 1^2 + 3 \times 1 = \frac{4}{3}$

$(1, \frac{4}{3})$



5

t

Max is 4/3m which is less than 3 so the skateboard park complies with the heigh regulations//

E7

ASSESSOR'S USE ONLY

QUESTION THREE

(a) The velocity v m s^{-1} of an object t seconds after it passes a fixed point can be modelled by the function

$$v(t) = 4t^3 - t^2 + 2t$$

Find the equation for the acceleration of the object.

$a(t) = 12t^2 - 2t + 2$

(b) Find the equation of the tangent to the curve $f(x) = x^3 - 2x^2 + x$ at the point (2,2) on the curve.

$f'(x) = 3x^2 - 4x + 1$

$f'(2) = 3 \times 2^2 - 4(2) + 1 = 5$

$y - 2 = 5(x - 2)$

$y - 2 = 5x - 10$

$y = 5x + 8$

6

r

(c) In an area surrounding a farming airstrip there is a height restriction for fireworks of 50 m.

The height h metres above the ground reached by a firework t seconds after it is fired, can be modelled by the function

$$h = 20t - 5t^2$$

Will the firework break the 50 m limit?

Use calculus methods to justify your answer.

ASSESSOR USE ONLY

(d) For a function $y = -ax^2 + bx + c$,

a, b, and c are positive numbers and $b = 2a$.

On the grid below, sketch the gradient function.

Show the value of all intercepts. The y'-intercept should be given in terms of b.

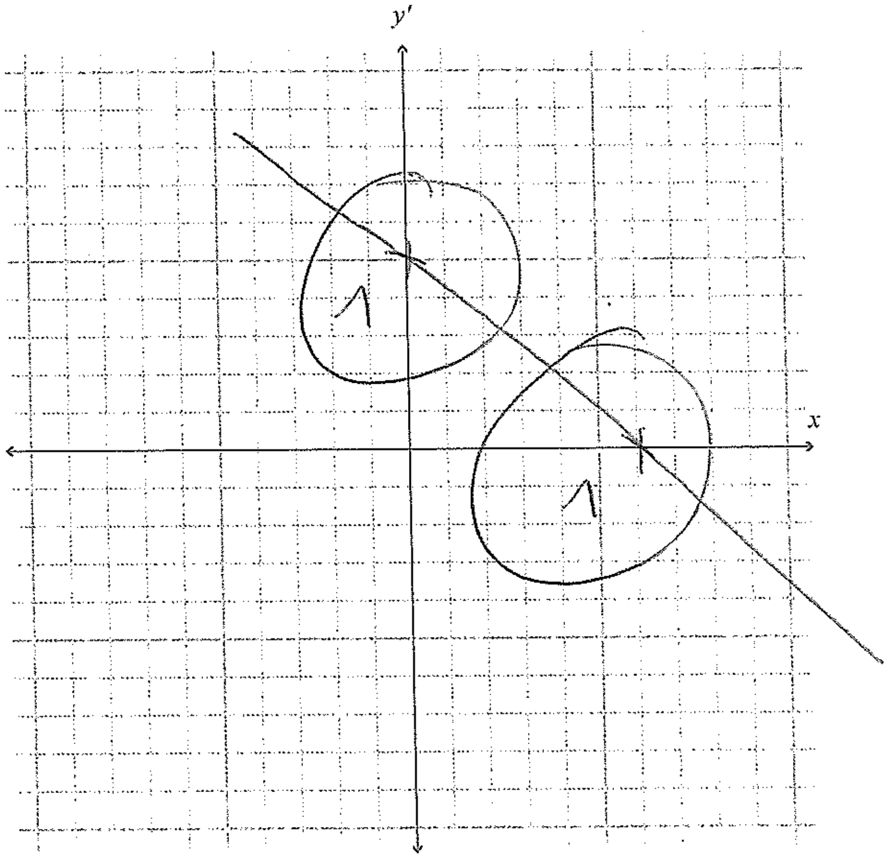
$y' = -2ax + b$

$y' = -2ax + 2a$

$y' =$

7

4



If you need to redraw this graph, use the grid on page 10.

ns

ASSESSOR'S USE ONLY

(e) y is the value of x after 3 has been subtracted and then the answer doubled, and x is between -0.5 and 3.

Find the maximum and minimum values of the product of x^2y.

Justify your answer.

MS

91262 2015

Apply Calculus methods in solving problems

Descriptions for the exemplar numbers:

NOTE: Top down marking was used in this standard.

GRADE = HIGH MERIT

1. Correct derivative found and x=-1 correctly substituted into the derivative to calculate the gradient.
2. Correct derivative given and the two values for x found. Region for function decreasing identified and justification demonstrated with the naming of the negative cubic and the graph.
3. Acceleration equation correctly equated to zero and t=8 second found. Correct velocity found.
4. Correct derivative recognised. Equated the gradient function to 0.5 and found the correct x value of 2.5. Substituted this into the height formula to find the height of 2.875m. Incomplete solution as the candidate did not subtract this solution for the original height of 3m (found in part (c) above).
5. Complete and accurate solution. Both x and y coordinates found for the turning points. A graph to demonstrate which of the turning points is a max and which is a min and an answer to the statement about the skate park being within the 3m regulations.
6. Correct derivative found and then x=2 substituted into the derived function to fine the gradient m=5. Correct use of the point gradient formula to find the equation of the tangent.
7. Correct derivative found but then no further progress towards the solution. Correct graph drawn but neither of the intercepts labelled.