

L3-CALCMF



NEW ZEALAND QUALIFICATIONS AUTHORITY  
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD  
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

## Tuanaki, Kaupae 3, 2020

9.30 i te ata Rāhina 23 Whiringa-ā-rangi 2020

### TE PUKAPUKA O NGĀ TIKANGA TĀTAI ME NGĀ TŪTOHI mō 91577M, 91578M me 91579M

Tirohia tēnei pukapuka hei whakatutuki i ngā tūmahi o ō Pukapuka Tūmahi, Tuhiinga hoki.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

**KA TAEA TĒNEI PUKAPUKA TE PUPURI HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.**

## TE PĀNGARAU – ĒTAHI TURE WHAI HUA

### TE TAURANGI

#### Ngā Whārite Pūrua

$$\text{Mēnā } ax^2 + bx + c = 0$$

$$\text{kāti } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Ngā Taupū Kōaro

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

#### Ngā Tau Matatini

$$z = x + iy$$

$$= r \operatorname{cis} \theta$$

$$= r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy$$

$$= r \operatorname{cis}(-\theta)$$

$$= r(\cos \theta - i \sin \theta)$$

$$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$$

$$\theta = \arg z$$

$$\text{ina } \cos \theta = \frac{x}{r}$$

$$\bar{a}, \sin \theta = \frac{y}{r}$$

#### Te Ture a De Moivre

Mēnā he tau tōpū a  $n$ , kāti,

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis}(n\theta)$$

### TE ĀHUAHANGA TAUNGA

#### Te Rārangi Torotika

$$\text{Whārite } y - y_1 = m(x - x_1)$$

### TE TUĀNAKI

#### Kimi Pārōnaki

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
$e^{ax}$	$ae^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

#### Ngā Tikanga Pāwhaitua

$f(x)$	$\int f(x) dx$
$x^n$	$\frac{x^{n+1}}{n+1} + c$ ( $n \neq -1$ )
$\frac{1}{x}$	$\ln x  + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x)  + c$

#### Te Pānga Tawhā

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

## MATHEMATICS – USEFUL FORMULAE

### ALGEBRA

#### Quadratics

If  $ax^2 + bx + c = 0$

then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

#### Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n \log_b x$

$\log_b x = \frac{\log_a x}{\log_a b}$

#### Complex numbers

$z = x + iy$

$= r \operatorname{cis} \theta$

$= r(\cos \theta + i \sin \theta)$

$\bar{z} = x - iy$

$= r \operatorname{cis}(-\theta)$

$= r(\cos \theta - i \sin \theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where  $\cos \theta = \frac{x}{r}$

and  $\sin \theta = \frac{y}{r}$

#### De Moivre's Theorem

If  $n$  is any integer, then

$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$

### COORDINATE GEOMETRY

#### Straight Line

Equation  $y - y_1 = m(x - x_1)$

### CALCULUS

#### Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
$e^{ax}$	$ae^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

#### Integration

$f(x)$	$\int f(x) dx$
$x^n$	$\frac{x^{n+1}}{n+1} + c$ ( $n \neq -1$ )
$\frac{1}{x}$	$\ln x  + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x)  + c$

#### Parametric Function

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \cdot \frac{dt}{dx}$

**Te Ture mō te Otinga Whakarau<sup>1</sup>**

$$(f \cdot g)' = f \cdot g' + g \cdot f' \quad \text{mēnā rānei } y = uv \quad \text{kāti } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

**Te Ture mō te Otinga Wehe**

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad \text{mēnā rānei } y = \frac{u}{v} \quad \text{kāti } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Te Ture Pānga Hiato, te Ture Mekameka rānei**

$$\left(f(g)\right)' = f'(g) \cdot g'$$

$$\text{mēnā rānei } y = f(u) \quad \bar{a}, u = g(x) \quad \text{kāti } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

**NGĀ TIKANGA TAU****Te Ture Taparara**

$$\int_a^b f(x) \, dx \approx \frac{1}{2} h \left[ y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1}) \right]$$

$$\text{ina } h = \frac{b-a}{n} \quad \bar{a}, y_r = f(x_r)$$

**Te Ture a Simpson**

$$\int_a^b f(x) \, dx \approx \frac{1}{3} h \left[ y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) \right]$$

$$\text{ina } h = \frac{b-a}{n}, y_r = f(x_r), \bar{a}, \text{ he taurua te } n.$$

