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91031



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# Level 1 Mathematics and Statistics, 2016

## 91031 Apply geometric reasoning in solving problems

9.30 a.m. Thursday 17 November 2016  
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Merit

TOTAL

17

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## THE SKY TOWER

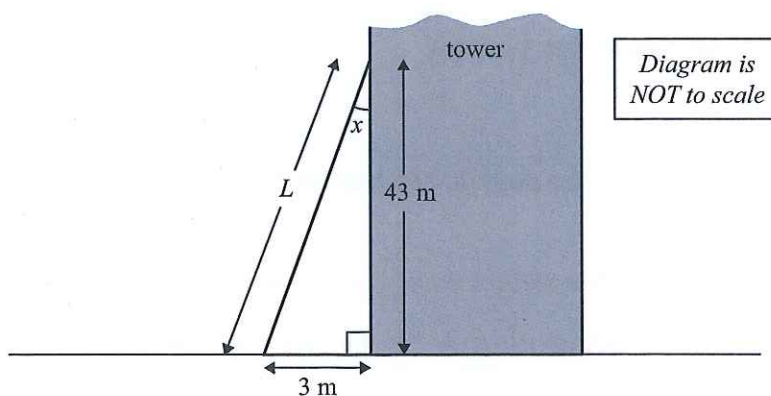
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Auckland's Sky Tower is the tallest man-made structure in the Southern Hemisphere.

## QUESTION ONE

- (a) The base of the tower is supported by 8 legs.  
These legs are  $L$  metres long and are 3 metres away from the tower at ground level.  
The legs join the tower 43 m above ground level.



- (i) Calculate the length,  $L$ , of the leg from the ground to the tower.

$$a^2 + b^2 = c^2$$

$$3^2 + 43^2 = L^2$$

$$L^2 = 1858$$

$$\sqrt{1858} = 43.1\text{m}$$

$$L = 43.1\text{m}$$

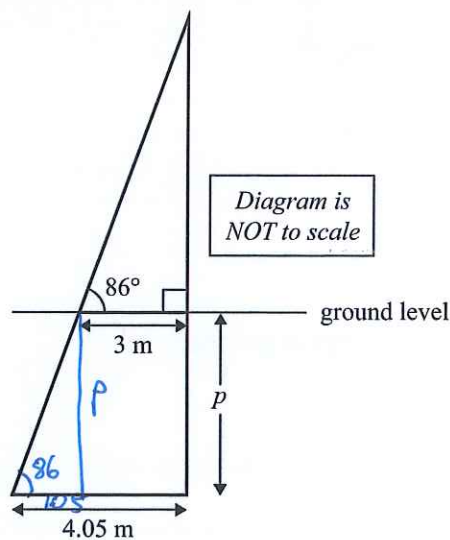
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- (ii) Use trigonometry to calculate the size of angle  $x$ , where the leg joins the tower.

SOH CAH TOA  
 $\tan x = \frac{O}{A}$   
 $\tan^{-1} \frac{3}{43} = 4$   
 $\angle x = 4^\circ$

- (iii) The legs of the tower go below ground level.

The horizontal distance from the tower to the bottom of the leg under the ground is 4.05 metres.



Calculate  $p$ , the vertical distance that the legs are built into the ground.

Show your working clearly.

