Starting vortex and lift on an airfoil

Chien-Cheng Chang, Ying-Chien Hsiau, and Chin-Chou Chu

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Chien-Cheng Chang, Ying-Chien Hsiau, and Chin-Chou Chu
Institute of Applied Mechanics, College of Engineering, National Taiwan University, Taipei 10764, Taiwan, Republic of China

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Consider an airfoil which is impulsively started from rest to a constant speed relative to an otherwise quiescent fluid at an angle of attack. The paper attempts to propose a precise definition for the "starting vortex" in viscous flow and investigate how an airfoil gains its lift through the starting vortex. It is found adequate to view the vortex itself as a source of the lift through the vorticity contained within it. Actually, the total lift on the airfoil is contributed by all the regions of nonvanishing vorticity in the flow. For an impulsive flow, the initial lift is due to the vortex sheet distributed along the airfoil; the lift is then rapidly dominated by the starting vortex near the trailing edge. In the next stage, the regions of vorticity next to the airfoil on both sides gradually become the major contributor to the lift. Relative importance of the starting vortex is assessed based on the angle of attack and the Reynolds number.

I. INTRODUCTION

"What is a starting vortex?" and "How is it related to the lift?" is quite a puzzle to the people of fluid mechanics. Theories for the starting vortex have been invariably based on inviscid assumption, while its generation is ascribed to the viscous effect. Potential flow theory predicts that the lift is proportional to the circulation around the airfoil; Kutta condition is typically assumed. The analysis based on the Euler's equations takes one further step by allowing sheet approximation of the starting vortex; see, e.g., Chow and Huang,[1] Graham,[2] and the references cited there. Neither of these previous studies and the references cited there concerns the effect of the starting vortex fully from the viewpoint of the (real) viscous flow since inviscid flows are assumed.

For a discussion of the starting vortex, consider the viscous flow about an impulsively started airfoil (NACA0012). Figure 1 is schematic of the physical problem. The fluid is assumed to have a constant density ρ and a kinematic viscosity ν. The angle of attack α is 5° and the Reynolds number is taken to be $\mathrm{Re}=5000$. The viscous theory indicates that vorticity is the only source of the forces: both lift and drag for such a flow. To quantify the contribution to the lift, we must therefore define the starting vortex from the viewpoint of vorticity. Figure 2 shows a sequence of numerical plots of contours of vorticity at different instants shortly after the flow is started. It is seen that the flow changes rapidly from an initial potential flow to a viscous one, possessing regions of substantial vorticity. Each of these plots reveals the existence of two connected regions of vorticity of opposite senses. One is a slender region of positive vorticity next to the leeward side of the airfoil, while the other is a negative region next to the windward side, extending somewhat downstream the trailing edge. The extended portion may loosely be defined as the starting vortex; there is, however, no clear cut between this portion and that right below the airfoil since the region of vorticity is connected. The vorticity is gradually shed into the flow. Two questions therefore arise naturally from these observations: "How does one define, without ambiguity, the starting vortex in viscous flow?" and "How does the new definition reconcile with the classical lift theory for the starting vortex which is ascribed to be the only source of the lift?" The present paper proposes one such definition and clarifies the role of the starting vortex as a major contributor to the lift (as well as drag) from the viewpoint of viscous flow. Based on the definition, our main findings include that (i) the initial lift is due to the vortex sheet distributed along the airfoil; (ii) the starting vortex is then the major contributor to the lift for a period of time shortly after the flow is started; (iii) the regions of vorticity next to airfoil on both sides gradually contribute significantly to the lift. The rest of the article is devoted to detailed discussion of the three stages and to examine their dependence on the angle of attack and the Reynolds number, according to extensive numerical results.

