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Sequential sampling plan using fuzzy SPRT

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Abstract. In this paper we introduce a new sequential sampling plan based on sequential probability ratio test for fuzzy hypotheses testing. In order to deal with uncertainty happened, the triangular fuzzy number (TFN) is used to express the fuzzy phenomenon of the sampling plan's parameters. In this plan the acceptable quality level (AQL) and the lot tolerance percent defective (LTPD) are TFN. For such a plan, we design a decision criterion of acceptance and rejection for every arbitrary λ-cut and a particular table of rejection and acceptance is calculated. This plan is well defined, since, if the parameters are crisp, it changes to a classical plan.

Keywords: Sequential sampling plan, fuzzy number, acceptable quality level, lot tolerance percent defective

1. Introduction

One of the most widely used statistical quality control (SQC) tools is the lot acceptance sampling plan (LASP), which can be applied in a variety of ways. In LASP area, several sampling schemes are available for the application of attribute quality characteristics, namely, single, double, multiple, chain and sequential sampling plan (SSP). In sequential sampling, we take a sequence of samples from the lot and allow the number of samples to be determined by the results of the sampling process. Under sequential sampling, samples are taken, one at the time, until a decision is made on the lot to accept, reject or continue sampling (take another sample and then repeat the decision process).

If the inspected sample size at each sequence of them is greater than one, the plan is called group sequential sampling, but if at each stage it is one, the plan is called item by item sequential sampling [5]. Sequential sampling was developed in1943 for use in rapid quality inspection in war research and production [15]. SSP has been studied by many researchers, and thoroughly elaborated on by Hardeo Sahai et al. [20].

Item by item sequential sampling plan, in traditional form, is based on the sequential probability ratio test (SPRT) with exact hypotheses. However, there are many different situations in which the parameters are imprecise. In the design sequential plan two levels quality AQL and LTPD, are crisp. In the other hand, sometimes the available qualities are not precise. To overcome this problem, first we are developed SPRT for fuzzy hypotheses where the imprecise parameters are as fuzzy numbers. Then item by item SSP is defined and extended based on fuzzy SPRT.

Testing fuzzy hypotheses was discussed by Arnold [2] and [3], Delgado et al. [17], Watanabe and Imaizumi [18], Taheri and Behboodian [19], Torabi and Behboodian [10], Torabi and Behboodian [11]. SPRT for the fuzzy hypotheses were discussed by Torabi and Mirhosseini [12], Torabi and Behboodian [9] and Akbari [16]. Sadeghpour Gildeh et al. [4] and Baloui Jamkhaneh et al. [6–8] considered acceptance sampling plan under the conditions of the fuzzy parameter.

We organize the matter in the following way: The next section introduces SPRT for fuzzy hypotheses. In the third section we design the fuzzy sequential sampling plan, considered broadly, and its limiting lines are computed. The results are summarized in the conclusions.

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2. SPRT for fuzzy hypothesis testing

In fuzzy hypothesis testing (FHT) with crisp data, fuzzy hypotheses are

$$\begin{cases} H_0 : \theta \, is \, H_0(\theta), \\ H_1 : \theta \, is \, H_1(\theta), \end{cases} \quad or \quad \begin{cases} H_0 : \tilde{\theta} = \tilde{\theta}_0 \\ H_1 : \tilde{\theta} = \tilde{\theta}_1 \end{cases} \tag{1}$$

where "$H_j : \theta \, is \, H_j(\theta),\ j = 0, 1$" implies that θ is a fuzzy set in Θ (the parameter space) with membership function $H_j(\theta)$ i.e. a function from Θ to $[0, 1]$. Note that the crisp hypothesis $H_j(\theta) : \Theta \to [0, 1],\ j = 0, 1$ is a fuzzy hypothesis with membership function $H_j(\theta) = 1$, $\theta \in \Theta_j$, and zero otherwise.

