1

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NZQA

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Level 1 Mathematics and Statistics, 2016

91028 Investigate relationships between tables, equations and graphs

9.30 a.m. Thursday 17 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.
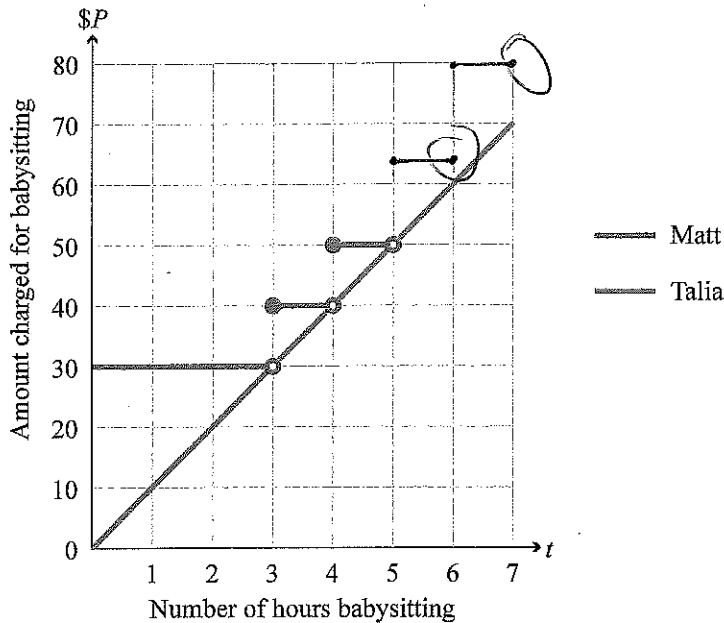
Merit

TOTAL **18**

ASSESSOR'S USE ONLY

QUESTION ONE

Tama and Pita have three different babysitters to choose from: Matt, Talia, and Sasha.

(a)   The graph of the amounts that Matt and Talia charge is shown below.

Matt

Talia

If you need to redraw your answer, use the graph on page 13.

(i)   How much would Matt be paid if he babysits for 4.5 hours?

$50

(ii)   Once Matt has babysat for 5 or more hours, he increases his charge for the additional hours to \$15 an hour or part of an hour that he babysits.

On the grid above, show the amount Matt would charge if he babysits for 5 or more hours.

(iii)   Find the average amount Matt charges per hour if he babysits for 6 hours.

$y = 50 + 15$

$y = 65$

$65

(iv)   Talia charges an **average** of \$10 per hour for any amount of time that she works.

This is shown on the graph above with the red line.

Give the equation of the graph.

$y = mx + c$

$y = 10x + 0$

$y = 10x$

(v) Sasha will babysit for up to 7 hours for $55.

Make recommendations on who Tama and Pita should have as their babysitter, based on the amount that each babysitter charges.

Tama and Pita want a babysitter who will charge the least amount for the time they want. For up to 3 hours, Talia would be the cheapest. At the end of 3 hours, she charges the same as Mall, $30. For up to 4 and 5 hours Talia is the cheapest but Mall costs the same at the end of 4 hours ($40) and the end of 5 hours ($50). At 5.5 hours, Talia and Sasha will cost the same ($55) but after that, Sasha will be the cheapest. (more at back)

(b) (i) Give the equation of the graph shown on the right.

$$y = a(x - 0.5)(x + 3)$$
$$4 = a(1 + 0.5)(1 + 3)$$
$$4 = a (0.5)(4)$$
$$4a = 2a$$
$$a = \frac{4}{2}$$
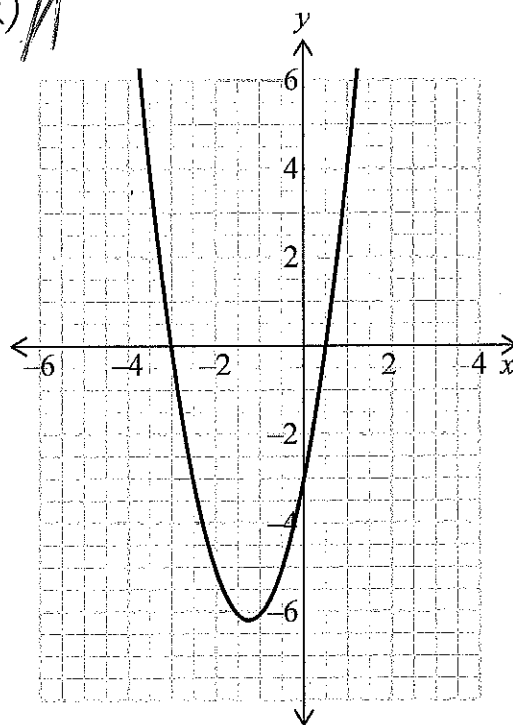$$a = 2$$
$$y = 2(x - 0.5)(x + 3)$$

(ii) The graph is then translated a units to the right and up b units.

