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The Commuting Carer Conundrum

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Abstract

The Commuting Carer Conundrum is a variation of the well-studied Travelling Salesperson Problem. It deals with the scheduling and routing of care staff visiting a number of a care company's clients. This is a problem faced by many care companies, including one based in Falmouth, United Kingdom.

An algorithm that uses graph theory and fuzzy clustering solves the Commuting Carer Conundrum in Falmouth. Several simulations are used to benchmark the algorithm and to showcase the stability of solutions for different starting conditions. They show that the allocation of clients to near-by carers is more stable than to those farther away. Additionally, changes to one carer's schedule also affect other carers, with the impact being greater the more clients a carer visits. On the other hand, changing the number of clients has as smaller effect on the carers' schedules.

In the future, synchronised visits, variation in carer skill and mode of transportation, as well as differing dwell times at client locations may be considered. The problem may also be expanded to encompass a working week instead of only a day.

Contents

1 Challenge and Aims

1.1 The (multiple) Travelling Salesperson Problem

The Travelling Salesperson Problem (TSP) is an NP-hard optimisation problem with a wide range of real-world applications. In essence, the route (or tour) a salesperson takes to travel to their clients is optimised by minimising a cost metric. Often the total travel time or distance is used as the cost metric. [1]. This type of problem is well-studied, and many exact and heuristic solutions are known [2].

A common extension of this problem is the inclusion of additional salespeople, leading to the multiple Travelling Salesperson Problem (mTSP) [2]. Its most basic version includes a set of m salespeople that start and end their tour at the same depot. The task then is to find tours for all salespeople so that each client is visited exactly once and the cost metric is minimised. Mail order deliveries are a common application for this type of problem.

Some variations of the problem use multiple depots instead of only one. These depots may serve as start and end points for the salespeople, provided that the total number of salespeople at each depot is the same at the start and the end of all tours. The number of salespeople itself can also be variable instead of being a fixed value. Alternatively, some clients may need to be visited at certain times, expanding the problem into a multiple Travelling Salesperson Problem with Time Windows (mTSPTW). Bus route or air traffic scheduling are examples for this type of problem [2].

Based on the intended application, further constraints may be imposed on the solution. When the scheduling of workers is involved for example, concerns regarding workload balance may be reflected by including a lower and upper bound on travel time for each salesperson [3].

1.2 The Home Healthcare Problem

One important area of application for the mTSPTW is in the home healthcare (HHC) sector. Here, a care company schedules its salespeople (carers) to travel to a number of their clients' homes, which are spread out over the company's operational area. Examples of care work tasks may include health care or supportive care, such as administration of medicine, cleaning, and meal preparation.

There are about 12,500 care companies in the UK alone, most of which are based in England. As of 2024, more than 950,000 people receive home care, but these numbers are projected to increase due to the UK's ageing population. On the other hand, the home care workforce has been steadily declining in recent years [4]. It is therefore important to ensure that, given a limited number of carers, as many clients as possible are cared for.

Literature discussing this problem varies in the exact formulation due to country-specific regulations, but there are many common elements that broadly characterise the HHC problem [5]. Most commonly, one carer is assumed to travel to each of their clients by car. However, some solutions include the option for carers to walk between near-by clients, or assume that carers are picked up and dropped off by a designated transport service [6]. These versions focus on the environmental impact of travelling carers over the optimisation of travel routes and neglect other aspects of the HHC problem mentioned below.

The timing of both client visits and carer shifts is an important aspect to consider. For example, a client that needs to take their medication in the morning and the evening cannot be visited in the middle of the day instead. Carers may also indicate a preference for their working hours that needs to be considered by the care company [7, 8]. Depending on the considerations of papers

including this aspect, these time windows may be flexible or fixed.
Continuity of care is another factor that many papers discussing the HHC problem focus on. Among a number of solutions, those that ensure that a client is visited by the same carer, either throughout the day or over multiple days, may be favoured [5, 8]. Other papers put emphasis on workload fairness by including considerations of work hour restrictions to ensure that carers are not overworked [7].
Carer skill is another element many solutions consider. Here, each carer possesses a number of skills that make them suitable to provide care work for certain clients [7, 8]. An alternative version of this problem assumes increased staff dissatisfaction if a highly-skilled carer is scheduled to provide lower-skilled care work. As such, those solutions prioritise matching the experience level of a carer with the difficulty of a client's care requirements.
A common and important aspect of the HHC problem are synchronisation issues, which refer to the necessity of at least two carers visiting the same client at the same time. In the literature, these types of visits are assumed to make up 10%-30% of all client visits, and often the number of carers is kept small due to the increased computation time caused by the added complexity [5, 7, 8]. Sometimes, a carer's preference to avoid a colleague is also taken into consideration [7], although this aspect is rare in the literature.
Finally, most versions of the HHC problem focus on the routing of carers over the course of a single day. For a multi-day formulation, clients may request care work that is spread over multiple days, and carers may work some or all days of a working week [5]. In this version of the problem, the cost metric may be calculated over the course of several days instead of a single day [9].
While there are several existing theoretical solutions for the HHC problem, many care companies still manually create their carers' schedules, which can lead to suboptimal solutions [5]. This may be due to the majority of the literature focussing on the theory behind their solutions and neglecting the reality of the problem they are attempting to solve.

1.3 The Commuting Carer Conundrum

The aim of this dissertation is the development of an algorithm capable of solving the HHC problem for a care company based in Cornwall, United Kingdom. A staff member of the care company was interviewed to learn about the specific constraints necessary for this application. A number of simulations are then run to benchmark the algorithm, as described in Section 3.

Remark 1.1. *While the care company operates in Cambourne, Redruth, Penryn, and Falmouth, only locations in Falmouth are used in the simulations. The algorithm can easily encompass the other three areas, however.*

Carers start their day at home. An app provided by the care company displays their daily itinerary, including every client's location, whether a visit is synchronised with another carer, and the tasks to be carried out for each client. These tasks involve cleaning, meal preparations, providing aid while the client showers or gets dressed, changing catheters or stoma bags, and/or administering oral medication. Unlike assumptions made in the literature, the skill level of all carers is assumed to be largely equal; only the care for a client that has a percutaneous endoscopic gastrostomy to ingest food (referred to as PEG feeding) requires special training.
The majority of care visits require only one carer; however, some care tasks may require two carers for safety reasons. As such, synchronisation issues need to be addressed in the ideal solution.

Remark 1.2. *Due to increased complexity, the simulations discussed in this dissertation do not include this synchronisation aspect and assume that all care tasks require only one carer. Considerations for the implementation of this aspect are discussed in Section 4.*

Dwell time at a client's house is based on the complexity of the care tasks. The most common dwell time is 30 minutes, but up to 2 hours and 30 minutes are possible.

Remark 1.3. *For the purposes of this dissertation, the dwell time at each client's home is assumed to be one hour.*

Allocation of clients to carers is largely based on carer availability and on the number of working hours a carer has requested (also referred to as their working hours requirement). An important aspect of the algorithm developed here is workload fairness, which aims to ensure that each carer fills their working hours requirement, potentially at the expense of overall efficiency.
Carers are typically scheduled in two shifts: between 7 am and 2 pm, and between 4 pm and 9 pm. This allows carers to take a lunch break or to only work one of the two shifts, based on preference.

Remark 1.4. *The simulations assume more flexible shifts that vary between carers. A carer may indicate a desired number of hours to work (up to 8 hours), and may indicate any time window between 7 am and 9 pm during which they may be scheduled to work (with a minimum time window length of one hour).*

Finally, carers which drive a car may have any client within the operational area allocated to them, whereas carers that can only reach clients on foot only provide care work for near-by clients.

Remark 1.5. *The simulations presented here assume that every carer reaches their clients by car. Inclusion of carers without cars is discussed in Section 4.*

This dissertation provides a thorough examination of each simulation's solution. It focusses on a variation of the HHC problem dubbed the Commuting Carer Conundrum (CCC), which is characterised by the constraints outlined above.