Let random variable X have the fuzzy probability density function (FPDF) $\tilde{f}(x;\tilde{\theta})$ where $\tilde{\theta}$ is a fuzzy parameter. Under the hypotheses $H_j,\ j = 0, 1$, the λ-cut of FPDF is as follows

$$\begin{aligned} \tilde{f}_j(x,\ \tilde{\theta}_j)[\lambda] &= \{f_j(x,\ \theta_j)\,|\theta_j \in \tilde{\theta}_j\ [\lambda]\} \\ &= [f_j^L(x)[\lambda],\ f_j^U(x)[\lambda]],\ j = 0, 1, \end{aligned} \tag{2}$$

where

$$\begin{aligned} f_j^L(x)[\lambda] &= \min\{f_j(x,\ \theta_j)\,|\theta_j \in \tilde{\theta}_j\ [\lambda]\}\,, \\ f_j^U(x)[\lambda] &= \max\{f_j(x,\ \theta_j)\,|\theta_j \in \tilde{\theta}_j\ [\lambda]\}, \end{aligned} \tag{3}$$

(For more details see [13]).

Let $X = (X_1, X_2, ..., X_n)$ is a random sample from a population with the $\tilde{f}(x;\tilde{\theta})$, and $\Phi(X)$ is a test function, it is the probability of rejecting H_0 provided that X = x is observed. The probability of type I and II errors for the testing of fuzzy hypothesis of Equation (1) are $\tilde{\alpha}_\Phi = \tilde{E}_0(\Phi(X))$ and $\tilde{\beta}_\Phi = 1 - \tilde{E}_1(\Phi(X))$ respectively, in which $\tilde{E}_j(\Phi(X))$ is the fuzzy expected value of $\Phi(X)$ over the FPDF of $\tilde{f}_j(x, \tilde{\theta})$, $j = 0, 1$ (For more details see [13]).

$$\begin{aligned} \tilde{E}_j(\Phi(X))[\lambda] &= \left\{ \int_x \Phi(x) f_j(x,\ \theta_j) dx \,|\theta_j \in \tilde{\theta}_j\ [\lambda] \right\} \\ &= [E_j^L(\Phi(X))[\lambda],\ E_j^U(\Phi(X))[\lambda]],\ j = 0, 1, \end{aligned}$$

where

$$E_j^L = \min \left\{ \int_x \Phi(x) f_j(x,\ \theta_j) dx \,|\theta_j \in \tilde{\theta}_j\ [\lambda] \right\},$$

$$E_j^U = \max \left\{ \int_x \Phi(x) f_j(x,\ \theta_j) dx \,|\theta_j \in \tilde{\theta}_j\ [\lambda] \right\}.$$

Now we propose the sequential probability ratio test for FHT. Let $X_1, X_2, ...$ denote a sequence of the iid random variables, from a population with FPDF $\tilde{f}(x;\tilde{\theta})$. First compute sequentially $\tilde{R}_1, \tilde{R}_2, ...$ for an arbitrary λ -cut, where

$$\begin{aligned} \tilde{R}_n(x) &= \left\{ \frac{L_{\tilde{\theta}_0}(x_1, ..., x_n\,)}{L_{\tilde{\theta}_1}(x_1, ..., x_n)} \,|\theta_j \in \tilde{\theta}_j\ [\lambda],\ j = 0, 1 \right\}, \\ &= \left\{ \Pi_{i=1}^n \frac{\tilde{f}_0(x_i, \tilde{\theta}_0)}{\tilde{f}_1(x_i, \tilde{\theta}_1)} \,|\theta_j \in \tilde{\theta}_j\ [\lambda],\ j = 0, 1 \right\}, \\ &= \left[\frac{\Pi_{i=1}^n f_0^L(x_i, \theta_0^L)}{\Pi_{i=1}^n f_1^U(x_i, \theta_1^U)}, \frac{\Pi_{i=1}^n f_0^U(x_i, \theta_0^U)}{\Pi_{i=1}^n f_1^L(x_i, \theta_1^L)} \right], \\ &= \left[\frac{L_{\tilde{\theta}_0}^L}{L_{\tilde{\theta}_1}^U},\ \frac{L_{\tilde{\theta}_0}^U}{L_{\tilde{\theta}_1}^L} \right], \\ &= [R_n^L(x)\,,\ R_n^U(x)], \end{aligned} \tag{4}$$