Give:

• the equation of the translated graph

• the x-value at the vertex.

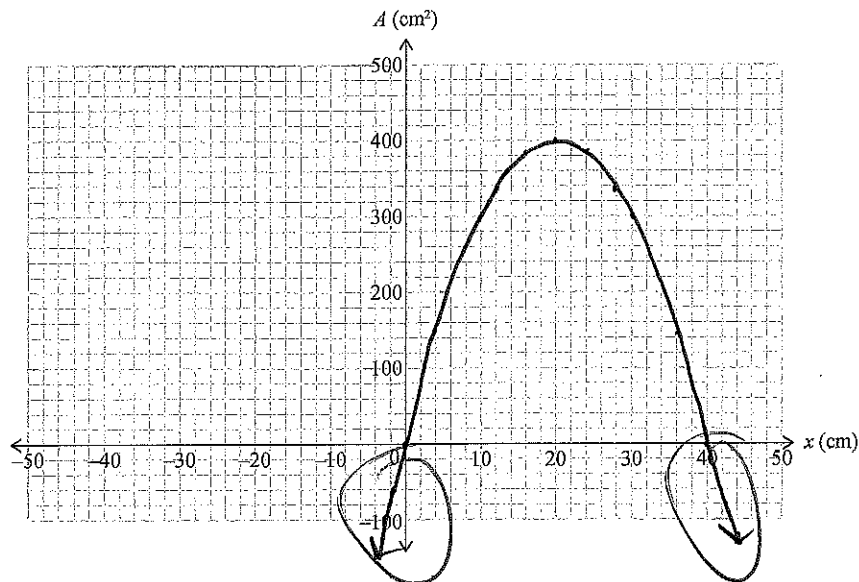• $y = 2(x - 0.5 - a)(x + 3 - a) + b$

The x value at the vertex for this graph before translation is $x = 1.5$ (in the middle of the x-intercepts $x0.5$ and $x = -3$) so the new x value at the vertex

QUESTION TWO

(a) (i) Maria is investigating a set of rectangles that have an area modelled by $A = -(x^2 - 40x)$.

Sketch the graph of the possible range of areas of the rectangles as the value of x changes.

If you need to redraw this graph, use the grid on page 13.

(ii) What is the maximum possible area of the rectangles?

$A = -(20^2 - 40x)$ $A = -(20^2 - 40 \times 20)$

$A = -(-400)$

$A = 400$ $400 \, cm^2$

(iii) For what values of x are the areas less than 300 cm²?

$300 = -(x^2 - 40x)$

~~[crossed out]~~

$0 = -(x - 10)(x - 30)$

$x < 10$ and $x > 30$

(iv) What is the maximum area of another set of rectangles that have an area, $A = -(x^2 - mx)$?