II. FORCE DECOMPOSITION

For the impulsive flow, we shall take the frame fixed with the airfoil relative to an oncoming stream with velocity U. Choose the half chord $a=c/2$ of the airfoil to be the reference length, a/U be the reference time, and the pressure is normalized by ρU^2. Nondimensional time is denoted by t, velocity by $\mathbf{u}$, space variable by $\mathbf{x}$, vorticity by $\boldsymbol{\omega}$, and the pressure by P, while the Reynolds number is defined by $\mathrm{Re}=Uc/\nu$. Throughout the article, the flow is assumed to be laminar and two dimensional. This is most likely in the early stages of flow development, which are exactly what we are concerned with in the present study. The detailed information of flow quantities is provided by carefully solving the Navier–Stokes and continuity equations with checking the grid independence by a deterministic hybrid vortex method (cf. Chang and Chern[3]).

Let L and D denote the lift and drag exerted on the airfoil per unit spanlength. The lift and drag coefficients are then defined respectively by $C_L=L/\rho U^2 a$ and $C_D=D/\rho U^2 a$. As shown in Fig. 1, the unit vector in the drag direction is taken to be $\mathbf{i}$, while the unit vector in the lift direction is taken to be $\mathbf{j}$. Let $\mathbf{e}$ denote the unit vector either in the lift ($\mathbf{j}$) or drag ($\mathbf{i}$) direction. Since the airfoil

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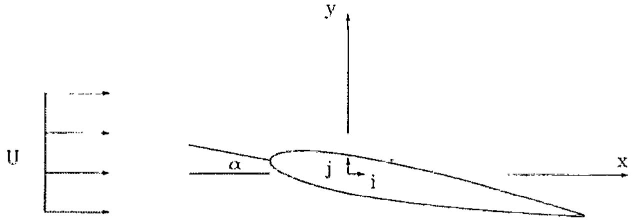

FIG. 1. Schematic of the physical problem.

is immersed in a uniform oncoming stream of constant speed, the only source of forces is conceived to be the vorticity. Actually, Chang[4] has recently proposed to decompose the lift and drag coefficients into

$$C_L = C_{LV} + C_{LS}, \quad C_D = C_{DV} + C_{DS}, \tag{1}$$

according to the following formula:

$$C_L, C_D = -\int_V \mathbf{u} \times \boldsymbol{\omega} \cdot \nabla\phi \, dV + \frac{2}{\mathrm{Re}} \int_S \mathbf{n} \times \boldsymbol{\omega} \cdot (\nabla\phi + \mathbf{e}) dA, \tag{2}$$

where S denotes the airfoil surface and n is the unit vector pointing toward the inside of S. Further, ϕ is an auxiliary acyclic potential flow for the airfoil moving with *unit velocity* $\mathbf{e}$ in the negative direction of lift or drag in an otherwise quiescent fluid. More precisely, if $\mathbf{e}$ is taken to be $-\mathbf{j}$, the first and second terms on the right side of the formula (2) are exactly C_{LV} and C_{LS}, respectively. For $\mathbf{e} = -\mathbf{i}$, we then have C_{DV}, C_{DS} on the right side of (2). For each choice $\mathbf{e}$, the field $\nabla\phi$ decays rapidly away from the airfoil, and at great distances is given by (cf. Landau and Lifshitz,[5] p. 27)

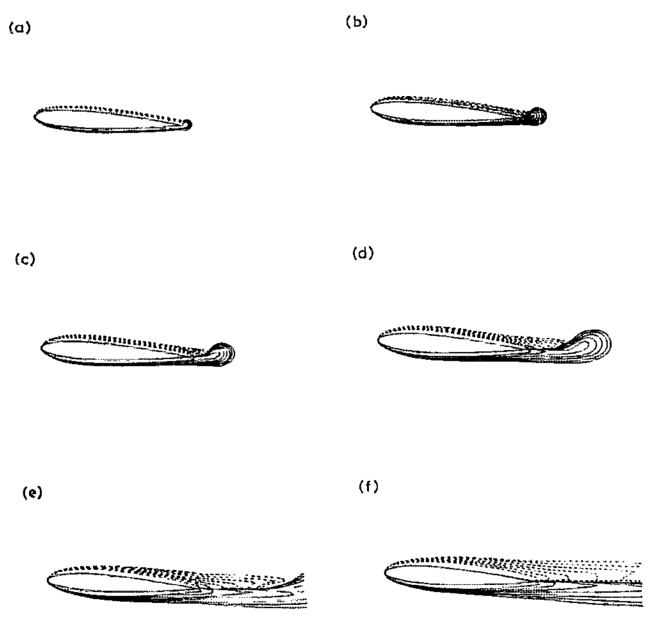

FIG. 2. Contours of vorticity at different instants for Re=5000, $\alpha=5°$: (a) t=0.1, (b) 0.3, (c) 0.5, (d) 1.0, (e) 2.0, (f) 4.0.

