Numerical solutions for a two-dimensional airfoil undergoing unsteady motion

WU Fu-bing, ZENG Nian-dong, ZHANG Liang, and WU De-ming

School of Shipbuilding Engineering, Harbin Engineering University, Harbin 150001, China

Abstract: Continuous vorticity panels are used to model general unsteady inviscid, incompressible, and two-dimensional flows. The geometry of the airfoil is approximated by series of short straight segments having endpoints that lie on the actual surface. A piecewise linear, continuous distribution of vorticity over the airfoil surface is used to generate disturbance flow. The no-penetration condition is imposed at the midpoint of each segment and at discrete times. The wake is simulated by a system of point vortices, which move at local fluid velocity. At each time step, a new wake panel with uniform vorticity distribution is attached to the trailing edge, and the condition of constant circulation around the airfoil and wake is imposed. A new expression for Kutta condition is developed to study (i) the effect of thickness on the lift build-up of an impulsively started airfoil, (ii) the effects of reduced frequency and heave amplitude on the thrust production of flapping airfoils, and (iii) the vortex-airfoil interaction. This work presents some hydrodynamic results for tidal-stream turbine.

Keywords: unsteady flows; vorticity panels; kutta condition; flapping airfoil; vortex-airfoil interaction

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1 INTRODUCTION

Giesing[1,2] as well as Basu and Hancock[3] has used surface singularity methods to solve the potential incompressible flow about two-dimensional airfoils undergoing unsteady motion. The difference between them was in the application of the Kutta condition.

Kim and Mook[4] used vorticity panels to model general unsteady inviscid, and incompressible, two-dimensional lifting flows. A piecewise linear, continuous distribution of vorticity over the approximated surface is used to generate the disturbance flow. The Kutta condition invoked by Kim and Mook[4] is that vortex strengths of upper and lower surface at the trailing edge are equal to zero and a point vortex of unknown strength is placed at the trailing edge. However the zero values are only consistent with the requirement of steady flow around airfoils of nonzero trailing-edge angles.

In this paper the same singularity distribution as Kim and Mook[4] is used, but a quite different Kutta condition is invoked. The method is applied to study (i) the effect of thickness on the lift build-up of an impulsively started airfoil, (ii) the effects of reduced frequency and heave amplitude on the thrust production of flapping airfoils, and (iii) the vortex-airfoil interaction.

2 MATHEMATICAL FORMULATION

Consider incompressible inviscid flow over an airfoil of arbitrary geometry which may execute an arbitrary motion. The problem is formulated in a space-fixed reference frame (Figure 1), which will simplify the computation effort when examining the wake and the interaction of several bodies. A body-fixed reference frame is chosen to model the body geometry. The flow is irrotational, the velocity potential $\Phi(\boldsymbol{R}, t)$ at

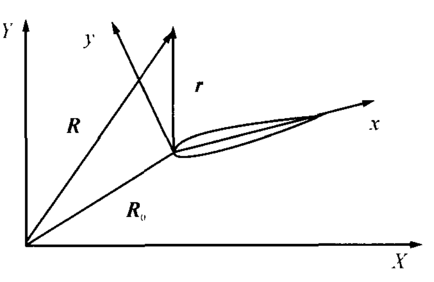

Fig. 1 Fixed and moving frame

point $\boldsymbol{R}$ and time t satisfies Laplace equation

$$\nabla^2 \Phi = 0, \tag{1}$$

and the fluid velocity $\boldsymbol{V} = \nabla \Phi$. Equation (1) governs the velocity potential variation throughout the flowfield and the temporal dependency must be introduced

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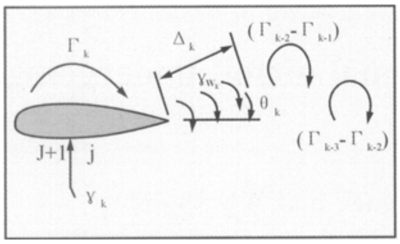

Fig. 2 Schematic of wake model

through the boundary conditions.

