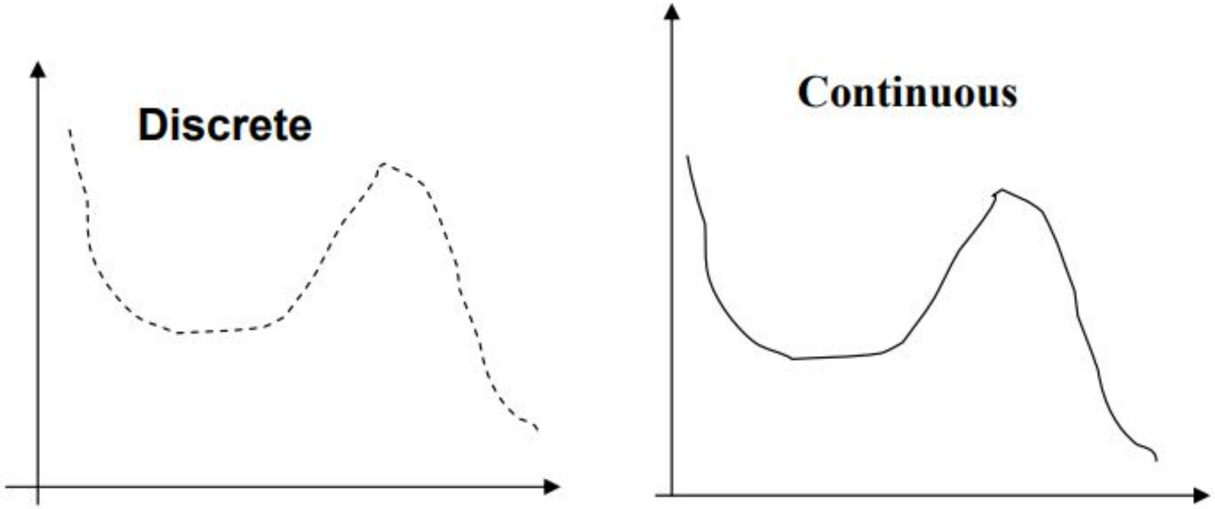
Discrete Structures

SOFTWARE ENGINEERING/DATA SCIENCE

Semester II, Batch 2019

Lec-01



Set of Integers:

3 -2 -1 0 1 2

Set of Real Numbers:

-3 -2 -1 0 1 2 3

What is Discrete Mathematics?

Discrete Mathematics concerns processes that consist of a sequence of individual steps.

LOGIC:

Logic is the study of the principles and methods that distinguish between a valid and an invalid argument.

SIMPLE STATEMENT:

A statement is a declarative sentence that is either true or false but not both.
A statement is also referred to as a **proposition**

EXAMPLES:

a. 2+2 = 4,
b. It is Sunday today

If a proposition is true, we say that it has a **truth value** of "**true**".
If a proposition is false, its truth value is "**false**".
The truth values **"true"** and **"false"** are, respectively, denoted by the letters **T** and **F**.

EXAMPLES:

Propositions

1) Grass is green.
2) $4 + 2 = 6$
3) $4 + 2 = 7$
4) There are four fingers in a hand.

Not Propositions

1) Close the door.
2) x is greater than 2.
3) He is very rich

Rule:

If the sentence is preceded by other sentences that make the pronoun or variable reference clear, then the sentence is a statement.

Example:

$x = 1$
$x > 2$
"$x > 2$" is a statement with truth-value FALSE.

Example

Bill Gates is an American
He is very rich
"He is very rich" is a statement with truth-value TRUE.

UNDERSTANDING STATEMENTS

1)	$x + 2$ is positive.	Not a statement
2)	May I come in?	Not a statement
3)	Logic is interesting.	A statement
4)	It is hot today.	A statement
5)	$-1 > 0$	A statement
6)	$x + y = 12$	Not a statement

COMPOUND STATEMENT:

Simple statements could be used to build a compound statement.

LOGICAL CONNECTIVES

EXAMPLES:

1. "3 + 2 = 5" **and** "Lahore is a city in Pakistan"
2. "The grass is green" **or** " It is hot today"
3. "Discrete Mathematics is **not** difficult to me"

AND, OR, NOT are called LOGICAL CONNECTIVES.

