91262M

912625

2

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tohua tēnei pouaka mēnā kāore he tuhituhi i roto i tēnei pukapuka

Te Pāngarau me te Tauanga, Kaupae 2, 2020

91262M Te whakahāngai tikanga tuanaki hei whakaoti rapanga

9.30 i te ata Rāpare 19 Whiringa-ā-rangi 2020
Ngā whiwhinga: Rima

Paetae	Kaiaka	Kairangi
Te whakahāngai tikanga tuanaki hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro whaipānga hei whakaoti rapanga.	Te whakahāngai tikanga tuanaki mā te whakaaro waitara hōhonu hei whakaoti rapanga.

Tirohia mēnā e rite ana te Tau Ākonga ā-Motu (NSN) kei runga i tō puka whakauru ki te tau kei runga i tēnei whārangi.

Me whakamātau koe i ngā tūmahi KATOA kei roto i tēnei pukapuka.

Tirohia mēnā kei a koe te Puka Tikanga Tātai L2–MATHMF.

Tuhia ō mahinga KATOA.

Ki te hiahia koe ki ētahi atu wāhi hei tuhituhi whakautu, whakamahia te wāhi wātea kei muri i te pukapuka nei.

Me mātua whakaatu e koe te whakamahi tuanaki i ō tuhinga mō ngā tūmahi katoa i tēnei pepa.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2-31 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

ME HOATU RAWA KOE I TĒNEI PUKAPUKA KI TE KAIWHAKAHAERE Ā TE MUTUNGA O TE WHAKAMĀTAUTAU.

TAPEKE

MĀ TE KAIMĀKA ANAKE

TŪMAHI TUATAHI

MĀ TE KAIMĀKA ANAKE

(a) Ka tohua he pānga mā te $f(x) = x^3 - 2x^2 + 5$.

Tātaihia te rōnaki o te kauwhata kei te pūwāhi $x = 4$.

(b) Ka tohua tētahi atu pānga mā te $h(x) = 0.5x^2 + 3x - 1$.

Whiriwhiria te taunga-x o te pūwāhi i te kauwhata o tēnei pānga, ko te 5 te rōnaki.

(c) Whiriwhiria te whārite o te pātapa ki te kōpiko o $y = x^2 + 5x$ i te pūwāhi (2,14).

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function is given by $f(x) = x^3 - 2x^2 + 5$.

Find the gradient of the graph at the point where $x = 4$.

(b) Another function is given by $h(x) = 0.5x^2 + 3x - 1$.

Find the x-coordinate of the point on the graph of this function where the gradient is 5.

(c) Find the equation of the tangent to the curve of $y = x^2 + 5x$ at the point (2,14).

(d) Ka tohua te pānga rōnaki o tētahi kōpiko mā te $f'(x) = px - 4$, ina ko te p he tau pūmau.
Ka pā te kōpiko ki ngā pūwāhi (4,12) me (–6,2).

Whiriwhiria te whārite o te kōpiko.

MĀ TE KAIMĀKA ANAKE

(d) The gradient function of a curve is given by $f'(x) = px - 4$, where p is a constant.
The curve passes through the points (4,12) and (–6,2).

Find the equation of the curve.

ASSESSOR'S USE ONLY

(e) 80 km te tawhiti o tētahi poti hī ika mai i tōna tauranga waka ina tae atu ki ngā taunga ika.

I te taenga atu ki ngā taunga ika, ka whakatere ake te poti ki tētahi rārangi torotika mai i te tauranga ina hopu ana i ngā ika. Ka tohua te whakaterenga o te poti mā te $a(t) = 0.5$ km h^{-2}, ina ko t te maha o ngā haora mai i te tīmatanga o te hī.

Ko te tere o te poti he 3 km h^{-1} i te tīmatanga ki te hī ika.

Ko tēhea te haora i tere atu te poti ki te 11.75 km? Me mātua whakamahi e koe te tuanaki i roto i tō tuhinga.

MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) A fishing boat is 80 km from its port when it reaches its fishing grounds.

Having reached its fishing grounds, the boat accelerates in a straight line directly away from its port as it catches fish. The acceleration of the boat is given by $a(t) = 0.5$ km h^{-2}, where t is the number of hours since it started fishing.

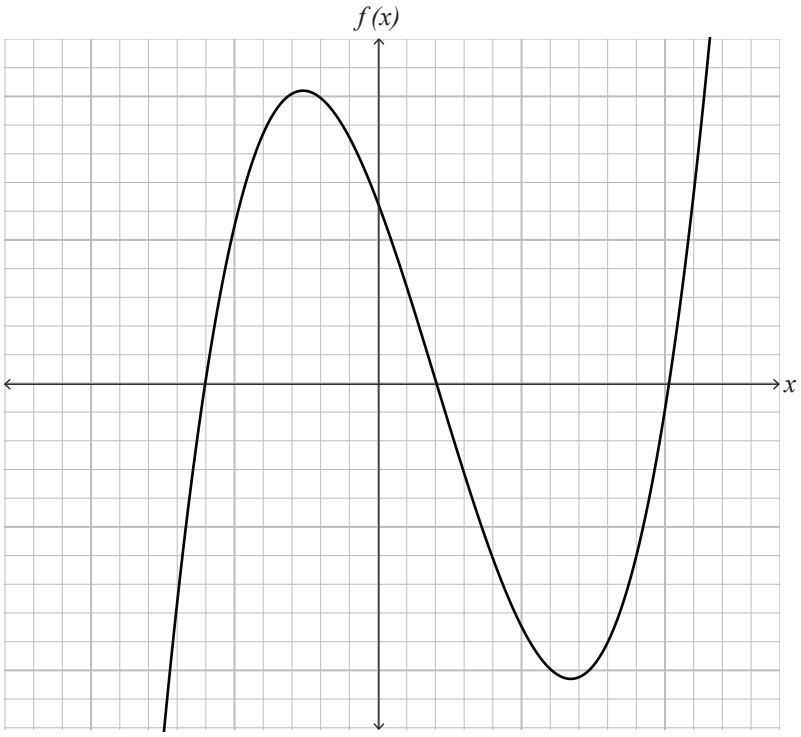
The speed of the boat is 3 km h^{-1} when it starts fishing.

During which hour did the boat travel 11.75 km? You must use calculus to find your answer.