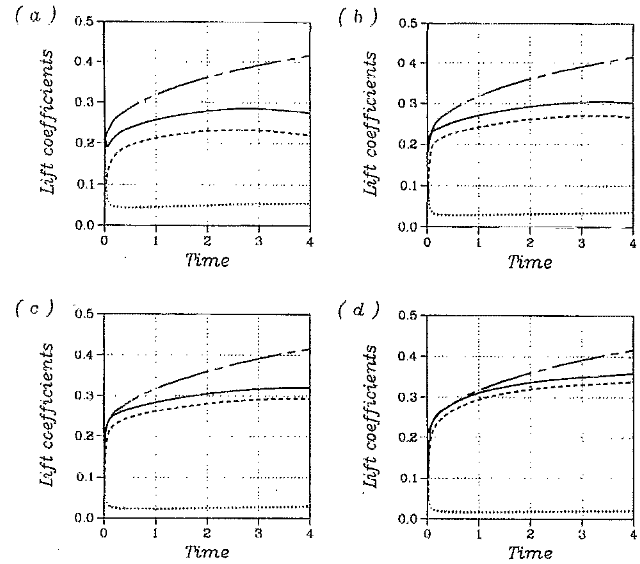

FIG. 3. Decomposition of the lift coefficient at $\alpha=5°$: $C_L = C_{LV} + C_{LS}$,—, C_L; ---, C_{LV}; $\cdots$, C_{LS} for (a) Re=1000, (b) 3000, (c) 5000, (d) 10 000. – – –, lift curve obtained from the inviscid solution of Giesing (Ref. 8) with smooth extrapolation to $t=0^+$ is included for comparison.

$$\nabla\phi = \frac{2(\mathbf{A}\cdot\hat{\mathbf{x}})\hat{\mathbf{x}} - \mathbf{A}}{|\mathbf{x}|^2} \quad \text{for large } |\mathbf{x}|, \tag{3}$$

where $\hat{\mathbf{x}}$ is the unit vector in the direction of $\mathbf{x}$, and the vector $\mathbf{A}$ depends on the shape and the hypothetical motion of the airfoil. The origin of the coordinates is chosen to be somewhere inside the airfoil. Notice also that on the right side of (2), the first and second terms come, respectively, from the vorticity within the flow and on the airfoil surface. Actually, one may show that the terms associated with $\nabla\phi$ (volume and surface) are due to pressure and the term associated with $\mathbf{e}$ (surface) is due to friction. For the first term, the triple products $-\mathbf{u}\times\boldsymbol{\omega}\cdot\nabla\phi$ will be called the volume lift element $E_L(\mathbf{x})$ or drag element $E_D(\mathbf{x})$, depending whether $\mathbf{e}$ is in the negative direction of lift or drag. The potential functions ϕ introduced for the lift and drag coefficients thus play the role of distributing the lift and drag elements, respectively. Notably, owing to the emergence of $\nabla\phi$, the force elements decay also rapidly away from the airfoil, provided $\boldsymbol{\omega} = \boldsymbol{\omega}(\mathbf{x})$ is compact or decays like $|\mathbf{x}|^{-\beta}$, $\beta > 0$. This indicates that the volume integral of (2) may converge within only several lengths of the airfoil, and which facilitates greatly the evaluation of the lift and drag through the lift and drag elements. The latter nice property of the new decomposition distinguishes itself from the classical theory, based on fluid impulse which is clearly explained in the text by Lighthill[6] (p. 213).