For a body moving with a space-fixed reference frame through an otherwise quiescent fluid, the far-field boundary condition is:

$$\lim_{r=|\boldsymbol{R}-\boldsymbol{R}_0|\to\infty} \nabla\Phi = 0, \tag{2}$$

and the no-penetration condition that should be imposed on the body surface is:

$$(\nabla\Phi - \boldsymbol{V}_s)\cdot\boldsymbol{n} = 0, \tag{3}$$

where $\boldsymbol{n}$ is the normal vector. This velocity of the airfoil surface motion $\boldsymbol{V}_s$ is given as

$$\boldsymbol{V}_s = \boldsymbol{V}_0 + \boldsymbol{\Omega}\times\boldsymbol{r}, \tag{4}$$

where the body moves with translating velocity $\boldsymbol{V}_0$, angular velocity $\boldsymbol{\Omega}$ and $\boldsymbol{r}$ is the position of the fluid particle relative to the moving frame.

The time dependency in the problem is introduced through Eqs. (3) and (4). An additional constraint on the temporal evolution of the flowfield is introduced by Kelvin's theorem, which states that the total circulation around a fluid curve remains constant,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \frac{\mathrm{d}\Gamma_B}{\mathrm{d}t} + \frac{\mathrm{d}\Gamma_W}{\mathrm{d}t} = 0. \tag{5}$$

Therefore any change in circulation around the body $\frac{d\Gamma_B}{dt}$ will be balanced by a change in circulation of the wake $\frac{d\Gamma_W}{dt}$.

3 CALCULATION PROCEDURE

The solution of the flow about an airfoil undergoing an arbitrary time dependent motion, which started at $t = 0$, is calculated at successive time intervals of time $t_k (k = 1, 2, \cdots)$. This unsteady model is shown in Figure 2 at time t_k.

The airfoil surface is divided into N straight-line elements. A vorticity is distributed on each element, and its strength varies linearly from $(\gamma_i)_k$ to $(\gamma_{i+1})_k$ across the element, where $i = 1, 2, \cdots, N$. The subscript k refers to time t_k. A small straight-line wake element, with uniform vorticity distribution $(\gamma_u)_k$, length Δ_k and inclined angle θ_k to the X-axis is attached to the trailing edge. The values of Δ_k and θ_k can be obtained as part of the solution and they are determined by the condition that this wake element is tangential to local resultant velocity and its length is proportional to the local resultant velocity, i. e.,

$$\theta_k = \arctan[(V_u)_k/(U_u)_k], \tag{6a}$$

and

$$\Delta_k = [(V_u)_k^2 + (U_u)_k^2]^{1/2}(t_k - t_{k-1}), \tag{6b}$$

where $(U_u)_k$ and $(V_u)_k$ are the induced velocity components in X- and Y-axis at the midpoint of the wake element. A downstream wake shed at earlier times is assumed to be concentrated into discrete vortices and convected by the resultant velocities calculated at the center of each vortex at each successive time interval. Thus, the strength and position of the downstream discrete vortices are regarded as known at time t_k.

At time t_k there are $N + 1$ vorticity strengths $(\gamma_j)_k$, $(j = 1, 2, \cdots N+1)$, Δ_k, θ_k, and $(\gamma_u)_k$, i. e., $N + 4$ unknowns. Thus, the following equations must be solved.

(I) The N no-penetration boundary conditions, Eq. (3), are imposed at the external midpoint of each airfoil element

$$\sum_{j=1}^{N+1} a_{ij}\gamma_j + T_i\gamma_u = C_i, \tag{7}$$

where a_{ij} is the normal component of the velocity at the collocation point of element i induced by unit vorticity strength at node j, T_i is the influence coefficient due to the wake element, and C_i represents the net induced normal velocity at the i th collocation point due to the airfoil motion and wake vortices.

(II) The Kelvin's circulation conservation condition, Eq. (5), is given by

$$\Delta_k(\gamma_u)_k + \Gamma_k = \Gamma_{k-1}, \tag{8}$$

where Γ_k represents the circulation around the airfoil at time t_k.

(III) The Kutta condition invoked in this paper is

$$\gamma_1 + \gamma_{N+1} = \gamma_w, \tag{9}$$

where γ_1 and γ_{N+1} are distributed vorticity strengths of lower and upper surface at the trailing edge.