TŪMAHI TUARUA

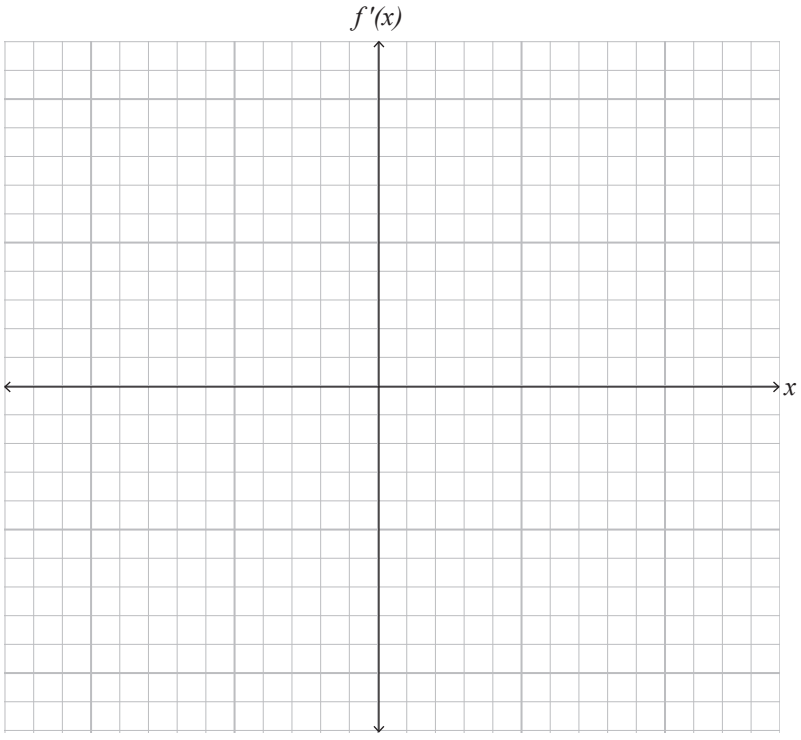
MĀ TE KAIMĀKA ANAKE

(a) E whakaatuhia ana te kauwhata o te pānga $y = f(x)$ ki ngā tuaka i raro nei.



Tuhia te kauwhata o te pānga rōnaki $y = f'(x)$ ki ngā tuaka o raro.

He ōrite te āwhata huapae o ngā huinga tuaka e rua.

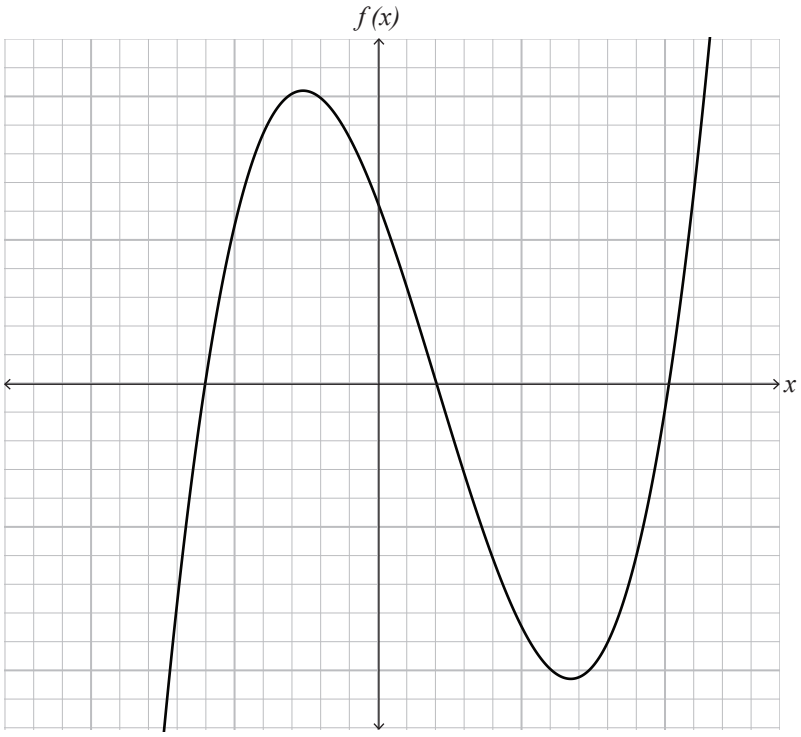


Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku kei te whārangi 26.

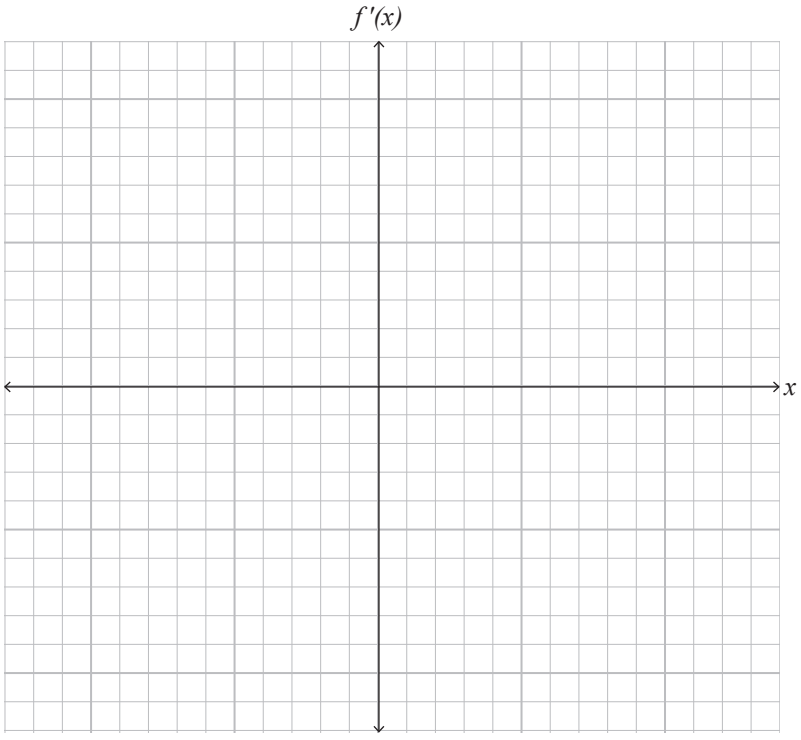
QUESTION TWO

ASSESSOR'S USE ONLY

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.
Both sets of axes have the same horizontal scale.



If you need to redraw this graph, use the grid on page 27.

(b) Whiriwhiria te taunga-x o te tihi o te kōpiko i tohua mā te $y = 2x^3 - 42x^2 + 240x + 8$.
Whakamāramahia mai he pēhea koe i mōhio ai ko tēnei pūwāhi te tihi, ehara i te whāruarua.

MĀ TE KAIMĀKA ANAKE

(c) Ka tohua te tere o tētahi ahanoa mā te $v(t) = 3t^2 - 5t$ m s^{-1}, ina ko t ka inea mā ngā hēkona.

He aha te whakaterenga o te ahanoa ina ko t = 2?