2 Methodology

2.1 Graphs

2.1.1 Graph Theory

The HHC problem is an extension of the mTSP and can therefore be solved in similar ways. In the literature, the mTSP is defined on a *graph* $G = (V, A)$, with V being a set of n *nodes* (vertices) and A being a set of *edges* (arcs) [2]. Every edge $e_{ij} \in A$ connects two nodes i and j. Connected nodes are called *adjacent*. A graph is *complete* if every node is adjacent to every other node. The *degree* $deg(v)$ of a node v is the number of edges that start or end at that node [10].

Remark 2.1. *In the context of the HHC problem, the nodes represent a set of locations of carer and client homes, and the arcs represent the routes that may be taken by a carer to travel from one node to the next.*

A graph G can be *directed* or *undirected*. In an undirected graph, the edge connecting two nodes may be traversed in either direction. On the other hand, an edge may only be traversed in one direction in a directed graph. An undirected graph can therefore be understood as a directed graph in which every edge has been duplicated and reversed [10].

Remark 2.2. *For the HHC, graphs are complete and generally assumed to be undirected, since a carer may directly travel from client 1 to client 2, or vice versa.*

The *adjacency matrix* A_G of a graph G represents the nodes and edges of a graph, with $A_{G,ij} = 1$ if the nodes i and j are adjacent, and 0 otherwise. The adjacency matrix is called *symmetric* if $A_{G,ij} = A_{G,ji}, \forall i, j \in V$, and *asymmetric* otherwise [2]. The adjacency matrix of an undirected graph is always symmetric, since the edge from node i to j is the same as the edge from j to i. For a directed graph however, an edge starting at node i and ending at node j is represented as $A_{G,ij} = 1$ and $A_{G,ji} = 0$.
Every edge in G is assigned an *edge weight* or *edge length* [10]. For the HHC, the edge weight represents the real-world time or distance needed to travel from one node to the other. The weight of all edges is then expressed in the form of a *cost matrix* C, with $c_{ij} \geq 0$ being the edge weight of e_{ij}. If nodes i and j are not adjacent, then $c_{ij} = 0$. The cost matrix may therefore replace the adjacency matrix, since $c_{ij} > 0$ denotes that i and j are adjacent nodes.

Remark 2.3. *Unlike adjacency matrices, the cost matrix of an undirected graph may still be asymmetrical if $c_{ij} \neq c_{ji}$, $c_{ij} > 0$, $c_{ji} > 0$. This may happen due to real-world obstacles such as one-way streets changing the travel time from node i to j compared to travelling from j to i. In this case, the graph may be interpreted as a directed graph, with two differently-oriented edges connecting each node-pair.*

A *path* in a graph is the sequence of nodes and edges that need to be traversed to get from one node to another. There are generally no repeated nodes and edges in a path, as this represents back-tracking or traversing loops [10]. Since the graphs for the mTSP and the HHC problem are typically complete graphs, a number of paths exist between any two client nodes. When solving the HHC problem, paths that minimise the total edge weight are therefore favoured.
Only Hamiltonian paths are relevant for the HHC problem. A Hamiltonian path is a path through a graph that visits each node exactly once [11], which is equivalent to a carer visiting each of their clients exactly once throughout a workday.

Remark 2.4. *In an undirected, complete graph, the set of Hamiltonian paths is equivalent to the set of all permutations of the nodes of the graph. In other words, every possible path through the graph that visits each node only once is a Hamiltonian path.*

2.1.2 Client-Carer Graphs

In the context of the mTSP, each client and depot is represented as a node in a complete, undirected graph. There are k salespeople (agents) that traverse the graph. Paths are found for every agent so that the total cost of routes travelled is kept to a minimum and every client node is visited exactly once [7].
As discussed in Section 1.2, there are a number of additional constraints regarding the HHC problem which may be reflected in the corresponding graphs. Here, each client node is associated with a dwell time (reflecting how long care work for this client is expected to take), as well as a time window between which the care work may begin [7, 8, 12].
Lunch breaks are frequently scheduled by including a mandatory "lunch break" node that every agent needs to pass. This node has an associated time window between which this visit must take place, as well as a dwell time that reflects how long the break should be [5, 7, 9, 12].

Remark 2.5. *The CCC already accounts for lunch breaks because of how the carers' work hours are defined. There is therefore no need to include a break node.*

To address the CCC, the problem of routing and scheduling k carers is split into k separate TSP, so that each of these k TSPs consists of a graph with one carer node and a number of client nodes. This pre-assignment of clients to carers is achieved using a clustering method (see Section 2.2) and has been shown to be a viable step for solving the mTSP [7].

Client and carer locations are approximated using postcodes. Coordinates of each postcode are determined using the Ordnance Survey's CodePoint-Open's dataset [13]. The dataset includes the eastings and northings of every UK postcode, which are converted into a latitude-longitude format using the convertbgn Python package [14]. Using these coordinates, the distance and travel time between each location pair is determined using the OpenRouteService Python package [15]. In the asymmetrical cost matrix of the corresponding graph, the travel times are the costs associated with travelling from one node to the next.

Each client-carer graph also represents a specific *time window* t during which care work may begin for a client. The time windows are 7 am – 9 am, 9 am – 12 pm, 12 pm – 3 pm, 3 pm – 6 pm, and 6 pm – 8 pm.

The planning horizon for the CCC encompasses only one working day. However, expansion of this solution to a working week is possible by determining the daily availability and workload of each carer and solving the problem for each working day.

Due to the small number of nodes and resulting Hamiltonian paths in each graph, all possible paths are explored. The path(s) with the shortest total travel time are then selected as the solution for that time window's client-carer graph.

Remark 2.6. *In a three hour long time window, a carer can care for a maximum of three clients, assuming that the dwell time at each location is one hour. Therefore, the maximum number of Hamiltonian paths in every client-carer graph is six.*

However, every additional client node may increase the complexity of a client-carer graph and therefore the number of viable paths. Exploring all these paths would increase the runtime and is therefore not feasible. Given the definition of the CCC, this scenario does not happen and is therefore not accounted for. However, this problem may become relevant when expanding the problem by allowing shorter dwell times and longer time windows.

Remark 2.7. *In such situations, the client-carer graph may be simplified by removing a number of the edges with the greatest weight, as they are unlikely to be part of the optimal solution. This reduces the number of Hamiltonian paths for the algorithm to explore. To avoid distant and unconnected nodes, the degree of each node would be monitored to ensure it was greater than one at all times.*

2.2 Clustering

2.2.1 Fuzzy, Density-Based Clustering

Clustering is an unsupervised learning technique that groups objects based on their similarity [16]. A number of clustering algorithms exist for various areas of application. One of these is density-based clustering, which, unlike other algorithms, does not require the number of clusters to be predefined. Clusters can be of any shape and are identified based on areas of high point density [17].

Many popular density-based clustering algorithms only provide hard clusters. In hard clusters, every point is either part of only one cluster or is classified as an outlier. In fuzzy clusters on the other hand, every point belongs to each cluster with a certain probability. Final clusters may then

be determined based on the most likely cluster of each point, or based on predefined thresholds. Outliers are characterised by having a low probability of belonging to any cluster [18].
HDBSCAN (hierarchical density-based spatial clustering of applications with noise) is a fuzzy density-based clustering algorithm that combines the two features outlined above [18]. The algorithm accepts a pre-computed distance matrix (such as the cost matrix of a graph) and assigns each point to the cluster it most likely belongs to.

2.2.2 Client-Carer Clusters

HDBSCAN is used to group carers and clients due to the algorithm's flexibility in determining the number and shape of clusters. For every time window, the set of available carers is determined based on their shift times and working hours requirements. Of all clients, only those that may be visited during the given time window are considered. The distance matrix is calculated based on the real-world travel time between each pair of points, which includes all viable carers and clients. Initial clusters are then determined based on this distance matrix.