III. RESULTS AND DISCUSSION

Consider the present lift decomposition at the angle $\alpha=5°$ for the four Reynolds numbers Re=1000, 3000, 5000, and 10 000. In order to resolve the initial field, the time step Δt is taken to be of the order 1/Re for t up to 0.01, and then Δt is increased gradually to 0.01 by t=0.25. Figure 3 shows that the surface vorticity is the major con-

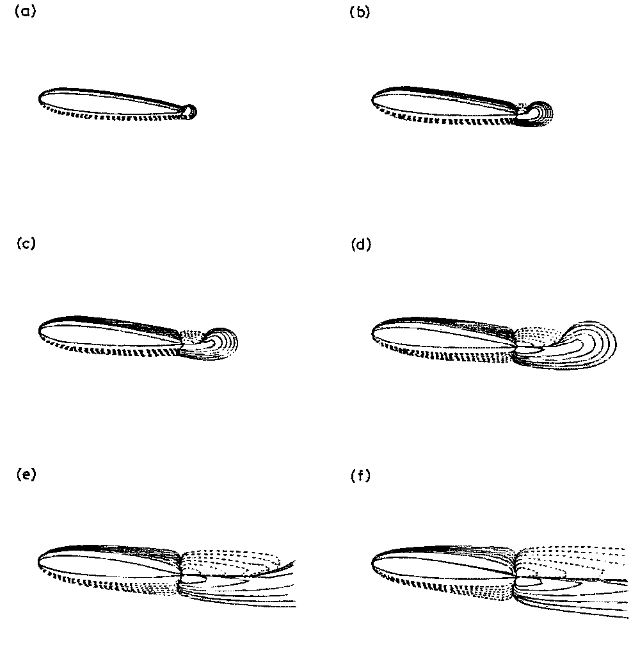

FIG. 4. Distribution of the lift elements at different times for the case: Re=5000 and $\alpha=5°$: (a) $t=0.1$, (b) 0.3, (c) 0.5, (d) 1.0, (e) 2.0, (f) 4.0.

tributor to the lift in an initial period of time right after the flow is started. The lift drops rapidly to a moderately small value during this period of time, which shrinks as the Reynolds number increases, while the (volume) vorticity in the flow becomes relatively important as a contributor to the lift. Indeed, previous asymptotic analysis of flow about (slender) elliptic cylinder at an angle of attack shows that the lift is proportional to $\sin 2\alpha/\sqrt{t\,\mathrm{Re}}$ at small times (cf. Wang[7]). It is therefore expected that in the limit of $\mathrm{Re}\to\infty$, the lift is due to the surface vorticity only very initially; the vorticity shed into the flow becomes immediately the major contributor to the lift (as well as the drag). Figure 3 reveals indeed this situation: C_{LS} due to the surface vorticity decays rapidly as t increases, while C_{LV} increases quickly so that the total lift is increasing steadily except in an initial period of time of the order 1/Re.

The lift due to the vorticity within the flow can further be analyzed by investigating the distributions of the lift elements $E_L(\mathbf{x})$ defined in Sec. II. Figure 4 shows a sequence of numerical plots of the lift elements at different times for the case: $\alpha=5°$ and Re=5000. Since our major concern is the role of vorticity in the lift coefficient, we define *the starting* vortex as the region of negative vorticity with positive lift elements near the trailing edge. Clearly from these plots, there are four connected regions of lift elements within the flow, two positive and two negative. Refer to Fig. 4(c). In the leeward side, there is a positive region next to the airfoil surface, denoted by BS(+); in contrast, there is a negative region in the windward side, denoted by BS(−). There are two regions near the trailing edge; the positive one, denoted by TE(+), is exactly the starting vortex we just defined, while the negative one is denoted by TE(−). Comparison between Figs. 2 and 4 reveals that each of the two connected regions of vorticity is divided to two regions of lift elements of different signs. The dividing lines of these regions include the lines separating different regions of vorticity and the lines on which $\mathbf{u}\times\omega$ and $\nabla\phi$ are perpendicular to each other.