These nonlinear equations can be solved by the iterative procedure: The values of Δ_k and θ_k are guessed. Thus there are $N+1$ linear equations consisting of Eqs. (7) ~ (9). Eqs. (7) ~ (9) are solved to yield the vorticity strengths of the airfoil elements and the wake element. Thus, Δ_k and θ_k can be calculated from Eq. (6) when the $N+1$ vorticity strengths of airfoil elements are known. The procedure are repeated until Δ_k and θ_k have converged to the desired accuracy.

Once the vorticity strengths are determined, the velocity distribution on the airfoil surface can be calculated. The pressure coefficients follow from the unsteady Bernoulli's equation, namely

$$C_p = -2\frac{\partial \Phi}{\partial t} - 2\boldsymbol{V}\cdot(\boldsymbol{V}_0 + \boldsymbol{\Omega}\times\boldsymbol{r}) - \boldsymbol{V}\cdot\boldsymbol{V}, \tag{10}$$

where Φ is the potential function expressed in terms of $\boldsymbol{r}$. The force and moment coefficients are obtained by direct integration of the pressure distributions.

Once the solution at time t_k has been determined, the distribution on the wake element at time t_k is assumed to be concentrated into a vortex of strength $(\gamma_w)_k\Delta_k$ and at time t_{k+1} shed to the position at

$$\begin{cases} X_{k+1} = (X_{TE})_k + \frac{1}{2}\Delta_k\cos\theta_k + (U_w)_k(t_{k+1}-t_k), \\ Y_{k+1} = (Y_{TE})_k + \frac{1}{2}\Delta_k\sin\theta_k + (V_w)_k(t_{k+1}-t_k). \end{cases} \tag{11}$$

The total velocity of the other concentrated vortices in the wake can be calculated from the solution at time t_k, the positions of the vortices at time t_{k+1} can be determined as well.

4 RESULTS AND DISCUSSION

In this section we provide a number of calculations to compare against previous results and demonstrate the use of present numerical scheme.

4.1 Impulsively started airfoil

When an airfoil starts impulsively from rest at an angle of attack, a circulation is generated around the airfoil and a vortex sheet is shed in the wake. The problem of a flat-plate has been solved by Wagner[5], whose method is based upon the assumption of an undistorted flat wake sheet. His result shows that the initial lift of the airfoil is one-half of the final steady value.

In the first example, we compare our result with linear theory for an impulsively started flat plate at small angle of attack. The angle of attack, represented by α, is two degrees. Figure 3 shows how the numerically predicted force converges to the theoretical result. In the figure, U is the reference velocity, c is the chord, C_L is transient lift coefficient, Ut/c is non-dimensional time, and C_{LS} represents the lift coefficient for steady case.

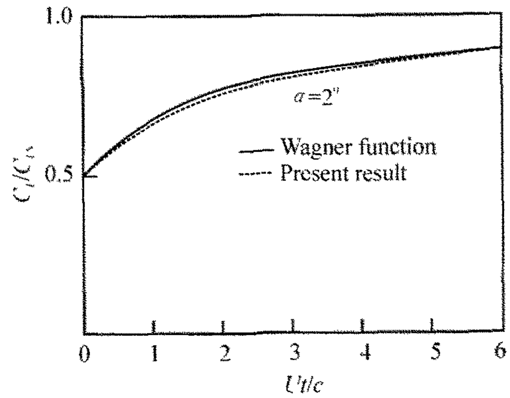

Fig. 3 Lift history of a flat plate

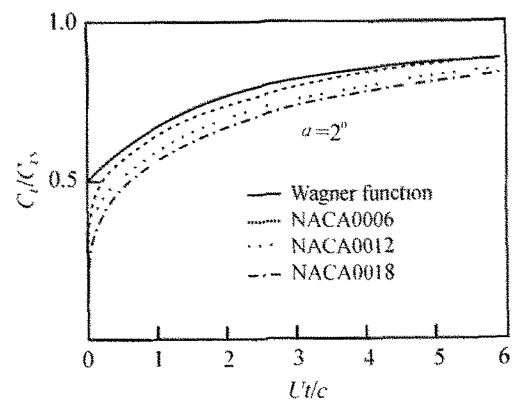

Fig. 4 Effect of thickness on Wagner function

Figure 4 shows the effect of thickness on Wagner function. Our result for three NACA airfoils with different thickness reveals that increasing thickness will decrease the initial lift and retard the build-up of lift.

