2

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91261

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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91261 Apply algebraic methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement

TOTAL 14

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Annotated Exemplar Template

Achieved exemplar for 91261 2015		Total score	14
Q	**Grade score**	**Annotation**	
1	M5	The two quadratic solutions in 1aiii provided only r evidence as the exclusion of x = -2 (can't have a negative base) was overlooked. Other questions which could have contributed r evidence were incorrect or incomplete.	
2	N2	Question 2a provided u evidence, the cancelation was correct but the factor of 2 was dropped from the denominator.	
3	E7	Candidate solution to 3c provided t evidence, subsequent working to find roots was ignored. In 3d sufficient evidence for r was given as the discriminant with inequality were correct and one constraint was given. No roots to the original equation were found.	

QUESTION ONE

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(a) (i) Find the value of $\log_2 1024$.

$2^x = 1024$

$x = 10$

$\log = 10$

u

(ii) Solve the equation $\log_4(3w+1) = 2$.

$4^2 = 3w+1$ $\quad w = 5$

$16 = 3w+1$

$3w = 16-1$

$3w = 15$

u

(iii) Luka says that the equation $\log_x(4x+12) = 2$ has only one solution.

Is he correct?

Find the solution(s), justifying your answer.

$x^2 = 4x+12$

$x^2 - 4x = 12$ He is wrong as

$x^2 - 4x - 12 = 0$ it formed a quadartic

$(x-6)(x+2)$ formula giving

$x = 6 \quad x = -2$ 2 x values 6 and

-2.

r

(b) Make x the subject of the equation $a^{2x} = b^{x+1}$.

$2x \log a = (x+1) \log b$

$2x - x = \log a$

$3x = \log A \quad \log b$

$\frac{\log A}{\log B}$

$x = 3\left(\frac{\log A}{\log B}\right)$

u

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(c) The market value of Sue's house has been increasing at a constant exponential rate of 3% per annum since she bought it sixteen years ago at the start of 1999. At the start of 2015 it was worth \$350 000.

(i) Assuming the exponential growth is of the form $y = A\,r^t$, what was the value of the house at the start of 1999 when she bought it?

350000 x 3^16

350 000 = A x 3^16

3/350000

~~16 = A x 350000~~ A = 1200

y = 350000 x 3^16 16^0.3

~~y = 350000 x 0.03^16~~

n.

(ii) A friend also bought a house at the start of 1999 that cost \$200 000.

Its market value also has been steadily increasing, but at a slightly higher exponential rate of 3.5%.

Its value, \$y, t years after the start of 1999, is given by the function

$$y = 200\,000 \times (1.035)^t$$

If the houses continue to keep increasing in value at the original rates, in which year will the two houses be worth the same amount?

= 200 000 x (1.035)^t

n

M5

QUESTION TWO

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(a) Simplify $\dfrac{2x^2+7x-4}{2x^2-32}$

x^2+7x-8 $\qquad \dfrac{(2x-1)(x+4)}{2(x^2-16)}$

$(x-\frac{1}{2})(x+\frac{8}{2})$

$(x+4)(x-4)$ $\qquad \dfrac{(2x-1)(\cancel{x+4})}{(\cancel{x+4})(x-4)}$ $\qquad \dfrac{2x-1}{2-4}$ //

u

(b) If $a = y^{\frac{3}{4}}$, find an expression for a^7 in terms of y.

$y^{3/4} = a$

$y = (\sqrt[4]{a^3})^7$

$y = (\sqrt[4]{a^3})^7$ //

n

(c) Solve the equation $2u^{\frac{2}{3}} + 7u^{\frac{1}{3}} = 4$

a b c

$2u^{0.7} + 7u^{0.3} - 4 = 0$ //

n

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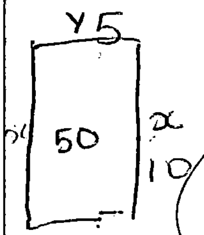
(d) Talia used timber to form the exterior sides of her rectangular garden. The length of the garden is x metres, and its area is 50 m^2.

(i) Show that the perimeter of the garden is given by $2x+\frac{100}{x}$

$50 = xy$
$A = xy$



$P = x + x + y + y$

$A = xy$

$P = 5 + 5 + 10 + 10$

$= 30$

$30 = \frac{2x^2}{x} + \frac{100}{x}$

n

(ii) If she uses 33 m of timber to build the sides, find the dimensions of the garden.

n

(e) David and Sione are competing in a cycle race of 150 km.

Sione cycles on average 4 km per hour faster than David, and finishes half an hour earlier than David.

Find David's average speed.

You MUST use algebra to solve this problem. (Hint: average speed = $\frac{distance}{time}$*)*

$S = 4D$ $\quad 4 \times 60 = 240 \quad \frac{1}{2}h + 4H$

$\frac{150}{\frac{1}{2}h + 4h}$

$= \frac{150}{\frac{9}{2}h}$

$4.5h = 150$

$h = \frac{150}{4.5}$

$h = 33.3$ $\quad \frac{150}{\frac{1}{2}(33.3) + 4(33.3)}$

$4D = S$ $\quad \frac{150}{2217.78} = 0.068$

$D = S/4$

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N2

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QUESTION THREE

(a) Simplify, giving your answer with positive exponents:

(i) $\left(\frac{a^{10}}{4a^5}\right)^{-2}$

$\left(\frac{4a^5}{16a^{10}}\right)^2 = \frac{16a^{10}}{a^{20}} = \cancel{16a^{10}}$

$\frac{16}{a^{10}}$ //

r

(ii) $\sqrt[5]{\left(\frac{32}{x^5}\right)^3}$

$\sqrt[5]{\frac{32768}{x^{10}}} = \frac{8}{x^2}$

$\left(\frac{32}{x^5}\right)^{3/5}$

u

(b) Solve the following equation for t:

$$\frac{1}{t(t-1)}-\frac{1}{t}=\frac{3}{t-1}$$

$\frac{1}{t^2-t} - \frac{1}{t} = \frac{3}{t-1}$

t

n

Question Three continues on the following page.

(c) For what value(s) of k does the graph of the quadratic function

$$y = x^2 + (3k - 1)x + (2k + 10)$$

a b c

never touch the x-axis?

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$b^2 - 4ac < 0$

$(3k-1)^2 - 4(1)(2k+10) < 0$

$(3k-1)(3k-1) - 4(2k+10) < 0$

$9k^2 - 3k - 3k + 1 - 8k - 40 < 0$

$9k^2 - 14k - 39 < 0$

$k = 3 \quad k = -1.4$

$(3k-1)(3k-1)$

$9k^2 - 3k - 3k + 1$

$9k^2 - 6k + 1$

$-1.4 < k < 3$

values never touch x axis //

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3k - 1 \pm \sqrt{(3k-1)^2 - 4(1)(2k+10)}}{2(1)}$

$x = \frac{-3k - 1 + \sqrt{(9k+13)(k-3)}}{2}$

$x = \frac{-3k - 1 - \sqrt{(9k+13)(k-3)}}{2}$

P

t

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(d) The quadratic equation

$$mx^2 - (m+2)x + 2 = 0$$

has two positive real roots.

Find the possible value(s) of m, and the roots of the equation.

$b^2 - 4ac > 0$

$(m+2)(m+2) - 4 \times m \times 2$

$m^2 + 2m + 2m + 4 - 4m \times 2 > 0$

$m^2 + 4m + 4 - 8m > 0$

$m^2 - 4m + 4 > 0$

$(m-2)(m-2) > 0$

$m \neq 2$

$2x^2 - 4x + 2 = 0$

r

E7