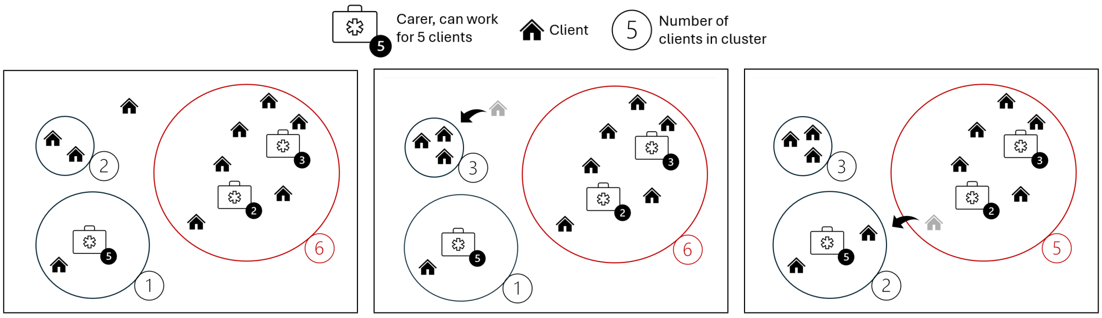
Remark 2.8. *While synchronised carer visits are not considered in this version of the CCC, the use of fuzzy clusters through HDBSCAN allows for this aspect to be included in the future. In this case, a client that needs to be visited by two carers is then assigned to two clusters instead of only one.*

Due to the nature of density-based algorithms, some points are occasionally classified as outliers. These points are then reclassified and added to their nearest cluster (Figure 1b). Proximity to clusters is determined as the average distance of a point to all other points in that cluster. Initial clusters also occasionally consist of only clients, which are then reassigned to the nearest cluster that contains at least one carer (Figure 1d). Clusters are then split so that each cluster contains only one carer (Figure 1e). The maximum size of each cluster is restricted by the workload indicated by each carer.

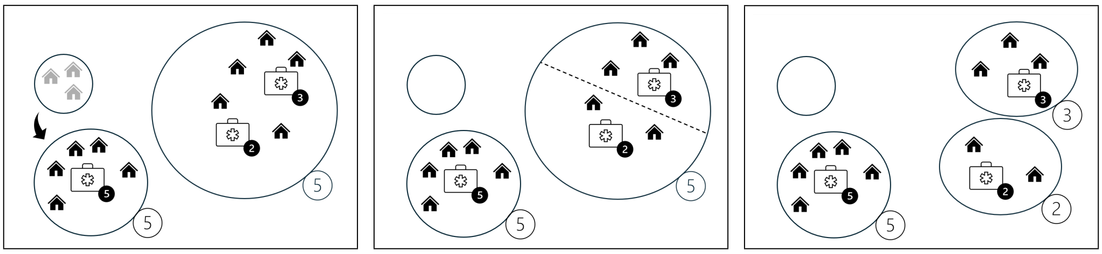
Remark 2.9. *For instance, a carer may indicate they would like to work for two hours during a three hour long time window. The maximum number of clients that may be assigned to this carer can therefore be no greater than two. Additionally, given that the dwell time at each client's location is assumed to be one hour, the maximum number of clients in one cluster is three during a three hour time window, regardless of a carer's overall workload preferences.*

When the splitting of clusters requires the removal of clients from clusters, those that can still be visited at a later point in the day are unassigned first to ensure that less flexible clients are given priority. Similarly, when a cluster containing two carers needs to be split, clients are primarily allocated to the carer with the lowest number of remaining working hours.

Remark 2.10. *For example, a cluster contains two carers and three clients. Carer A's shift ends at the end of the current time window, whereas carer B is still available later in the day. When splitting the cluster into two, clients are primarily allocated to carer A because carer B can still meet their working hours requirement during a later time window.*



(a) Initial clustering results with three clusters and one outlier.
(b) Outliers are added to the nearest cluster.
(c) Clients are moved out of overfilled clusters.



(d) Client-only clusters are merged with carer-clusters.
(e) Clusters containing multiple carers are split.
(f) Each client-carer cluster contains exactly one carer.

Figure 1: Steps following initial fuzzy clustering results. Clusters are represented by a circle. A red outline indicates that there are more clients in a cluster than the carers can care for.

2.3 Linear Programming

Most known solutions of the mTSPTW and the HHC problem utilise Linear Programming, which is briefly discussed in the following.
Linear Programming (LP) is a mathematical optimisation method. It seeks to find optimal values for a set of variables $(x_1, x_2, ..., x_n)$ by maximising or minimising an objective function z, given a number of linear constraints [19]. LP problems are typically expressed as follows:

$$\text{Maximise } z = \sum_j c_j x_j$$

$$\text{subject to } \sum_j a_{ij} x_j \leq b_i \ (i = 1, 2, ..., n)$$

$$x_j \geq 0 \ (j = 1, 2, ..., n)$$

Integer Linear Programming (ILP) refers to LP problems in which variables may only take on whole numbers. Mixed Linear Programming (MLP) on the other hand is an LP problem where variables may take on continuous values.

2.3.1 Solutions for the multiple Travelling Salespeople Problem

There are various LP formulations of the mTSP. In the assignment-based formulation, $x_{ij} = 1$ if edge e_{ij} is used in a tour, and set to 0 otherwise. Constraints ensure that m agents depart from and return to one depot, and that subtours among intermediate nodes are avoided. Especially the number of these latter constraints increases exponentially with the number of nodes. Some authors also include a lower bound of the number of nodes visited by an agent [2].
For a tree-based formulation of the mTSP, the depot node is considered the centre of a tree with

k adjacent edges. The set of edges is then partitioned into subsets, including a set of edges that are part of a solution. The objective of this formulation is to minimise the overall cost of edges that are part of the solution. Additional constraints ensure that the solution consists of connected edges [2].

Alternatively, a flow-based formulation uses a three-index notation, where $x_{ij}^k = 1$ represents an agent k traversing an edge e_{ij}. Flow conservation constraints ensure that an agent arriving at one node also departs from that node. However, the large number of variables for even moderately-sized mTSPs make this approach impractical [2].

2.3.2 Solutions for the Home Healthcare Problem

When applied to the HHC problem, many authors build on solutions for the mTSP problem and impose additional constraints and variables.

One exact solution utilises a binary variable $x_{ij}^k = 1$ if carer k performs a service at client node i and immediately afterwards at node j. The variable $t_i \in [a_i, b_i]$ denotes a service at client i being allowed to begin between an earliest time a_i and latest time b_i. A number of constraints are used to find the solution of the HHC problem with synchronisation of carers. When running computational experiments with these constraints, the runtime of each iteration was limited to 10 minutes. Depending on the complexity of initial conditions, 52% to 100% of solutions were explored [7].

A different solution utilises three-indexed MLP to solve the problem on a daily level by considering available staff, staff changes due to illness, and changes to the set of clients. In this version, each carer has a skill set and works either full-time or part-time. Unlike in the CCC, every carer begins and ends their tour at the care company's office [8].

Remark 2.11. *There is no need to include a shared start and end point for the CCC, since all carers may view their daily schedule on an app. They may therefore start their working day at home and finish it once they have cared for their final client.*

In the three-indexes MLP version, every client is associated with a time window during which care work must begin, as well as the duration of the care work. However, this solution includes an additional metric that reflects client and carer preferences. Such a metric is weighed against the total travel time and may be positive if one client is cared for by the same carer, therefore rewarding continuity of care. Prior to solving the problem, each visit is associated with a carer. The MLP formulation finds a heuristic solution in 2 minutes for small instances, and in 10 minutes for large instances. This heuristic solution is achieved by running the MLP model until one solution is found. Whether this solution is optimal is not investigated [8].

A more complicated formulation of the problem involves scheduling carer visits over the course of a working week. In this version, the weekly travel distance is minimised. Carers may be scheduled to work for clients based on their location and/or their availability. Each carer is also associated with a binary variable $S_i^k = 1$ if carer k has the necessary skills to care for client i [9].

Remark 2.12. *While different skill levels are not considered in this version of the CCC, this aspect may be included in future works by the inclusion of such a binary variable.*

Using an ILP model, a solution for the weekly scheduling of carers without synchronised visits was found within two hours. However, just as with the previous example, whether this solution is the optimal one was not investigated [9].

2.3.3 Regarding Linear Programming

One thing the LP solutions for the mTSP and HHC have in common is the long run-time required to execute the associated algorithms, which can span hours. These solutions are often approximate, not exact, and found by manually terminating the search. LP is a well established method however, making it suitable for studying variations of the problem and examining how initial conditions affect the solution.
However, one reason for the long runtime is that every possible combination of variables is explored, even when many combinations are infeasible. The solution presented in this dissertation therefore does not use LP. Instead, graph theory and clustering are utilised to address various aspects of the CCC while preventing infeasible solutions from being explored. This reduces the runtime of the algorithm while still finding feasible solutions, making this approach more suitable for the real-world scheduling of carers.