$\angle a = 86^\circ$  because corresp  $\angle$  on parallel lines are equal

$$4.05 \text{ m} - 3 \text{ m} = 1.05 \text{ m}$$

SOH CAT TOA

TOA  $\tan 86 = \frac{O}{A}$

$$\tan 86^\circ = \frac{O}{1.05}$$

$$\tan 86 \times 1.05 = O = 15.01 \text{ m}$$

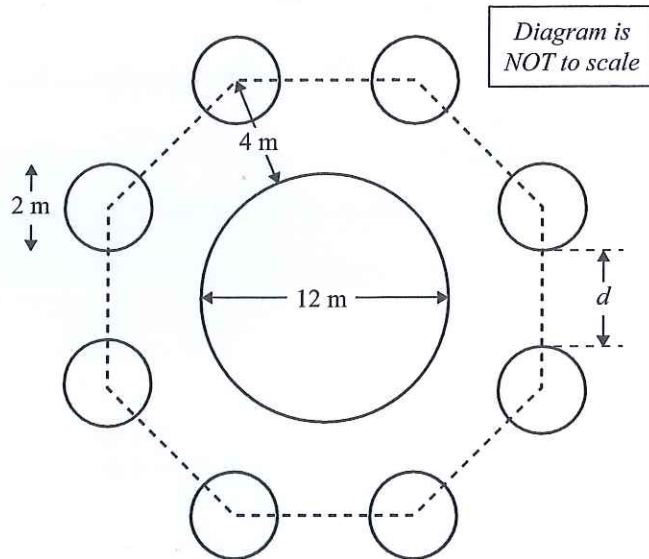
$$O = P$$

$$P = 15.01 \text{ m}$$

- (b) The centres of the 8 circular legs form a regular octagonal shape.

The tower has a diameter of 12 metres and each leg has a diameter of 2 metres.

The distance from the outside edge of the tower to the centre of the legs at the ground is 4 metres.

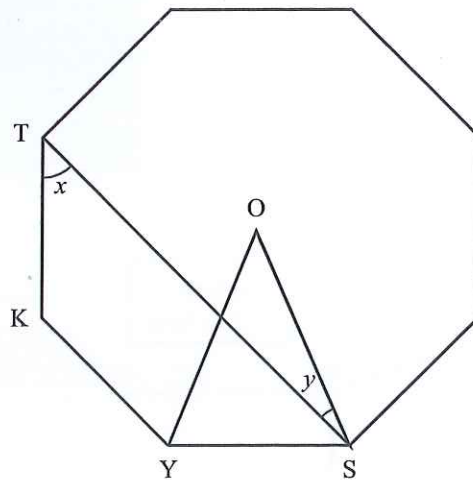


Calculate the shortest distance,  $d$ , between adjacent legs at ground level.

Show your working clearly.



- (c) A simplified diagram of the position of the legs is shown below as a regular octagon. Point O is at the centre of the octagon.



Show that angle  $y$  is half the size of angle  $x$ .

Justify your answer with clear geometric reasoning.

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M5

## QUESTION TWO

Below the Sky Tower is a car park made of ramps.

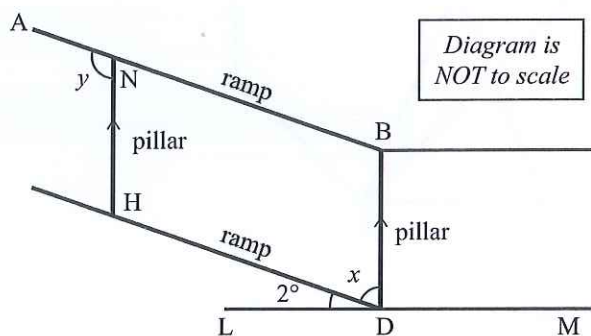
The ramps are at a  $2^\circ$  angle.

There are vertical pillars regularly placed along the ramps for strength.



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USE ONLY

- (a) All pillars are parallel to each other. LM is horizontal.



- (i) Calculate the size of angle  $x$  in the diagram above.

Justify your answer with clear geometric reasoning.

$$\angle LDB = 90^\circ$$

$$90^\circ - 2^\circ = 88^\circ$$

$x = 88^\circ$  because angles in a right angle add to  $90^\circ$

- (ii) Calculate the size of angle  $y$  in the diagram above.

