Interaction of two-dimensional impulsively started airfoils

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Abstract: Continuous vorticity panels were used to model general unsteady inviscid, incompressible, two-dimensional flows. The geometry of the airfoil was approximated by series of short straight segments having endpoints that lie on the actual surface. A piecewise linear, continuous distribution of vorticity over the airfoil surface was used to generate disturbance flow. The no-penetration condition was imposed at the midpoint of each segment and at discrete times. The wake was simulated by a system of point vortices, which moved at local fluid velocity. At each time step, a new wake panel with uniform vorticity distribution was attached to the trailing edge, and the condition of constant circulation around the airfoil and wake was imposed. A new expression for Kutta condition was developed to study the interference effect between two impulsively started NACA0012 airfoils. The tandem arrangement was found to be the most effective to enhance the lift of the rear airfoil. The interference effect between tidal turbine blades was shown clearly.

Key words: unsteady flows; vorticity panels; kutta condition; interference effect
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1 INTRODUCTION

The potential flow about 2D airfoils undergoing unsteady motion at low speeds has been studied by Giesing[1] and Basu [2] using the panel methods. Both of them approximated the airfoil by a series of short straight segments. They used a uniform source distribution whose strength varies from element to element and a uniform vorticity distribution on the airfoil. These two methods differ in the application of the Kutta condition. Kim and Mook[3] developed a continuous linear vorticity over the surface to generate disturbance flow. They placed a discrete point vortex of unknown strength at the trailing edge to satisfy the Kutta condition.

When attempting to model airfoils with cusped trailing edges, most methods have problems near the trailing edge because of the very tight placing of the collocation points. The linear-strength vortex method with the Neumann boundary condition seems to be the only method that is not sensitive to this cusped trailing-edge problem [4]. The Kutta condition invoked by Kim and Mook[3] is that vortex strengths of upper and lower surface at the trailing edge are equal to zero and a point vortex of unknown strength is placed at the trailing edge.

Here the basic numerical approach of Kim and Mook[3] is used, but the conditions in the region of the trailing edge are modified. The modified Kutta condition is met exactly at the trailing edge. Since this approach is a numerical one, the Kutta condition(s) have to be formulated for consistency with the mathematical model. In the present solution it is ensured that the flow separates from the trailing edge, that there is zero loading across the vorticity shed from the trailing edge. This paper presents the application of the modified Kutta condition to the flow about airfoil undergoing unsteady motion and interference effect of two impulsively started airfoils.

2 FLUID DYNAMIC MODEL

Given the motion of the airfoil, the problem is formulated in a space-fixed reference frame (Fig. 1) unlike most previous methods which were formulated in a body-fixed system. This will ease the computational effort when examining the wake and the interaction of several airfoils. By using a time-stepping procedure, lift history and the shape of trailing wake will be determined .

The fluid is assumd to be inviscid, imcompressible and the flow is irrotational. The velocity $\boldsymbol{V}$ is obtained by solving the Laplace equation for the velocity potential Φ, i. e. ,

$$\nabla^2 \Phi = 0 \text{ , where } \boldsymbol{V} = \nabla \Phi, \qquad (1)$$

where Φ is the function of $\boldsymbol{R}$ and t.

The loads are obtained by integrating the pressure over the airfoil surface, which is given by the unsteady Bernoulli's equation. The equation must be written with reference to a rotating and translating frame as:

$$C_p = -2\frac{\partial \varphi}{\partial t} - 2\boldsymbol{V}\cdot(\boldsymbol{V}_0 + \boldsymbol{\Omega}\times\boldsymbol{r}) - \boldsymbol{V}\cdot\boldsymbol{V}, \qquad (2)$$

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where V_0 is the absolute velocity of the origin of the moving frame, Ω the angular velocity of the moving frame, r the position vector in moving frame, and φ is the potential function expressed in terms of r and t. Equation (2) is used to compute the loads.

For the moving airfoil the no-penetration boundary condition takes the form,

$$(V - V_s) \cdot n = 0 , \tag{3}$$

on the surface of the airfoil, where V_s is the absolute velocity of a point on the surface of the airfoil, and n a vector normal to the surface. If the airfoil is considered a rigid body, V_s is given as:

$$V_s = V_0 + \Omega \times r_s , \tag{4}$$

where r_s is the relative position of the point on the surface.

The total circulation $\Gamma(t)$ in the flowfield is governed by Kelvin's theorem, which states that the total circulation around a fluid curve remains constant,

$$\frac{d\Gamma}{dt} = 0 . \tag{5}$$

Therefore any change in circulation around the airfoil will be balanced by a change in circulation of the wake.

