

Research on Multi-Objective Multidisciplinary Design Optimization Based on Particle Swarm Optimization

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Abstract—Complex systems consist of many disciplines or components, which are often difficult to the design optimize as a overall. They need to be broken down into different components, and then coordinate the links between different parts. ATC (Analysis Target Cascade) - one of the multidisciplinary design optimization methods, is an effective way to solve such intricate problems. In the traditional multidisciplinary design optimization methods, there is only one objective function. But the multi-objective optimization problems are often emerged in practical engineering problems. So, we will focus on the multi-objective optimization problems in multidisciplinary design optimization, and solve them with particle swarm optimization. The original problem is firstly decomposed into multiple coupled sub-problems and then coordinate the relation between each sub-problems by ATC method. The system-level sub-problem is a multi-objective optimization problem and the other subsystems are the general single-objective optimization problems, the MOPSO method and the sequence quadratic programming (SQP) method will be used to solve them respectively. The final optimization result is consistent with the optimization result before the original problem is decomposed. Finally, we used two examples to demonstrate the feasibility of particle swarm optimization (PSO) method to get the solution of the multi-objective problems with ATC method.

Keywords: multidisciplinary design optimization; analysis target cascade; multi - objective optimization; particle swarm optimization algorithm

I. INTRODUCTION

A complex system often has the following characteristics: (1) consist of many disciplines or components; (2) complex interactions between them, e.g. the output (input) of one subsystem is input (Output) of another subsystem; (3) need different departments and positions of technical staff or experts to solve it for its distribution. Sobiesczanski-Sobieski proposed a method of multidisciplinary design optimization method (MDO) to these problems [1]. MDO is an optimized design method that analyzes the interaction of different disciplines in the design process of complex system and makes full use of the interactions between them to optimize the system [2]. ATC (Analytical Target Cascading) [3], is one method of

MDO, and has been successfully applied in many fields. ATC decomposes the problem into a multi-layer structure, and there is no data transfer between the same level, allowing only the exchange of data between the parent and its offspring. In ATC, the number of the decomposition levers is not limited. More importantly, the equivalence and the convergence between the original problem and the decomposed problem have already been demonstrated [4, 5].

At present, the research on MDO is basically about the single objective optimization, which is inconsistent with many practical engineering problems. MOO (Multi-Objective Optimization) can provide a guidance program for the decision-makers with an efficient way and meeting the conflicting conditions simultaneously [6]. In recent years, few scholars have begun to focus on multi-objective problems in MDO. In [7], the particle swarm optimization and fuzzy decision method was used to study the multi-objective optimization problem in CO (Collaborative optimization), and obtained a satisfactory Pareto optimal solution. In [8], the MOMDO problem in the design of UAV was studied. In [9], a hybrid multi-objective optimization method based on user's preference for the quality characteristics was proposed.

In 1995, Kennedy and Eberchart [10] proposed the ‘Particle Swarm Optimizer’ method, which is a parallel and random optimization algorithm based on group intelligence. Their idea comes from the study of foraging behavior of the biological population of birds. It is envisaged that a group of birds randomly search for food in an area and do not know the location information of the food. The strategy taken by the group is through the synthesis of their own information and the information from companions to adjust the location and speed. The entire population continues to move to the place where there is the most abundant food. Particle swarm algorithm is a simple and easy way to solve the multi-objective optimization problems. In [11], the State-of-the-Art of MOPSO had been investigated and summarized, and it is very suitable for researchers who are interested in this field.

II. ATC

A. The definition of ATC

ATC (Analysis Target Cascade) is a decomposed method for the complex system. The whole system is decomposed in accordance with the hierarchical structure, as shown in Figure 1. The set E_i is a set of elements at i -th lever. In Figure 1, $E_0 = \{A\}, E_1 = \{B, C\}, E_2 = \{D, E, F, G\}$. C_{ij} is the set of elements of the j -th element in set E_i . $C_{0A} = \{B, C\}, C_{1B} = \{D, E\}, C_{1C} = \{F, G\}$

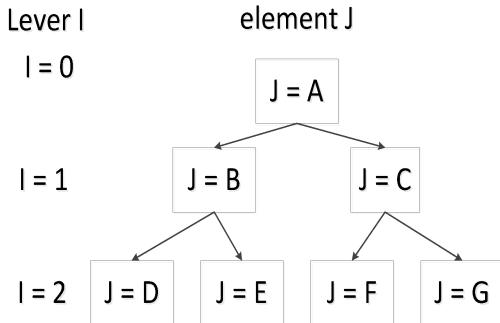


Figure 1. The hierarchy of ATC

The information exchange only exists between the father and the offspring. At the same layer, the offspring is mutual independent, no communication between them. The parent passes the target value to the offspring, and the child returns the corresponding response value to the parent. There are four steps in the process of ATC, shown in Figure 2: (1) Specify the overall goal; (2) Assign the overall goal to a lower level of system, subsystem, and components; (3) Design systems, subsystems, and components to achieve their respective goals; (4) Verify that the final design can achieve the overall goal [12].