2.4 Solving the Commuting Carer Conundrum

The algorithm solving the CCC requires two Excel files as input: one file containing client and carer information, and one file encoding each postcode in Falmouth in eastings and northings. The latter file can be accessed through the Ordnance Survey's CodePoint Open dataset [13].

Remark 2.13. *While only locations in Falmouth are considered in this dissertation, the solution presented here can be applied to any location in the UK by using the corresponding CodePoint Open dataset.*

The file containing client and carer information consists of two sheets. The first sheet contains the postcode of each client, during which time window they can be visited (see Section 2.1.2), and a unique ID for each client. The carer sheet also contains each carer's unique ID, the postcode of their home, the number of hours the carer has requested to work that day, and the hours during which they may be scheduled to work. Client and carer information was randomised prior to the initial simulation.

Remark 2.14. *For further work on this problem, the client sheet may include dwell time, whether two carers are needed for a visit, or if the client requires PEG feeding. More information on the carer sheet may include their qualification regarding PEG feeding and whether they commute on foot or by car. A client requiring several visits in a day and a carer available for two non-consecutive shifts in a day may be represented by two separate dummy-entries.*

For each time window $t = [t_0, t_1)$ (see Section 2.1.2), the algorithm first finds the subset of available carers and eligible clients. Let $s^k = \left[s_0^k, s_1^k\right)$ be the shift of carer k, with s_0^k being the start and s_1^k being the end of the shift. The carer is available during a time window t if $s_0^k < t_1$ and $s_1^k > t_0$. Furthermore, let $w^k \geq 0$ be the working hours requirement of the carer, which is reduced by any amount of time spent commuting or providing care work. A carer is available during a time window if $w^k > 0$.
Let $f_i \in [0, 4]$ be the *flexibility* of a client i, which is the number of additional time windows during which they can be visited after the current time window t. The smaller the number of time windows, the less flexible the client is, and the higher their *priority* to be allocated to a carer in a given time window. Of all eligible clients, those with the lowest flexibility are selected first, until a sufficient number of clients are selected (see Remark 2.9).

Remark 2.15. *For example, client A can only be visited in the current time window, whereas client B can be visited during another time window later in the day. Client A is therefore less flexibile and is selected first.*

A time window is skipped if no carers are available or no clients eligible. Otherwise, the location of each carer and client is approximated via their postcodes, and the distance matrix between each point used as the basis for the initial clustering. The final output of the clustering step is a number of clusters equal to the number of carers, with each cluster containing a maximum number of clients equal to the carer's workload (see Section 2.2.2).
These clusters are then used to solve the TSP problem. For each cluster, an undirected, complete graph consisting only of client nodes is created. The travel time of every possible path through the graph is calculated as the sum of edge weights when following the path plus the travel time from the carer's current location to the first node in the path. The path with the shortest total travel time is then selected as the solution; in the event of multiple viable solutions, a random one is selected.

Remark 2.16. *This random selection of a viable solution is unproblematic in this version of the CCC. However, when considering the synchronisation of visits, all viable solutions should be considered and evaluated. This is because one of these solutions may be favoured when considering restrictions imposed by another carer's optimal route(s).*

Each carer's schedule is then displayed in text form and their routes visualised using the Folium package for Python [20].

3 Simulations

Several scenarios are considered to highlight aspects of the algorithm and to examine the stability of the solutions. Scenario 1 shows the solution found for 20 carers and 50 clients. Based on these results, more carers are first recruited in Scenario 2, and then additional clients added to the clientele in Scenario 3. Scenarios 4 and 5 both follow the results from Scenario 3 and examine how the solution changes after the removal of carers or clients, respectively.
Each scenario includes a summary that reflects the work schedules of all carers. A more detailed look is then offered through the case study of carer 11, who is available for most of the working day and can visit many clients. Changes in all carers' schedules are therefore most likely reflected by carer 11. Finally, the overall schedule is examined and any non-ideal aspects discussed.

3.1 Scenario 1: Starting Conditions

The first scenario assumes that 20 carers work for a fictional care company based in Falmouth. All carers are equally skilled and have indicated preferences regarding the number of working hours (their working hours requirement) as well as their availability throughout the day. There are also 50 clients in and around Falmouth that each require one hour of care work that may start at any time between 7 am and 8 pm. The location of every carer and client is visualised in an interactive Folium map (Figure 2).
Scheduling and routing of all carers in this scenario takes 4 seconds. Of the 20 carers, 15 are required to work overtime, with the longest amount of overtime being 15 minutes. While the scheduling of overtime should be discouraged, it is unavoidable given that the travel time between visits is often more than 0 minutes. Scheduled overtime should therefore be kept to a minimum for

each carer; in this case, 15 minutes is a reasonable amount of overtime. Any amount of overtime is compensated by the fictional care company.

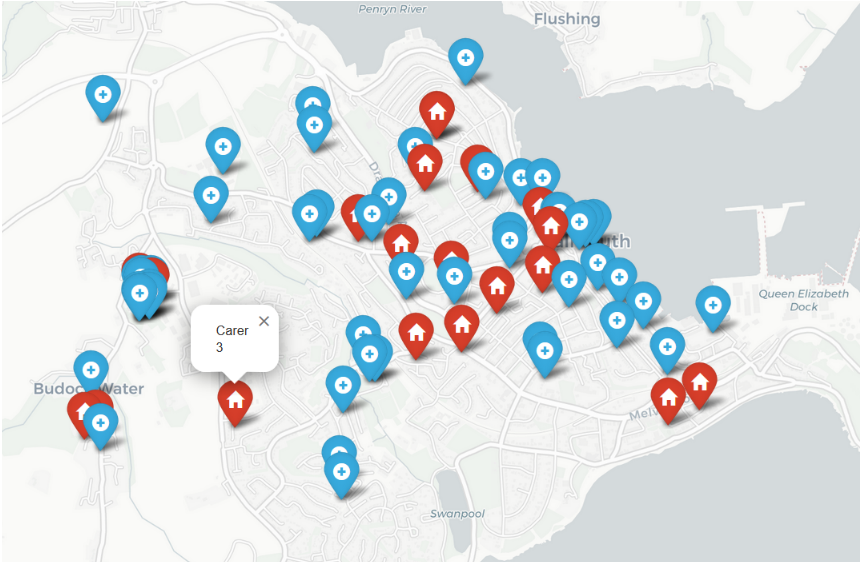


Figure 2: Map of Falmouth showing the location of 20 carers' (red) and 50 clients' (blue) homes. Clicking on an icon shows the carer's or client's ID in a pop-up.

Of the 20 carers, 3 work about one hour less than their indicated preference. This is also not ideal, since a lower number of working hours may negatively impact the income of care workers. An ideal solution ensures that all care workers meet their working hours requirement; however, as discussed below, this is not possible in this simulation.

3.1.1 Case Study: Carer 11

The individual daily schedule of each carer is available both in table format and as a route on a Folium map. An example for carer 11's schedule is given below (see Table 1 and Figure 3). The carer travels a total distance of 3.2 km.

Remark 3.1. *The use of postcodes is an approximation for the sake of this dissertation. A real care company may instead choose to use the full address of the carer and clients.*

Table 1: Schedule for carer 11, showing the arrival and departure times at each client's home as well as the travel time between clients. Clients are identified by their ID. The first row instead contains the carer's ID, starting location, and travel time to their first client. The carer starts work at 11.00 am and finishes at 5.09 pm.

ID	Postcode	Arrival	Departure	Travel Time	Next ID
11	TR11 3LT	-	11.00	2 minutes	21
21	TR11 3BG	11.02	12.02	0 minutes	40
40	TR11 3PG	12.02	13.02	4 minutes	66
66	TR11 3HU	13.06	14.06	3 minutes	47
47	TR11 4RA	14.07	15.07	1 minute	51
51	TR11 4QZ	15.08	16.08	1 minute	70
70	TR11 3HR	16.09	17.09	-	-

Remark 3.2. *A travel time of 0 minutes by car does not exist. Based on the travel times returned by the OpenRouteService package, client locations that are very close may have a travel time of 0 minutes between them. In reality, a carer walks between such near-by locations. Travel of foot is not considered in this version of the CCC, however.*

Figure 3: Step-by-step view of the scheduled route for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to each step.