4.2 Influencing factors for thrust production of flapping airfoil

When an airfoil advances a constant velocity while executing a sinusoidal heaving motion, the average value of drag is negative, indicating that the heaving motion of the airfoil produces a small thrust.

The classical linear approach considered a flat-plate airfoil heaving with infinitesimal amplitude, at a specified frequency and with a non-deforming, planar wake. Here we concentrate on the nonlinear effect on the thrust production of flapping airfoil. The heaving motion is defined as

$$Y_0(\bar{t}) = h \sin(k\bar{t}) \quad \text{and} \quad k = \omega c/U, \tag{13}$$

where h is heaving amplitude in terms of chord, $\bar{t}$ is non-dimensional time, k is reduced frequency, and ω is circular frequency. Thrust coefficient C_t is equal to minus drag coefficient.

In figure 5, the heaving amplitude is varied for a NACA0012 airfoil, and it is clear that the heaving amplitude has a strong influence on the thrust. In figure 6, the reduced frequency is varied for a NACA0012 airfoil, while holding heaving amplitude constant, the thrust shows a strong dependency on reduced frequency. As the heaving amplitude or reduced frequency is increased, the thrust is increased accordingly.

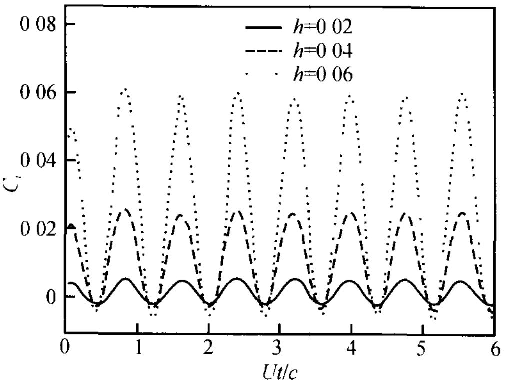

Fig. 5 Effect of heaving amplitude on C_t

NACA0012, $K = 4.0$

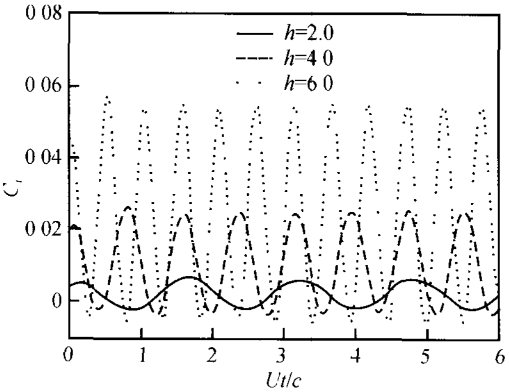

Fig. 6 Effect of reduced frequency on C_t

NACA0012, $H = 0.04$

4.3 Vortex-airfoil interaction

The last example is the case of vortex-airfoil interaction. Vortex-airfoil interaction is a well-known phenomenon that occurs in the flowfield about a tubine. In this model, a discrete vortex is released at a point upstream of a stationary airfoil.

For this problem, a vortex having a circulation $\Gamma/Uc = 0.2$ (positive in the clockwise direction), initially placed below the airfoil at different values of $(y/c)_0$, and passed an NACA0012 airfoil at zero angle of attack. In the general vortex-airfoil interaction problem, the initial position of the vortex above (or below) the airfoil is an important variable. All calculations were started at $(x/c)_0 = -5.0$ and $\alpha = 0$.

The results are shown in Figs. 7 and 8 where the effect of the initial vortex height is shown. As expected, the close encounters result in large fluctuations in the aerodynamic coefficients. For upstream and down-