where

$$f_j^L(x_i, \theta_j^L) = \min\{f_j(x_i,\ \theta_j)\,|\theta_j \in \tilde{\theta}_j\ [\lambda]\},\ \ j = 0, 1,$$

$$f_j^U(x_i, \theta_j^U) = \max\{f_j(x_i,\ \theta_j)\,|\theta_j \in \tilde{\theta}_j\ [\lambda]\},\ \ j = 0, 1.$$

For fixed k_0 and k_1 satisfy $0 < k_0 < k_1$, adopt the following procedure: Take observation x_1 and compute $\tilde{R}_1$; if $R_1^U \le k_0$, reject H_0; if $R_1^L \ge k_1$; accept H_0; and if $R_1^L < k_1$ and $R_1^U > k_0$, take observation x_2, and compute $\tilde{R}_2$; if $R_2^U \le k_0$, reject H_0; if $R_2^L \ge k_1$, accept H_0; and if $R_2^L < k_1$ and $R_2^U > k_0$, take observation x_3, etc. The idea is to continue sampling as long as $R_n^L < k_1$ and $R_n^U > k_0$ and stop as soon as $R_n^U \le k_0$ or $R_n^L \ge k_1$, rejecting H_0 if $R_n^U \le k_0$ and accepting H_0 if $R_n^L \ge k_1$. Consequently, the critical region of the described SPRT for fuzzy hypotheses testing of Equation (1) is $C = \cup_{n=1}^{\infty} C_n$, where

$$\begin{aligned} C_n = \{(x_1, ... x_n) \Big|\ & R_j^L < k_1 \text{ and} \\ & R_j^U > k_0\,,\ j = 1, ..., n-1,\ R_n^U \le k_0\}. \end{aligned} \tag{5}$$

Similarly, the acceptance region can be as $A = \cup_{n=1}^{\infty} A_n$, where

$$\begin{aligned} A_n = \Big\{(x_1, ...x_n) \Big|\ & R_j^L < k_1 \text{ and} \\ & R_j^U > k_0\,,\ j = 1, ..., n-1,\ R_n^L \ge k_1 \Big\} \end{aligned} \tag{6}$$

Therefore in the SPRT for fuzzy hypotheses, the probability of type I and II errors are calculated by

$$\begin{aligned}\tilde{\alpha} &= \left\{ \sum_{n=1}^{\infty} \int_{C_n} L_{\tilde{\theta}_0}(X)dX \,\middle|\, \theta_0 \in \tilde{\theta}_0[\lambda] \right\}, \\ \tilde{\beta} &= \left\{ \sum_{n=1}^{\infty} \int_{A_n} L_{\tilde{\theta}_1}(X)dX \,\middle|\, \theta_1 \in \tilde{\theta}_1[\lambda] \right\}.\end{aligned} \tag{7}$$

Remark. Let $\tilde{A}[\lambda] = [A^L[\lambda], A^U[\lambda]]$ and $\tilde{B}[\lambda] = [B^L[\lambda], B^U[\lambda]]$ be λ-cut of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively. In one way of ordering fuzzy numbers, we are defining $\tilde{A} \precsim \tilde{B}$ if and only if $A^U[\lambda] < B^L[\lambda]$ for all $\lambda \in [0, 1]$.