Remark 3.3. *In Figure 3, the first two clients' locations are very close together. Because of this, the first client's marker partially obscures the second client's marker.*

3.1.2 Discussion of Results

Out of all 50 clients, 6 are not seen by a carer in this solution. This is due to a mismatch in carer and client availability: most of these clients' only eligible visit times are in the early morning when few to no carers are available (Figure 6).

Remark 3.4. *Situations when no carer is available to visit certain clients may also occur for real care companies. In such cases, flexible "back-up carers" are employed that may be scheduled at any time to ensure all clients are cared for. The use of such back-up carers should be kept to a minimum, however, as they may result in higher operating costs for the care company. Alternatively, other regular carers may be asked to work overtime to cover these shifts. This practice should also be discouraged, since it can reduce staff satisfaction.*

Location of client homes is a less important factor than their availability throughout the day. Clients that are not visited by a carer in a given day are often near clients that are visited that day (Figure 4), which indicates that location plays a less important role.

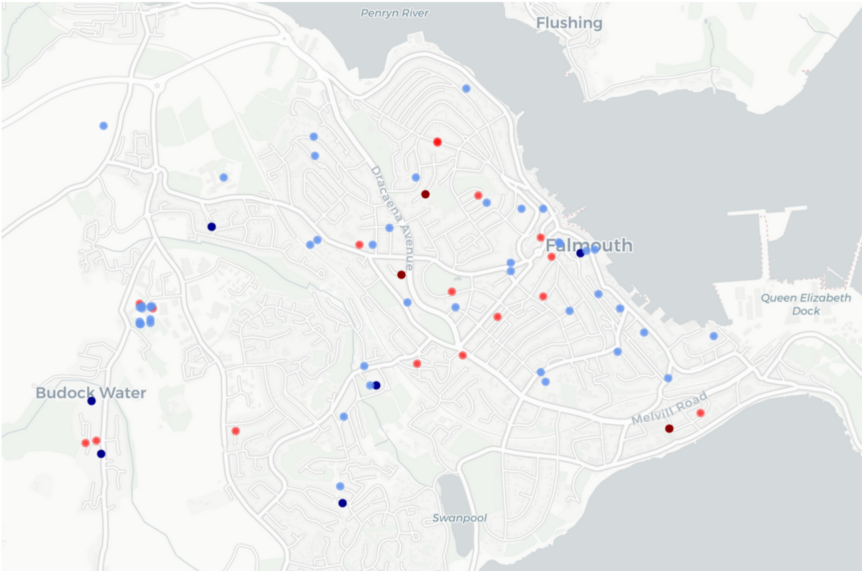
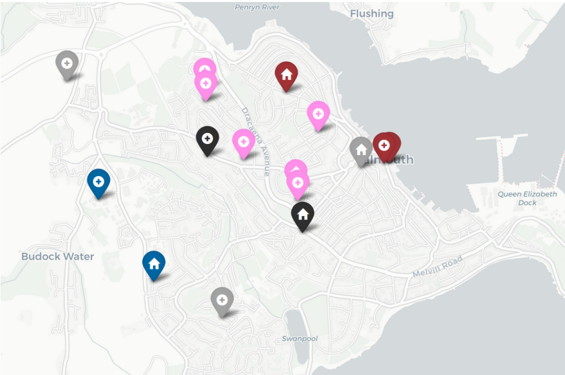


Figure 4: Map of Falmouth showing the location of carer and client homes. Light blue = a client that is visited by a carer throughout the day. Dark blue = a client that is not visited by a carer. Light red = a carer whose working hours requirement is fulfilled. Dark red = a carer whose working hours requirement is not fulfilled.

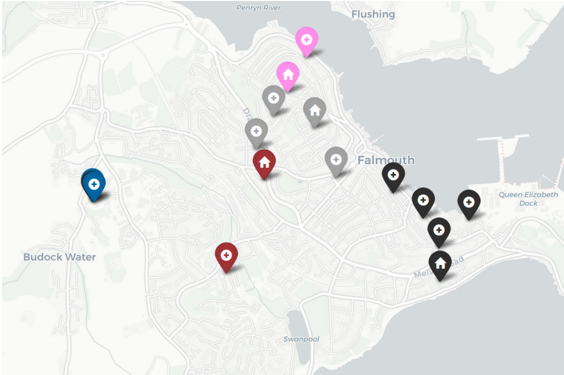
Clients are primarily allocated to carers based on their availability in a given time window, and secondarily based on proximity. This may then result in far-away clients being a part of carer's cluster when near-by clients are not (Figure 5). For instance, the clients allocated to carer 11 are in close proximity to each other and the carer (Figure 5c). On the other hand, most of the clients that carer 10 visits are nearby; however, one client they are required to visit lives further away to the west near Budock Water (Figure 5b). This is because of when carers 10 and 11 are availabile; while both work during the 12 pm – 3 pm and 3 pm – 6 pm time windows and stop at similar times, carer 11 starts work at 11 am, allowing them to be scheduled to care for an additional client during the earlier time window (Figure 6). This results in carer 11 meeting their working hours requirement by visiting only nearby clients, whereas carer 10 needs to be scheduled to a more distant client in an effort to meet their working hours requirement.

Another factor in the formation of clusters is proximity to other points. Clients are primarily assigned to the carer closest to them. However, if a given carer's working hours requirement is already fulfilled by visiting near-by clients, more distant clients may then be assigned to any other available carers. This is why the clients that carer 2 visits are further spread out than carer 11's clients (Figures 5a and 5c).

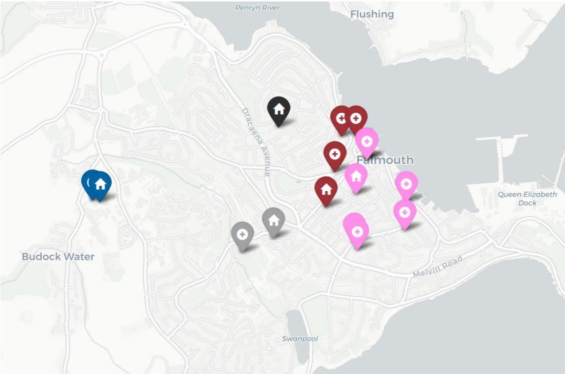
The location of carers is therefore also not the primary reason for why certain carers are unable to meet their working hours requirement. Instead, a mismatch between carer availability and client visit eligibility is the main reason for this suboptimal scheduling outcome (Figure 6).



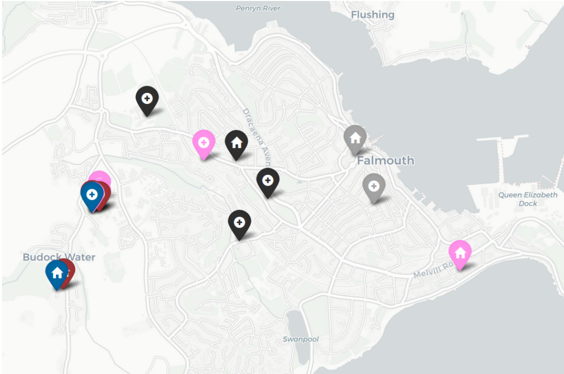
(a) Clients allocated to carer 1 (pink), carer 2 (grey), carer 3 (blue), carer 4 (red), and carer 5 (black).



(b) Clients allocated to carer 6 (pink), carer 7 (grey), carer 8 (blue), carer 9 (red), and carer 10 (black).



(c) Clients allocated to carer 11 (pink), carer 12 (grey), carer 13 (blue), carer 14 (red), and carer 15 (black).



(d) Clients allocated to carer 16 (pink), carer 17 (grey), carer 18 (blue), carer 19 (red), and carer 20 (black).

Figure 5: Map of Falmouth showing the allocation of clients to carers based on fuzzy clustering. Clusters are shown in groups of fives to aid interpretation; the same colour being used in two different subfigures does not indicate identical cluster affiliation.