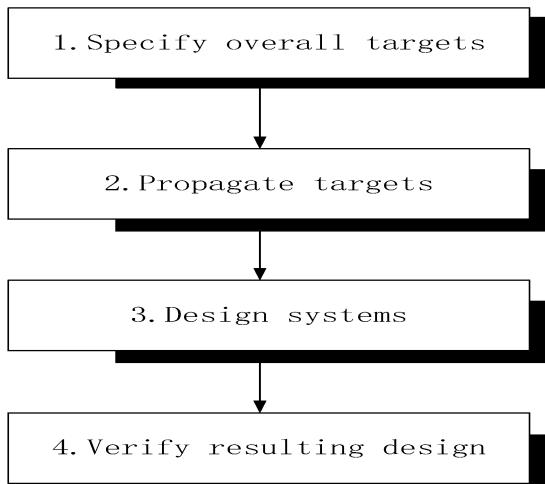


Figure 2. The four steps of the ATC process

B. The principle of ATC

The ATC method is a modular, hierarchical optimization method. In ATC, there are two types of modules: the

optimization design module and the analysis module. The optimization design module performs optimization, and the analyze module calculate the response. The input of the optimization design module includes parameters, local design variables, response of the offspring elements, and the response of the analysis module to the optimization design module [13]. The principle of the sub-problem formulation in ATC is shown in Figure 3.

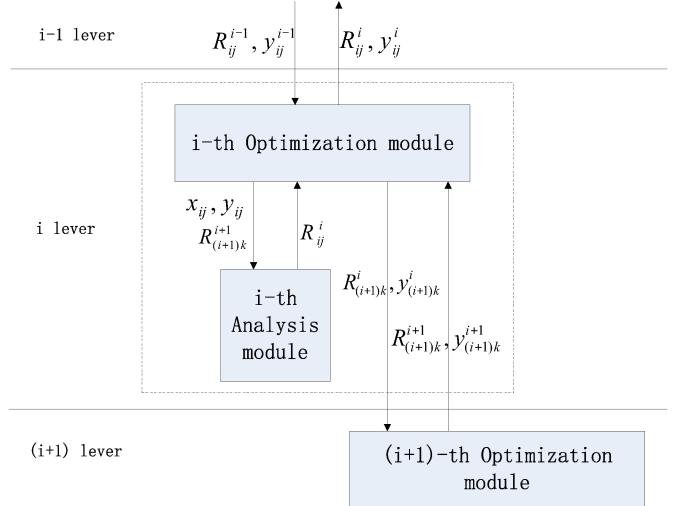


Figure 3. The principle of the ATC method

In ATC, the mathematical model of the element P_{ij} is as follows:

$$\begin{aligned} \min f_{ij} = & \|W_y^R \circ (R_{ij}^i - R_{ij}^{i-1})\|_2^2 + \|S_j W_{ip}^y \circ (S_j y_{ip}^{i-1} - y_{ij}^i)\|_2^2 + \varepsilon_{ij}^R + \varepsilon_{ij}^y \\ \text{s.t. } & \begin{cases} \sum_{k \in C_{ij}} \|W_{(i+1)k}^R \circ (R_{(i+1)k}^i - R_{(i+1)k}^{i+1})\|_2^2 \leq \varepsilon_{ij}^R \\ \sum_{k \in C_{ij}} \|S_k W_{(i+1)j}^y \circ (S_k y_{(i+1)j}^i - y_{(i+1)j}^{i+1})\|_2^2 \leq \varepsilon_{ij}^y \\ g_{ij}(\bar{x}_{ij}) \leq 0 \\ h_{ij}(\bar{x}_{ij}) = 0 \end{cases} \end{aligned} \quad (1)$$

Where $R_{ij}^i = r_{ij}(\bar{x}_{ij})$,

$$\bar{x}_{ij} = \{R_{(i+1)k_1}^i, R_{(i+1)k_2}^i, \dots, R_{(i+1)C_{ij}}^i, x_{ij}^i, y_{ij}^i\}^T$$