Lemma 1. Let k_0 and k_1 be defined so that the SPRT for fuzzy hypotheses has the fixed probability of type I and II errors $\tilde{\alpha}$ and $\tilde{\beta}$. Then k_0 and k_1 can be approximated by $k'_0 = \frac{\alpha^U}{1-\beta^U}$ and $k'_1 = \frac{1-\alpha^U}{\beta^U}$.

Proof. If $R_i^U \le k_0$ then null hypothesis reject, it means that in critical region we have

$$\frac{L^U_{\tilde{\theta}_0}}{L^L_{\tilde{\theta}_1}} \le k_0 \Rightarrow L^U_{\tilde{\theta}_0} \le k_0 L^L_{\tilde{\theta}_1} \Rightarrow \tilde{L}_{\tilde{\theta}_0} \lesssim k_0 \tilde{L}_{\tilde{\theta}_1}$$

then

$$\begin{aligned}\tilde{\alpha} = \sum_{n=1}^{\infty} \int_{C_n} \tilde{L}_{\tilde{\theta}_0}(X)dX &\lesssim \sum_{n=1}^{\infty} \int_{C_n} k_0 \tilde{L}_{\tilde{\theta}_1}(X)dX \\ &= k_0(1 - \tilde{\beta})\end{aligned}$$

therefore

$$\frac{\alpha^U}{1 - \beta^U} \le k_0 \tag{8}$$

If $R_i^L \ge k_1$ then null hypothesis accepted, it means that in acceptance region we have

$$\frac{L^L_{\tilde{\theta}_0}}{L^U_{\tilde{\theta}_1}} \ge k_1 \Rightarrow L^L_{\tilde{\theta}_0} \ge k_1 L^U_{\tilde{\theta}_1} \Rightarrow \tilde{L}_{\tilde{\theta}_0} \gtrsim k_1 \tilde{L}_{\tilde{\theta}_1}$$

then

$$\begin{aligned}1 - \tilde{\alpha} &= \sum_{n=1}^{\infty} \int_{A_n} \tilde{L}_{\tilde{\theta}_0}(X)dX \\ &\gtrsim \sum_{n=1}^{\infty} \int_{A_n} k_1 \tilde{L}_{\tilde{\theta}_1}(X)dX = k_1 \tilde{\beta},\end{aligned}$$

therefore

$$k_1 \le \frac{1 - \alpha^U}{\beta^U}. \tag{9}$$

Finally, we have

$$0 < \frac{\alpha^U}{1 - \beta^U} \le k_0 < k_1 \le \frac{1 - \alpha^U}{\beta^U}. \tag{10}$$

According to Equation (10) proof is obvious. ■

Lemma 2. If $\tilde{\alpha}'$ and $\tilde{\beta}'$ are the error size of the SPRT defined by k'_0 and k'_1, then $\alpha'^U + \beta'^U \le \alpha^U + \beta^U$ for arbitrary λ-cut.

Proof:

$$\begin{aligned}\tilde{\alpha}' &= \sum_{n=1}^{\infty} \int_{C'_n} \tilde{L}_{\tilde{\theta}_0}(X)dX, \\ &\lesssim \sum_{n=1}^{\infty} \int_{C'_n} k'_0 \tilde{L}_{\tilde{\theta}_1}(X)dX = \frac{\alpha^U}{1 - \beta^U}(1 - \tilde{\beta}'), \\ &\Rightarrow \tilde{\alpha}'(1 - \beta^U) \lesssim \alpha^U(1 - \tilde{\beta}'),\end{aligned} \tag{11}$$

and

$$\begin{aligned}1 - \tilde{\alpha}' &= \sum_{n=1}^{\infty} \int_{A'_n} \tilde{L}_{\tilde{\theta}_0}(X)dX, \\ &\gtrsim \sum_{n=1}^{\infty} \int_{A'_n} k'_1 \tilde{L}_{\tilde{\theta}_1}(X)dX = \frac{1 - \alpha^U}{\beta^U} \tilde{\beta}', \\ &\Rightarrow (1 - \alpha^U)\, \tilde{\beta}' \lesssim (1 - \tilde{\alpha}')\beta^U.\end{aligned} \tag{12}$$