Specifically carers 9, 10, and 15 do not meet their working hours requirement. This is represented by a dark red line that shows the time those carers are available to work but are not scheduled to care for clients. During those times, all clients have either already been scheduled to be cared for by another carer (light blue) or are not eligible to be visited at that time. Similarly, clients 23, 24, 36, 39, 58, and 59 are not visited by a carer during the day. Four of those clients can only be visited between 7 am and 9 am when no carers are available. The remaining two clients can be visited between 7 am and 12 pm, but the only two carers that are available during that time are already scheduled to visit other clients (Figure 6).

To ensure a minimum number of back-up carers are used for a solution, the fictional care company would be incentivised to recruit carers that are available in the early morning. Location is less important, although recruitment of carers living in the west and south of Falmouth would reduce travel time and therefore scheduled overtime for these new carers.

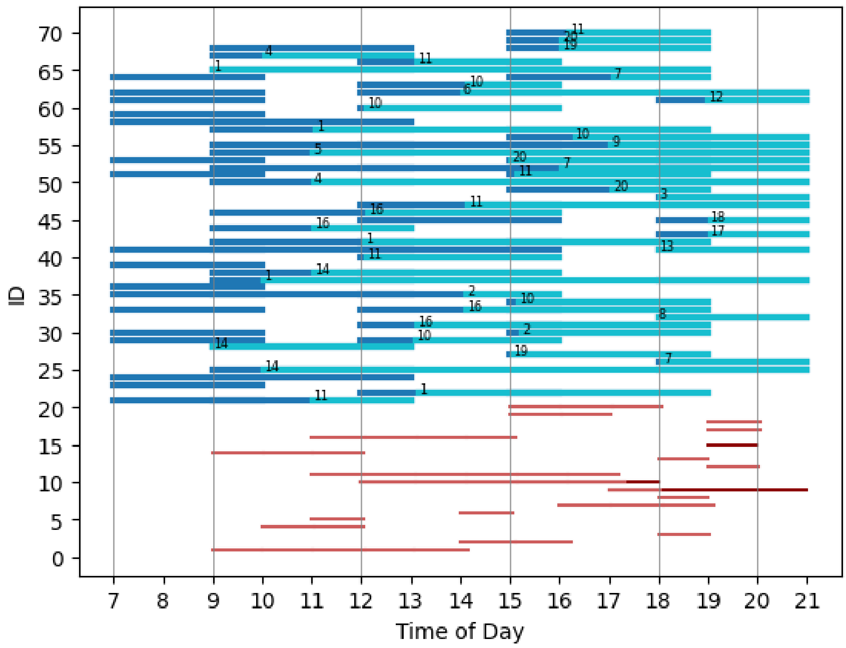


Figure 6: A graph showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20) or a client (21-70). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work. Vertical lines separate time windows.

Remark 3.5. *The working day ends at 9 pm, despite the latest time window for a client visit ending at 8 pm. This is because the latest possible client visit begins at 8 pm and ends at 9 pm.*

3.2 Scenario 2: Additional Carers

After examining the results from Scenario 1, the fictional care company recruits three additional carers that are available in the early morning and live in the west or south of Falmouth. All of these carers' other characteristics are randomised to reflect the real requirements the new staff members might have.

These carers are identified by their IDs 71, 72, and 73. Carer 71 lives in the west and would like to work for 6 hours at any time between 7 am and 6 pm. Carer 72 lives in the south, is available between 7 am and 12 pm, and has a working hours requirement of 2 hours. Finally, carer 73 lives in the southwest near Budock Water, has a working hours requirement of 5 hours, and is available between 7 am and 3 pm (Figure 7). If these three carers are scheduled for two hours of work in the early morning, they should be able to care for all six clients not visited by a carer in Scenario 1.

Remark 3.6. *Since the old carers' IDs range from 1 to 20 and the existing clients' IDs from 21 to 70, identifying new carers by IDs starting at 71 allows for direct comparison with past results.*

Scheduling takes 4 seconds. Of the 20 carers, 14 carers work overtime, including the 3 new hires. The maximum amount of overtime is 15 minutes, which is identical to Scenario 1.

On the other hand, 7 carers do not meet their working hours requirement by up to 2 hours, compared to only 3 carers in Scenario 1. This is unsurprising, since the overall workload has not increased to match the larger workforce. In total, about 11 hours of work are missing to allow all carers to meet their working hours requirements.

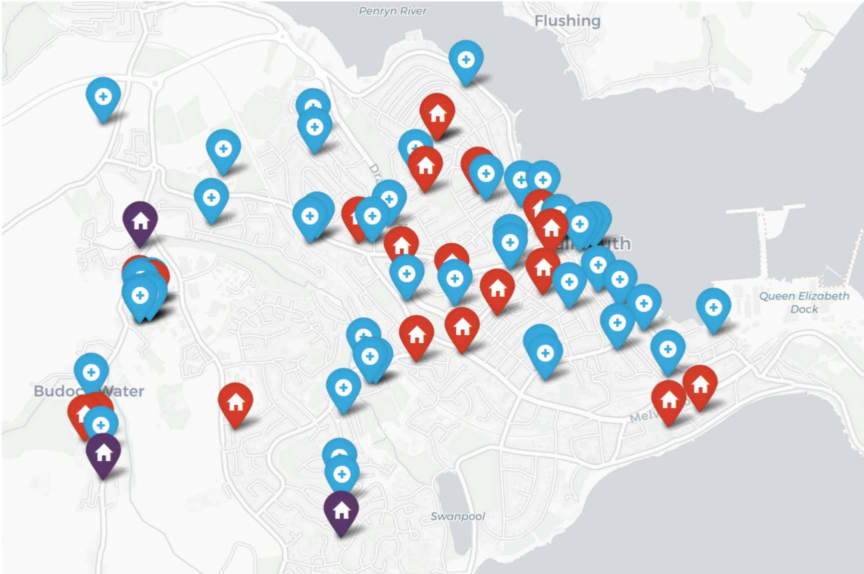
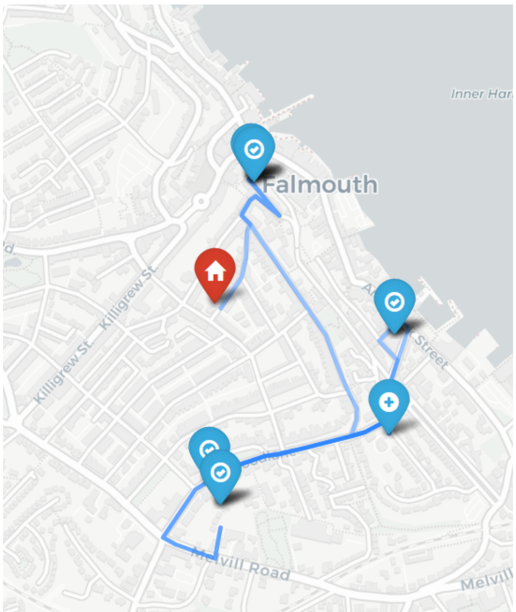


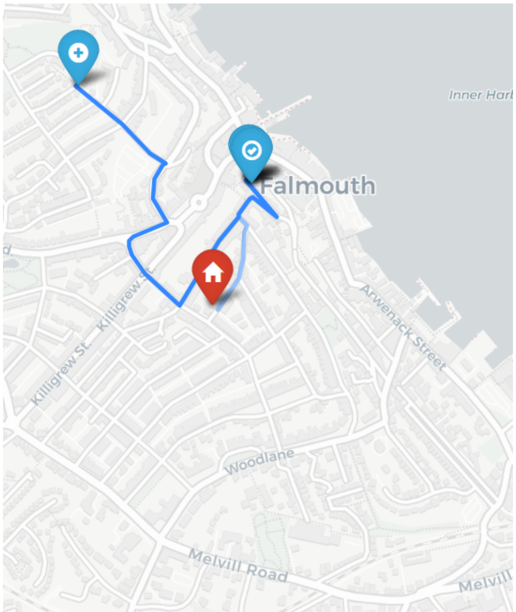
Figure 7: Map of Falmouth showing the location of 23 carers' (red and purple) and 50 clients' (blue) homes. New carers shown in purple.

3.2.1 Case Study: Carer 11

The inclusion of additional carers also affects the route and schedule of other carers. For example, the route carer 11 now takes is very different compared to Scenario 1. The carer works three hours less, finishing work at 2.07 pm instead of 5.07 pm, which results in their working hours requirement not being met (Figure 10). The carer now also visits a client they did not visit in Scenario 1 (Table 2). The carer travels 1.5 km in total.