$A = -(x(x - m))$

x intercepts would be $x = 0$ and $x = m$

Middle of that would be $m/2$

$A = -(m/2^2 - m + (m/2))$

$A = -(m/2)^2 + 2m/2$

$A = -(m/2)^2 + m$

$A = -2m/2 + m$

6

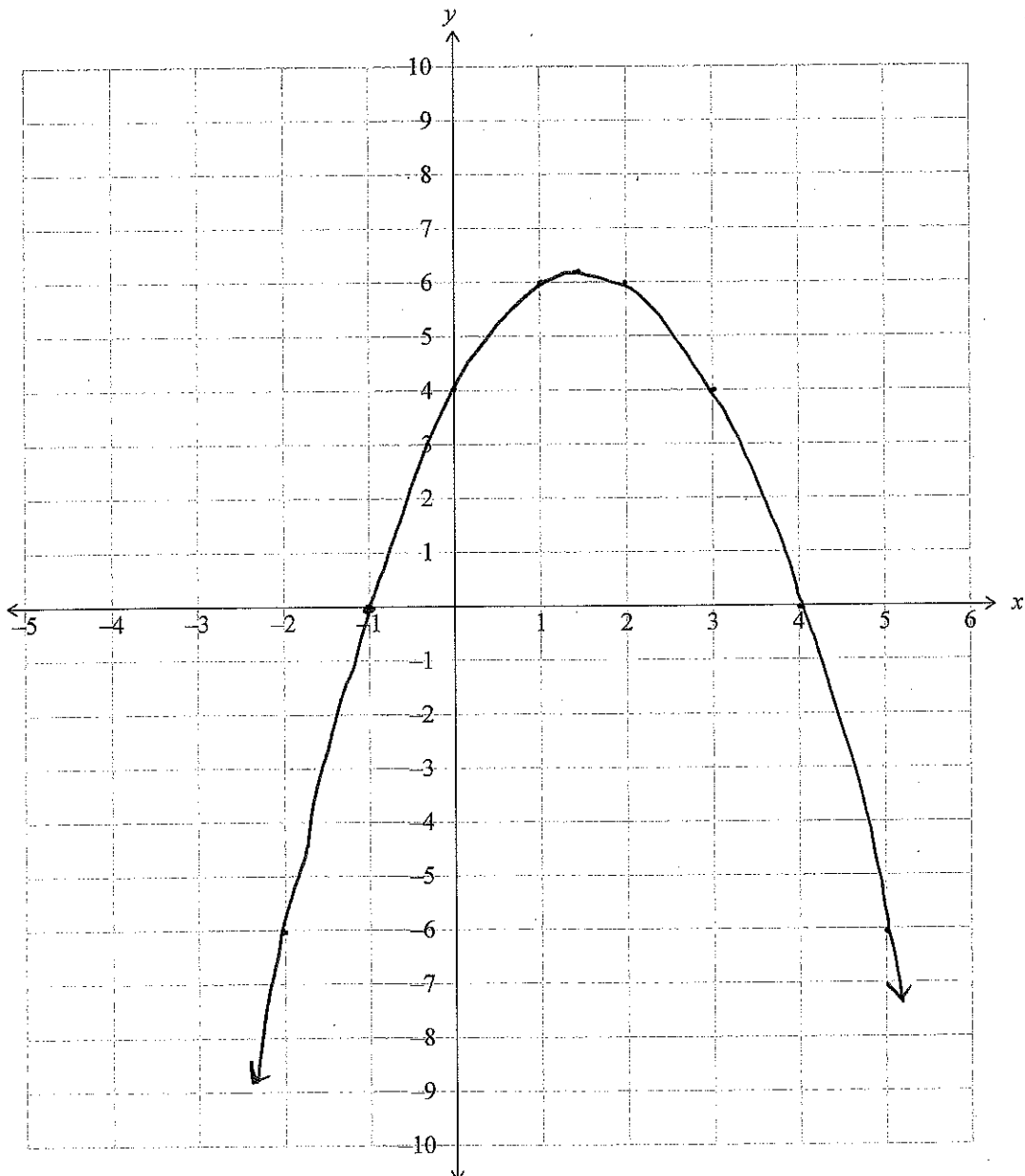(b) The points listed in the table below lie on a parabola.

x	y
−2	−6
−1	0
3	4
5	−6

(i) Sketch the parabola represented by these points, and give the coordinates of the intercepts and the vertex.

Intercepts, x = 4, x = -1, y = 4

Vertex, (1.5, 6.25)

If you
need to
redraw this
graph, use
the grid on
page 14.