TABLE I. DESCRIPTION OF TERMINOLOGY IN (1)

Terminology	Description
C_{ij}	the set of children of P_{ij}
R_{ij}^i	the response of P_{ij}
R_{ij}^{i-1}	the target for P_{ij} from its parent
$R_{(i+1)k}^i$	the target for $P_{(i+1)k}$ from P_{ij}

$R_{(i+1)k}^{i+1}$	the response for P_{ij} from $P_{(i+1)k}$
x_{ij}^i	the local variable of P_{ij}
y_{ij}^i	the linking variable of P_{ij}
y_{ip}^{i-1}	the coordinated linking variable that the parent of P_{ij} sets for all of its offspring
$y_{(i+1)j}^i$	the coordinated linking variable that P_{ij} sets for all of its offspring
$y_{(i+1)k}^{i+1}$	the linking variable uploaded by the offspring of P_{ij}
w_{ij}^R	the weight coefficient of the deviation for P_{ij}
$w_{(i+1)k}^R$	the weight coefficient of the deviation for $P_{(i+1)k}$
W_{ip}^y	the weighting coefficient of linking variable deviation that set by the parent of P_{ij}
$W_{(i+1)j}^y$	the weighting coefficient of linking variable deviation set by the offspring of P_{ij}
S_k	the binary selection matrix from $y_{(i+1)j}^i$ to $y_{(i+1)k}^{i+1}$
S_j	the binary selection matrix from y_{ip}^{i-1} to y_{ij}^i
ε_{ij}^R	the tolerance of response bias coordinating all offspring of P_{ij}
ε_{ij}^y	the tolerance of linking bias coordinating all offspring of P_{ij}
\circ	the multiplication of two vectors, (e.g. $(a,b) \circ (c,d) = (ac, bd)$)

III. MULTI - OBJECTIVE OPTIMIZATION PROBLEM

A. MO problem

Almost all decision-making issues in social life should be dealt with several conflicting or incomplete goals under different constraints at the same time. For example, we often want the cost of a product is relatively low, and hope that the quality is relatively high. It is necessary to provide effective methods for the decision makers as much as possible, which is the multi-objective optimization problem that should be addressed by the system analysts. In other literatures, several terms have the same meaning of "Multi-objective optimization" problem: multi-performance optimization [14], vector optimization [15], and multi-scale optimization [16].

The multi-objective optimization problem can be described as follows:

$$\begin{aligned} \min F(X) &= (f_1(X), f_2(X), f_3(X), \dots, f_m(X))^T \\ s.t. & \begin{cases} g_i(X) \leq 0, & i = 1, 2, 3, \dots, q \\ h_j(X) = 0, & j = 1, 2, 3, \dots, p \end{cases} \\ \text{where } & x_k \in [x_{k\min}, x_{k\max}], \quad k = 1, 2, 3, \dots, n \\ X &= (x_1, x_2, \dots, x_n) \in \Omega \end{aligned} \quad (2)$$

In (2), m is the numbers of objective functions; q and p is the number of inequalities and equality constraints respectively; n is the dimension of decision variables; in the decision space Ω , $x_{k\min}, x_{k\max}$ is the minimum and maximum values of the decision variables respectively.

B. Pareto Optimal

In 1896, the French economist Pareto first proposed the "multi-objective optimization problem" from the perspective of political economy. In order to express his contribution to the multi-objective optimization problem, scholars named the optimal solution of the multi-objective problem "Pareto optimal solution". Through a large number of theoretical and experimental studies, it is found that the solution of multi-objective optimization problem is a set of independent solutions, not a single solution, called "Pareto optimal solution".

The related concepts about Pareto [17]:

Definition 1 (domination): The objective function vector $F(x^*) \in Z$ is non-dominant, when there are no other vectors $F(x) \in Z$, so that $F(x) \leq F(x^*)$ is satisfied. In addition, there is at least one $f_i(x) < f_i(x^*)$. And vice versa, it is said to be dominated.

Definition 2 (Pareto optimal solution): $x^* \in X$ is the Pareto optimal solution when there is no such point that satisfies $F(x) \leq F(x^*)$, and at least one $f_i(x) < f_i(x^*)$ is established.

Definition 3 (Pareto front): A set of all Pareto optimal solutions corresponding to the objective vector, called the "Pareto front", as shown in Figure 4.