ASSESSOR'S USE ONLY

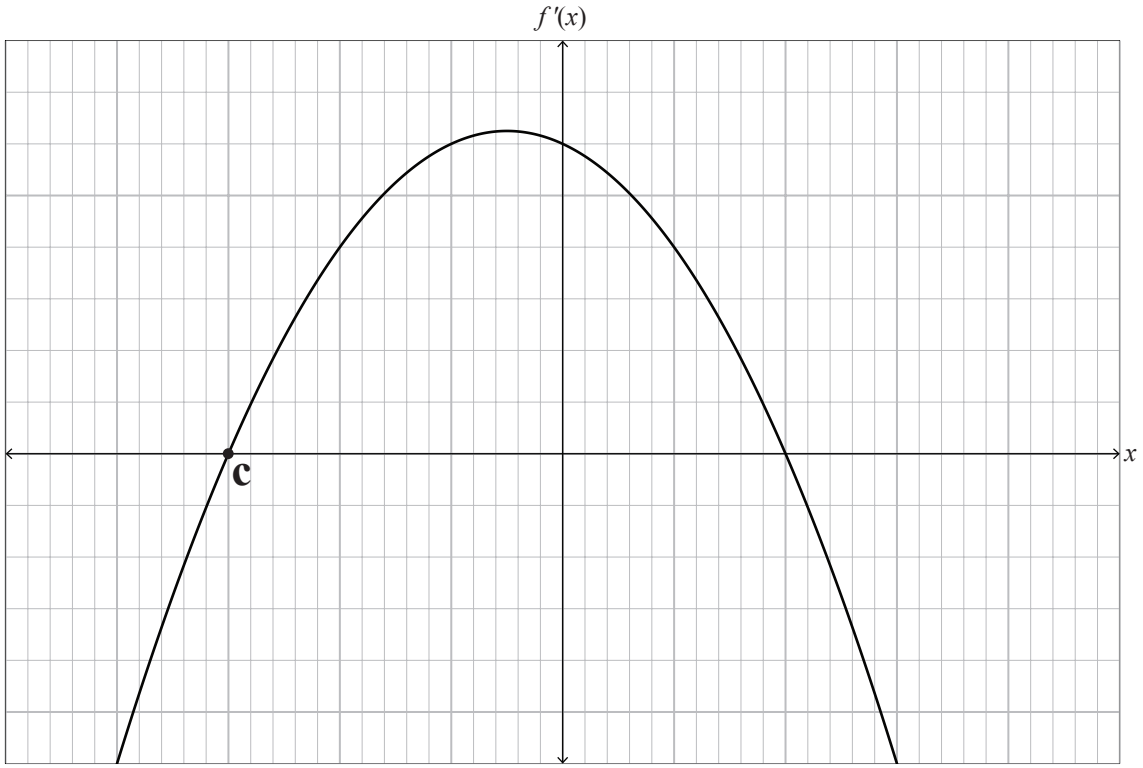
(b) Find the x-coordinate of the maximum point on the curve given by $y = 2x^3 - 42x^2 + 240x + 8$. Explain how you know that this point is the maximum, not the minimum, point.

(c) The speed of an object is given by $v(t) = 3t^2 - 5t$ m s^{-1}, where t is measured in seconds.

What is the object's acceleration when t = 2?

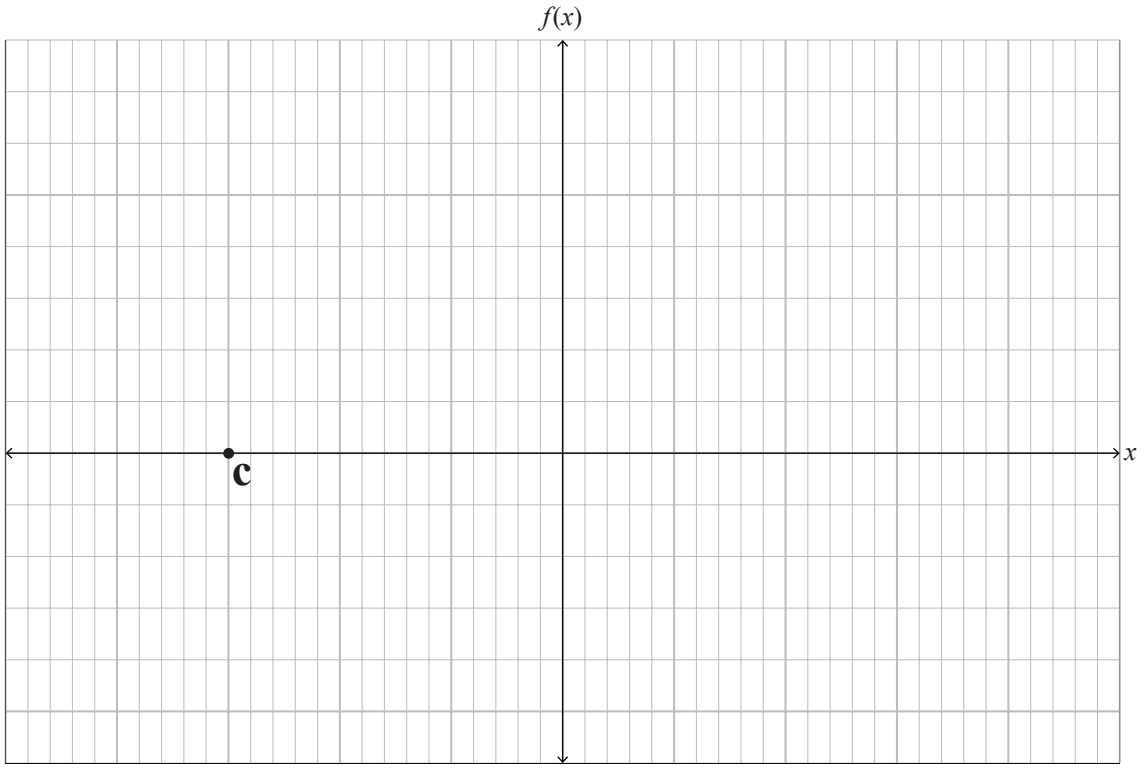
MĀ TE KAIMĀKA ANAKE

(d) E whakaatuhia ana te kauwhata o te pānga rōnaki, $f'(x)$, ki ngā tuaka i raro nei. Kei te kauwhata te pūwāhi (c,0).



Tātuhia te kauwhata o te pānga $f(x)$ ki ngā tuaka i raro, ina ko $f(c) = 0$.

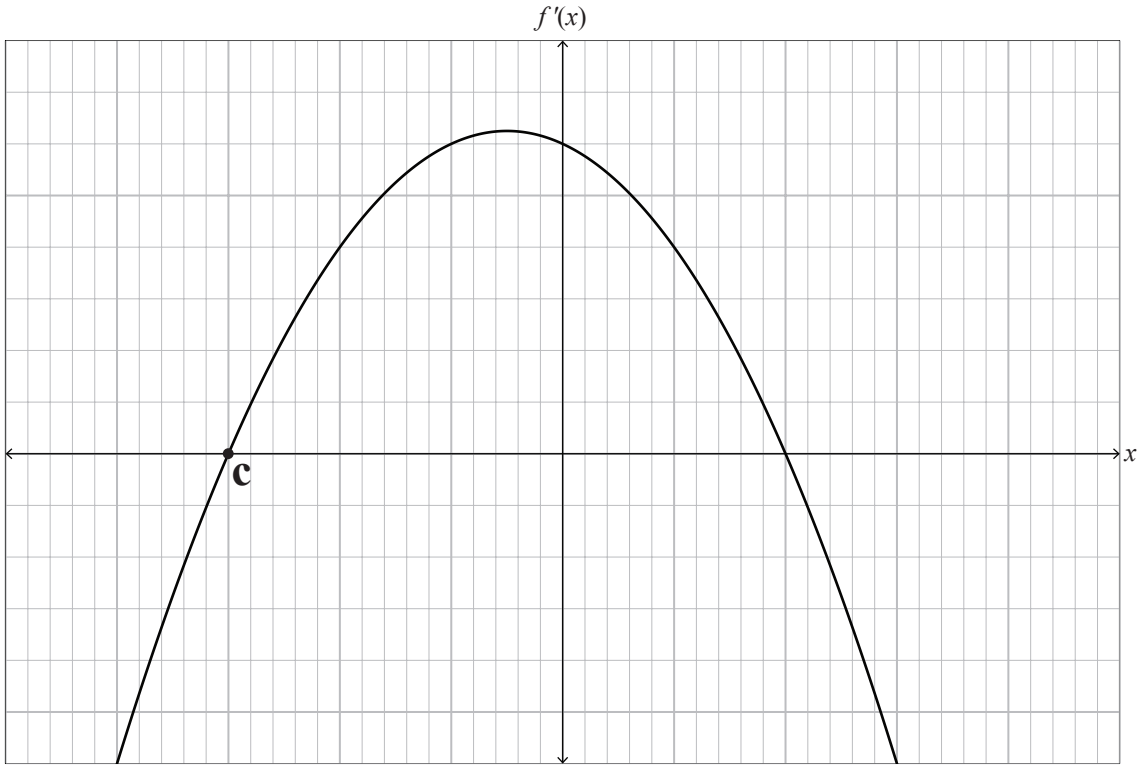
He ōrite te āwhata huapae o ngā huinga tuaka e rua.



