1. split the sum

$$= \frac{1}{N}\left(\sum_{i=a}^{n} a + \sum_{i=1}^{N} bx_i\right)$$

since a/b constants

$$= \frac{1}{N}\left(Na + b\sum_{i=1}^{N} x_i\right)$$

$$= a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

$$= a + b\,m(X)$$

2.

$$cov(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2$$

def of simple var

$$s^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

Therefore $(X,X) = s^2$

3.

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))[(a+by_i) - m(a+bY)]$$

part (1)

$$m(a+bY) = a + b\,m(Y)$$

inside sum

$$(a+by_i) - (a+b\,m(Y)) = b(y_i - m(Y))$$

Thus:

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X)) \cdot b(y_i - m(Y))$$

Factor out b.

$$= b \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$$

$$= b\,\mathrm{cov}(X, Y)$$

4. Show that

apply part 3 twice

$$\mathrm{cov}(a+bX, a+bY) = b\,\mathrm{cov}(X, a+bY)$$

from part (3)

$$\mathrm{cov}(X, a+bY) = b\,\mathrm{cov}(X, Y)$$

So:

$$b \cdot b \cdot cov(X, Y) = b^2 cov(X, Y)$$

Showing

$cov(bX$

5.

Let $IQR(X) = Q_3(X) - Q_1(X)$

Than

$Q_3(a+bX) = a + bQ_3(X)$, $Q_1(a+bX) = a + bQ_1(X)$

So.

$IQR(a+bX) = b(Q_3(X) - Q_1(X)) = b\,IQR(X)$

So addative cancels out $a + b\,IQR(X)$

6. Let $X = \{1, 4\}$

Squre example $m(X) = \frac{1+4}{2} = 2.5$

$(m(X))^2 = 6.25$

Now Compute

$X^2 = \{1, 16\} \Rightarrow m(X^2) = \frac{1+16}{2} = 8.5$

$m(X^2) \neq m(X)^2$