In Figure 4, the point C and D is in the feasible and non-feasible domains respectively; A and B are on the Pareto front; F(1) and F(2) are two objective functions.

The evaluation of non-dominated solution set:

(1) The distance between the non-dominated solution set and the Pareto front;

(2) The distribution of non-dominated solution set;

(3) The covered degree of the Pareto front by the non-dominated solutions.

When the distance between the non-dominated solutions and the Pareto frontier is smaller, the covered degree is larger,

and the distribution ratio is more uniform, the non-dominated solutions is better.

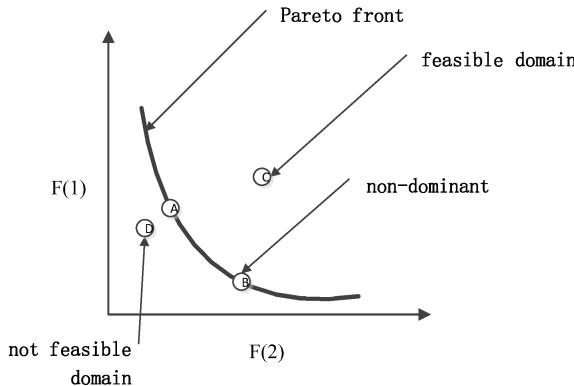


Figure 4. Pareto front

C. Methods for MO Problem

There are two kinds of methods to solve the multi-objective optimization problem: one is based on single target and the other is the heuristic multi-objective optimization method.

The method based on the single target is also called the traditional optimization method. The basic idea is to convert the multi-objective optimization problem into a single objective optimization problem according to certain artificial methods. Commonly, the methods of hierarchical sequence method, restraint method, efficacy coefficient method (target planning method), and evaluation function method can be used.

The principle of hierarchical sequence method is to order the multi-targets in accordance with the importance. First, find the solution of the opening goal, and then find the solution of the second goal based on the first solution, and so on. The solution process is executed sequentially, rather than in parallel. Constraint method is to select one of the sub-targets in the multi-objective optimization problem as the objective function of the new optimization problem, and transform the other sub-targets into the constraint condition. The function coefficient method is also called the objective planning method. When the optimization is carried out, the decision maker sets a predetermined target level for each objective function $z_i (i=1, \dots, m)$, and then gets the offset variable for each target level. The new objective function combines the target level with the original objective. For the minimization problem, the target is in the form $f_i(X) \leq z_i$. The value beyond target level is σ_i . We can describe it as:

$$\begin{aligned} & \min \sum_i^m \omega_i \sigma_i \\ & s.t. \begin{cases} f_i(X) + \sigma_i \geq z_i, i = 1, \dots, m \\ \sigma_i \geq 0, i = 1, \dots, m \end{cases} \end{aligned} \quad (3)$$

The evaluation function method includes the linear weighting method, and the ideal point method and so on. This method uses an evaluation function to reflect the importance of

different goals and then minimizes this evaluate function to get the optimal solution. The linear weighting method is the simplest method, gaining inertia weights based on the importance of the objective functions in the mind of the decision makers, and then forming a new objective function combining these inertia weights with the original objective function. This algorithm has the following shortcomings: on one hand, a small change of the weight parameter can cause the significant change of the objective vector; on the other hand, the significant change of different weight parameters may get the similar solution vector. Thus, the uniform distribution of the weight generally does not produce a uniformly distributed Pareto solutions.

The methods based on single target have been applied in many fields and have achieved some effect, but these methods have the following obvious shortcomings:

(1) Because the physical and measurement units of different objectives are often different in actual problems, the objective may not be compared or weighted directly. Although it can be solved by the dimensionless processing of the objective function, this increases the complexity and causes change in the objective space. And the decision information cannot be used normally.

(2) These algorithms require users to provide accurate information, but usually the users cannot provide the decision-making information in line with actual needs. So it cannot accurately model the optimization problems.

(3) Most algorithms only get a local optimal solution. To avoid falling into local optimum, it is a common way to enlarge the neighborhood size. However, the addition of neighborhood, increases the complexity of the algorithm.

(4) Many traditional algorithms can only be applied to relatively small problems. So, they are not very extensive.

The heuristic multi-objective optimization method is proposed by researchers simulating various phenomena of nature. The characteristics of modern heuristic algorithm: (1) it is effective to avoid falling into local optimality; (2) it is not keen to the continuity and shape of the Pareto front; (3) it's highly generalized without being limited to a specific field; (4) it can get a potential Pareto solution set in a run.