By using Equations (11) and (12) we have

$$\tilde{\alpha}' + \tilde{\beta}' \lesssim \alpha^U \tilde{1} + \beta^U \tilde{1} \Rightarrow \alpha'^U + \beta'^U \le \alpha^U + \beta^U. \quad ■$$

If

$$\begin{aligned}\tilde{Z}_i &= [Z_i^L,\ Z_i^U], \\ &= \left[Ln \frac{f_0^L(x_i, \theta_0^L)}{f_1^U(x_i, \theta_1^U)},\ Ln \frac{f_0^U(x_i, \theta_0^U)}{f_1^L(x_i, \theta_1^L)} \right],\end{aligned} \tag{13}$$

with observed one sample and using Equation (13), an equivalent test to the SPRT for fuzzy hypotheses is given by the following: Continue sampling as long as $Ln(k'_0) < \sum_{i=1}^{n} Z_i^U$ and $\sum_{i=1}^{n} Z_i^L < Ln(k'_1)$, otherwise stop sampling, if $\sum_{i=1}^{n} Z_i^L \ge Ln(k'_1)$ then we will accept H_0 and if $\sum_{i=1}^{n} Z_i^U \le Ln(k'_0)$ then we will reject H_0. Let

N be the random variable denoting the sample size of SPRT for fuzzy hypothesis testing, then

$$\tilde{E}_0^{(l)} \precsim \tilde{E}(N\,|H_0\,is\,true) \precsim \tilde{E}_0^{(u)}, \tag{14}$$

where

$$\tilde{E}_0^{(l)} = \frac{\tilde{\alpha} L_n \frac{\alpha^U}{1-\beta^U} + (1-\tilde{\alpha}) L_n \frac{1-\alpha^U}{\beta^U}}{\tilde{E}(Z_i^U\,|H_0\,is\,true)},$$

$$\tilde{E}_0^{(u)} = \frac{\tilde{\alpha} L_n \frac{\alpha^U}{1-\beta^U} + (1-\tilde{\alpha}) L_n \frac{1-\alpha^U}{\beta^U}}{\tilde{E}(Z_i^L\,|H_0\,is\,true)},$$

and

$$\tilde{E}_1^{(l)} \precsim \tilde{E}(N\,|H_1\,is\,true) \precsim \tilde{E}_1^{(u)} \tag{15}$$

where

$$\tilde{E}_1^{(l)} = \frac{(1-\tilde{\beta}) L_n \frac{\alpha^U}{1-\beta^U} + \tilde{\beta} L_n \frac{1-\alpha^U}{\beta^U}}{\tilde{E}(Z_i^U\,|H_1\,is\,true)},$$

$$\tilde{E}_1^{(u)} = \frac{(1-\tilde{\beta}) L_n \frac{\alpha^U}{1-\beta^U} + \tilde{\beta} L_n \frac{1-\alpha^U}{\beta^U}}{\tilde{E}(Z_i^L\,|H_1\,is\,true)},$$

where

$$\tilde{E}(Z_i^L\,\big|H_j\,is\,true) = \left\{\int_x L_n \frac{P_0^L(x,\theta_0^L)}{P_1^U(x,\theta_1^U)} f_j(x,\theta_j)dx\,\big|\theta_j \in \tilde{\theta}_j\,[\lambda]\right\}, \tag{16}$$

and

$$\tilde{E}(Z_i^U\,\big|H_j\,is\,true) = \left\{\int_x L_n \frac{P_0^U(x,\theta_0^U)}{P_1^L(x,\theta_1^L)} f_j(x,\theta_j)dx\,\big|\theta_j \in \tilde{\theta}_j\,[\lambda]\right\}. \tag{17}$$

3. Fuzzy sequential sampling plans

The acceptance and rejection lines are decision criterion in an item by item SSP. These two limit lines are plotted in terms of the total number of items thus far selected, and show the total number of observed nonconforming items. Figure 1 illustrates the operation of such a sampling plan.

