The Effectiveness of Optimization Algorithms in Shape and Topology Discrete Optimisation of 2-D Composite Structures

A. Muc
Institute of Mechanics & Machine Design Cracow University of Technology
Kraków, Poland
olekmuc@mech.pk.edu.pl

I. Sanetra
Institute of Mechanics & Machine Design Cracow University of Technology
Kraków, Poland
izabela.sanetra@mech.pk.edu.pl

Abstract - The aim of the paper is to discuss the formulation and solution of optimisation problems for composite structures. A special attention is focused on the coding problems of design variables. The appropriate discrete coding allows us to use the same optimisation algorithms for different class of problems, i.e. shape and topology optimisation using both deterministic and fuzzy approaches.

Key words - Probabilistic Algorithms, Plates, Shells, Composites, Fuzzy Optimisation.

I. INTRODUCTION

Recently, an increasing trend to the application of different variants of evolutionary algorithms in searching for the optimal solutions in various classes of optimisation problems is observed. It is mainly caused by the simplicity of the proposed algorithm and on the other hand by the certainty that the found solution is better than the initial one.

In order to use anisotropic properties of composite materials in an appropriate manner a lot of efforts have been put into introduction and application of effective optimisation algorithms in the design of 2-D laminated plated and shell structures, particularly in the sense of searching for a global optimum. The latter problem is also associated with the imprecise definition of geometrical and material properties of composite materials what is taken into account via membership functions and so-called optimisation in a fuzzy enviroment. The second still open problem in this area is associated with the effective definition of discrete design variables describing real continuous ones. The above-mentioned problems are discussed herein and various numerical examples are demonstrated. They are connected with the topology and shape optimisation.

The effectiveness of probabilistic algorithms is evaluated and determined in the sense of following factors:

– the accuracy of computed global optimum,
– the computational time required in the evaluation of the global optimum,
– the similarity (identity) of optimal design for various probabilistic algorithms.

Since for composite structures, especially in the discrete optimisation problems, a lot of local extrema exists, it is necessary to compare results with the use of various algorithms and in addition using different selection procedures. Both for shape and topology optimisation problems the optimisation procedures (probabilistic algorithms) are conjugated with the finite element analysis that is used for the numerical derivation of a required objective function – see Refs [1], [2]. However, they are completely separated from the FE analysis. In order to verify the effectiveness of the proposed algorithms three different approaches have been applied herein, i.e.: different variants of genetic algorithms (GA), simulated annealing (S.A.) algorithm and the variant of GA proposed in the EVOLVER package.

II. OBJECTIVES OF OPTIMISATION

If any of design variables, constraints or even objective functions are not known precisely (uniquely – a crisp form) the formulation of the optimisation problem should be changed. It may be conducted using statistical analysis (not discussed herein) or the fuzzy formulation. In Table 1 there

978-1-5090-5454-1/17/$31.00 ©2017 IEEE

are compared two optimisation problems deterministic and fuzzy. Of course, the minimum design problem may be simply reformulated to the maximum problem. As it may be noticed the definition of the constraints is the fundamental difference in both formulations. However, as it will be demonstrated below the significant difference lies also in the optimisation algorithms and the reformulation of the fuzzy optimum design problems, since in a fuzzy environment searching for the optimal solutions is carried out not for the values of design variables s (a crisp problem) but for the membership functions μ.

TABLE I. COMPARISON OF THE DETERMINISTIC AND FUZZY FORMULATION OF OPTIMIZATION PROBLEMS

	Deterministic formulation	**Fuzzy formulation**
Objective	Find: Min f(s)	Find: Min f(s)
Constraints	$g_j(s) \le b_j$, j = 1,2,...,m	$g_j(s) \in G_j$ (fuzzy sets), j=1,2,...m

Thus, using the fuzzy formulation the initial minimum problem is formu-lated as a MinMax problem. The latter problem is reformulated to the fol-lowing maximum problem introducing an additional objective λ:

$$\text{Find Max } \lambda \qquad (1)$$

subject to the constraints

$$\lambda \le \mu_f(s), \quad \lambda \le \mu_{gj}(s), \; j=1,2,...,m. \qquad (2)$$