SYMBOLIC REPRESENTATION

Statements are symbolically represented by letters such as ***p, q, r,...***

EXAMPLES:

p = "Islamabad is the capital of Pakistan"
q = "17 is divisible by 3"

CONNECTIVE	MEANINGS	SYMBOLS	CALLED
Negation	not	$\sim$	Tilde
Conjunction	and	$\wedge$	Hat
Disjunction	or	$\vee$	Vel
Conditional	if…then…	$\rightarrow$	Arrow
Biconditional	if and only if	$\leftrightarrow$	Double arrow

EXAMPLES

p = "Islamabad is the capital of Pakistan"
q = "17 is divisible by 3"
$p \wedge q$ = "Islamabad is the capital of Pakistan and 17 is divisible by 3"
$p \vee q$ = "Islamabad is the capital of Pakistan or 17 is divisible by 3"
$\sim p$ = "It is not the case that Islamabad is the capital of Pakistan"
or simply "Islamabad is not the capital of Pakistan"

TRANSLATING FROM ENGLISH TO SYMBOLS

Let p = "It is hot", and q = " It is sunny"

SENTENCE	**SYMBOLIC FORM**
1.It is **not** hot.	$\sim p$
2.It is hot **and** sunny.	$p \wedge q$
3.It is hot **or** sunny.	$p \vee q$
4.It is **not** hot **but** sunny.	$\sim p \wedge q$
5.It is **neither** hot **nor** sunny.	$\sim p \wedge \sim q$

EXAMPLE

Let ***h*** = "Zia is healthy"
w = "Zia is wealthy"
s = "Zia is wise"

Translate the compound statements to symbolic form:

1) Zia is healthy and wealthy but not wise. $(h \wedge w) \wedge (\sim s)$
2) Zia is not wealthy but he is healthy and wise. $\sim w \wedge (h \wedge s)$
3) Zia is neither healthy, wealthy nor wise. $\sim h \wedge \sim w \wedge \sim s$

TRANSLATING FROM SYMBOLS TO ENGLISH:

Let m = "Ali is good in Mathematics"
c = "Ali is a Computer Science student"

Translate the following statement forms into plain English:

1) $\sim c$ Ali is **not** a Computer Science student
2) $c \vee m$ Ali is a Computer Science student **or** good in Maths.
3) $m \wedge \sim c$ Ali is good in Maths **but not** a Computer Science student

A convenient method for analyzing a compound statement is to make a truth table for it.

Truth Table

A **truth table** specifies the truth value of a compound proposition for all possible truth values of its constituent propositions.

NEGATION (~):

If ***p*** is a statement variable, then negation of ***p***, ***"not p"***, is denoted as **"~p"**
It has opposite truth value from p i.e., if p is true, then ~ p is false; if p is false, then ~ p is true.

TRUTH TABLE FOR ~ p

p	$\sim p$
T	F
F	T

CONJUNCTION (∧):

If ***p*** and ***q*** are statements, then the conjunction of ***p*** and ***q*** is ***"p and q"***, denoted as ***"p ∧ q"***.

Remarks

- $p \wedge q$ is true only when both p and q are true.
- If either p or q is false, or both are false, then $p \wedge q$ is false.

TRUTH TABLE FOR p ∧ q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION (∨) or INCLUSIVE OR

If ***p*** & ***q*** are statements, then the disjunction of ***p*** and ***q*** is ***"p or q"***, denoted as ***"p ∨ q"***.

Remarks:

- ***p ∨ q*** is true when at least one of p or q is true.
- ***p ∨ q*** is false only when both p and q are false.

TRUTH TABLE FOR p ∨ q

p	*q*	*p ∨ q*
T	T	T
T	F	T
F	T	T
F	F	F

Note it that in the table F is only in that row where both p and q have F and all other values are T. Thus for finding out the truth values for the disjunction of two statements we will only first search out where the both statements are false and write down the F in the corresponding row in the column of p ∨ q and in all other rows we will write T in the column of p ∨ q.

Remark:

Note that for Conjunction of two statements we find the T in both the statements, But in disjunction we find F in both the statements. In other words, we will fill T in the first row of conjunction and F in the last row of disjunction.

SUMMARY

1. What is a statement?
2. How a compound statement is formed.
3. Logical connectives (negation, conjunction, disjunction).
4. How to construct a truth table for a statement form.