Discrete Structures

SOFTWARE ENGINEERING/DATA SCIENCE

Semester II, Batch 2019

Lec-07

Venn diagram

UNION:

Let A and B be subsets of a universal set U. The union of sets A and B is the set of all elements in U that belong to A or to B or to both, and is denoted $A \cup B$.

Symbolically:

$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$$

EMAMPLE:

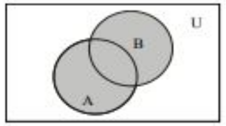
Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$, $B = \{d, e, f, g\}$

Then $A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$

$= \{a, c, d, e, f, g\}$

VENN DIAGRAM FOR UNION:



A ∪ B is shaded

REMARK:

1. $A \cup B = B \cup A$ that is union is commutative you can prove this very easily only by using definition.
2. $A \subseteq A \cup B$ and $B \subseteq A \cup B$

The above remark of subset is easily seen by the definition of union.

MEMBERSHIP TABLE FOR UNION:

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

REMARK:

This membership table is similar to the truth table for logical connective, disjunction ($\vee$).

INTERSECTION:

Let A and B subsets of a universal set U. The intersection of sets A and B is the set of all elements in U that belong to both A and B and is denoted $A \cap B$.

Symbolically:

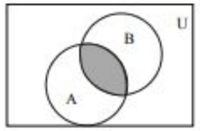
$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$$

EXMAPLE:

Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$, $B = \{d, e, f, g\}$

Then $A \cap B = \{e, g\}$



VENN DIAGRAM FOR INTERSECTION:

$A \cap B$ is shaded

REMARK:

1. $A \cap B = B \cap A$
2. $A \cap B \subseteq A$ and $A \cap B \subseteq B$
3. If $A \cap B = \phi$, then A & B are called disjoint sets.

MEMBERSHIP TABLE FOR INTERSECTION:

A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

REMARK:

This membership table is similar to the truth table for logical connective, conjunction ($\wedge$).

DIFFERENCE:

Let A and B be subsets of a universal set U. The difference of "A and B" (or relative complement of B in A) is the set of all elements in U that belong to A but not to B, and is denoted $A - B$ or $A \setminus B$.

Symbolically:

$$A - B = \{x \in U \mid x \in A \text{ and } x \notin B\}$$

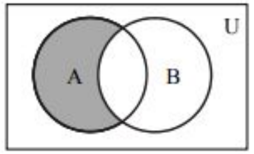
EXAMPLE:

Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$, $B = \{d, e, f, g\}$

Then $A - B = \{a, c\}$

VENN DIAGRAM FOR SET DIFFERENCE:



A-B is shaded

REMARK:

1. $A - B \neq B - A$ that is Set difference is not commutative.
2. $A - B \subseteq A$
3. $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets.

MEMBERSHIP TABLE FOR SET DIFFERENCE:

A	B	A – B
1	1	0
1	0	1
0	1	0
0	0	0

REMARK:

The membership table is similar to the truth table for $\sim (p \rightarrow q)$.

COMPLEMENT:

Let A be a subset of universal set U. The complement of A is the set of all element in U that do not belong to A, and is denoted AN, A or Ac

Symbolically:

$$A^c = \{x \in U \mid x \notin A\}$$

EXAMPLE:

Let $U = \{a, b, c, d, e, f, g]$
$A = \{a, c, e, g\}$
Then $A^c = \{b, d, f\}$

VENN DIAGRAM FOR COMPLEMENT:

A^c is shaded

REMARK :

1. $A^c = U - A$
2. $A \cap A^c = \phi$
3. $A \cup A^c = U$

MEMBERSHIP TABLE FOR COMPLEMENT:

A	A^c
1	0
0	1

REMARK

This membership table is similar to the truth table for logical connective negation (~)

EXERCISE:

Let $U = \{1, 2, 3, \ldots, 10\}$, $X = \{1, 2, 3, 4, 5\}$

$Y = \{y \mid y = 2x, x \in X\}$, $Z = \{z \mid z^2 - 9z + 14 = 0\}$

Enumerate:

(1) $X \cap Y$ (2) $Y \cup Z$ (3) $X - Z$

(4) Y^c (5) $X^c - Z^c$ (6) $(X - Z)^c$

Firstly we enumerate the given sets.