(a) Original route taken in Scenario 1.



(b) New route taken in Scenario 2.

Figure 8: Final scheduled routes for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

Table 2: Schedule for carer 11, showing the arrival and departure times at each client's home as well as the travel time between clients. Clients are identified by their ID. The first row instead contains the carer's ID, starting location, and travel time to their first client. The carer starts work at 11.00 am and finishes at 2.07 pm. Clients that are not part of the solution in Scenario 1 are highlighted in bold

ID	Postcode	Arrival	Departure	Travel Time	Next ID
11	TR11 3LT	-	11.00	2 minutes	21
21	TR11 3BG	11.02	12.02	0 minutes	40
40	TR11 3PG	12.02	13.02	5 minutes	**22**
22	**TR11 2BJ**	13.07	14.07	-	-

This change in schedule is also apparent when comparing the new route carer 11 now takes (Figure 8b) with the route taken in Scenario 1 (Figure 8a).

3.2.2 Discussion of Results

Only client 23, who can only be visited early in the morning, is not seen by a carer in this scenario. They live in the northeast of Falmouth and are therefore farthest away from the newly recruited carers (Figure 9).

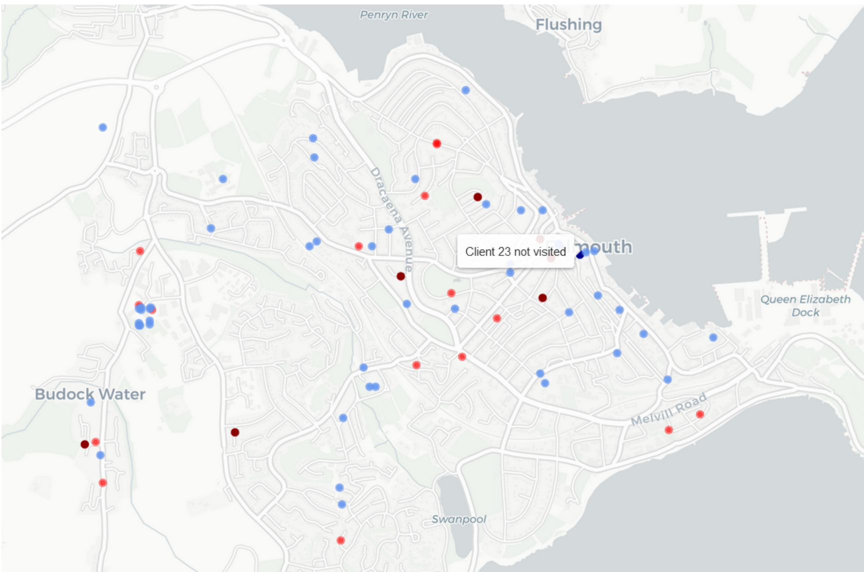


Figure 9: Map of Falmouth showing the location of carer and client homes. Light blue = a client that is visited by a carer throughout the day. Dark blue = a client that is not visited by a carer. Light red = a carer whose working hours requirement is fulfilled. Dark red = a carer whose working hours requirement is not fulfilled. A tool-tip shows that a carer visit of client 23 has not been scheduled.

Evidently, recruiting based on availability alone does not ensure that every client is cared for. So while carer and client location is a minor factor in the scheduling of carers, it is still important and should not be neglected. The fictional care company may therefore decide to recruit a fourth carer living near client 23 that is available in the morning.

This result highlights the way the algorithm handles the time aspect of this problem. For each time window, a selection of clients is made to ensure that the total working hours requirement for the available carers is met. In this case, to ensure that the 3 carers work enough hours in the 7 am – 9 am time window, at least 6 clients need to be selected. This selection is primarily based on the flexibility of the clients, with less flexible clients receiving a higher priority. However, in this instance, there are only 4 clients with the highest priority, resulting in 8 slightly more flexible clients being selected as well.

As a result of this, 12 clients and 3 carers are considered for the clustering step of the algorithm. This step primarily allocates clients to carers based on proximity and secondarily on priority, which results in no care assignment for client 23.

The inclusion of 3 additional carers also impacts the allocation of clients to the remaining carers. Because more carers are now available in the morning, many clients are being visited earlier in the day compared to Scenario 1. As a result of this, especially carers that are available later in the day fail to meet their working hours requirement (Figure 10).

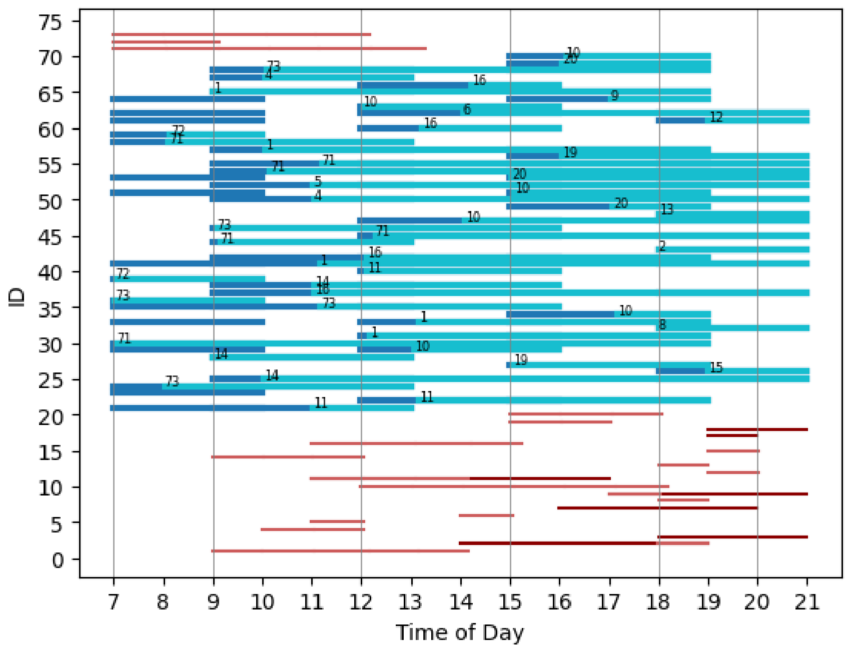


Figure 10: A graph showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20 and 71-73) or a client (21-70). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work and needs to meet working hour requirements. Vertical lines separate time windows.

In summary, the recruitment of 3 additional carers ensures that more clients are visited throughout the day. The schedule of the remaining carers is also impacted by this expansion of the workforce, which results in more carers failing to meet their working hours requirement by a total of 11 hours. To combat this, a care company may take on 11 additional clients to ensure enough work for the carers.

3.3 Scenario 3: Additional Clients

Since the number of people needing home care is steadily increasing [4], the fictional care company has no problems taking on additional clients to provide work for care staff that do not meet their working hours requirement. Criteria for accepting a client are based on their location and the time windows during which they need to be visited. A client should only be added to the clientele if there is an available carer who lives near-by. In this case, the fictional care company identifies how many additional clients each underworked carer can take on, when they can be scheduled for a visit, and where the carer lives (Table 3).

Table 3: Carers who do not meet their working hour requirements in Scenario 2, including the number of additional hours required and the availability of each carer throughout the day.

ID	Postcode	Hours	Availability
2	TR11 3BB	1	2 pm – 7 pm
3	TR11 4PX	1	6 pm – 9 pm
7	TR11 2BQ	3	4 pm – 8 pm
9	TR11 2EQ	1	5 pm – 9 pm
11	TR11 3LT	3	11 am – 5 pm
17	TR11 3PQ	1	7 pm – 8 pm
18	TR11 5EL	1	7 pm – 9 pm

The care company then takes on 11 additional clients based on these restrictions (Figure 11). These clients are identified by IDs spanning from 74 to 84.