Justify your answer with clear geometric reasoning.

$$180 - 88 = 92^\circ$$

$\angle NBD = 92^\circ$  because co-interior  $\angle$ 's on  $\parallel$  lines add to  $180^\circ$

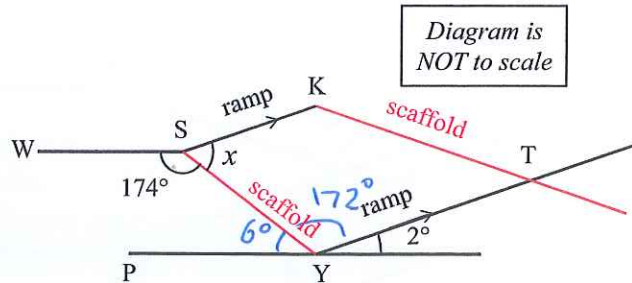
$\angle y = 92^\circ$  because corresp  $\angle$ 's on  $\parallel$  lines are equal.

$$\angle y = 92^\circ$$

- (iii) Part of the ramp had extra scaffolding added for support, as shown in the diagram below. The lines SK and YT are parallel.

Angle WSY is  $174^\circ$ .

The lines WS and PY are both horizontal.



Calculate the size of angle  $x$  in the diagram above.

Justify your answer with clear geometric reasoning.

$\angle PYT = 6^\circ$  because co-interior on  $\parallel$  lines  
add to  $180^\circ$

$$180 - 8 = 172^\circ$$

$\angle SYT = 172^\circ$  because  $\angle$ s on a straight line  
add to  $180^\circ$

$$180 - 172^\circ = 8^\circ$$

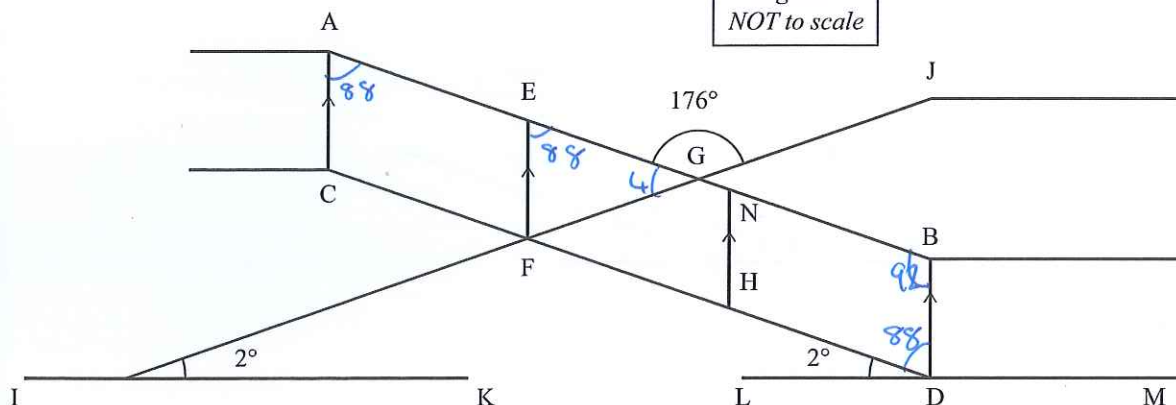
$\angle x = 8^\circ$  because co-interior  $\angle$ s on  $\parallel$  lines  
add to  $180^\circ$

$$\angle x = 8^\circ$$



- Angle EGJ is  $176^\circ$ .

Diagram is  
NOT to scale



*Justify your answer with clear geometric reasoning.*

$$90 - 2 = 88$$

$\angle LDB = 88^\circ$  because right angle is  $90^\circ$

$$180 - 176 = 4^{\circ}$$

LEGF =  $4^\circ$  because  $\angle$ s ~~at~~ on straight line add to  $180^\circ$ .