III. CODING OF THE DESIGN VARIABLES

Dependly on the analysed optimisation problem (Table 3) the binary coding of design variables is used with the aid of the incidence matrix introduced in the form given in Table 2. The total number of columns represents the length of the chromosome. Each gen in the chromosome describes the coded information in rows, i.e. if *a(1,1)*=1 then the ply orientation is equal to 0^0, and if *a(2,1)*=*a(3,1)*=0 then the ply orientation is not equal to 45^0 (or 90^0, rsp.) etc. Shape optimisation is connected with searching for optimal structural boundaries, whereas topology optimisation is mainly associated with the minimum mass (volume) problems taking into account the area occupied by a structure, its thickness and/or density of fibres in the construction.

TABLE II. THE FORM OF THE INCIDENCE MATRIX *a(i,j)*

1	0	0	0	0
0	0	1	1	0
0	1	0	0	0

TABLE III. THE PHYSICAL SENSE OF THE TERMS IN THE INCIDENCE MATRIX

Optimisation Problem	**Row**	**Column**
Stacking sequence	Orientation	Ply Number
Shape	Position	Number of Key Point
Topology	Existence of FE	Number of FE

a. Shape optimisation

In the shape optimisation problem the required boundary curve is evaluated with the use of the Bezier splines – see Fig.1. Each keypoint (six in Fig.1) is represented by the radius (denoted by the position in Table 3). The radii are defined on a discrete set. Dependly on the Bezier splines construction the convexity of the curves may be prescribed in advance together with the required values of derivatives at the curve ends and other constraints (e.g. the total area under the curve).

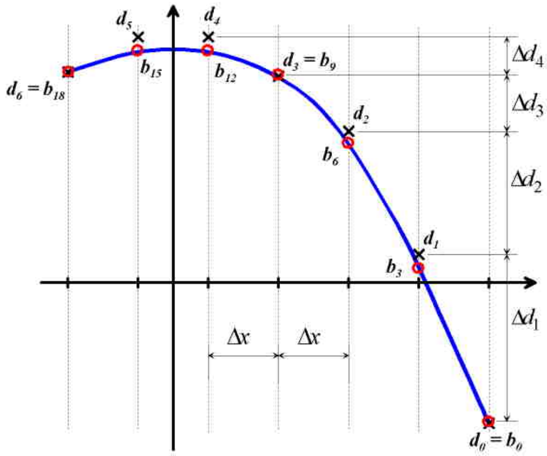

Fig. 1. Construction of a curve describing the required shape

b. Topology optimisation

Topology optimisation problems are based on the elimination of sets of FE from the initial mesh. The total length of the chromosome is composed of a finite number of parts, and each of them corresponds to a curve joining the base points. Two independent methods have been introduced: (1) the variation of the base points – Fig. 2 and (2) the elimination of domains enclosing the given set of curves – Fig.3. In the first case (1) the base points are connected with the use of straight lines and the lines constitute the row of FE having the prescribed properties. In the second case (2) the area between the curved lines is filled by generated FE.

