# Multi-Objective EM-Based Design Optimization of Compact Branch-Line Coupler

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Abstract—This work addresses a problem of multi-objective design optimization of a computationally expensive compact branch-line coupler. Circuit miniaturization is achieved here primarily by using intricate slow-wave structures instead of conventional transmission lines. The presented approach exploits a point-by-point Pareto set exploration with consecutive trade-off designs found by applying adjusted design specifications and executing a surrogate-based optimization routine with a lowfidelity model of the slow-wave structure composed of duplicated electromagnetic simulation data blocks of its constitutive element. The optimization engine is a trust-region-embedded gradient search with Jacobian estimation. A set of six Paretooptimal designs representing trade-offs between the coupler layout area and its bandwidth is obtained at the cost corresponding to less than forty high-fidelity simulations of the entire coupler.

Keywords—bandwidth maximization; branch-line couplers; electromagnetic simulations; multi-objective design; size minimization.

# I. INTRODUCTION

Equal-split branch-line couplers (BLCs) are essential microwave components finding numerous applications in balanced-type circuits, such as amplifiers [1], mixers [2], antenna feeding networks [3], and others [4]. Contemporary coupler design is continuously challenged with ever more stringent requirements imposed on both the layout area and the electrical performance (typically operational bandwidth) of the considered device [5]-[7]. However, circuit area minimization and bandwidth maximization are conflicting objectives that can be simultaneously handled only through multi-objective optimization [8]. The goal of this process is to identify a socalled Pareto set, which contains the best possible trade-offs between non-commensurable design objectives [9]. Normally, this is accomplished by executing a massively time-consuming and CPU-expensive population-based metaheuristic routine, such as an evolutionary algorithm [8], [10]. On the other hand, the aforementioned approach becomes prohibitive when the objective function is evaluated through accurate but numerically demanding electromagnetic (EM) simulations: a typical metaheuristics algorithm run requires thousands of such evaluations [11]. This high cost issue is particularly pertinent to

microwave couplers miniaturized by means of geometrically intricate slow-wave structures inserted in place of conventional transmission lines (TLs) [12]. A preliminary work on the subject [5] investigates only simple slow-wave structures (i.e., folded TLs) as building blocks of a compact coupler. This allows for executing a surrogate-based optimization scheme utilizing a space-mapping-corrected equivalent circuit model of the entire coupler to obtain the trade-off designs point by point. Unfortunately, equivalent circuit models of more complex slow-wave structures (e.g., [6], [12]) are too inaccurate to provide reliable results without the necessity of further EM-based fine-tuning.

In this work, an area-bandwidth trade-off curve is found by using a point-by-point Pareto front identification scheme of [5]; however, due to the lack of an accurate equivalent circuit model, we use circuit decomposition and construct a reliable model from duplicated EM-simulated data blocks of its constitutive element. Given the above, each Pareto-optimal point is identified by means of surrogate-based optimization (SBO) with trust-region-embedded local gradient search method. In addition, moving along the Pareto front is realized by adjusting design specifications instead of applying thresholds for one of the objective functions as in [5].

### II. DESIGN PROBLEM: COMPACT BRANCH-LINE COUPLER

The multi-objective design optimization problem pertaining to compact couplers is demonstrated here using the application example reported in [14]. In more detail, we consider a compact two-section BLC developed by replacing lowimpedance TLs of an original coupler (cf. Fig. 1) with a multielement slow-wave structure of Fig. 2. The latter can be sufficiently described by a vector of eight designable parameters  $\mathbf{x} = [l_1 \ l_2 \ l_3 \ w_1 \ w_2 \ w_3 \ w_4 \ g]^T$ . The remaining highimpedance TLs are folded to the interior of the circuit to keep its dimensional small. In consequence, the coupler area depends only on x and is denoted as A(x). The goal here is to determine the best possible trade-off designs between conflicting objectives regarding coupler layout area and its bandwidth. For more details on slow-wave structures and the related miniaturization concept, the interested reader is referred to [12].