Currently, many heuristic methods have already been researched and applied, for instance multi-objective genetic method, multi-objective simulated annealing method, and multi-objective ant colony method. Because the multi-objective particle swarm method is relatively easy to implement, it will be used in this paper to solve the MOMDO problem.

IV. MOPSO

A. Particle Swarm Optimization

Group intelligence shows the characteristics of complex intelligent behavior through the cooperation of simple and intelligent individual, and can achieve group wisdom that goes

beyond the best individual wisdom. At present, the theory of group intelligence is related to the interdisciplinary aspects of many disciplines, including sociology, computer science, economics, organization and management, and philosophy. With deep research, the combination of different disciplines has also formed many new research fields, promoting the development of relevant disciplines overall [18]. The group intelligent methods can effectively solve the problems that are difficult to be described accurately in many complex systems, and provide a general technical framework for the solution of complex and difficult problems.

PSO is an intelligent method. In the standard PSO algorithm, the velocity and position is updating as follows:

$$\begin{cases} v_{id}^{(k+1)} = \omega \cdot v_{id}^{(k)} + c_1 \cdot r_1 \cdot (p_{id}^k - x_{id}^k) + c_2 \cdot r_2 \cdot (p_{gd}^k - x_{id}^k) \\ x_{id}^{(k+1)} = x_{id}^{(k)} + v_{id}^{(k+1)} \end{cases} \quad (4)$$

Where $|v_{id}^{(k+1)}| \leq V_{\max}$ (5)

$$p_{id}^{(k+1)} = \begin{cases} x_i^{(k+1)}, f(x_i^{(k+1)}) \leq f(p_{id}^{(k)}) \\ p_{id}^{(k)}, f(x_i^{(k+1)}) > f(p_{id}^{(k)}) \end{cases} \quad (6)$$

$$p_{id}^{(k)} \in \{x_{id}^{(k)}, \dots, x_{md}^{(k)} | f(x_{id}^{(k)})\} = \min \{f(x_{id}^{(k)}), \dots, f(x_{md}^{(k)})\} \quad (7)$$

$$p_{gd}^{(k)} \in \{p_{id}^{(k)}, \dots, p_{md}^{(k)} | f(p_{id}^{(k)})\} = \min \{f(p_{id}^{(k)}), \dots, f(p_{md}^{(k)})\} \quad (8)$$

TABLE II. THE DESCRIPTION OF TERMINOLOGY

Terminology	Description
ω	inertia factor
k	iteration cycle
$x_{id}^{(k)}$	current position of the individual particle
$v_{id}^{(k)}$	current velocity of the individual particle
$p_{id}^{(k)}$	optimal position of individual
$p_{gd}^{(k)}$	optimal position of group
c_1, c_2	acceleration factor
r_1, r_2	random numbers with interval (0,1)
m	the number of particles
$f(\cdot)$	fitness function(objective function)

The flow of standard PSO algorithm:

- 1) Initialization. Set the dimension of the particle swarm, inertia factor, and acceleration factor;
- 2) The velocity and position is randomly initialized in the search space. The individual optimal position is set as the position of the current particle and the optimal position of the group is calculated according to the formula (6);
- 3) Updating the position and velocity according to (4), and constraining the maximum velocity according to (5). Resetting the location of the particles that exceed the search space;

4) Calculating the objective function of each particle and updating the historical optimal position of each particle and the whole group according to the formula (7) and (8) respectively;

5) If satisfying the termination condition, stop the search, and output the search results. Otherwise return to 3 to continue the search.

B. Multi-objective Particle Swarm Optimization

Applying standard PSO method to multi-objective optimization, the following problems will emerge:

1) How to choose the individual optimal? For the single objective optimization, we can get the better one after comparison. But for multi-objective problems, we cannot know the better one after comparison of two particles. If each objective of a particle is better, the particle is better. If uneven, it is very difficult to compare.

2) How to choose the group optimal? For the single objective problem, there is only one optimal individual in the population. But for multi-objective problems, there are many optimal individuals. For the PSO, each particle can only choose one as the optimal individual (leader).

For the first question, MOPSO randomly selects one of the objective function values as the historical optimal when we cannot infer which one is better. For the second question, MOPSO chooses a leader in the optimal set (archive) based on the degree of congestion. Try to choose particles that are not so dense (the grid method is used). MOPSO applied the adaptive grid method when selecting the leader and updating the archive [19].

MOPSO algorithm flow is shown in Figure 5.