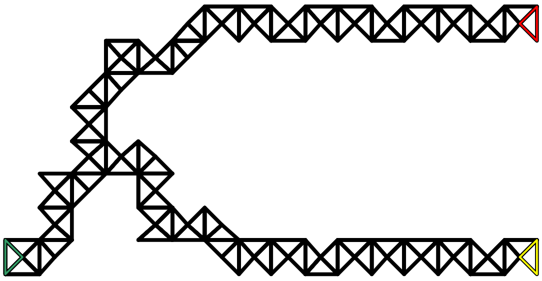

Fig. 2. Variation of the base points

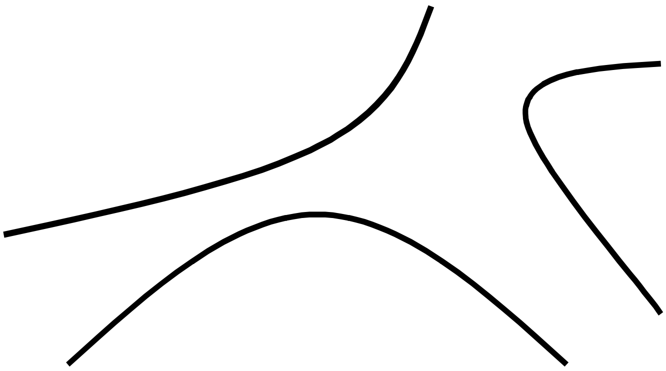

Fig. 3. Elimination of domains

The curves enclosing the selected domains vary in the optimisation problems. They are described in the similar manner as for the shape optimisation problems via a finite number of the key points.

IV. Effectiveness of Optimisation

a. Benchmarks

For shape and topology optimisation problems the effectiveness of the optimisation procedures and methods of coding have been determined using the known theoretical solutions given in a theoretical form.

i. Shape Optimisation

For shape optimisation problems the accuracy in the boundary curve evaluation is the most important problem. To verify it the following well-known problem have been solved: *to find the maximal area A surrounded by the curve with the fixed length* $\tilde{L}$. The problem has been formulated in the following way:

$$Max\left[Area - pen*\left(\tilde{L} - L\right)\right] \qquad (3)$$

The results of the numerical solution are shown in Fig. 4. They demonstrate an excellent accuracy. The error between theory and numerical analysis is equal to 0.66% in the evaluation of the area A, and 0.11% - the length L.

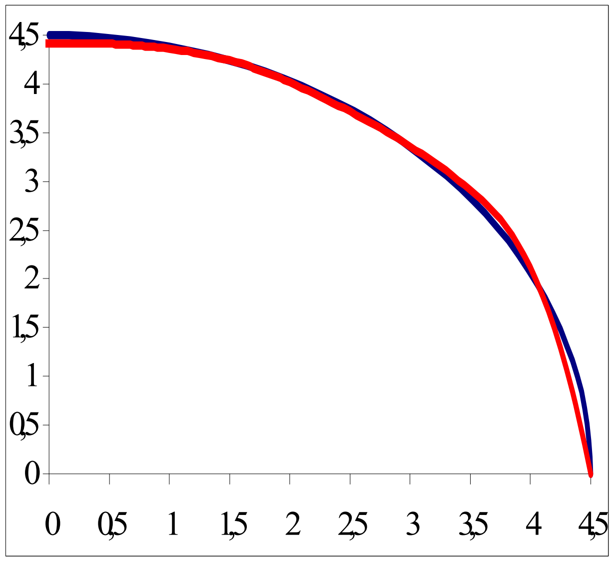

Fig. 4. Comparison of numerical and theoretical results (red curve – analytical results)

ii. Topology Optimisation

Let us consider a composite beam reinforced by short fibres, loaded at the center (Fig. 5 – three point bending test) and having a variable density of fibres $\rho_f(x)$ along the length.

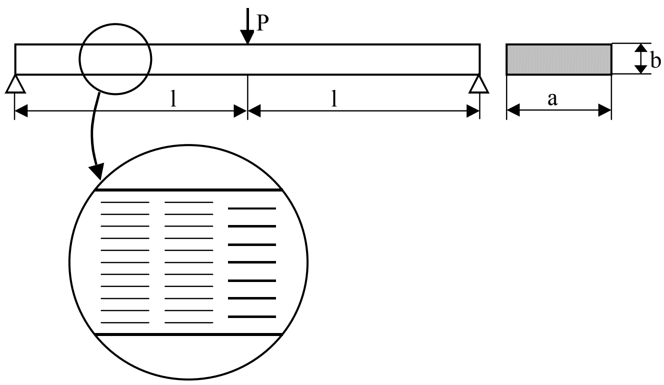

Fig. 5. Bending of a simply supported beam having a variable fibre volume fraction distribution

The objective of the optimisation is following: *to find the minimal mass of the beam satisfying the equality constraint imposed on the displacement parameter and inequality (upper and lower bounds) constraint - on the* *fibre volume density fraction* V_f. – the strict formulation is given by Banichuk et al. [3]. The optimisation problem deals with the topology optimisation since we are looking for the optimal material distribution. The results are presented in Fig. 6 and show a very good correlation between numerical and analytical studies.