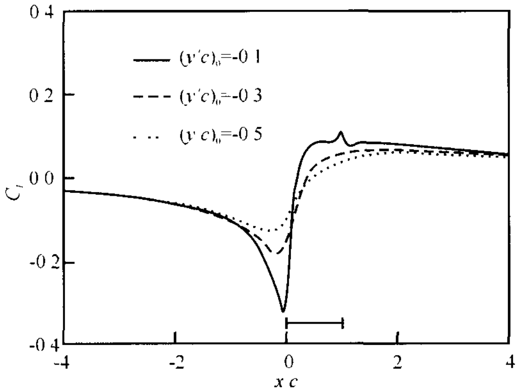

Fig. 7 Effect of initial vortex height on C_L

$\Gamma/U_c = 0.2, \alpha = 0.0$

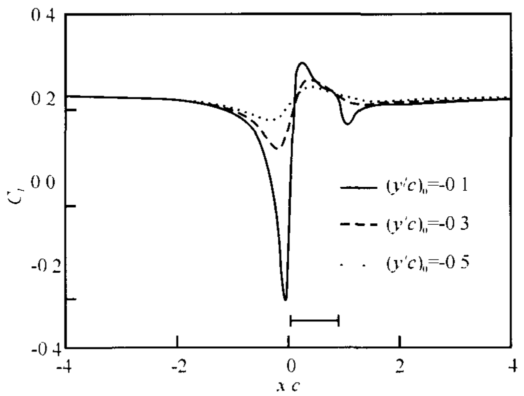

Fig. 8 Effect of initial vortex height on C_d

$\Gamma/U_c = 0.2, \alpha = 0$

stream of the airfoil, the results are virtually identical. In fact, the results are almost identical up to the point where the traveling vortex is in line with the leading edge. As the vortex passes the airfoil, larger differences become apparent. The drag is largely negative (vortex exerts a forward thrust on the airfoil) during the interaction.

5 CONCLUSIONS

A numerical scheme for modeling general unsteady two-dimensional lifting flows is developed. In order to imitate the trailing edge flow, a new expression for Kutta condition is derived. Based upon present numerical scheme, we study the effect of thickness on the lift build-up of an impulsively started airfoil and compare our result with the linear theory. Increasing thickness retards the lift build-up. For flapping airfoil, increasing reduced frequency and heaving amplitude will increase the thrust. As the last example, calculations for a vortex-airfoil interaction are performed for several initial vortex positions. It was found that the main features of lift and drag results could be characterized by incremental coefficients.

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WU Fubing, born in 1970, is studying for a doctorate of fluid mechanics in Harbin Engineering University. He graduated as a bachelor of petroleum engineering in Daqing Petroleum Institute in July, 1992. Then he graduated as a master of petroleum engineering in Daqing Petroleum Institute in March, 2000. His dissertation for the Degree of M. Eng is Second order pertubation solution of power-law fluid through helical pipe, and supervised by Cui Haiqing. He has put forward 2 theses in domestic famous publication. The main research directions: numerical simulation of non-Newtonian fluid flow; nonlinear free surface; theoretical modeling of flapping airfoil propulsion.

ZENG Niandong, born in 1976, is studying for his doctor degree of fluid mechanics in Harbin Engineering University. He graduated as a bachelor of mechanical design and manufacture in Daqing Petroleum Institute in 1997. Then he graduated as a master of mechanical design and theory in Daqing Petroleum Institute in 2000. His dissertation for the Degree of M. Eng is Flowfield characteristics investigation of hydorcyclone, and supervised by Jiang Minghu. He has put forward 2 theses in domestic famous publication. The main research directions: design and numerical simulation of hydrocyclone; large eddy simulation; bionic propulsion.

ZHANG Liang, born in 1959, doctor of engineering, professor, doctoral supervisor of fluid mechanics. Having graduated as a bachelor of mechanical design in Inner Mongolia Polytechnic University, he finished a 2.5 years study of fluid mechanics in Tsinghua University. He got his Msc and PhD of marine hydrodynamics in Harbin Shipbuilding Engineering Institute in 1989 and 1992 respectively. The main research interest: marine hydrodynamics; marine current energy utilization; wave-current-body interaction; hydrodynamic interference of two underwater bodies. He obtained many rewards in teaching and research. The achievement on the marine current turbine and energy conversion research has been acknowledged by many specialists worldwide. He has published more than 30 papers. Besides teaching work, he is now undertaking the contracted research of National 973 Program, National Hi-Tech. R&D Program, National Natural Science Foundation Program, and Sino-Italy Bilateral Program of Science and Technology.

WU Deming, born in 1938, professor, doctoral supervisor of fluid mechanics. He graduated as a bachelor of aerodynamics in Harbin Military Institute in 1962. Since then, he has been working in Harbin Military Institute, Harbin Institute of Architect Engineering, and Harbin Engineering University as a teacher.