Some remarks can now be made about the effect of viscosity. Potential theory based on the Kutta condition predicts the steady lift coefficient to be $C_L^S=2\pi\alpha$ which is equal to 0.548 for $\alpha=5°$. Inviscid theory may further give the time history of the lift. For a comparison, we choose the 8.4%-thick von Mises airfoil, for which Giesing[8] (also Basu and Hancock[9]) was able to obtain the initial history of C_L/C_L^S from the inviscid solution at small angles of attack. Employing $C_L^S=2\pi\alpha$ for $\alpha=5°$ and using the data C_L/C_L^S from Giesing, we obtain the inviscid curve in each plot of Fig. 3. Notice that the inviscid curve has a finite initial value (extrapolated) in contrast to the infinite value for flow at any finite Reynolds number. Basically the inviscid curve is higher than all the viscous lift curves, except perhaps in a small initial period. The viscous curves, however, appear to indicate that the lift increases toward its inviscid value with increasing the Reynolds number (say, based on the data at $t=4$). On one hand, the viscosity enforces the no-slip condition so that vorticity must be generated along the airfoil, which may then be diffused or shed into the flow. This is true for flow at all the Reynolds numbers. On the other hand, the effect of viscous diffusion weakens the vorticity right after being generated, and therefore the lift elements. The lift elements of each region shown in the plots of Fig. 4 would be weaker at lower Reynolds number, and stronger at higher Reynolds number. Moreover, Fig. 3 indicates that the lift elements (for C_{LV}) are the major contributor to the lift at moderately high Reynolds number. These facts explain how the viscosity effects the change of the lift, and thus accounts for why the value of the lift coefficient decreases with decreasing the Reynolds number. Since potential and any inviscid models do not take into consideration of the viscous diffusion, the lift at relatively low Reynolds number deviates greatly from its inviscid value.

Figure 5 shows the individual lift contributions from the four regions of lift elements. In the following discussion, we shall take the viewpoint by summing up the lifts due to BS(+), BS(−), and TE(−) and compare the resultant sum to the lift due to TE(+) in order to single out the role of the starting vortex. To enhance the lift, we must have regions of concentrated vorticity attached to the airfoil; the lift will decrease rapidly as the region of vorticity in the leeward side is detached from the surface. It is seen that the region of vorticity in front (and near the nose) of the airfoil contributes negatively to the lift. This is reasonable because the existence of vorticity reduces the force in the direction of lift relative to the corresponding potential flow. In order to understand the detailed distribution of the lift elements, we plot the auxiliary potential ϕ for lift decomposition in Fig. 6. Since for the lift, ϕ corresponds to the motion of the airfoil in the $-\mathbf{j}$ direction, the surface is not a streamline and thus $\nabla\phi$ is not necessarily parallel to the airfoil surface. Concerning the local contribution, the lift element is most significant when the vector

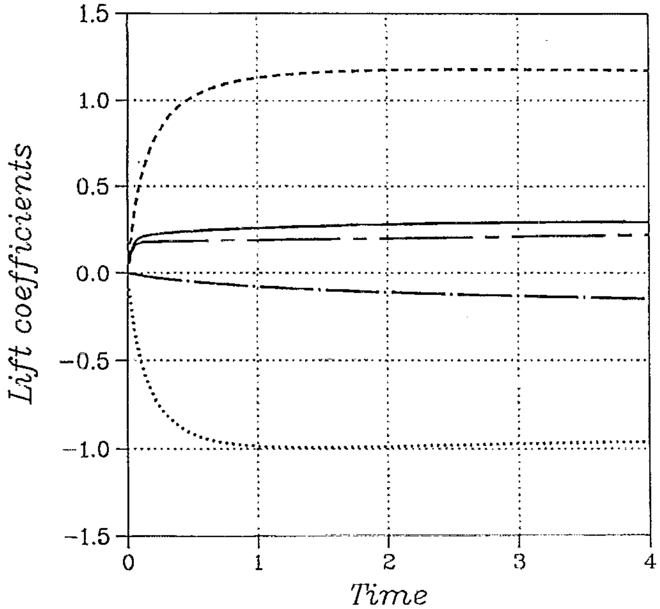

FIG. 5. Contributions to the lift from individual regions of lift elements for Re=5000 and $\alpha=5°$. —, C_{LV}; ---, C_L[BS(+)]; ···, C_L[BS(−)]; – – –, C_L[TE(+)]; -·-, C_L[TE(−)].

