1)

x	1	3	4	6
y	-7	5	8	14

$$P_3(x) = f(x_0) + f(x_0,x_1)\cdot(x-x_0) + f(x_0,x_1,x_2)\cdot(x-x_0)(x-x_1) + f(x_0,x_1,x_2,x_3)\cdot(x-x_0)(x-x_1)(x-x_2), \text{ где}$$

x	y			
x_0	y_0	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
x_1	y_1	$f(x_1,x_2)$	$f(x_1,x_2,x_3)$	
x_2	y_2	$f(x_2,x_3)$		
x_3	y_3			

$$f(x_0,x_1) = \frac{y_1-y_0}{x_1-x_0},$$

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$$

$$f(x_0,x_1,x_2,x_3) = \frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$$

x	y			
1	-7	6		
3	5	3	-1	
4	8	3	0	1/6
6	14			

$$P_3(x) = -7 + 6(x-1) - 1(x-1)(x-3) + \frac{1}{6}(x-1)(x-3)(x-4)$$

2)

$$\int_1^2 \frac{dx}{x} = \ln|x|\Big|_1^2 = \ln|2| - \ln|1| = 0{,}69315 - 0 = 0{,}69315 = I$$

$$n = 5 \Rightarrow h = \frac{2-1}{5} = \frac{1}{5} = 0{,}2$$

$$\tilde{I} = h\left(\frac{1}{2}f(x_0) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}f(x_n)\right)$$

$$\tilde{I} = 0{,}2\left(\frac{1}{2}\cdot 1 + 0{,}8333 + 0{,}7143 + 0{,}625 + 0{,}5556 + 0{,}25\right) = 0{,}69558$$

$$\varepsilon = |I - \tilde{I}| = |0{,}69315 - 0{,}69558| = 0{,}00243.$$

3)

x	3	4	5
y	2	-1	6

$$y'(x_0) = \frac{y_1 - y_{-1}}{2h} = \frac{6-2}{2\cdot 1} = \frac{4}{2} = 2$$

$x_0 = 4;\ h = 5-4 = 1$

4) $y' = \frac{y-x}{y+x};\ y(0) = 1;\ h = 0{,}1;\ x_i = h\cdot i,\ i = \overline{0,3}$

$$y_1 = y_0 + h\left(\frac{y_0 - x_0}{y_1 + x_0}\right) = 1 + 0{,}1\left(\frac{1-0}{1+0}\right) = 1{,}1$$

$$y_2 = y_1 + h\left(\frac{y_1 - x_1}{y_1 + x_1}\right) = 1{,}1 + 0{,}1\left(\frac{1{,}1 - 0{,}1}{1{,}1 + 0{,}1}\right) = 1{,}1083$$

$$y_3 = y_2 + h\left(\frac{y_2 - x_2}{y_2 + x_2}\right) = 1{,}1083 + 0{,}1\left(\frac{1{,}1083 - 0{,}2}{1{,}1083 + 0{,}2}\right) = 1{,}174$$

$$y_4 = y_3 + h\left(\frac{y_3 - x_3}{y_3 + x_3}\right) = 1{,}174 + 0{,}1\left(\frac{1{,}174 - 0{,}3}{1{,}174 + 0{,}3}\right) = 1{,}2333$$

Ответ: $y_1 = 1{,}1$
$y_2 = 1{,}1083$
$y_3 = 1{,}174$
$y_4 = 1{,}2333$