Discrete Structures

SOFTWARE ENGINEERING/DATA SCIENCE

Semester II, Batch 2019

Lec-02

Truth Tables for:

1. $\sim p \wedge q$
2. $\sim p \wedge (q \vee \sim r)$
3. $(p \vee q) \wedge \sim (p \wedge q)$

Truth table for the statement form $\sim p \wedge q$

p	q	~p	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

Truth table for $\sim p \wedge (q \vee \sim r)$

p	q	r	~r	$q \vee \sim r$	~p	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	T	T	T

Truth table for $(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

USAGE OF "OR" IN ENGLISH

In English language the word **OR** is sometimes used in an inclusive sense (p or q or both).

Example: I shall buy a pen or a book.
In the above statement, if you buy a pen or a book in both cases the statement is true and if you buy both pen and book, then statement is again true. Thus we say in the above statement we use or in inclusive sense.

The word **OR** is sometimes used in an exclusive sense (p or q but not both). As in the below statement

Example: Tomorrow at 9, I'll be in Lahore or Islamabad.
Now in above statement we are using **OR** in exclusive sense because if both the statements are true, then we have F for the statement.

While defining a disjunction the word **OR** is used in its inclusive sense. Therefore, the symbol **∨** means the "inclusive **OR**"

EXCLUSIVE OR:

When **OR** is used in its exclusive sense, The statement "p or q" means "p or q but not both" or "p or q and not p and q" which translates into symbols as $(p \vee q) \wedge \sim(p \wedge q)$
It is abbreviated as $p \oplus q$ or **p XOR q**

TRUTH TABLE FOR EXCLUSIVE OR:

p	*q*	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

TRUTH TABLE FOR $(p \vee q) \wedge \sim(p \wedge q)$

p	q	p∨q	p∧q	~(p ∧ q)	(p∨q) ∧ ~(p ∧ q)
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Note: Basically

$$\begin{aligned} \mathbf{p \oplus q} &\equiv (p \wedge \sim q) \vee (\sim p \wedge q) \\ &\equiv [p \wedge \sim q) \vee \sim p] \wedge [(p \wedge \sim q) \vee q] \\ &\equiv (p \vee q) \wedge \sim (p \wedge q) \\ &\equiv (p \vee q) \wedge (\sim p \vee \sim q) \end{aligned}$$

LOGICAL EQUIVALENCE

If two logical expressions have the same logical values in the truth table, then we say that the two logical expressions are logically equivalent. In the following example, $\sim(\sim p)$ is logically equivalent p. So it is written as $\mathbf{\sim(\sim p) \equiv p}$

Double Negative Property $\mathbf{\sim(\sim p) \equiv p}$

p	~p	~(~p)
T	F	T
F	T	F

Example

Rewrite in a simpler form:

"It is not true that I am not happy."

Solution:

Let $\mathbf{p}$ = "I am happy"
then $\sim \mathbf{p}$ = "I am not happy"
and $\sim \mathbf{(\sim p)}$ = "It is not true that I am not happy"
Since $\sim(\sim \mathbf{p}) \equiv \mathbf{p}$
Hence the given statement is equivalent to "**I am happy**"

Example

Show that $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent

Solution:

p	q	~p	~q	p∧q	~(p∧q)	~p ∧ ~q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Different truth values in row 2 and row 3

DE MORGAN'S LAWS

1) The negation of an **AND** statement is logically equivalent to the **OR** statement in which each component is negated.

Symbolically $\sim(p \wedge q) \equiv \sim p \vee \sim q$

2) The negation of an **OR** statement is logically equivalent to the **AND** statement in which each component is negated.

Symbolically $\sim(p \vee q) \equiv \sim p \wedge \sim q$

Truth Table of $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Same truth values

APPLICATION:

Give negations for each of the following statements:

a) The fan is slow **or** it is very hot.
b) Akram is unfit **and** Saleem is injured.

Solution:

a) The fan is **not** slow **and** it is **not** very hot.
b) Akram is **not** unfit **or** Saleem is **not** injured.