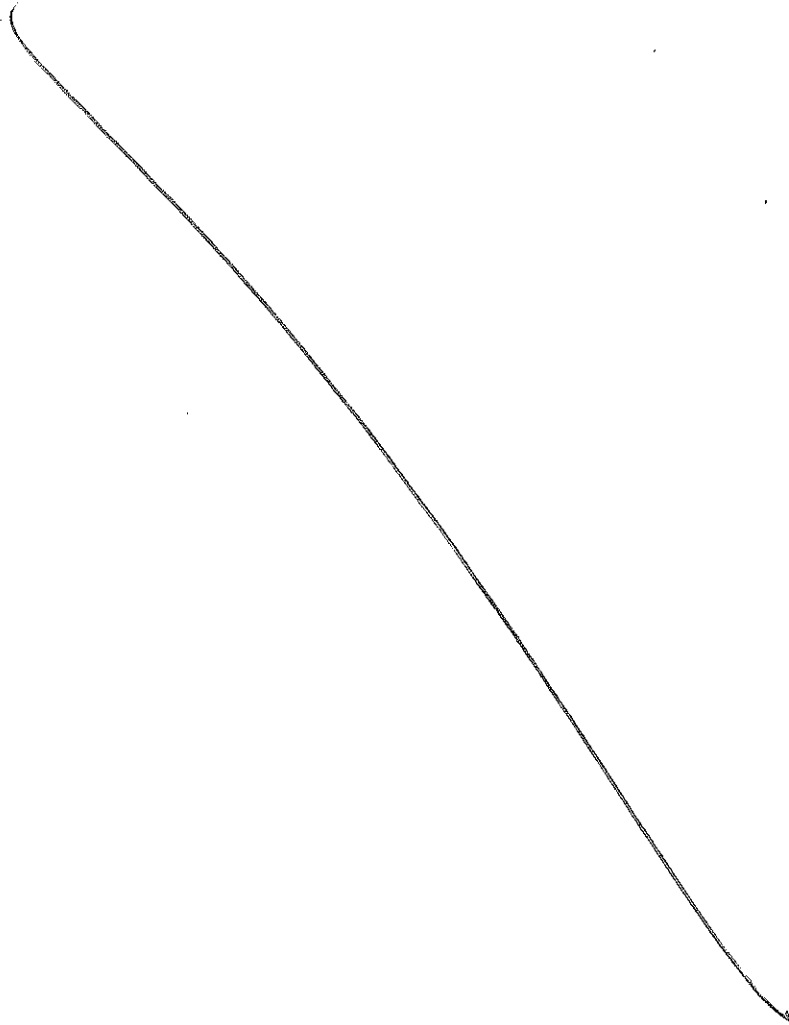(ii) Give the equation of the graph.

$y = a(x-4)(x+1)$

$6 = a(1-4)(1+1)$

$6 = a(-3)(2)$

$6 = -6a$

$a = 6/-6$

$a = -1$

$y = -(x-4)(x+1)$ ///

QUESTION THREE

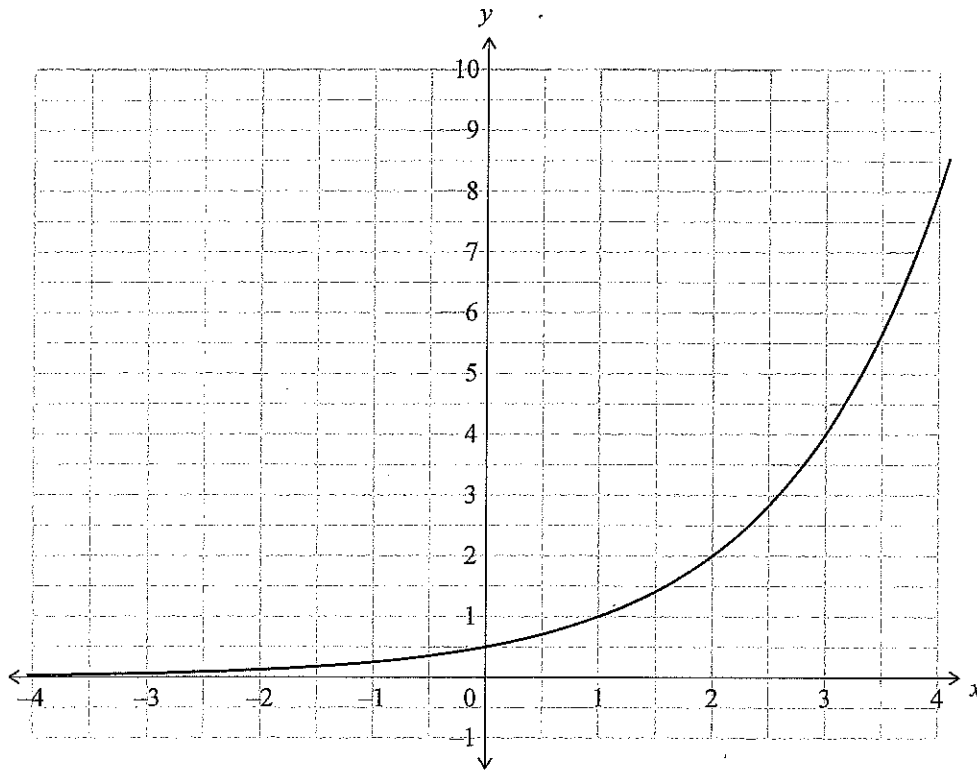(a)　(i)　Give the equation of the graph below.