V. CASE STUDY

In this paper, two examples will be used to verify the effectiveness of our methods: one is a geometric programming problem [20] and the other an engineering problem -three bar truss [21]. Each of them is originally a single-goal optimization problem. Here, we added an objective function, which constituted a multi-objective optimization problem. There is no correlation between the two objective functions, such as positive correlation or negative correlation, otherwise there is no need for a multi-objective optimization process.

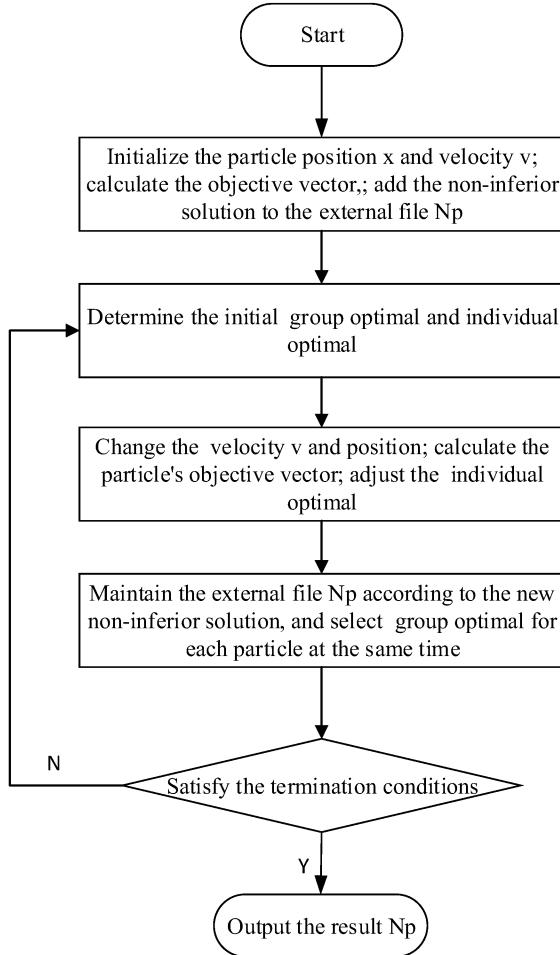


Figure 5. The flow of MOPSO algorithm

1) Geometric Programming Problem

The optimization problem of geometric programming in [20] is changed to be a multi-objective optimization problem. There are 14 design variables, four equality constraints, and six inequality constraints. In (9), f_1 and f_2 are the two objective functions; x_i is the design variable; h_i is the equality constraint; and g_i is the inequality constraint.

$$\begin{aligned}
 & \min f_1 = x_2^2 - x_1^2, \quad f_2 = (x_1^2 + x_2^2)/100 \\
 & \text{s.t.} \quad \begin{cases} g1 = x_3^{-2} + x_4^{-2} - x_5^2 \leq 0 \\ g2 = x_5^{-2} + x_6^{-2} - x_7^2 \leq 0 \\ g3 = x_8^{-2} + x_9^{-2} - x_{11}^2 \leq 0 \\ g4 = x_8^{-2} + x_{10}^{-2} - x_{11}^2 \leq 0 \\ g5 = x_{11}^{-2} + x_{12}^{-2} - x_{13}^2 \leq 0 \\ g6 = x_{11}^{-2} + x_{12}^{-2} - x_{14}^2 \leq 0 \\ h1 = x_1^2 - x_3^2 - x_4^{-2} - x_5^2 = 0 \\ h2 = x_2^2 - x_5^2 - x_6^2 - x_7^2 = 0 \\ h3 = x_3^2 - x_8^2 - x_9^{-2} - x_{11}^2 = 0 \\ h4 = x_6^2 - x_{11}^2 - x_{12}^2 - x_{13}^2 - x_{14}^2 = 0 \end{cases} \quad (9)
 \end{aligned}$$

where $x_i \in [0.5, 10], i = 3, 4, 5, 6, 7, 11$

Using the two-layer ATC decomposition method, the whole decomposition process of problem (9) is shown in Figure 6.