Given

$U = \{1, 2, 3, \ldots, 10\}$,

$X = \{1, 2, 3, 4, 5\}$

$Y = \{y \mid y = 2x, x \in X\} = \{2, 4, 6, 8, 10\}$

$Z = \{z \mid z^2 - 9z + 14 = 0\} = \{2, 7\}$

(1) $X \cap Y = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\}$

$= \{2, 4\}$

(2) $Y \cup Z = \{2, 4, 6, 8, 10\} \cup \{2, 7\}$

$= \{2, 4, 6, 7, 8, 10\}$

(3) $X - Z = \{1, 2, 3, 4, 5\} - \{2, 7\}$

$= \{1, 3, 4, 5\}$

(4) $Y^c = U - Y = \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

(5) $X^c = \{6, 7, 8, 9, 10\}$

$Z^c = \{1, 3, 4, 5, 6, 8, 9, 10\}$

$X^c - Z^c = \{6, 7, 8, 9, 10\} - \{1, 3, 4, 5, 6, 8, 9, 10\}$

$= \{7\}$

(6) $(X - Z)^c = U - (X - Z)$

$= \{1, 2, 3, \ldots, 10\} - \{1, 3, 4, 5\}$

$= \{2, 6, 7, 8, 9, 10\}$

NOTE $(X - Z)^c \neq X^c - Z^c$

EXERCISE:

Given the following universal set U and its two subsets P and Q, where

$U = \{x \mid x \in Z, 0 \le x \le 10\}$

$P = \{x \mid x \text{ is a prime number}\}$

$Q = \{x \mid x^2 < 70\}$

(i) Draw a Venn diagram for the above

(ii) List the elements in $Pc \cap Q$

SOLUTION:

First we write the sets in Tabular form.

$U = \{x \mid x \in Z,\ 0 \le x \le 10\}$

Since it is the set of integers that are greater then or equal 0 and less or equal to 10. So we have

$U = \{0, 1, 2, 3, \ldots, 10\}$

$P = \{x \mid x \text{ is a prime number}\}$

It is the set of prime numbers between 0 and 10. Remember Prime numbers are those numbers which have only two distinct divisors.

$P = \{2, 3, 5, 7\}$

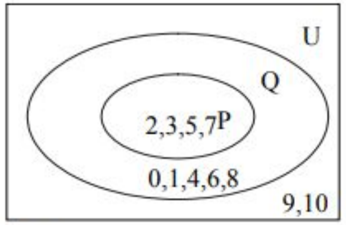
$Q = \{x \mid x^2 < 70\}$

The set Q contains the elements between 0 and 10 which have their square less or equal to 70.

$Q = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Thus we write the sets in Tabular form.

VENN DIAGRAM:



(i) $\mathbf{P^c \cap Q = ?}$

$P^c = U - P = \{0, 1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$
$= \{0, 1, 4, 6, 8, 9, 10\}$

and

$P^c \cap Q = \{0, 1, 4, 6, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
$= \{0, 1, 4, 6, 8\}$

EXERCISE:

Let

$U = \{1, 2, 3, 4, 5\}, \quad C = \{1, 3\}$

and A and B are non empty sets. Find A in each of the following:

(i) $A \cup B = U, \quad A \cap B = \phi$ and $B = \{1\}$

(ii) $A \subset B$ and $A \cup B = \{4, 5\}$

(iii) $A \cap B = \{3\}, \quad A \cup B = \{2, 3, 4\}$ and $B \cup C = \{1,2,3\}$

(iv) A and B are disjoint, B and C are disjoint, and the union of A and B is the set $\{1, 2\}$.

(v)

(i) $\mathbf{A \cup B = U, \quad A \cap B = \phi}$ **and** $\mathbf{B = \{1\}}$

SOLUTION:

Since $A \cup B = U = \{1, 2, 3, 4, 5\}$

and $A \cap B = \phi$,

Therefore $\mathbf{A = B^c = \{1\}^c = \{2, 3, 4, 5\}}$

(ii) $\mathbf{A \subset B}$ **and** $\mathbf{A \cup B = \{4, 5\}}$ **also** $\mathbf{C = \{1, 3\}}$

SOLUTION:

When $A \subset B$, then $A \cup B = B = \{4, 5\}$

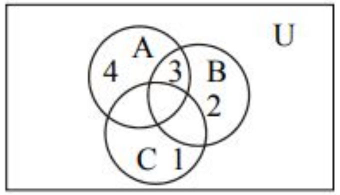
Also A being a proper subset of B implies