INEQUALITIES AND DEMORGAN'S LAWS:

Use DeMorgan's Laws to write the negation of

$-1 < x \leq 4$ for some particular real number x

Here, $-1 < x \leq 4$ means $x > -1$ **and** $x \leq 4$

The negation of ($x > -1$ **and** $x \leq 4$) is ($x \leq -1$ **OR** $x > 4$).

We can explain it as follows:

Suppose $p: x > -1$

$q: x \leq 4$

$\sim p: x \leq -1$

$\sim q: x > 4$

The negation of $x > -1$ **AND** $x \leq 4$

$\equiv \sim(p \wedge q)$

$\equiv \sim p \vee \sim q$ by DeMorgan's Law,

$\equiv x \leq -1$ **OR** $x > 4$

EXERCISE:

1. Show that $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
2. Are the statements $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ logically equivalent?

TAUTOLOGY:

A tautology is a statement form that is always true regardless of the truth values of the statement variables. A tautology is represented by the symbol "**t**".

EXAMPLE: The statement form $p \vee \sim p$ is tautology

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

$$p \vee \sim p \equiv t$$

CONTRADICTION:

A contradiction is a statement form that is always false regardless of the truth values of the statement variables. A contradiction is represented by the symbol "c".

So if we have to prove that a given statement form is **CONTRADICTION,** we will make the truth table for the statement form and if in the column of the given statement form all the entries are F, then we say that statement form is contradiction.

EXAMPLE:

The statement form $p \wedge \sim p$ is a contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Since in the last column in the truth table we have F in all the entries, so it is a contradiction i.e. $p \wedge \sim p \equiv c$

REMARKS:

- Most statements are neither tautologies nor contradictions.
- The negation of a tautology is a contradiction and vice versa.
- In common usage we sometimes say that two statement are contradictory. By this we mean that their conjunction is a contradiction: they cannot both be true.

LOGICAL EQUIVALENCE INVOLVING TAUTOLOGY

1. Show that $p \wedge t \equiv p$

p	t	$p \wedge t$
T	T	T
F	T	F

Since in the above table the entries in the first and last columns are identical so we have the corresponding statement forms are Logically equivalent that is

$$p \wedge t \equiv p$$

LOGICAL EQUIVALENCE INVOLVING CONTRADICTION

Show that $p \wedge c \equiv c$

p	c	$p \wedge c$
T	F	F
F	F	F

There are same truth values in the indicated columns, so $p \wedge c \equiv c$

EXERCISE:

Use truth table to show that $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ is a tautology.

SOLUTION:

Since we have to show that the given statement form is Tautology, so the column of the above proposition in the truth table will have all entries as T. As clear from the table below

p	q	$p \wedge q$	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	F	T	T	F	T	T

Hence $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q)) \equiv t$

EXERCISE:

Use truth table to show that $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

SOLUTION:

Since we have to show that the given statement form is Contradiction, so its column in the truth table will have all entries as F. As clear from the table below.

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

LAWS OF LOGIC

1) **Commutative Laws**

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

2) **Associative Laws**

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

3) **Distributive Laws**

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

4) **Identity Laws**

$$p \wedge t \equiv p$$
$$p \vee c \equiv p$$

5) **Negation Laws**

$$p \vee {\sim}p \equiv t$$
$$p \wedge {\sim}p \equiv c$$

6) **Double Negation Law**

$${\sim}({\sim}p) \equiv p$$

7) **Idempotent Laws**

$$p \wedge p \equiv p$$
$$p \vee p \equiv p$$

8) **DeMorgan's Laws**

$${\sim}(p \wedge q) \equiv {\sim}p \vee {\sim}q$$
$${\sim}(p \vee q) \equiv {\sim}p \wedge {\sim}q$$

9) **Universal Bound Laws**

$$p \vee t \equiv t$$
$$p \wedge c \equiv c$$

10) **Absorption Laws**

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

11) **Negation of t and c**

$${\sim}t \equiv c$$
$${\sim}c \equiv t$$