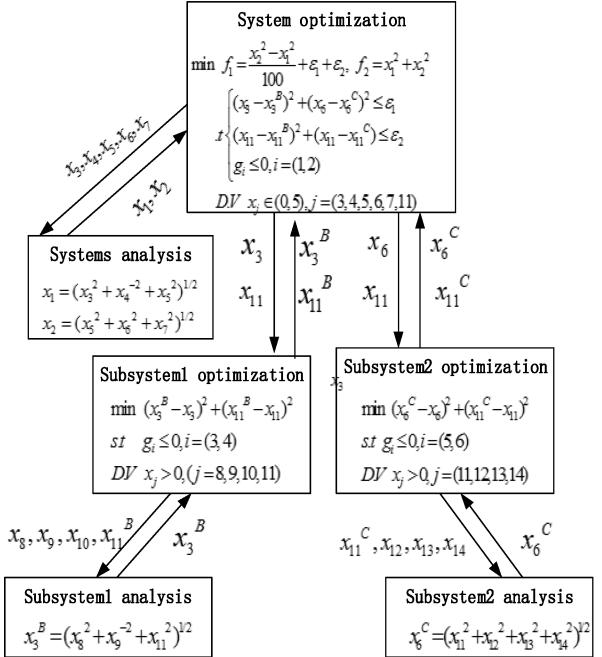


Figure 6. The decomposition process of problem 1

At the system-level, the MOPSO method is used. At the subsystem-level, sequential quadratic programming method is used. Figure 7 shows the final optimization results.

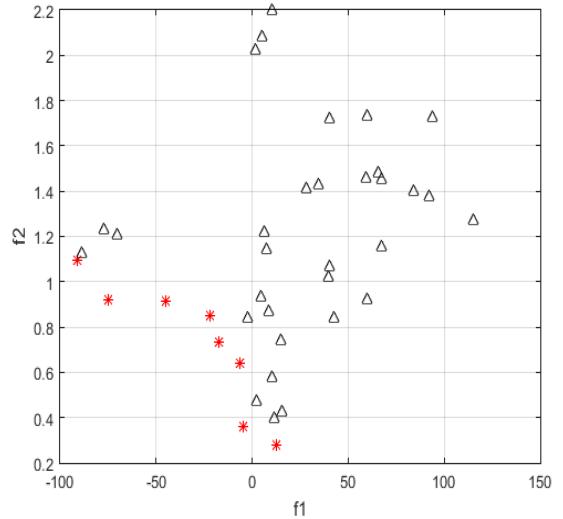


Figure 7. The value of f1 and f2

In Figure 7, f_1 and f_2 are the two objective function values to be optimized. The red points marked with '*' are the non-dominated solutions (Pareto solutions). The grey points marked with '△' are the dominated solutions. We can easily see that the Pareto solutions are at the edge of all the solutions, forming the Pareto front. Depending on the choice of parameters, the

number of solutions obtained will be different. Here only part of the Pareto solutions, if we want to get more solutions, we need to modify the corresponding parameters, and this will increase the calculation time. So we only got part of the Pareto solutions.

2) Three Bar Truss

A Three Bar Truss problem was proposed in [21]. Three tubes form a truss structure which is shown in Figure 8. The force applied in the horizontal direction is p , and in the vertical direction is $8p$. One objective function is the weight f_1 , and the other is a function f_2 that we added, while satisfying stress and displacement constraints. The inner radius and thickness are the design variables. Because of symmetric, just considering the left (or right) tube and middle tube. There are only four design variables are needed [21].

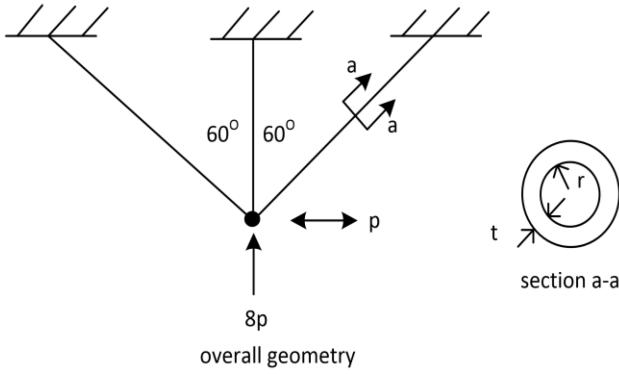


Figure 8. Three Bar Truss Design Problem

It can be formulated as:

$$\begin{aligned} \min_{x_1, x_2, w_1, w_2} \quad & f_1 = \frac{\rho l}{\sigma} (4x_1 + x_2), \quad f_2 = \frac{\rho l}{\sigma} (4x_1 - x_2) \\ \text{s.t.} \quad & g_1(x) = 16p - 0.25x_1 - x_2 \leq 0 \\ & g_2(x) = \frac{p\sqrt{3}}{3x_1} + \frac{2p}{x_2 + 0.25x_1} - 1 \leq 0 \\ & g_{11}(x, w_1) = -4.814 \times 10^{-4} \frac{x_1}{pw_1} + \frac{p\sqrt{3}}{3x_1} + \frac{2p}{x_2 + 0.25x_1} \leq 0 \\ & g_{12}(x, w_1) = -616.9w_1 + \frac{p\sqrt{3}}{3x_1} + \frac{2p}{x_2 + 0.25x_1} \leq 0 \\ & g_{21}(x, w_2) = -4.814 \times 10^{-4} \frac{x_2}{pw_2} + \frac{8p}{x_2 + 0.25x_1} \leq 0 \\ & g_{22}(x, w_2) = -2467w_2 + \frac{8p}{x_2 + 0.25x_1} \leq 0 \end{aligned}$$

where $1 \leq x_1, x_2 \leq 400$, $0 < w_1, w_2 \leq 1$, $p \in [18, 22]$

$$\rho = 7.85 \times 10^{-6}, l = 1000, \sigma = 0.2$$

This problem can be decomposed into three sub-problems, as shown in Figure 9, one system problem and two subsystem problems. There is no coupling between the two subsystems, but there is a coupling relationship with the system-level problems.

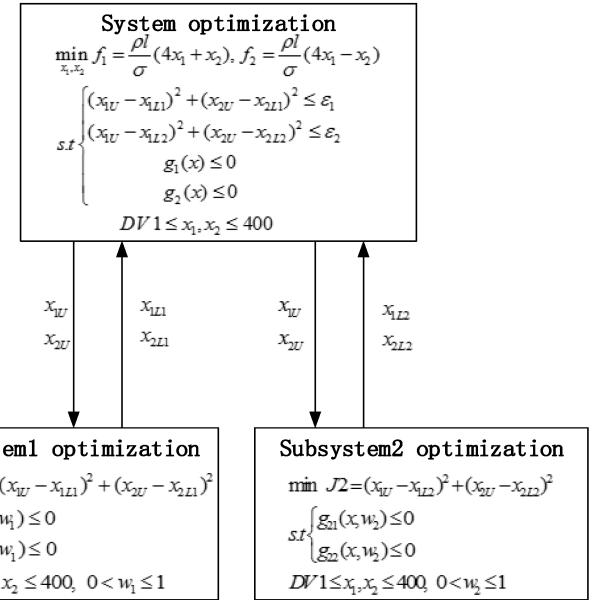


Figure 9. The decomposition process of problem2

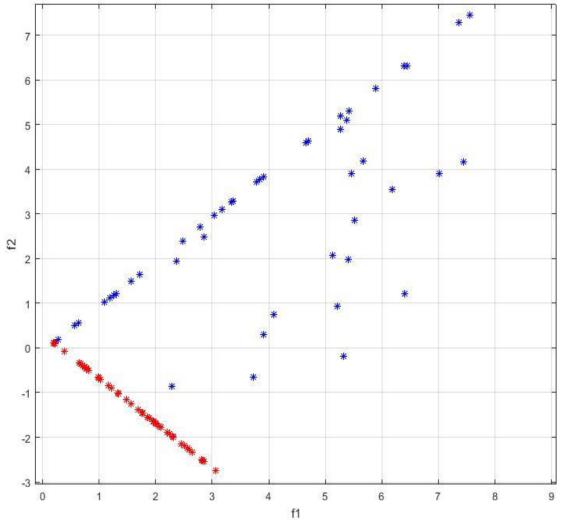


Figure 10. The value of f1 and f2

The optimization result is shown in Figure 10, f_1 and f_2 are the two objective function values to be optimized. The red points are the Pareto solutions (non-dominated solutions). The blue points are the dominated solutions. It is easily to see that in the result of this problem, we get many Pareto solutions, and the distribution is evenly distributed on the graphics. So the optimization result is very good.

VI. CONCLUSION

In this paper, we use the decomposed strategy of ATC to solve MDO problem, which decomposes the original problem into a two-layer optimization problem. The system layer contains two objective functions, and the particle swarm optimization method is used to solve it. The subsystem layer is

a single objective problem, which is solved by the sequential quadratic programming method. Finally, the feasibility of PSO to solve multi-objective problem in MDO is verified by solving a geometric programming problem and an engineering problem -three bar truss with two objectives. Compared with the second problem, the number of optimization results obtained in the first problem is relatively small. So in the future, we will further improve the optimization algorithm to get more uniformly distributed Pareto solutions.

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