<sup>1</sup> whakarea

**Product Rule**

$$(f \cdot g)' = f \cdot g' + g \cdot f' \quad \text{or if } y = uv \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

**Quotient Rule**

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad \text{or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Composite Function or Chain Rule**

$$(f(g))' = f'(g) \cdot g'$$

$$\text{or if } y = f(u) \text{ and } u = g(x) \text{ then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

**NUMERICAL METHODS****Trapezium Rule**

$$\int_a^b f(x) \, dx \approx \frac{1}{2} h [y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\text{where } h = \frac{b-a}{n} \text{ and } y_r = f(x_r)$$

**Simpson's Rule**

$$\int_a^b f(x) \, dx \approx \frac{1}{3} h [y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\text{where } h = \frac{b-a}{n}, y_r = f(x_r) \text{ and } n \text{ is even.}$$

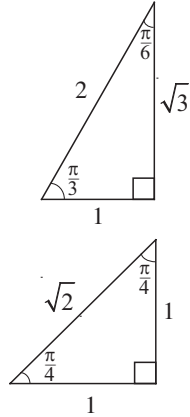
**TE PĀKOKI**

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

**Te Ture Aho**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**Te Ture Whenua**

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Ngā Whārite ka Pono Ahakoa  
ngā Uara Ka Whakaurua Atu**

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

**Ngā Otinga Whānui**

$$\text{Mēnā } \sin \theta = \sin \alpha \text{ kāti } \theta = n\pi + (-1)^n \alpha$$

$$\text{Mēnā } \cos \theta = \cos \alpha \text{ kāti } \theta = 2n\pi \pm \alpha$$

$$\text{Mēnā } \tan \theta = \tan \alpha \text{ kāti } \theta = n\pi + \alpha$$

ko te  $n$ , he tau tōpū ahakoa

**Ngā Koki Hiato**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

**Ngā Koki Rearua**

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

**Ngā Otinga Whakaraui**

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

**Ngā Otinga Tāpiri**

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

**TE INE****Te Tapatoru**

$$\text{Horahanga} = \frac{1}{2} ab \sin C$$

**Te Taparara**

$$\text{Horahanga} = \frac{1}{2} (a+b)h$$

**Te Pewanga**

$$\text{Horahanga} = \frac{1}{2} r^2 \theta$$

$$\text{Te roa o te pewa} = r\theta$$

**Te Rango**

$$\text{Rōrahi} = \pi r^2 h$$

$$\text{Horahanga mata kōpiko} = 2\pi rh$$

**Te Koeko**

$$\text{Rōrahi} = \frac{1}{3} \pi r^2 h$$

$$\text{Horahanga mata kōpiko} = \pi r l \text{ ina ko te } l \text{ te teitei o te tītaha}$$

**Te Poi**

$$\text{Rōrahi} = \frac{4}{3} \pi r^3$$

$$\text{Horahanga mata} = 4\pi r^2$$

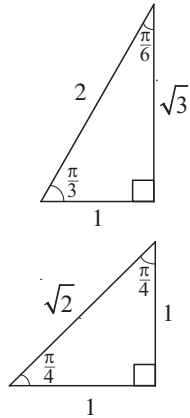
## TRIGONOMETRY

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$



### Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

### Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

### General Solutions

$$\text{If } \sin \theta = \sin \alpha \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$\text{If } \cos \theta = \cos \alpha \text{ then } \theta = 2n\pi \pm \alpha$$

$$\text{If } \tan \theta = \tan \alpha \text{ then } \theta = n\pi + \alpha$$

where  $n$  is any integer

### Compound Angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

### Double Angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

## Products

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

## Sums

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

## MEASUREMENT

### Triangle

$$\text{Area} = \frac{1}{2} ab \sin C$$

### Trapezium

$$\text{Area} = \frac{1}{2} (a+b)h$$

### Sector

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$\text{Arc length} = r\theta$$

### Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi r h$$

### Cone

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi r l \text{ where } l = \text{slant height}$$

### Sphere

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$

*English translation of the wording on the front cover*

L3-CALCMF

## Level 3 Calculus, 2020

9.30 a.m. Monday 23 November 2020

### FORMULAE AND TABLES BOOKLET for 91577, 91578 and 91579

Refer to this booklet to answer the questions in your Question and Answer booklets.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

**YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.**