Vector Operations

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Definition

Vectors

A vector is an ordered list of numbers:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

Common vector operations

Addition of vectors

The sum of two vectors $\mathbf{u}$ and $\mathbf{v}$ is given by:

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix}$$

Scalar multiplication

The product of a scalar c and a vector $\mathbf{v}$ is given by:

$$c\mathbf{v} = c \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} cv_1 \\ cv_2 \\ \vdots \\ cv_n \end{bmatrix}$$

Vector norm

The norm of a vector $\mathbf{v}$ is given by:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

L^p norm

The L^p norm of a vector $\mathbf{v}$ is given by:

$$\|\mathbf{v}\|_p = \left(\sum_{i=1}^{n} |v_i|^p \right)^{\frac{1}{p}}$$

Dot product

The dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ is given by:

$$\mathbf{u} \cdot \mathbf{v} = \sum_{i=1}^{n} u_i v_i = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

Cross product

The cross product of two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ is given by:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Vector calculus

Gradient

The gradient of a scalar function $f(x, y, z)$ is given by:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$

Divergence

The divergence of a vector field $\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$ is given by:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Curl

The curl of a vector field $\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$ is given by:

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

Properties

Properties of Vector Operations

- **Commutativity of Addition**: $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- **Associativity of Addition**: $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
- **Distributivity of Scalar Multiplication**: $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
- **Associativity of Scalar Multiplication**: $c(d\mathbf{v}) = (cd)\mathbf{v}$
- **Identity Element of Addition**: $\mathbf{v} + \mathbf{0} = \mathbf{v}$
- **Inverse Element of Addition**: $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$
- **Identity Element of Scalar Multiplication**: $1\mathbf{v} = \mathbf{v}$
- **Distributivity of Scalar Multiplication over Addition**: $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
- **Distributivity of Scalar Multiplication over Vector Addition**: $(c + d)\mathbf{v} = c\mathbf{v} + d\mathbf{v}$