Ki te hiahia koe ki te tuhi anō i tēnei kauwhata, whakamahia te tukutuku kei te whārangi 28.

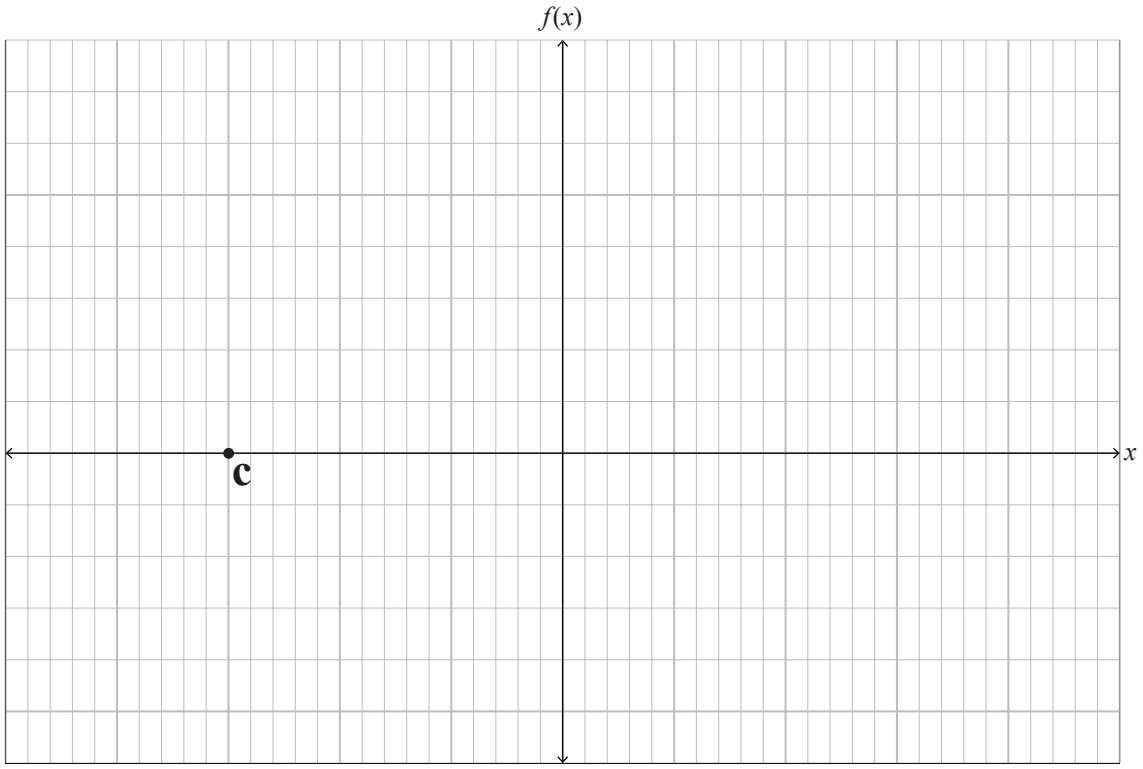
ASSESSOR'S USE ONLY

(d) The graph of a gradient function, $f'(x)$, is shown on the axes below. The point (c,0) is on this graph.



Sketch the graph of the function $f(x)$ on the axes below, given that $f(c) = 0$.

Both sets of axes have the same horizontal scale.



If you need to redraw this graph, use the grid on page 29.

(e) Ko te pānga pūtoru, $f(x) = ax^3 + bx^2 + cx + d$, he pūwāhi huri kei $x = 3$ me $x = -5$.

Whiriwhiria he kīanga mō c e pā ana ki a.

MĀ TE KAIMĀKA ANAKE

(e) A cubic function, $f(x) = ax^3 + bx^2 + cx + d$, has turning points that occur at $x = 3$ and $x = -5$.

Find an expression for c in terms of a.

ASSESSOR'S USE ONLY

TŪMAHI TUATORU

MĀ TE KAIMĀKA ANAKE

(a) Ko $f'(x) = 3x^2 - 2x - 4$ te pārōnaki o tētahi pānga f.

Ka whakawhiti te kauwhata o $f(x)$ mā te pūwāhi (3,10).

Whiriwhiria te kīanga mō $f(x)$.

(b) Ka puea ake he tohorā ki te moana aiō. I te pueatanga ake o te tohorā, ka whānui haere atu te ripo porohita ki te tere pūmau, kia tukuna ai te pūtoro o te porohita mā te $r = 0.7t$ mita t hēkona i muri i te pueatanga ake o te tohorā.

He aha te auau e nui haere ake te horahanga o te wai i roto i te ripo porohita, i te 20 hēkona i muri i te pueatanga ake o te tohorā?

QUESTION THREE

ASSESSOR'S USE ONLY

(a) $f'(x) = 3x^2 - 2x - 4$ is the derivative of a function f.

The graph of $f(x)$ passes through the point (3,10).

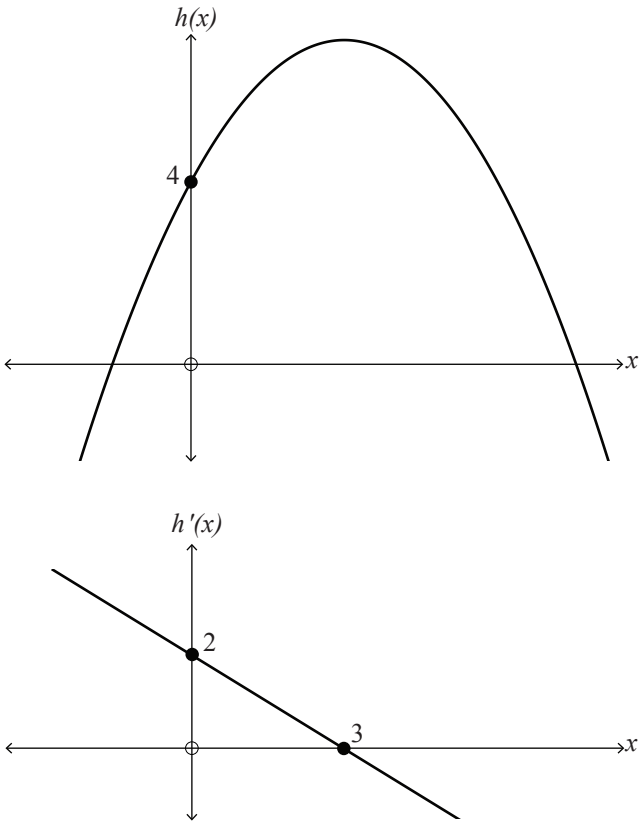
Find an expression for $f(x)$.