$\mathbf{A = \{4\}}$ or $\mathbf{A = \{5\}}$

(iii) $A \cap B = \{3\}$, $A \cup B = \{2, 3, 4\}$ and $B \cup C = \{1,2,3\}$

Also $C = \{1, 3\}$

SOLUTION

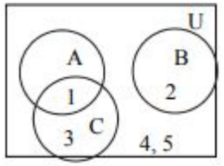


Since we have 3 in the intersection of A and B as well as in C so we place 3 in common part shared by the three sets in the Venn diagram. Now since 1 is in the union of B and C it means that 1 may be in C or may be in B, but 1cannot be in B because if 1 is in the B then it must be in $A \cup B$ but 1 is not there, thus we place 1 in the part of C which is not shared by any other set. Same is the reason for 4 and we place it in the set which is not shared by any other set. Now 2 will be in B, 2 cannot be in A because $A \cap B = \{3\}$, and is not in C. So $A = \{3, 4\}$ and $B = \{2, 3\}$

(iv) $A \cap B = \phi$, $B \cap C = \phi$, $A \cup B = \{1, 2\}$.

Also $C = \{1, 3\}$

SOLUTION



$A = \{1\}$

EXERCISE:

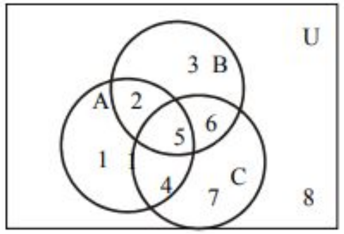
Use a Venn diagram to represent the following:

(i) $(A \cap B) \cap C^c$

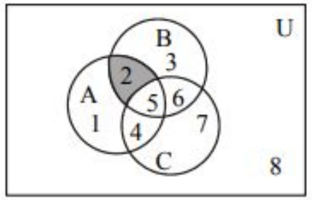
(ii) $A^c \cup (B \cup C)$

(iii) $(A - B) \cap C$

(iv) $(A \cap B^c) \cup C^c$

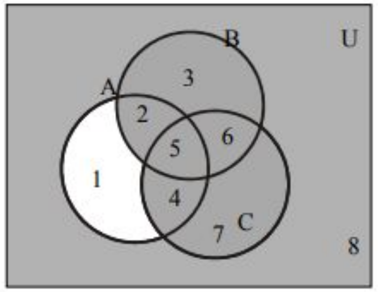


(i) $(A \cap B) \cap C^c$

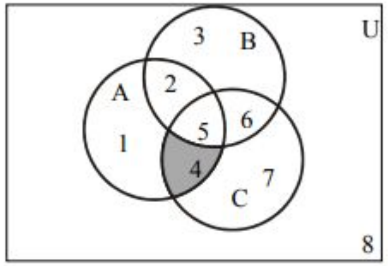


$(A \cap B) \cap C^c$ is shaded

(ii) $A^c \cup (B \cup C)$ is shaded.

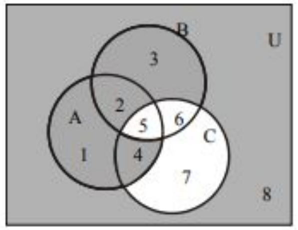


(iii) $(A - B) \cap C$



$(A - B) \cap C$ is shaded

(iv) $(A \cap B^c) \cup C^c$ is shaded.



PROVING SET IDENTITIES BY VENN DIAGRAMS:

Prove the following using Venn Diagrams:

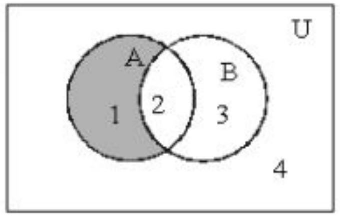
(i) $A - (A - B) = A \cap B$

(ii) $(A \cap B)^c = A^c \cup B^c$

(iii) $A - B = A \cap B^c$

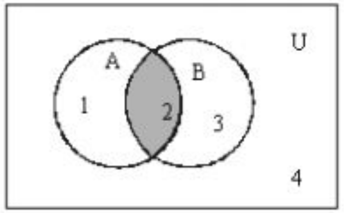
SOLUTION (i)

$$\mathbf{A - (A - B) = A \cap B}$$



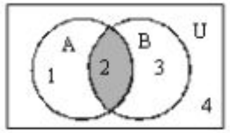
A – B is shaded

$A = \{ 1, 2 \}$
$B = \{ 2, 3 \}$
$A - B = \{ 1 \}$



A – (A – B) is shaded

$A = \{ 1, 2 \}$
$A - B = \{ 1 \}$
$A - (A - B) = \{ 2 \}$



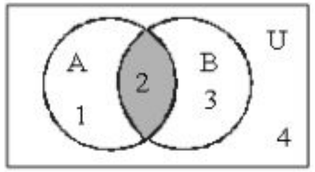
$A \cap B$ is shaded

$A = \{ 1, 2 \}$
$B = \{ 2, 3 \}$
$A \cap B = \{2\}$

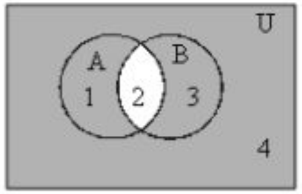
RESULT: $A - (A - B) = A \cap B$

SOLUTION (ii)