For each point in this figure, the horizontal axis is the total number of items selected up to that time, and the vertical axis is the total number of observed nonconforming items. Then the operation procedure is given in the following,

(i) If the plotted point falls within the limit lines the process continues by drawing another sample,
(ii) When plotted point falls on or above the upper line, the lot is rejected,
(iii) When plotted point falls on or below the lower line, the lot is accepted [5].

Accepting or rejecting a lot in the SSP analogous to not rejecting or rejecting the null hypotheses in a hypotheses test. The hypotheses for SSP as a kind of statistical test are [14]:

H_0: The lot is of acceptable quality level (AQL)
H_1: The lot is of reject able quality level (RQL)

The AQL presents the poorest level of quality for the vender's process that the consumer would consider to be acceptable as a process average [5]. Alternate name for the RQL is lot tolerance percent defective (LTPD). The LTPD is the poorest level of quality that the consumer is willing to accept in an individual lot. Probability of type I and II errors α and β for this hypotheses test is as follows

$$\begin{aligned} \alpha &= P(\text{rejected } H_0\,|H_0 \text{ is true}), \\ \beta &= P(\text{not rejected } H_0\,|H_0 \text{ is false})\,. \end{aligned} \tag{18}$$

A lot may be rejected that should be accepted and the risk of doing this is called the producer's risk (α). The second error is that a lot may be accepted that should be rejected and the risk of doing so is called the consumer's risk (β).

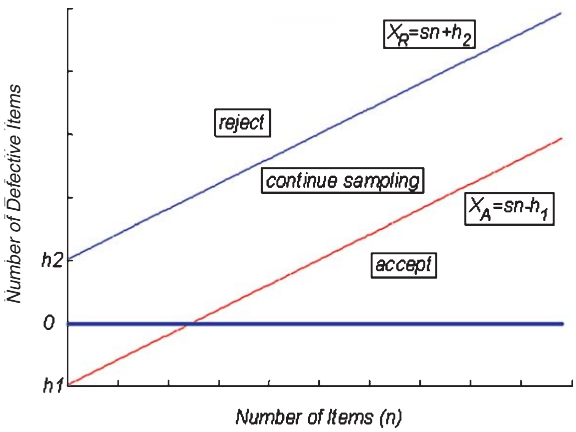

Fig. 1. Graphical performance of sequential sampling plan.

If $AQL = p_0$ and $LTPD = p_1$, $(p_0 < p_1)$ then the equivalent hypothesis is given by the following

$$\begin{cases} H_0 : p = p_0, \\ H_1 : p = p_1. \end{cases} \tag{19}$$

When designing an item by item sequential sampling plan, four parameters of the AQL, the producer's risk α (the probability of rejecting a lot with AQL quality), LTPD and the consumer's risk β (the probability of accepting a lot with LTPD quality) must be determined prior to determining the acceptance and rejection lines. In the design SSP two levels of quality are crisp, but sometimes these are not exact and certain. So, we face a fuzzy hypothesis as follows

$$\begin{cases} H_0 : p \approx p_0, \\ H_1 : p \approx p_1. \end{cases} \tag{20}$$