(b) A whale surfaces on a still sea. As the whale surfaces, a circular ripple expands outwards at a constant speed, so that the radius of the circle is given by $r = 0.7t$ metres t seconds after the whale surfaces.

At what rate is the area of water within the circular ripple increasing, 20 seconds after the whale surfaces?

(c) Ka tukuna i raro ko te kauwhata o te pānga pūrua $h(x)$ me te pānga rōnaki $h'(x)$.

MĀ TE KAIMĀKA ANAKE

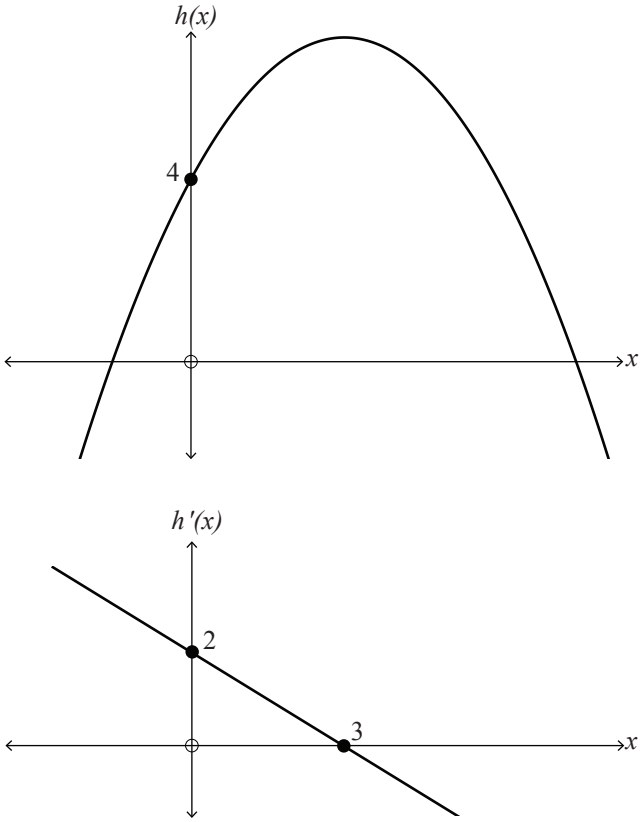


Whiriwhiria te kīanga mō $h(x)$.

Me mātua whakamahi e koe te tuanaki i roto i tō tuhinga.

ASSESSOR'S USE ONLY

(c) The graph of the quadratic function $h(x)$ together with that of its gradient function $h'(x)$ are given below.



Find an expression for $h(x)$.

You must use calculus to obtain your answer.

(d) Kei te pānga o $f(x) = kx^3 + 9x$ tētahi pātapa me te rōnaki o te 15 ina ko $x = 2$.

Whiriwhiria te uara o k.

MĀ TE KAIMĀKA ANAKE

(d) The function $f(x) = kx^3 + 9x$ has a tangent with a gradient of 15 where $x = 2$.

Find the value of k.

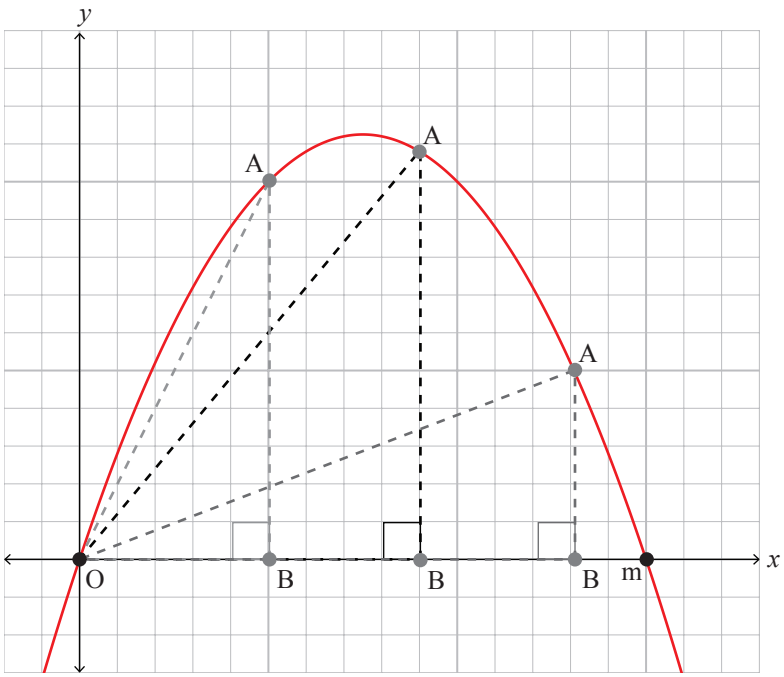
ASSESSOR'S USE ONLY

(e) Ka tātuhia he tapatoru hāngai OAB i roto i te unahi, e whakaaturia ana ki te hoahoa i raro. E toru ngā tapatoru OAB pea kua tātuhia.

MĀ TE KAIMĀKA ANAKE

Ko te pūwāhi O ko te pūtake (0,0), ā, ka taea te pūwāhi A te noho ki tētahi wāhi ahakoa kei hea ki tēnei unahi i runga ake i te tuaka-x.

Ko te āhua o te whārite o te unahi ko $y = mx - x^2$, ina ko m he tau pūmau tōrunga.



Me whakamahi te tuanaki hei kimi i tētahi kīanga, e pā ana ki m, mō te horahanga mōrahi ka taea o te tapatoru OAB.

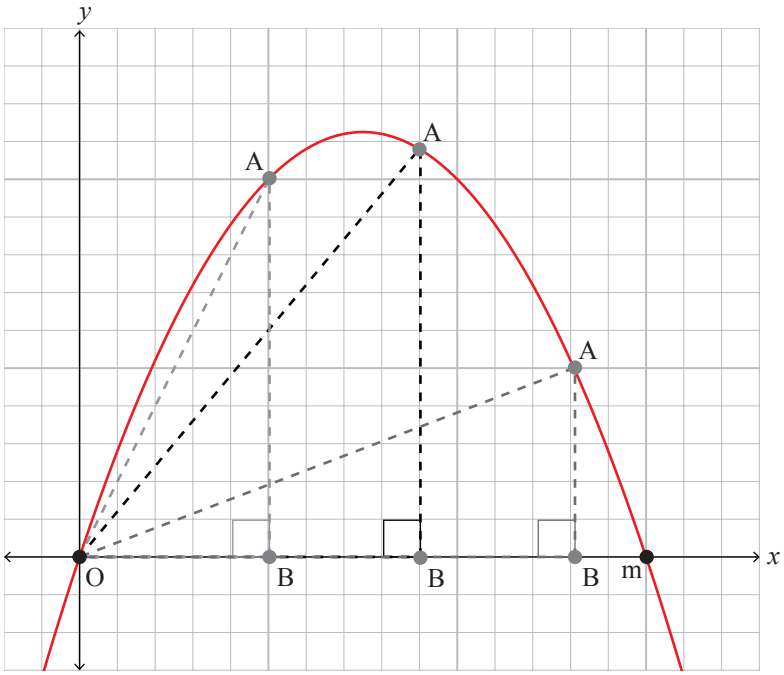
MĀ TE KAIMĀKA ANAKE

ASSESSOR'S USE ONLY

(e) A right-angled triangle OAB is drawn within a parabola, as shown in the diagram below. Three possible triangles OAB have been drawn.