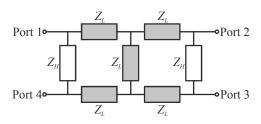


Fig. 1. Circuit model of a two-section BLC composed of quarter-wavelength low-impedance  $(Z_L)$  TLs (grey components) and high-impedance  $(Z_H)$  TLs (white components) [5]. Port description: (1) input port, (2) direct port, (3) coupled port, and (4) isolated port.

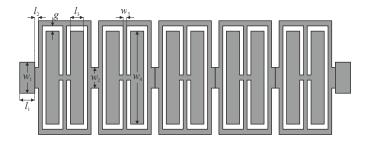


Fig. 2. A parameterized layout of a five-element slow-wave structure [6].

### III. MULTI-OBJECTIVE DESIGN OPTIMIZATION METHODOLOGY

In this section, we describe the proposed design optimization technique. On one hand, we aim at identifying a set of designs representing the best possible trade-offs between the coupler bandwidth and its footprint area. On the other hand, we want to reduce the cost of finding these Pareto-optimal designs in order to make the entire procedure computationally feasible.

# A. Pareto Front Generation

For the coupler circuit of Fig. 1, obtaining an equal power division between the direct and coupled ports with the return loss and isolation kept below -20 dB (all at the operating frequency  $f_0$ ) is possible for an infinite number of solutions. This is evident by inspecting the design chart reported in [14], which illustrates the coupler bandwidth as the function of impedances  $Z_L$  and  $Z_H$ . This information can be used to determine the Pareto set; each Pareto-optimal design  $\mathbf{x}^{(j)}$  is found by adjusting the reference impedance  $Z_L = Z_L^{(j)}$  and minimizing the coupler size (depending only on  $\mathbf{x}$ ) while satisfying the following requirements for the slow-wave structure of Fig. 2:

- Obtaining a central-frequency quarter-wavelength phase shift, i.e.,  $arg(S_{21}) = -90^{\circ}$  at  $f_0$ ;
- Maintaining the maximum return loss level at the acceptable value (here, -30 dB) over the bandwidth of interest. Note that for each Pareto-optimal point the slow-wave structure is loaded by Z<sub>L</sub><sup>(j)</sup>.

EM-driven design of a folded high-impedance TL (cf. Fig. 1) is realized in a straightforward fashion by satisfying the above conditions using  $Z_H^{(j)}$  loads corresponding to  $Z_L^{(j)}$ .

### B. Coupler Optimization for Minimum Size

All of the goals stated in Section III.A can be achieved by minimizing the expression

$$U(\mathbf{x}) = A(\mathbf{x}) + \beta_1 [(P(\mathbf{x}) + 90)/90]^2 +$$

$$+ \beta_2 [\max(S(\mathbf{x}) + 30, 0)/30]^2$$
(1)

where P(x) and S(x) are the phase shift (at  $f_0$ ) and the return loss of the slow-wave structure (over a frequency band of interest), both evaluated from EM-simulated model of the structure. The first penalty term in (1) enforces a required value of  $-90^{\circ}$  for the transmission phase; the second term allows for maintaining the in-band return loss below or around the prescribed level of -30 dB;  $\beta_k$ , k=1, 2, are penalty coefficients.

# C. Surrogate-Assisted Optimization Algorithm

Direct minimization of (1) using a fine EM model the slow-wave structure  $R_f$  is expensive. To alleviate this issue, a trust-region gradient search [16] is applied that generates a series of approximations  $x^{(k)}$  of the minimum of (1) as follows

$$\mathbf{x}^{(k+1)} = \arg\min_{\mathbf{x}: ||\mathbf{x} - \mathbf{x}^{(k)}|| \le \delta^{(i)}} G^{(k)}(\mathbf{x})$$
 (2)

where  $\delta^{(i)}$  is the trust region radius at iteration k updated using the standard rules [16]. The model  $G^{(k)}$  is defined as