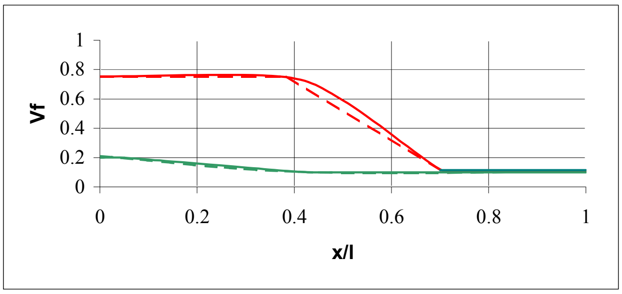

Fig. 6. Comparison of analytical and numerical (a continuous line) results

b. Convergence of probabilistic algorithms

The similarity (identity) of optimal design for various probabilistic algo-rithms is understood in the sense of the convergence of the numerical results to the global optimum. The detailed analysis deals with the topology optimi-sation problem of a composite rectangular plate subjected to the action of a shearing point load at the plate center. We are looking for the minimal mass of the plate satifying the additional constraint conditions determined for the values of allowable displacements and the effective stresses – see Ref [1].

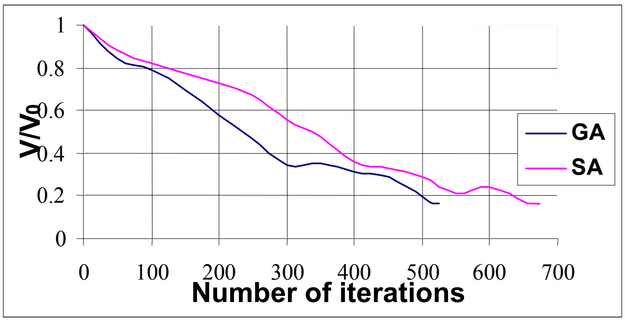

Fig. 7. Variation of the base points

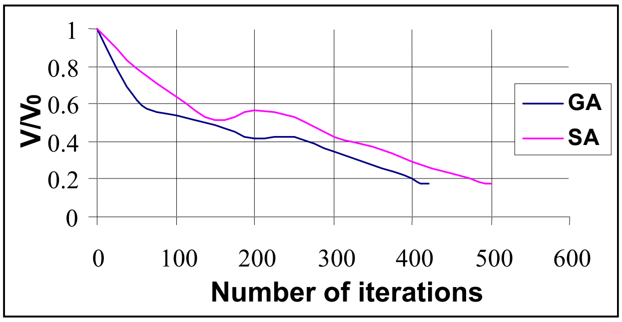

Fig. 8. Elimination of domains

Figures 7 and 8 demonstrate the effectiveness of the optimisation problems in the sense of number of iterations required in the analysis. The obtained topologies are almost the same in the sense of the final volume (area) of the structure both for the simulating annealing algorithms as well as genetic algorithms. For all analysed numerically cases with the different degrees of anisotropy in the composite plates the differences (of the value of optimal volume) between different variants of GA and S.A. do not exceed 2%. However, the total number of iterations is always higher for problems solved with the use of S.A. algorithm.