The point O is the origin (0,0), and point A can lie anywhere on this parabola above the x-axis.

The equation of the parabola is of the form $y = mx - x^2$, where m is a positive constant.



Use calculus to find an expression, in terms of m, for the maximum possible area of triangle OAB.

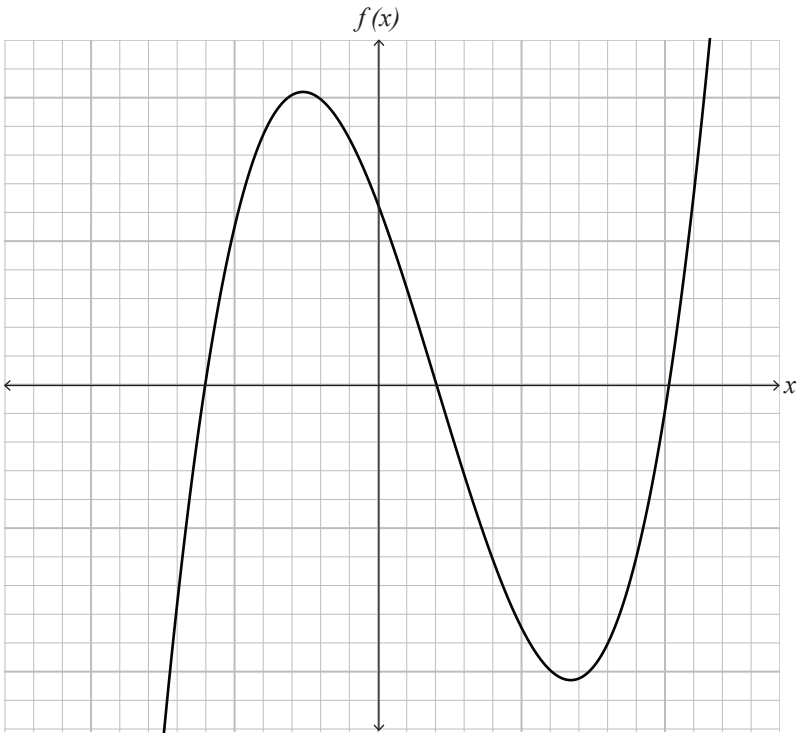
ASSESSOR'S USE ONLY

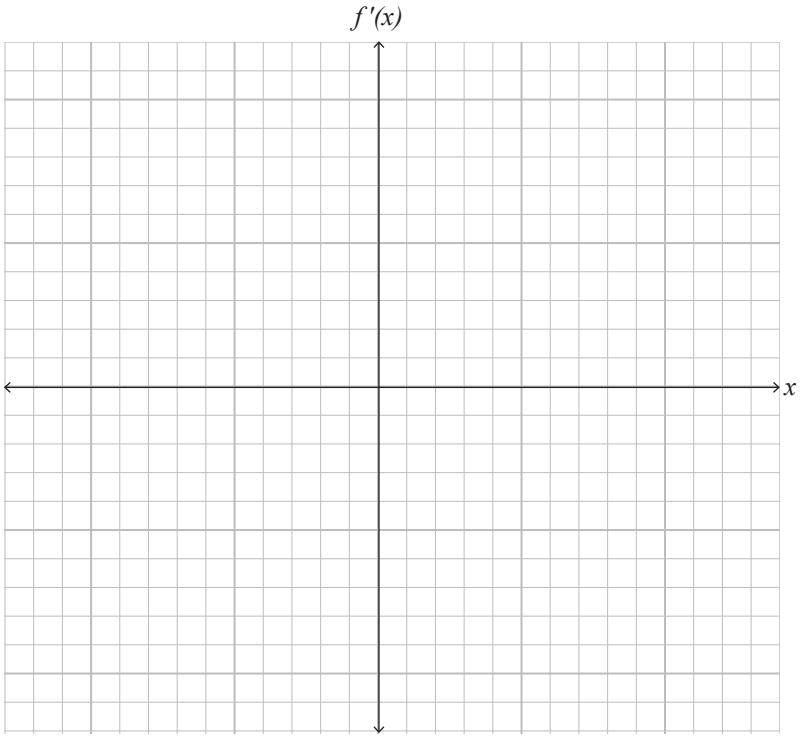
NGĀ TUKUTUKU TĀPIRI

MĀ TE KAIMĀKA ANAKE

Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuarua (a), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

(a)