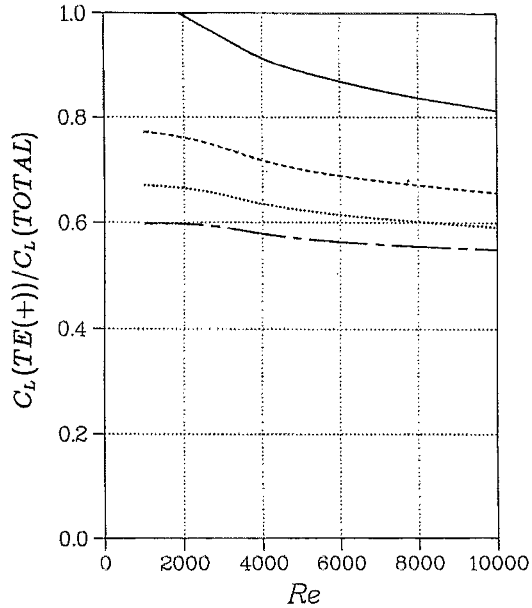

FIG. 7. Relative importance of the starting vortex as a contributor to the lift versus the Reynolds number at $t=0.5$. —, $\alpha=3°$; ---, 5°; ···, 7°; – – –, 10°.

$\mathbf{u}\times\boldsymbol{\omega}$ is parallel to the auxiliary potential velocity $\nabla\phi$. Typically, the contribution to the lift from the starting vortex soon attains a maximum when the vorticity remains concentrated near the trailing edge. The starting vortex then diffuses, having a substantial portion of the vorticity convected away from the trailing edge gradually. Hence, the contribution to the lift from the starting vortex also reduces gradually. On the other hand, vorticity continues to accumulate in the leeward side, increasing its contribution to the lift, in the meanwhile, the vorticity in the windward side diffuses, reducing therefore its contribution to the lift. Furthermore, the region TE(−) grows substantially in size due to vorticity flux from the region BS(+) and local

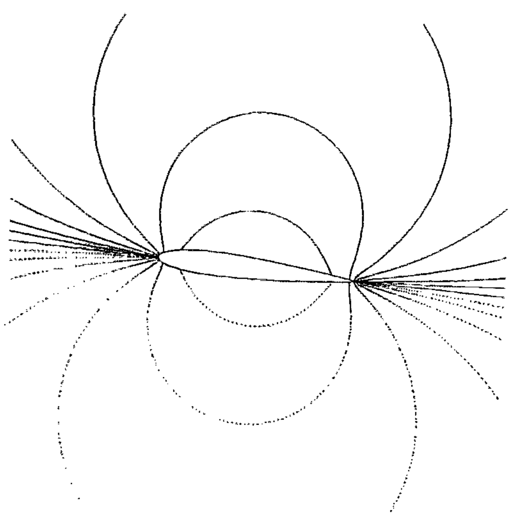

FIG. 6. The auxiliary acyclic potential flow ϕ. The airfoil moving (down) hypothetically with the unit velocity $-\mathbf{j}$ in the negative direction of lift.

distribution of the lift elements. As a consequence, the resultant contribution from TE(+) and TE(−) to the lift is getting smaller. Figure 5 shows that by $t=4$, the regions BS(+) and BS(−) together contribute to 72% of the total lift; this contribution appears to increase as time increases further. Since BS(+) and BS(−) are adjacent to the airfoil, we may say that these two regions combine to give the effect of a "bound vortex," which serves as the major contributor to the lift in the final stage.