We propose to design item by item SSP with fuzzy AQL and fuzzy LTPD. Such a plan is based on the concept of the SPRT for fuzzy hypotheses. The sampling distribution of the number of nonconforming items in every stage (X_i) is the Bernoulli distribution with fuzzy parameter $\tilde{p}$ (incoming quality level), because each item inspected is either defective or not. That is, $X_i = 1$, if the ith inspected item is defective, otherwise $X_i = 0$. In FHT with crisp data, fuzzy hypotheses are

$$\begin{cases} H_0 : p\, is\, H_0(p), \\ H_1 : p\, is\, H_1(p), \end{cases} \tag{21}$$

where membership function of $H_j(p)$ is considered as follows

$$H_j(p) = \begin{cases} \dfrac{p - p_j^{(1)}}{p_j^{(2)} - p_j^{(1)}}\ ,\ p_j^{(1)} \le p \le p_j^{(2)} \\[2ex] \dfrac{p_j^{(3)} - p}{p_j^{(3)} - p_j^{(2)}}\ ,\ p_j^{(2)} \le p \le p_j^{(3)} \end{cases}, \tag{22}$$

$\forall\, p \in (0, 1)$, $j = 0, 1$. Finally by using of Equations (4) and (22) for arbitrary λ-cut, we obtain

$$\begin{aligned} R_n^L(x) &= \frac{\Pi_{i=1}^n (p_0^L)^{x_i} (1 - p_0^U)^{1-x_i}}{\Pi_{i=1}^n (p_1^U)^{x_i} (1 - p_1^L)^{1-x_i}}, \\ &= \Pi_{i=1}^n \left(\frac{p_0^L (1 - p_1^L)}{p_1^U (1 - p_0^U)} \right)^{x_i} \left(\frac{1 - p_0^U}{1 - p_1^L} \right), \end{aligned} \tag{23}$$

and

$$\begin{aligned} R_n^U(x) &= \frac{\Pi_{i=1}^n (p_0^U)^{x_i} (1 - p_0^L)^{1-x_i}}{\Pi_{i=1}^n (p_1^L)^{x_i} (1 - p_1^U)^{1-x_i}}, \\ &= \Pi_{i=1}^n \left(\frac{p_0^U (1 - p_1^U)}{p_1^L (1 - p_0^L)} \right)^{x_i} \left(\frac{1 - p_0^L}{1 - p_1^U} \right). \end{aligned} \tag{24}$$

If

$$Z_i^L = X_i Ln \left(\frac{p_0^L (1 - p_1^L)}{p_1^U (1 - p_0^U)} \right) + Ln \left(\frac{1 - p_0^U}{1 - p_1^L} \right), \tag{25}$$

and

$$Z_i^U = X_i Ln \left(\frac{p_0^U (1 - p_1^U)}{p_1^L (1 - p_0^L)} \right) + Ln \left(\frac{1 - p_0^L}{1 - p_1^U} \right), \tag{26}$$

then at the nth stage of sampling,

i. Reject the lot if $\sum_{i=1}^n Z_i^U \le Ln \frac{\alpha^U}{1-\beta^U}$,
ii. Accept the lot if $\sum_{i=1}^n Z_i^L \ge Ln \frac{1-\alpha^U}{\beta^U}$,
iii. Continue sampling by taking an additional observation if $\sum_{i=1}^n Z_i^U > Ln \frac{\alpha^U}{1-\beta^U}$, and $\sum_{i=1}^n Z_i^L < Ln \frac{1-\alpha^U}{\beta^U}$.

Using rejection and acceptance regions, and the parameters $\tilde{p}_0$, $\tilde{p}_1$ $(\tilde{p}_0 \precsim \tilde{p}_1)$, $\tilde{\alpha}$ and $\tilde{\beta}$ the SSP using fuzzy SPRT is determined by the acceptance and rejection lines given as follows:

$$X_A = s^{(l)} n - h^{(l)} \text{ (Acceptance line)}, \tag{27}$$

$$X_R = s^{(u)} n + h^{(u)} \text{ (Rejection line)}, \tag{28}$$

here, we have

$$\begin{aligned} s^{(u)} &= \frac{Ln \left(\frac{1-p_0^L}{1-p_1^U} \right)}{k^{(u)}}, k^{(u)} = Ln \left(\frac{p_1^L (1 - p_0^L)}{p_0^U (1 - p_1^U)} \right), \\ s^{(l)} &= \frac{Ln \left(\frac{1-p_0^U}{1-p_1^L} \right)}{k^l},\ k^{(l)} = Ln \left(\frac{p_1^U (1 - p_0^U)}{p_0^L (1 - p_1^L)} \right), \\ h^{(l)} &= \frac{Ln \frac{1-\alpha^U}{\beta^U}}{k^l},\ h^{(u)} = \frac{Ln \frac{1-\beta^U}{\alpha^U}}{k^u}. \end{aligned} \tag{29}$$