The unsteady Kutta condition requires no pressure jump across the wake at the airfoil trailing edge, i. e., at the trailing edge,

$$\triangle C_p = 0 . \tag{6}$$

Various approaches in the past have implemented this condition in different ways. We apply Bernoulli's equation at the trailing edge along the upper and lower surfaces. It follows from Eq. (2) that:

$$C_{pl} - C_{pu} = 2\frac{\partial}{\partial t}(\varphi_u - \varphi_l) + V_u^2 - V_l^2 + 2(V_l - V_u) \cdot (V_0 + \Omega \times r) , \tag{7}$$

where the subscripts l and u denote the values the different quantities approach along the lower and upper surfaces. The requirement that $C_{pl} = C_{pu}$ leads to:

$$2\frac{d\Gamma}{dt} = V_l^2 - V_u^2 + 2(V_u - V_l) \cdot (V_0 + \Omega \times r) , \tag{8}$$

where Γ is the clockwise circulation around the airfoil.

Using the relation $V = V_0 + \Omega \times r + v$, where v is velocity with respect to the moving frame (the relative velocity), and rewriting Eq. (8) in terms of the velocity relative to the moving frame, one can derive:

$$2\frac{d\Gamma}{dt} = v_l^2 - v_u^2 . \tag{9}$$

For circulation around both airfoil and wake to remain constant, the rate of change of the circulation around the wake must be the negative of that given by Eq. (9).

3 NUMERICAL PROCEDURE

The solution for the flow about an airfoil undergoing an arbitrary time-dependent motion which started at $t = 0$ was calculated at successive steps of time

$$t_k (t_0 = 0,\ k = 1,2,3\cdots).$$

The surface of the airfoil at time t_k was divided into N straight-line elements. A vorticity distribution, whose strength varies linearly from $(\gamma_i)_k$ to $(\gamma_{i+l})_k$ across this element, was placed on the ith element ($i = 1,2,\cdots,N$) and the subscript k refers to the time t_k.

For unsteady flow, a model is required to simulate the continuous shedding of vorticity into the wake. The treatment of this vortex shedding process follows the approach of Basu and Hancock[2]. A small straight-line wake element of length $\triangle_k$ and inclined at an angle θ_k to the X-axis was attached to the trailing edge. The arbitrary values of $\triangle_k$ and θ_k dependent on the time t_k, were determined as part of the solution. The uniform vorticity distribution on the trailing-edge wake element was $(\gamma_w)_k$. A downstream wake of concentrated vortices was formed from the vorticity shed at earlier times, which was assumed to be concentrated into discrete vortices and convected according to the resultant velocities calculated at the center of each vortex at each successive time step. Thus the pattern of the downstream discrete vortices, their strengths and positions were regarded as known at time t_k.

Thus at time t_k there were $(N+1)$ unknown linearly varying vorticity strength $(\gamma_j)_k (j = 1,2,\cdots,N+1)$, $(\gamma_w)_k$, $\triangle_k$ and θ_k, i. e. $(N+4)$ unknowns. The basic set of equations can be formulated as follows.

(I) The N no-penetration boundary conditions, Eq. (3), were replaced by the following:

$$\sum_{j=1}^{N+1} (a_{ij}\gamma_j)_k + (T_i\gamma_w)_k = (V_0 + \Omega \times r - V_{wi})_k \cdot (n_i)_k , \tag{10}$$

for $i = 1,2,\cdots,N$. Here a_{ij} is the normal component of the velocity at the control point of element i induced by unit vorticity strength at node j, and T_l is the normal component of velocity induced by wake element of unit strength. V_{wj} is the velocity induced by all the vortices in the wake.

(II) According to Kelvin's theorem the condition of constant circulation around airfoil and wake can be expressed as:

$$(\gamma_w)_k \Delta_k + \Gamma_k - \Gamma_{k-1} = 0 . \qquad (11)$$

Thus the circulation on the wake element was the change in circulation around the airfoil between times t_{k-1} and t_k, assuming that Γ_{k-1} has already been evaluated.

