Probabilities and Distributions

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Common Probability Distributions

Binomial Distribution

The probability of getting exactly k successes in n independent Bernoulli trials is given by the binomial distribution:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where p is the probability of success on a single trial.

Common Uses:

- Modeling the number of successes in a fixed number of trials.
- Quality control and reliability testing.

Maximum Likelihood Estimation (MLE):

$$\hat{p} = \frac{k}{n}$$

Mean and Variance:

$$\text{Mean} = np, \quad \text{Variance} = np(1-p)$$

Normal Distribution

The probability density function of a normal distribution with mean μ and standard deviation σ is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Common Uses:

- Modeling natural phenomena (e.g., heights, test scores).
- Assumption in many statistical methods.

Maximum Likelihood Estimation (MLE):

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2$$

Mean and Variance:

$$\text{Mean} = \mu, \quad \text{Variance} = \sigma^2$$

Poisson Distribution

The probability of observing k events in a fixed interval of time or space is given by the Poisson distribution:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where λ is the average number of events per interval.

Common Uses:

- Modeling the number of events in a fixed interval of time or space.
- Queueing theory and telecommunications.

Maximum Likelihood Estimation (MLE):

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Mean and Variance:

$$\text{Mean} = \lambda, \quad \text{Variance} = \lambda$$