Matrix operations

Tyreek Alexander

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1 Matrix addition

If A and B are matrices of the same dimension, their sum $C = A+B$ is obtained by adding corresponding elements:

$$C = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

2 Matrix multiplication

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, their product $C = AB$ is an $m \times p$ matrix:

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{bmatrix}$$

where

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

3 Transpose of a matrix

The transpose of a matrix A is denoted by A^T and is obtained by swapping rows with columns:

$$A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

4 Determinant of a matrix

For a 2×2 matrix A, the determinant is given by:

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

5 Inverse of a matrix

For a 2×2 matrix A, the inverse is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

provided that $\det(A) \neq 0$.

6 Trace of a matrix

The trace of a matrix A is the sum of its diagonal elements:

$$\text{Trace of a matrix } A = \sum_{i=1}^{n} a_{ii}$$

7 Eigenvalues and eigenvectors

Eigenvalues λ of a matrix A are found by solving:

$$\det(A - \lambda I) = 0$$

Eigenvectors $\mathbf{v}$ corresponding to eigenvalue λ satisfy:

$$A\mathbf{v} = \lambda \mathbf{v}$$

8 Rank of a matrix

The rank of a matrix A is the maximum number of linearly independent rows or columns.

9 Symmetric matrix

A matrix A is symmetric if:

$$A = A^T$$

10 Orthogonal matrix

A matrix A is orthogonal if:

$$A^T A = AA^T = I$$

11 Matrix operations

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\begin{aligned}
A + B &= B + A, \\
(A + B) + C &= A + (B + C), \\
c(A + B) &= cA + cB, \\
c(dA) &= (cd)A, \\
A + 0 &= A, \\
A + (-A) &= 0, \\
1A &= A, \\
c(A + B) &= cA + cB, \\
A(B + C) &= AB + AC, \\
(A + B)C &= AC + BC, \\
A(BC) &= (AB)C, \\
A(B^T) &= (A^T)B^T, \\
(AB)^T &= B^T A^T, \\
(AB)^{-1} &= B^{-1}A^{-1}, \\
(A^T)^{-1} &= (A^{-1})^T, \\
\det(AB) &= \det(A)\det(B), \\
\det(A^T) &= \det(A), \\
\det(A^{-1}) &= \frac{1}{\det(A)}, \\
\mathrm{Trace}(A + B) &= \mathrm{Trace}(A) + \mathrm{Trace}(B), \\
\mathrm{Trace}(AB) &= \mathrm{Trace}(BA), \\
\mathrm{Trace}(A^T) &= \mathrm{Trace}(A), \\
\mathrm{Trace}(A^{-1}) &= \frac{1}{\mathrm{Trace}(A)}, \\
\mathrm{Rank}(A) &= \mathrm{Rank}(A^T), \\
\mathrm{Rank}(A) &= \mathrm{Rank}(A^T A), \\
\mathrm{Rank}(A) &= \mathrm{Rank}(AA^T)
\end{aligned}
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