$\Lambda\ \boxed{0.5}^{(x-1)}\ /\!\!/$

(ii)　Give the equation of the resulting graph if the graph above is reflected in the y axis.

$a \lessgtr 0.5^{(-x-1)}$

$y = \boxed{0.5}^{(-x-1)}\ /\!\!/$

(b) A new fun park was very popular when it opened. In the first three months, an average of 4000 people visited the park each month.

After the first three months, the attendance began to drop by approximately 15% each month for the next nine months.

After the first three months, the approximate number of visitors to the park can be modelled by:

$P = 4000 \times 0.85^{n-3}$, where n is the number of months since the park opened.
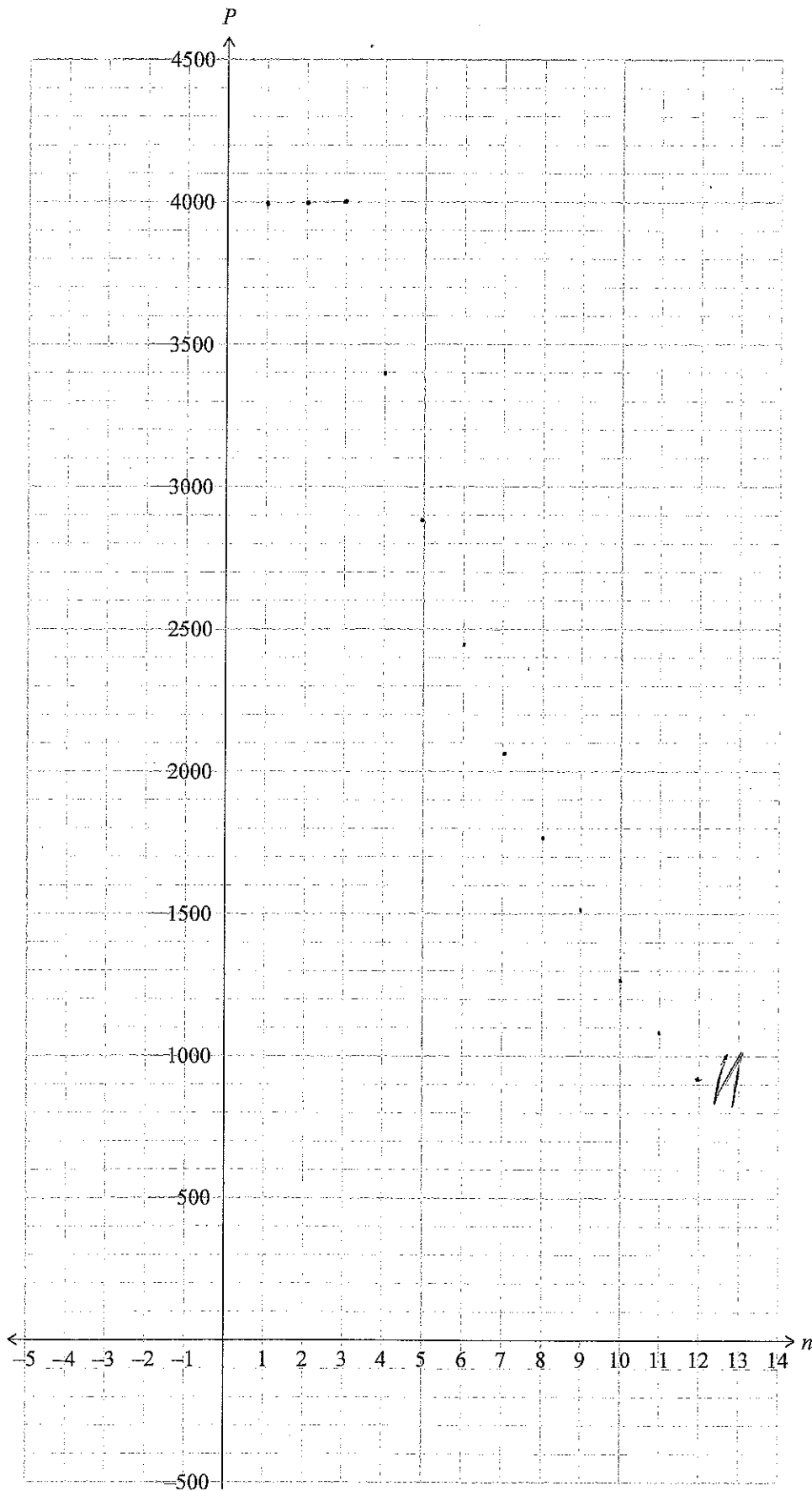
(i) Complete the table below showing the approximate number of people who visited the fun park during each month for the first year.