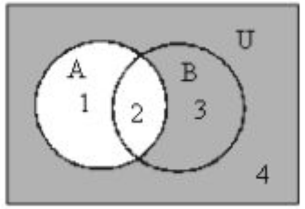
$(A \cap B)^c = A^c \cup B^c$



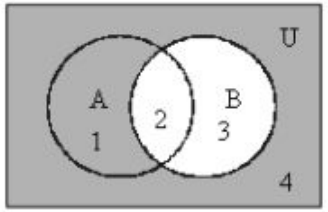
$A \cap B$



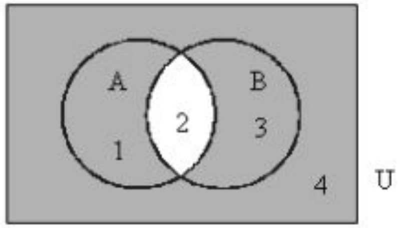
$(A \cap B)^c$ ----------------(a)



A^c is shaded.



B^c is shaded.



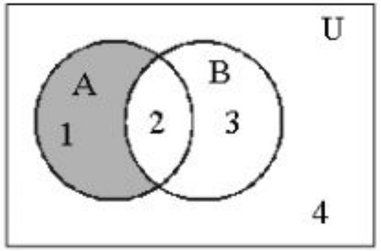
$A^c \cup B^c$ is shaded. --------------(b)

Now diagrams (a) and (b) are same hence

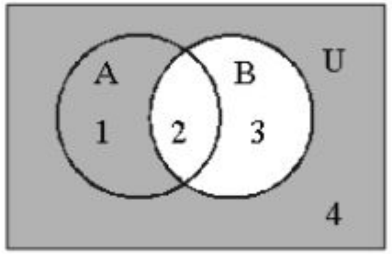
RESULT: $(A \cap B)^c = A^c \cup B^c$

SOLUTION (iii)

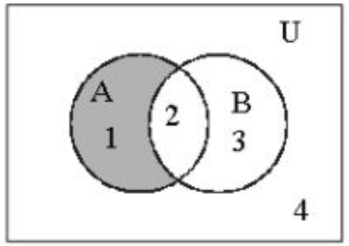
$$A - B = A \cap B^c$$



A – B is shaded. -------------(a)



B^c is shaded.



$A \cap B^c$ is shaded ------------------(b)

From diagrams (a) and (b) we can say

RESULT: $\mathbf{A - B = A \cap B^c}$

PROVING SET IDENTITIES BY MEMBERSHIP TABLE:

Prove the following using Membership Table:

(i) $A - (A - B) = A \cap B$

(ii) $(A \cap B)^c = A^c \cup B^c$

(iii) $A - B = A \cap B^c$

SOLUTION (i)

$$\mathbf{A - (A - B) = A \cap B}$$

A	B	A-B	A-(A-B)	A∩B
1	1	0	1	1
1	0	1	0	0
0	1	0	0	0
0	0	0	0	0

Since the last two columns of the above table are same hence the corresponding set expressions are same. That is

$$\mathbf{A - (A - B) = A \cap B}$$

SOLUTION (ii)

$$\mathbf{(A \cap B)^c = A^c \cup B^c}$$

A	**B**	**A∩B**	**$(A\cap B)^c$**	**A^c**	**B^c**	**$A^c \cup B^c$**
1	**1**	**1**	**0**	**0**	**0**	**0**
1	**0**	**0**	**1**	**0**	**1**	**1**
0	**1**	**0**	**1**	**1**	**0**	**1**
0	**0**	**0**	**1**	**1**	**1**	**1**

Since the fourth and last columns of the above table are same hence the corresponding set expressions are same. That is

$$(A \cap B)^c = A^c \cup B^c$$

SOLUTION (iii)

A	B	$A - B$	B^c	$A \cap B^c$
1	1	0	0	0
1	0	1	1	1
0	1	0	0	0
0	0	0	1	0