V. OPTIMISATION IN A FUZZY ENVIROMENT

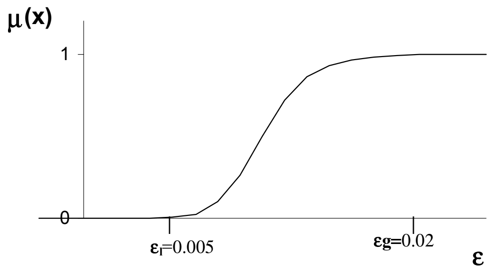

Fig. 9. The possible form of the membership function μ

A statistical distribution of mechanical properties is a standard phenomenon for composite materials. A fuzzy set analysis offers a powerful tool which may be applied in all problems where impreciseness, uncertainty, fuzziness play an important role and become significant in the evaluation of the output data understood in the sense of failure loads for composite structures.

For instance let us consider the experimental data characterizing the values of laminate allowable strains in tension which lie in the interval $[\varepsilon_l, \varepsilon_g]$. Using the fuzzy set approach it is possible to describe the gradual transition between the state of the lack of failure $\varepsilon<\varepsilon_l$ and the complete failure if $\varepsilon > \varepsilon_g$. It is described with the use of the membership function (see e.g. Fig. 9) where e.g. the statistical data may be used in the derivation of the particular membership function form.

To illustrate and demonstrate the sense of the introduced description of optimisation in a fuzzy enviroment the stacking sequence (topology) optimisation problem for rectangular bi-axially compressed and subjected to a uniform normal pressure multilayered composite plate have been solved. The objective function is given by eqn (1). We intend to maximize the loading parameter q. The objective is subjected to the set of six fuzzy constraints corresponding to the values of allowable strains in the longitudinal, perpendicular and transverse directions to fibres (the simplest FPF criterion in the form of the maximal strains) for tension and compression, independently. The form of the membership functions (six functions) is assumed to be identical to that plotted in Fig. 9. The different parameters β characterizing the slope of the curve have been analysed starting from 0 (the deterministic, crisp analysis) to 0.5 to describe various values of the fuzziness.

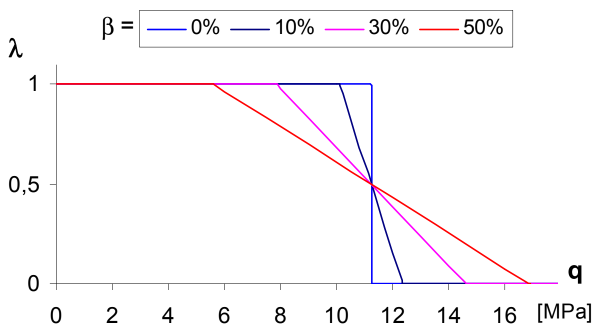

Fig. 10. The fuzziness of the optimal loading parameter q

Then with the use of classical relations for deformations of plates one may express in the analytical form the relations between strains in each individual ply, external loads and fibre orientations. Applying genetic algorithms one may find the optimal fibre orientations in plies as the function of the load parameter and of the fuzziness related to the slope of the curve plotted in Fig. 9. The EVOLVER package have been also used in numerical computations. For instance, for the laminated structure made of 20 plies (symmetric, balanced laminates) the optimal fibre orientation is: $[90_2°,90_2°,90_2°,90_2°,90_2°,90_2°,90_2°,90_2°,90_2°,0_2°]_S$. The optimal fibre orien-tations are a function of material and geometrical parameters as well as of the total numbers of layers in the laminate. Figure 10 demonstrates the fuzziness of the loading parameter q for various parameters β and λ. In the analysed case the optimal stacking sequence written above is independent on the fuzzy constraints, however the maximal loading parameter may vary.

VI. References

[1] Muc, A., Gurba, W., Genetic algorithms and finite element analysis in optimisation of composite structures, *Composite Structures*, **54**, pp. 275-281, 2001

[2] Baniczuk, N. W., Iwanowa, S. Ju., Szaranjuk, A.W. , *Dynamics of structures. Analysis and Optimisation*, Nauka, Moscow, 1989 (in Russian).

[3] Muc, A., Gurba, W., Optimization of volume for composite plated and shell structures, *Proc. 3rd Int.Conf. Thin-Walled Str.*, pp. 585-592, Elsevier, London., 2001