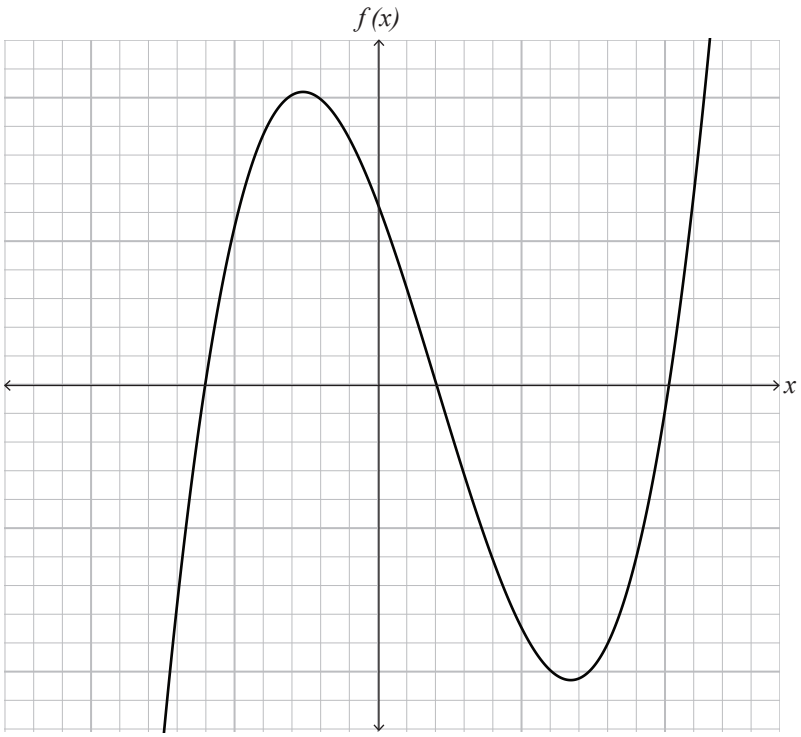


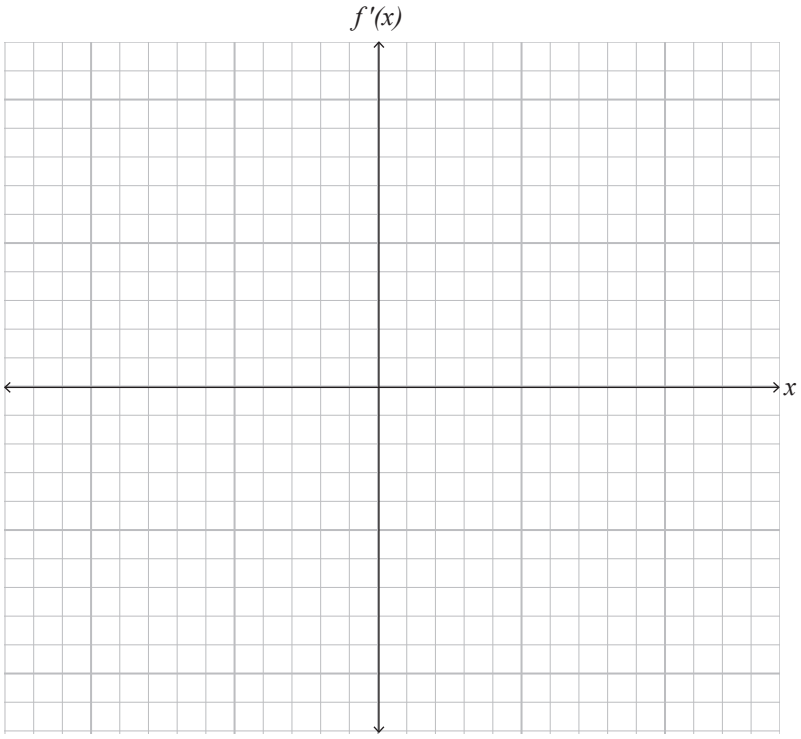
SPARE GRIDS

ASSESSOR'S USE ONLY

If you need to redo Question Two (a), use the grid below. Make sure it is clear which answer you want marked.

(a)

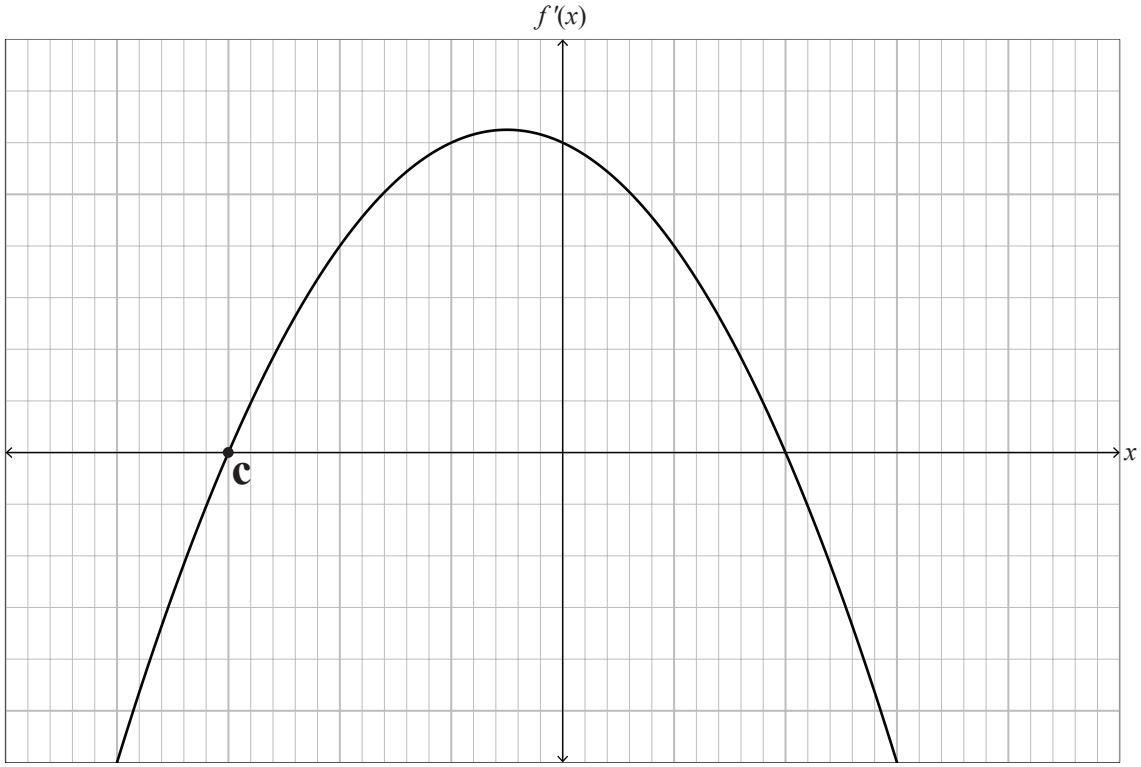


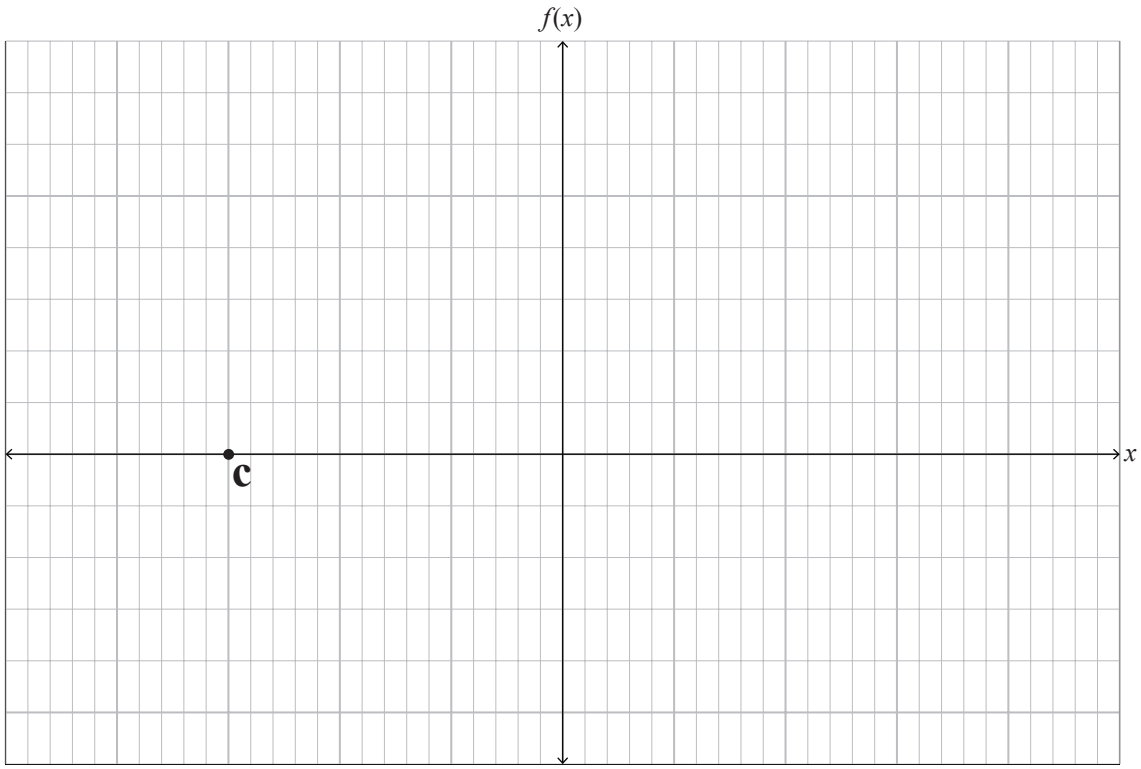


Ki te hiahia koe ki te tātuhi anō i tō urupare ki te Tūmahi Tuarua (d), whakamahia te tukutuku i raro nei. Kia mārama te tohu ko tēhea te tuhinga ka hiahia koe kia mākahia.