Month (n)	Approximate number of people visiting park this month (P)
1	4000
2	4000
3	4000
4	3400
5	2890
6	2457
7	2088
8	1775
9	1509
10	1282
11	1090
12	926

(ii) Draw the graph showing the approximate number of people visiting the fun park each month.

If you need to redraw this graph, use the grid on page 15.

(iii) At the end of the month that the number of visitors dropped below 2000 for the first time, the management decided to open only on weekends.

Find how many months of the year the park was open only on the weekends, and explain **by using the features of the graph**, how this information can be found.

When n is less than 2000, at the end of that month, from then on, the park would be open weekends. Since when n=8, y=1775, after that month, the park would only be open weekends so from month 9 and onwards, the park would only be open on weekends. This is 4 months of the year that the park was only open on weekends.//

(iv) In the second year, more people visit the park during the first three months.

As the year progresses, the number of people visiting the park declines at the same rate as it did for the first year.

The managers want to limit to a maximum of 2 months, the period when the park is running just on weekends.

What is the average number of people who would need to be visiting the park each month in the first three months if this was to be achieved?

The managers want to change the 4 months a year to 2 months a year when the park is only open on weekends. So they need to take off two months.

$N=3$, $P=4300$, needs to change to

$N=5$, $P=4000$ using this, then use the equation to go back to month 3...

$N=2$ (now 4), $P=4700$

$N=1$ (now 3) $(P=5537)$

at least

The first 3 months need to have an average number of 5537 people to limit the park running on weekends a maximum of 2 months in the year.//
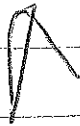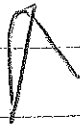
M6

Extra paper if required.
Write the question number(s) if applicable.

ASSESSOR'S
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QUESTION
NUMBER

Q1. a(v) So I would recommend Talia for up to 5.5 hours of babysitting and Sasha for more longer than that.

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Q1. a(v) So I would recommend Talia for up to 5.5 hours of babysitting and Sasha for more longer than that.

91028

Annotated Exemplar Template

Merit exemplar 2016

Subject:	Mathematics		Standard:	91028	Total score:	18
Q	**Grade score**	**Annotation**				
1	M6	a(i). Correct a(ii) Candidate has correctly identified steps but has missed the open circle so u instead of t. a(iii) A common error was using $65 instead of $80. If candidate had gone on to divide by 6 to get $10.83 they could have gained u. a(iv) Correct. a(v) Please note additional paper. Final statement of Talia up to 5.5 hours and Sasha after that gained r. To gain t candidates were required to recognise that Matt was always the most expensive. b(i) Correct equation. b(ii) Candidate has correctly substituted a and b to gain r but has not found x-value of the vertex.				
2	M6	a(i) Candidate has carried graph on below x-axis and has gained u because they have not recognised the situation. a(ii) Correct. a(iii). Candidate gained r for recognising x<10 and x>30 but not that x>0 and x<40. a(iv) Recognising m/2 gained u. This was not then correctly substituted. b(i) Candidate has correctly identified coordinates of vertex and correctly drawn the graph. To gain t the coordinates of the intercepts were required. b(ii) Correct equation.				
3	M6	a(ii) and (ii). Both equations incorrect. b(i) Table completed correctly. b(ii) Discrete points correctly plotted. b(iii) Candidate has correctly identified 4 months with justification. b(iv) Incorrect.				