(III) It followed from Eq. (9) and the requirement of constant circulation around airfoil and wake that, during a small time interval Δt, a vortex having approximately the circulation,

$$(\Delta\Gamma_w)_k = -0.5(v_l^2 - v_u^2)_k \Delta t , \qquad (12)$$

was added to the wake. From the discussion of Basu and Hancock[2], there exist following relations

$$(\Delta\Gamma_w)_k = (\gamma_w)_k \Delta_k, \text{ and } \Delta_k = 0.5(v_l - v_u)_k \Delta t . \qquad (13)$$

Considering $(v_l)_k = (-\gamma_1)_k$ and $(v_u)_k = (\gamma_{N+1})_k$ in body-fixed frame and substituting Eq. (13) into Eq. (12), we obtained the Kutta condition for present numerical procedure:

$$(\gamma_1)_k + (\gamma_{N+1})_k = (\gamma_w)_k . \qquad (14)$$

This expression, which is quite different from all the previous Kutta conditions, is linear and has clear physical meaning. Kim and Mook[3] also developed a linear Kutta condition which is satisfied by placing a point vortex of unknown strength at the trailing edge and let $(\gamma_1)_k = 0$ and $(\gamma_{N+1})_k = 0$.

(IV) The length and orientation of the trailing-edge wake element (i. e. Δ_k and θ_k are determined from the condition that the element is tangential to the local resultant velocity and that its length is proportional to the local resultant velocity. If $(U_w)_k$ and $(V_w)_k$ are the total component velocities induced at the midpoint of the trailing-edge wake element, excluding the effect of the element on itself, then,

$$\left.\begin{aligned} \tan\theta_k &= (V_w)_k/(U_w)_k \\ \Delta_k &= \sqrt{(U_w)_k^2 + (V_w)_k^2}\,[t_k - t_{k-1}] \end{aligned}\right\} . \qquad (15)$$

These equations can be solved by the iterative procedure: the values of Δ_k and θ_k are guessed, leaving $(N+2)$ linear equations from (10), (11) and (14). The $(N+2)$ linear equations are solved to give the vorticity strength $(\gamma_j)_k$, $(j = 1, 2, \cdots, N+1)$ and $(\gamma_w)_k$. Once $(\gamma_j)_k$ are known $(U_w)_k$ and $(V_w)_k$ can be calculated and substituted into (15) to find the new values of Δ_k and θ_k. The procedure is repeated until Δ_k and θ_k have converged to the desired accuracy.

Once the solution at time t_k has been determined, the distributed vorticity on the wake element at time t_k is now assumed to be concentrated into a vortex of strength $(\gamma_w)_k \Delta_k$ at time t_{k+1} situated at :

$$\left.\begin{aligned} x_{k+1} &= (x_{T.E.})_k + \frac{1}{2}\Delta_k\cos\theta_k + (U_w)_k(t_{k+1} - t_k) \\ y_{k+1} &= (y_{T.E.})_k + \frac{1}{2}\Delta_k\sin\theta_k + (V_w)_k(t_{k+1} - t_k) \end{aligned}\right\} . \qquad (16)$$

The resultant velocity at the center of each of the other concentrated vortices in the wake is calculated from the solution at time t_k, then the position of that vortex at time t_{k+1} follows directly.

The above numerical scheme can be easily extended to study unsteady airfoil interference effects. The two airfoil's surfaces are approximated by a large number of surface elements. Both airfoil surfaces are distributed vorticity whose strength varies across each element and a small straight-line wake element is attached to the trailing edge of each airfoil. The solution technique is similar to that of an airfoil. In the following calculations, NACA0012 airfoil is chosen, d/c represents the non-dimensional horizontal distance between leading edges, h/c the non-dimensional vertical distance between leading edges, U reference velocity, and C_{LS} steady lift coefficient of single airfoil. The arrangement is shown in Fig. 2.

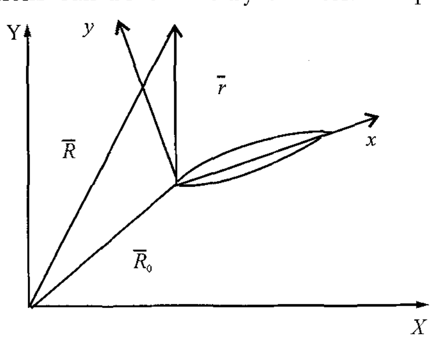


Fig. 1 Fixed and moving frame

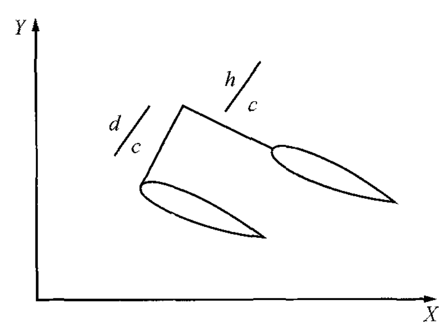


Fig. 2 Schematic of arrangement

4 NUMERICAL EXAMPLES

In this section we provide a number of example calculations to compare with previous results, demonstrate the use of the Kutta condition, and investigate the interference effect between two impulsively started airfoils. In order to verify the present Kutta condition, we first compare the results of transient lifts with those obtained by Platzer[5].