$$G^{(k)}(\mathbf{x}) = A(\mathbf{x}) + \beta_1 [(G_p^{(k)}(\mathbf{x}_c) + 90)/90]^2 + \beta_2 [\max(G_s^{(k)}(\mathbf{x}_c) + 30,0)/30]^2$$
(3)

with  $G_P^{(k)}$  and  $G_S^{(k)}$  being linear expansion models of P and S, respectively established at point  $\mathbf{x}^{(k)}$ . For the sake of computational efficiency, Jacobians of P and S are estimated using finite differentiation of a coarse slow-wave structure model  $\mathbf{R}_c$ , which is constructed by cascading EM simulation data blocks of its elements (cf. Fig. 2). The models  $\mathbf{R}_f$  and  $\mathbf{R}_c$  are obviously misaligned, but normally well correlated, which is sufficient to obtain reliable sensitivity estimation.

# IV. NUMERICAL RESULTS

The multi-objective design problem of the compact coupler described in Section II is solved here by using the methodology of Section III. We choose the Taconic RF-35 dielectric substrate with relative permittivity of 3.5 and height of 0.762 mm for circuit implementation realized in microstrip technology. The operating frequency is set to  $f_0 = 1$  GHz.

TABLE I. NUMERICAL RESULTS

Design Variables [mm]								Objectives	
<i>l</i> <sub>1</sub>	$l_2$	<i>l</i> <sub>3</sub>	<i>w</i> <sub>1</sub>	w <sub>2</sub>	w <sub>3</sub>	w <sub>4</sub>	g	BW [MHz]	A [mm <sup>2</sup> ]
4.68	0.1	0.1	1.16	4.71	0.59	8.93	0.1	350	1557
4.40	0.1	0.1	1.89	3.13	0.55	9.19	0.1	330	1500
3.10	0.1	0.46	3.14	1.67	0.43	8.08	0.1	320	1443
3.88	0.1	0.1	2.06	0.53	0.54	8.33	0.1	270	1346
3.12	0.1	0.38	2.83	0.66	0.41	7.82	0.1	240	1318
3.23	0.1	0.38	2.81	0.59	0.33	8.19	0.1	190	1205

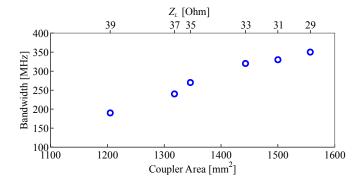


Fig. 3. A Pareto set representing the best possible design trade-offs.

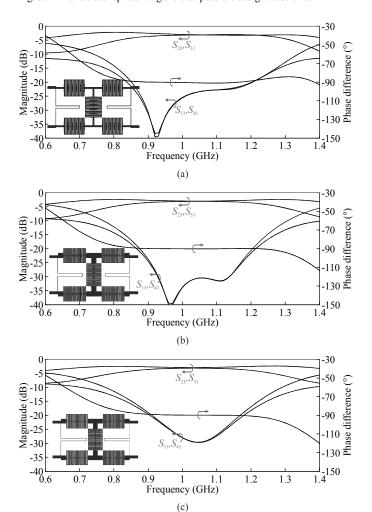


Fig. 4. Frequency characteristics for the selected Pareto-optimal designs, corresponding to couplers of 350, 320, and 240 bandwidths (in MHz).

EM models of the compact coupler, slow-wave structure and its constitutive elements are implemented in CST [17]. An average simulation time of the above listed circuits and components is 1800 s, 700 s, and 80 s. The presented method allows for obtaining six Pareto-optimal designs at a low computational cost corresponding to 38 high-fidelity EM simulations of the compact coupler structure. Numerical

results are listed in Table I. The bandwidth of each design is calculated as the frequency range for which the return loss and isolation curves are both below –20 dB. The Pareto front spans from 1200 mm² to almost 1600 mm² with respect to the coupler size and from almost 200 MHz to almost 350 MHz in terms of the circuit bandwidth. This gives a designer a wide range of options in selecting the design (as a trade-off between the geometrical and electrical performance). The EMvalidated frequency characteristics of the selected trade-off designs are shown in Fig. 4.

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