Remarks:

i. $k^{(l)} > 0, k^{(u)} > 0, s^{(u)} > 0, s^{(l)} > 0$.
ii. For low values of $\tilde{p}_0$ and $\tilde{p}_1$, it can be concluded that $k^{(l)} \geq k^{(u)}$, and hence we have $s^{(l)} \leq s^{(u)}$.
iii. The parameters uncertainty is less slop of rejection and acceptance lines are closer together. If the parameters are crisp, then the lines are parallel.
iv. For low values of $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{p}_0$ and $\tilde{p}_1$, it can be concluded that $h^{(l)} > 0$ and $h^{(u)} > 0$.

Instead of using the graph to decisions about of the lot, one can resort to generating tables of acceptance number and rejection number. Both acceptance numbers and rejection numbers must be integers. The acceptance number is the next integer less than or equal to $\mathrm{X_A}$, and the rejection number is the next integer greater than or equal to $\mathrm{X_R}$. For example, suppose we will be to find a SSP for which

$$\alpha^U[0] = 0.06,\ \tilde{p}_0 = (0.005,0.01,0.015),$$

$$\beta^U[0] = 0.11,\ \tilde{p}_1 = (0.055, 0.06, 0.065),$$

then, with using Equation (29) we have:

$$k^{(u)} = 1.3615,\ s^{(u)} = 0.0457,\ h^{(u)} = 1.9808,$$

$$k^{(l)} = 2.6029,\ s^{(l)} = 0.01593,\ h^{(l)} = 0.8242,$$

therefore the acceptance and rejection lines are

$$\begin{aligned} X_A &= 0.01593n - 0.8242, \\ X_R &= 0.0457n + 1.9808. \end{aligned} \tag{30}$$

Now for $n = 120$, according to Equation (30) we obtain

(i) Accept lot, if the number of nonconforming items is less than or equal to 1.
(ii) Continue sampling, if the number of nonconforming item is in (1, 8).
(iii) Reject lot, if the number of nonconforming items is greater than or equal to 8.

Table 1 shows that the lot cannot be accepted until at least 51 items have been tested.

Table 1
Sequential sampling plan
$p_0 \approx 0.01, \alpha^U[0] = 0.06, p_1 \approx 0.06, \beta^U[0] = 0.11$

n	Ac. Number	Re. Number	n	Ac. Number	Re. Number
1	a	b	88	0	7
2	a	b	...	...	...
3	a	3	109	0	7
4	a	3	110	0	8
...	...	...	...	...	...
22	a	3	114	0	8
23	a	4	115	1	8
...	...	...	...	...	...
44	a	4	131	1	8
45	a	5	132	1	9
...	...	...	...	...	...
51	a	5	153	1	9
52	0	5	154	1	10
...	...	...	...	...	...
66	0	5	175	1	10
67	0	6	176	1	11
...	...	...	177	1	11
87	0	6	178	2	11

a-means that acceptance not possible, b-means that rejection not possible.

4. Conclusions

In this paper, a new approach for SPRT based on fuzzy hypothesis was presented. Then, we have proposed a method to design SSP with fuzzy AQL and fuzzy LTPD based on our fuzzy SPRT. This plan is comprehensive since, if the AQL and LTPD are crisp, it changes to traditional SSP. The parameters uncertainty is less slop of rejection and acceptance lines are closer together. If the parameters are crisp, then the lines are parallel.

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