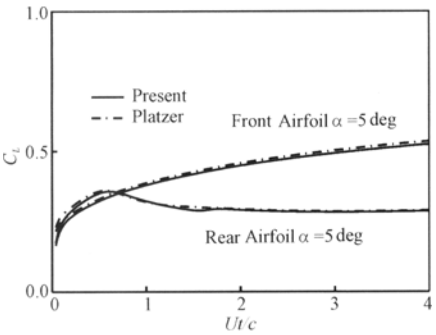


Fig. 3a Lift coefficients curve $d/c = 1.5$, $h/c = 0.2$

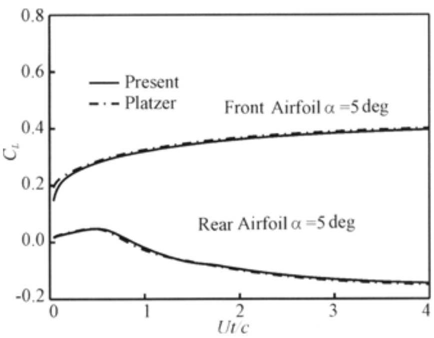


Fig. 3b Lift coefficients curve $d/c = 1.5$, $h/c = 0.2$

Fig. 3a and 3b compared the computed results of transient lifts with those of Platzer. The results obtained by the present numerical scheme agree well with those of Platzer.

For interference effects, the following arrangements were studied.

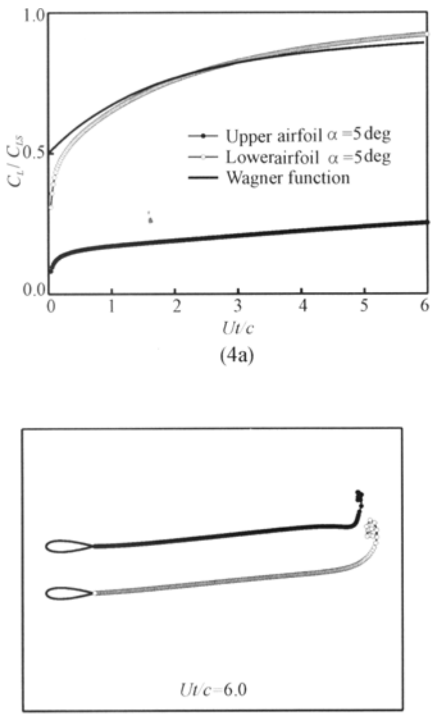


Fig. 4 Lift coefficients curve and wake pattern $d/c = 0$, $h/c = 0.5$

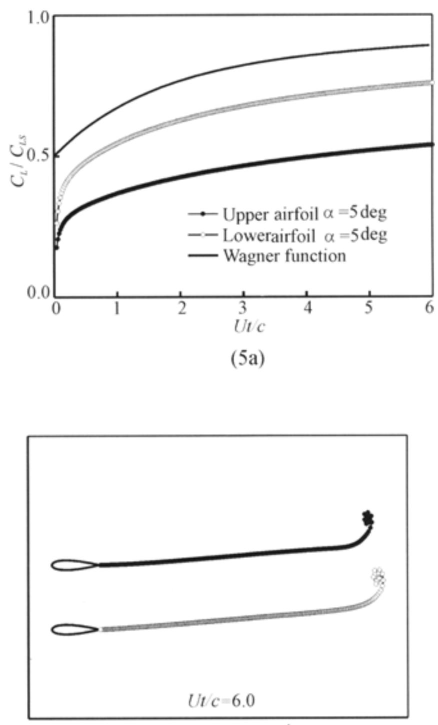
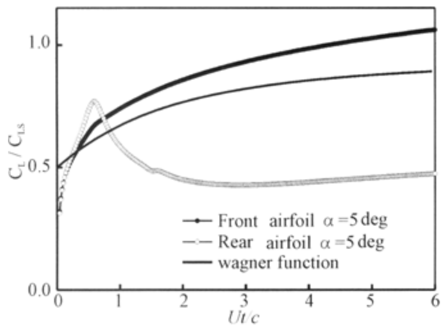
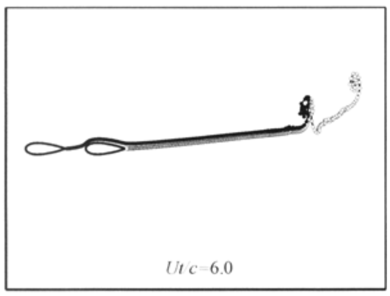