$\angle FEG = 88^\circ$  because base  $\angle$ s in  $\triangle$ s are  $=$

$\angle CAE = 88^\circ$  because corresp.  $\angle$ s on  $\parallel$  lines are =

$\angle NBD = 92^\circ$  because co-interior  $\angle$ 's on  $\parallel$  add to  $180^\circ$

$$88 + 92 = 180^\circ$$

$\angle HDB + \angle NBD = 180^\circ$  because co-interior  $\angle$ s

on // lines add to 180

therefore

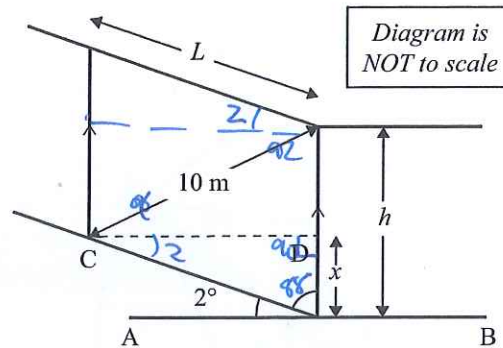
AC and CD are parallel.



- (b) The length along the slope between two pillars is  $L$  metres.

The diagonal distance between the top of one pillar and the base of the next higher pillar is 10 m.

AB and CD are horizontal.



- (i) Find the height,  $x$ , in terms of the length  $L$ .

Show your working clearly.

$$L = 10$$

- (ii) Calculate  $h$ , the height in metres of a pillar, in terms of  $L$ .

Show your working clearly.

*Handwritten mark*

*n*

*n*

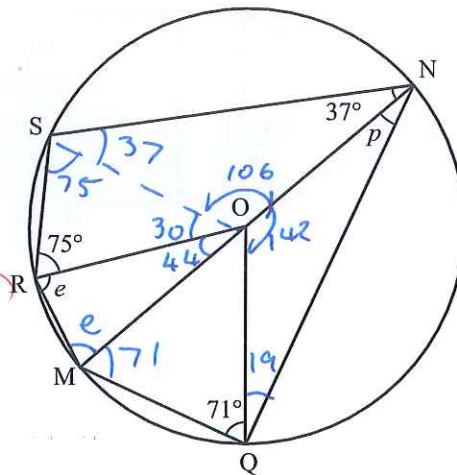
*M6*

## QUESTION THREE

ASSESSOR'S  
USE ONLY

- (a) In the diagram below, the line MN passes through the centre of the circle, O.  
Angle MQO is  $71^\circ$ , angle SNO is  $37^\circ$  and angle SRO is  $75^\circ$ .

Diagram is  
NOT to scale



- (i) Find the size of angle  $p$ .

Justify your answer with clear geometric reasoning.

$\angle MQN = 90^\circ$  because  $\angle$  in semi-circle is  $90^\circ$

$$90 - 71 = 19^\circ$$

$\angle p = 19^\circ$  because base  $\angle$ s in isos are =

- (ii) Find the size of angle  $e$ .

Justify your answer with clear geometric reasoning.

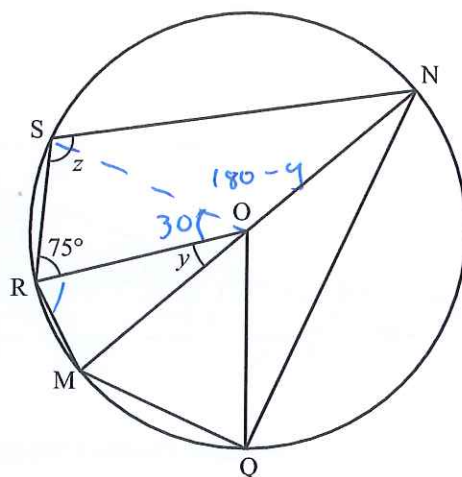
base angles of isos are =

$\angle ROM = 44^\circ$  because  $\angle$ s around a point add to  $360^\circ$

$\angle e = 68^\circ$  because  $\angle$ s in a  $\Delta$  add to  $180^\circ$

- (iii) In the diagram below, angle SRO is  $75^\circ$ .