Summarizing the above observations, we have three stages of lift contributions: (i) the vortex sheet as the major contributor right after the flow is started, (ii) the starting vortex as a major contributor in a period of time, and (iii) the lift elements next to the airfoil contributing significantly to the lift in a gradual manner. It would be interesting to know how much the starting vortex contributes to the total lift. Since this contribution maximizes soon after the flow is started, it is adequate to define $t=0.5$ as the time of reference. Figure 7 shows the ratio $C_L[\mathrm{TE}(+)]/C_L(\mathrm{TOTAL})$ at different Reynolds numbers with the angle of attack as a parameter. It is observed that for a fixed angle of attack, the relative importance of the starting vortex decreases gradually as the Reynolds number increases. For a fixed Reynolds number, the relative contribution becomes less significant as the angle of attack increases. Figure 8 shows another view of looking at the same aspect by drawing the level curves of $C_L[\mathrm{TE}(+)]/C_L(\mathrm{TOTAL})$ in the plane of Reynolds number-angle of attack. To maintain a given ratio, say, 90%, one must reduce the angle of attack α as the Reynolds number Re is increased. It is inferred that the angle of attack must be limited to very small values so that the starting vortex is the major contributor to the lift in the limit of Re→ ∞. On the other hand, Fig. 9 shows that the

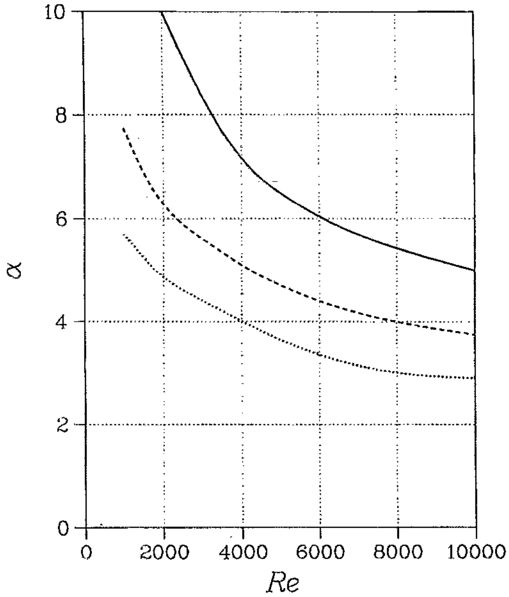

FIG. 8. $C_L[\mathrm{TE}(+)]/C_L(\mathrm{TOTAL})$—Relative importance of the starting vortex as a contributor to the lift, with respect to the angle of attack and the Reynolds number at $t=0.5$. —, 70%; ---, 80%; $\cdots$, 90%.

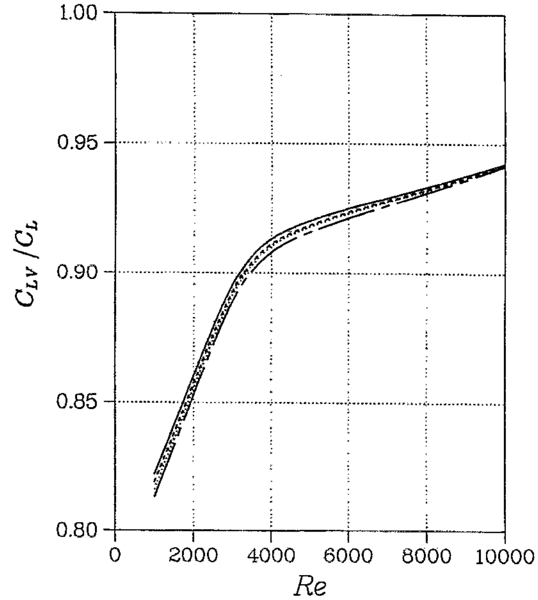

FIG. 9. C_{LV}/C_L—The ratio of the volume lift to the total lift versus the Reynolds number at $t=0.5$. —, $\alpha=3°$, ---, 5°; $\cdots$, 7°; – – –, 10°.

ratio of the volume lift to the total lift increases steadily as the Reynolds number increases; the difference of this ratio between different angles of attack is small. From the above observations, we may conclude that it would be insufficient to consider only the effect of "the starting vortex" in any real flow of finite Reynolds number, provided that one aims to obtain accurate values of the (initial) lift coefficients even for a body like the present airfoil (NACA0012) with a sharp trailing edge at only moderately small angles of attack. The Kutta condition, which is however a good approximation at large Reynolds numbers up to $\alpha=12°$ (cf. Anderson,[10] p. 210), is equivalent to incorporating the effects of all the regions of nonvanishing vorticity within the flow.

ACKNOWLEDGMENT

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