Fig. 5 Lift coefficients curve and wake pattern $d/c = 0$, $h/c = 0.8$

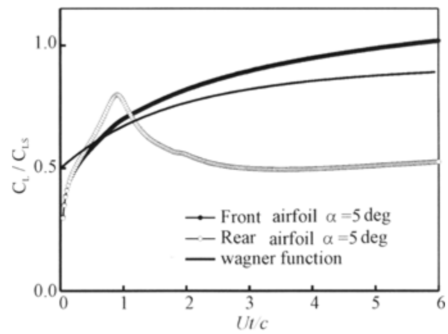


(6a)

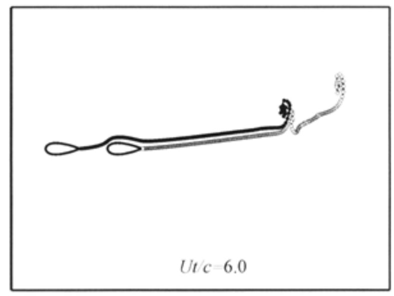


(6b)

Fig. 6 Lift coefficients curve and wake pattern $d/c = 1.5$, $h/c = 0.0$

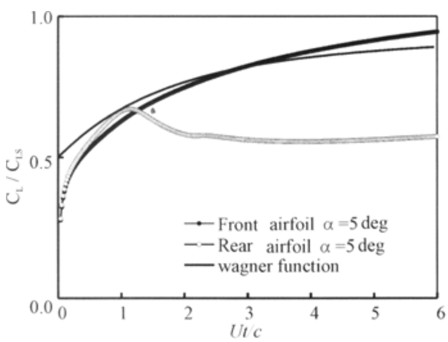


(7a)



(7b)

Fig. 7 Lift coefficients curve and wake pattern $d/c = 1.8$, $h/c = 0.0$



(8a)

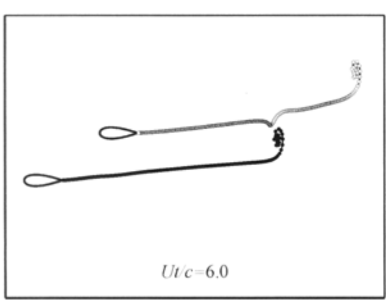
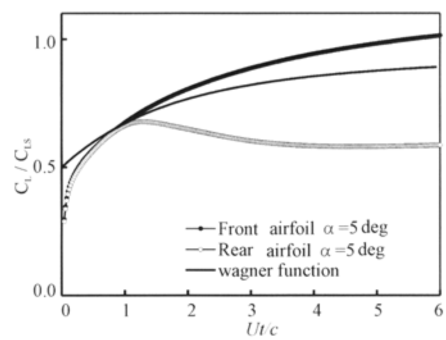


Fig. 8 Lift coefficients curve and wake pattern $d/c = 2.0$, $h/c = 0.5$



(9a)

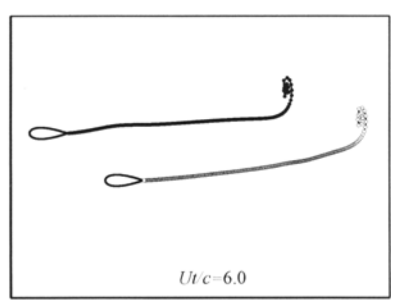


Fig. 9 Lift coefficients curve and wake pattern $d/c = 2.0$, $h/c = 0.5$

For unstaggered arrangement (Fig. 4 and Fig. 5), the interaction is comparatively weak, the lift generation of this system is not effective. As the vertical displacement is reduced, the interference effect is intensified and the lower airfoil gains excessive lift.

For tandem configuration (Fig. 6 and Fig. 7), it can be observed that when the main body of wake vortices of front airfoil arrives at the leading edge of rear airfoil, the lift of rear airfoil shows a peak. The lift of front airfoil is enhanced.

For staggered configuration (Fig. 8 and Fig. 9), whether the oncoming wake vortices going above or below the rear airfoil, the lift of rear airfoil shows the same change as tandem configuration. The lift of front airfoil is also enhanced, which is compatible with steady case.

5 CONCLUSIONS

A method for modeling general unsteady two-dimensional lifting flows is presented. The problem is imposed in a space-fixed frame of reference unlike most previous methods which were formulated in a body-fixed coordinate system. By analyzing the flow near the trailing edge, a new linear expression for Kutta condition is developed, which is quite different from all the previous Kutta conditions. The computed results show that the present approach is efficient and the Kutta condition is reasonable. For interference effect, the tandem configuration is found to be most effective for two impulsively started airfoils.

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