Diagram is  
NOT to scale



Find an expression for  $z$  in terms of  $y$ .

Justify your answer with clear geometric reasoning.

$$\angle O = 180 - y$$

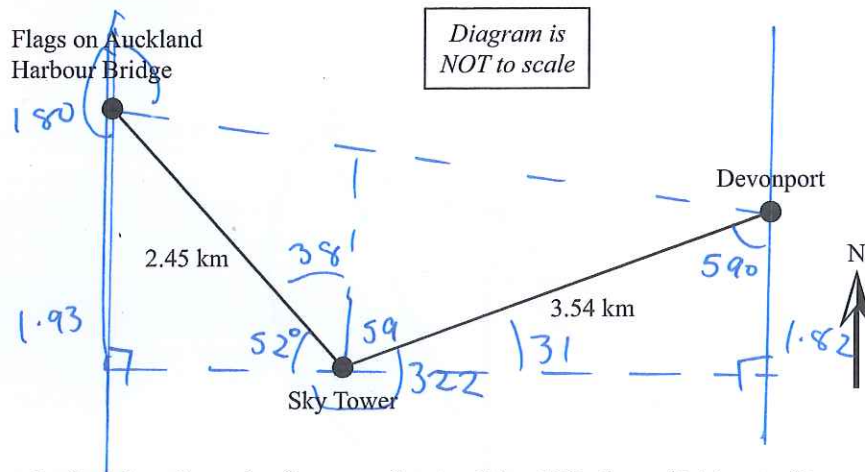
$\angle Z = 20$  because angle at centre of circle is twice angle at circumference

$$\angle Z = \frac{180 - y}{2}$$



- (b) Devonport is at a bearing of  $059^\circ$  and 3.54 km from the Sky Tower.

The flags on the Auckland Harbour Bridge are at a bearing of  $322^\circ$  and 2.45 km from the Sky Tower.



Calculate the bearing from the flags on the Auckland Harbour Bridge to Devonport.

Show your working clearly.

$$38 + 270 = 308$$

$$360 - 308 = 52$$

SOH

$$2.45 \times \sin 52 = 1.93 \text{ km}$$

$$\text{CAH} \quad \cos^{-1} 1.93 / 2.45 = 38^\circ$$

SOH

$$3.54 \times \sin 31 = 1.82$$

$$\text{CAH} \quad \cos^{-1} 1.82 / 3.54 = 59^\circ$$

$$360 - 180 - 59 - 38 = 83^\circ$$

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## Annotated Exemplar Merit

### Merit exemplar 2016

<b>Subject:</b>	<b>Mathematics</b>	<b>Standard:</b>	<b>91031</b>	<b>Total score:</b>	<b>17</b>
<b>Q</b>	<b>Grade score</b>	<b>Annotation</b>			
1	M5	<p>Candidate has used a range of methods to solve problems. Pythagoras and Trigonometry have been correctly used to find a side and an angle. Trigonometry has been used as a strategy to solve a) iii correctly.</p> <p>Candidate has been able to relate and communicate the solutions in context.</p> <p>To gain M6, the candidate would have needed to recognise and use the correct right angle triangle to use in 1(b) or make progress in a chain of reasoning in 1(c).</p>			
2	M6	<p>Candidate has used a range of methods in parallel line geometry and has reasoned correctly to find the angles in 2 (a) (i)–(iii). Each step towards solution is correctly reasoned.</p> <p>To gain a grade of E7, the candidate would need to develop a chain of logical reasoning to prove that the lines AB and CD are parallel or trigonometry in the abstract to find the heights in 2(b).</p>			
3	M6	<p>Candidate has used a range of methods in circle geometry to correctly calculate angle p and angle e in 3(a) (i) and (ii). Each step towards the solution is correctly reasoned.</p> <p>To gain a grade of E7, the candidate would need to correctly find an expression for z in terms of y correctly in 3(a) (iii) or correctly calculate the bearing in 3(b).</p>			