MĀ TE KAIMĀKA ANAKE

(d)

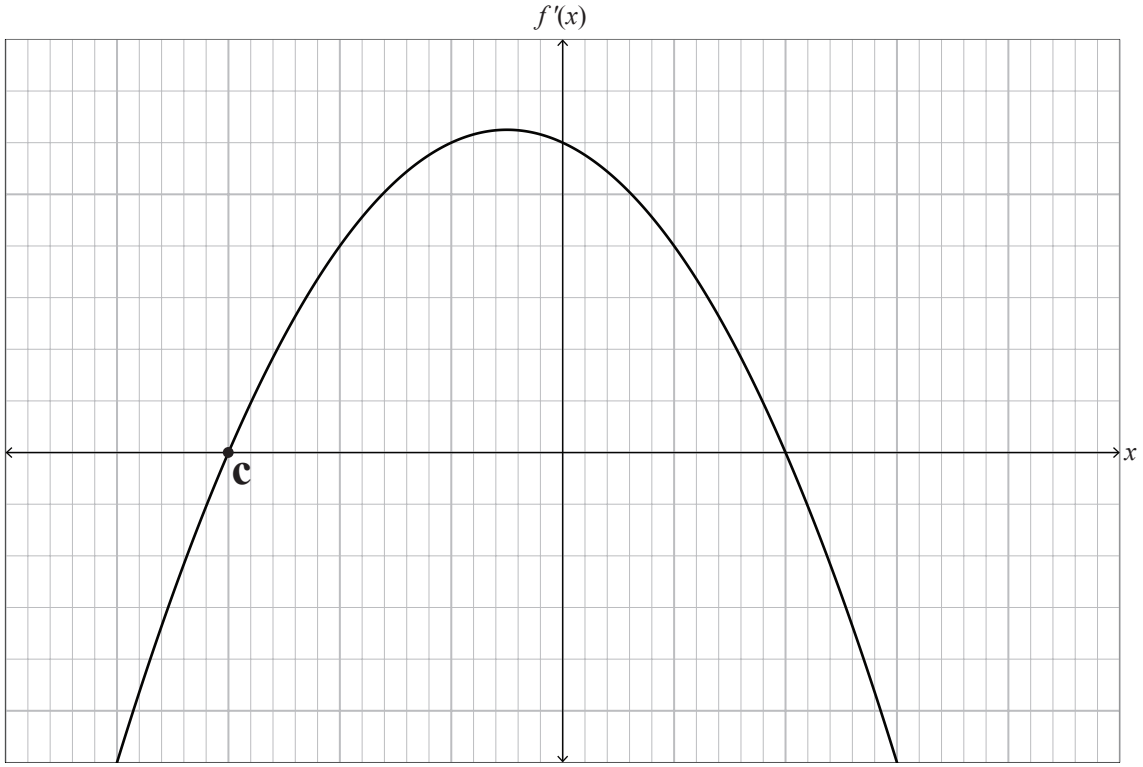


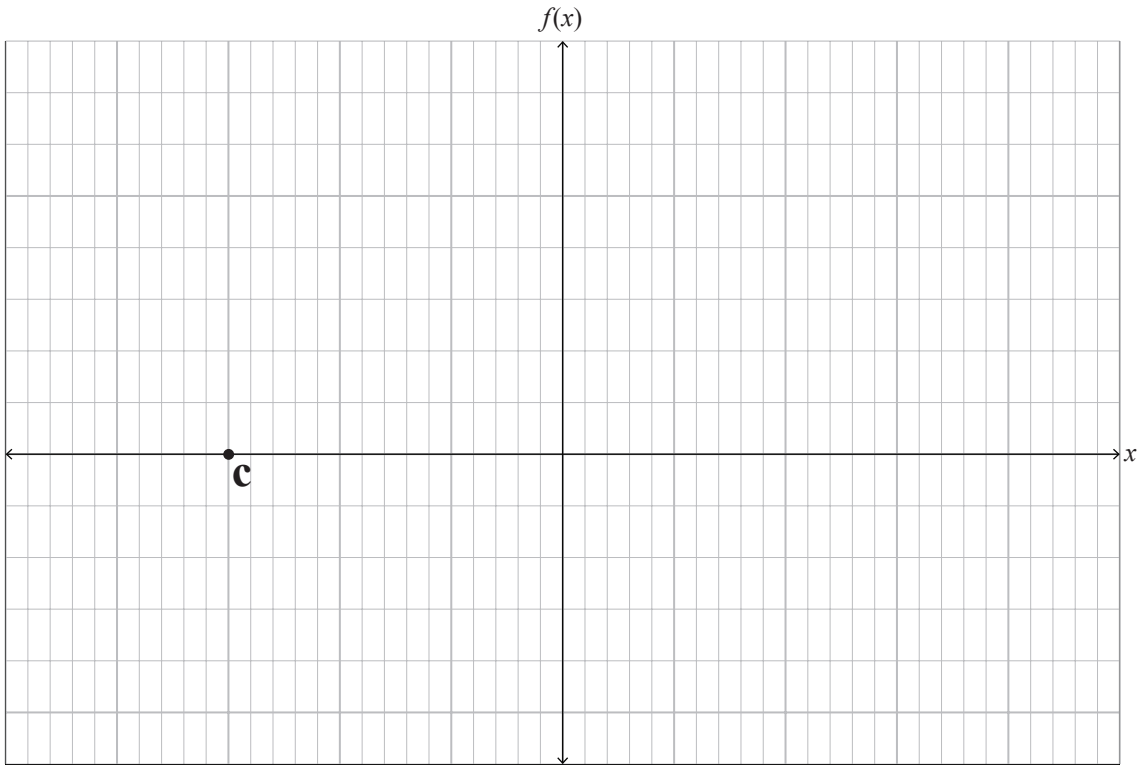


If you need to redo Question Two (d), use the grid below. Make sure it is clear which answer you want marked.

ASSESSOR'S USE ONLY

(d)





He whārangi anō ki te hiahiatia.
Tuhia te (ngā) tau tūmahi mēnā e tika ana.

TAU TŪMAHI

MĀ TE KAIMĀKA ANAKE

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

English translation of the wording on the front cover

Level 2 Mathematics and Statistics 2020

91262 Apply calculus methods in solving problems

9.30 a.m. Thursday 19 November 2020
Credits: Five

91262M

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHMF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–31 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.