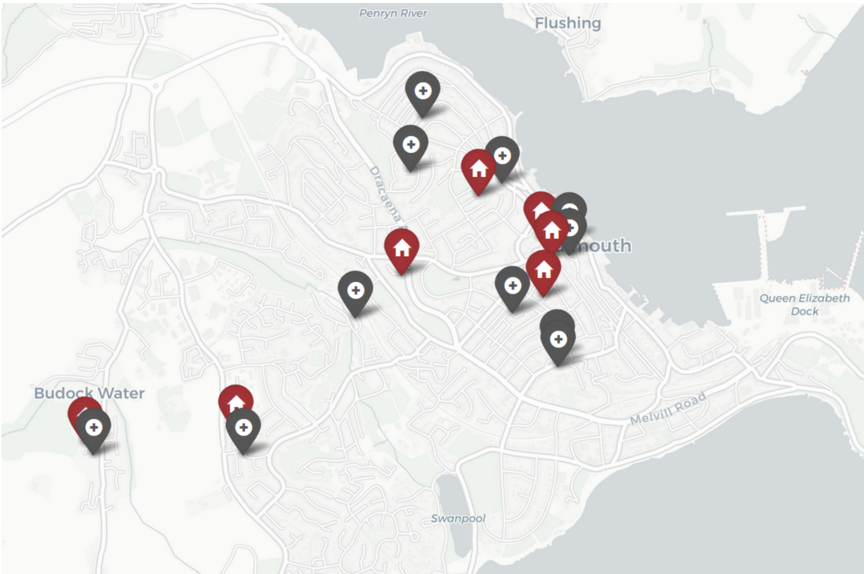


Figure 11: Map of Falmouth showing the location of carers that do not meet their working hours requirement in Scenario 2 (dark red) and new clients (grey).

Remark 3.7. *Client 23 was not visited by a carer in Scenario 2, which should have prompted the recruitment of an additional carer. To allow for direct comparisons between Scenarios 2 and 3 however, the number of carers is unchanged. This means that client 23 is not visited by a carer in this scenario as well.*

Scheduling of carers takes 5 seconds. Of the 23 carers, 20 work overtime, with the maximum amount of overtime being 12 minutes. Compared to Scenario 2, 6 more carers work overtime, but the maximum amount of overtime is reduced by 3 minutes.

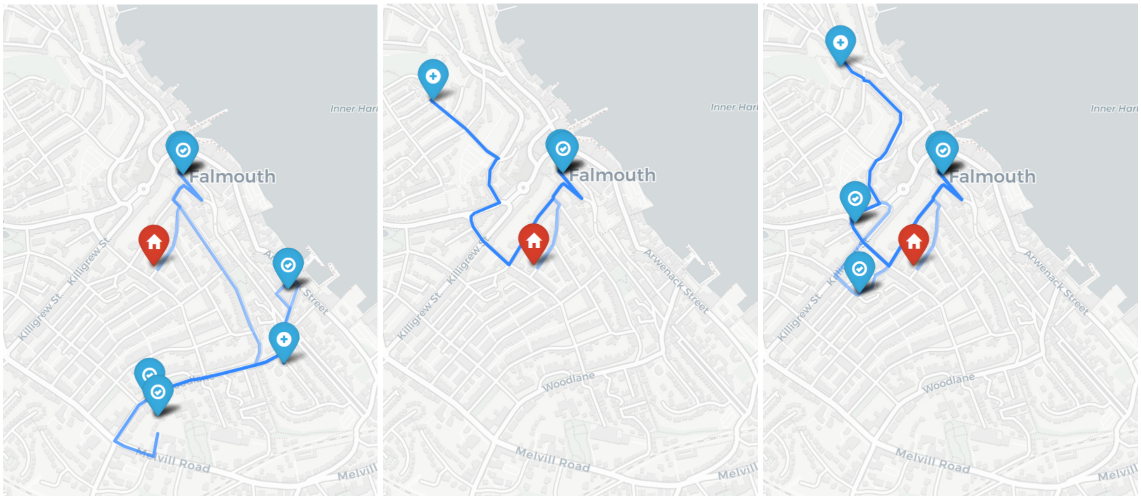
3.3.1 Case Study: Carer 11

The scheduled route of carer 11 differs from the routes taken in Scenarios 1 and 2. Here, the route includes the two new clients 78 and 81. Clients 21 and 40 however are part of that carer's schedule in all three scenarios, showing the robustness of certain parts of the solution (Table 4). The carer travels a total of 3.4 kilometres.

Table 4: Schedule for carer 11, showing the arrival and departure times at each client's home as well as the travel time between clients. Clients are identified by their ID. The first row instead contains the carer's ID, starting location, and travel time to their first client. The carer starts work at 11.00 am and finishes at 4.13 pm. Clients that are not part of the solution in Scenarios 1 or 2 are highlighted in bold.

ID	Postcode	Arrival	Departure	Travel Time	Next ID
11	TR11 3LT	-	11.00	2 minutes	21
21	TR11 3BG	11.02	12.02	3 minutes	**81**
81	**TR11 3BP**	12.05	13.05	1 minute	**52**
52	**TR11 3TH**	13.06	14.06	3 minutes	40
40	TR11 3PG	14.09	15.09	4 minutes	**78**
78	**TR11 2AQ**	15.13	16.13	-	-

The carer visits more clients in this scenario compared to the previous two. Because of a higher number of available clients, the route for any specific carer can include more near-by clients (see Figure 12c compared to Figures 12a and 12b).



(a) Route taken in Scenario 1. (b) Route taken in Scenario 2. (c) New route taken in Scenario 3.

Figure 12: Final scheduled routes for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

3.3.2 Discussion of Results

Only one carer, carer 11, does not meet their working hours requirement by about an hour, instead of 7 carers in Scenario 2. In addition to client 23 (see Remark 3.7), client 63 is also not visited by a carer in this scenario (Figure 13).

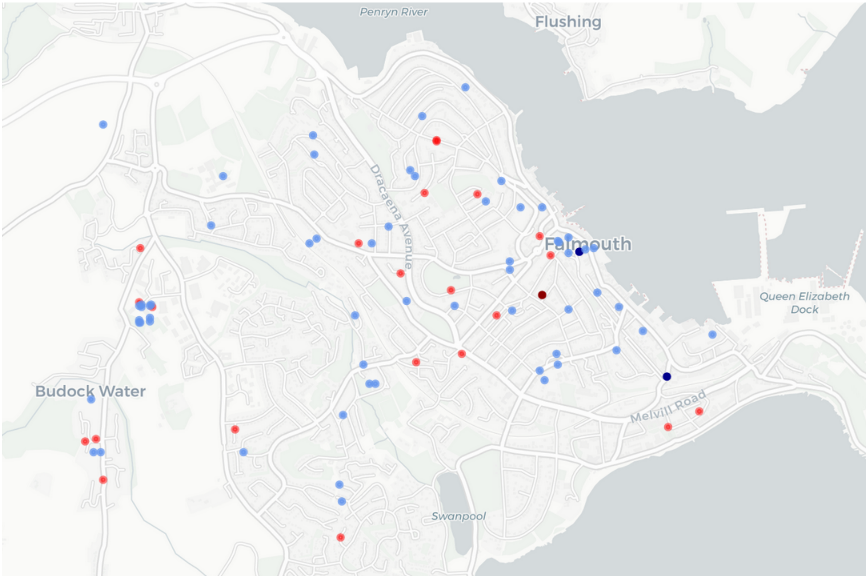


Figure 13: Map of Falmouth showing the location of carer and client homes. Light blue = a client that is visited by a carer throughout the day. Dark blue = a client that is not visited by a carer. Light red = a carer whose working hours requirement is fulfilled. Dark red = a carer whose working hours requirement is not fulfilled.

Similarly to Scenario 1, the reason that client 63 is not visited and that carer 11 does not meet their working hours requirement is a mismatch between all carers' and clients' availability. No carer is available when client 63 can be visited, and not enough clients are available during carer 11's working hours (Figure 14).

The aim of accepting additional clients was to provide more work for previously underworked care staff. This was successful: carers 2, 3, 7, 9, 11, and 18 all visited some of the new clients. Overall, accepting new clients improved workload fairness among the care staff of the fictional care company by reducing the number of underworked staff. However, the solution is still not ideal, as evidenced by the underworked carer and the two unvisited clients that remain in this scenario.

In summary, while accepting additional clients improves workload fairness, new clients cannot be accepted based on carer location and availability alone. Instead, before accepting a new client, the care company should consider a hypothetical schedule which includes the new client. A client can only be accepted if their inclusion improves workload fairness for the employees.

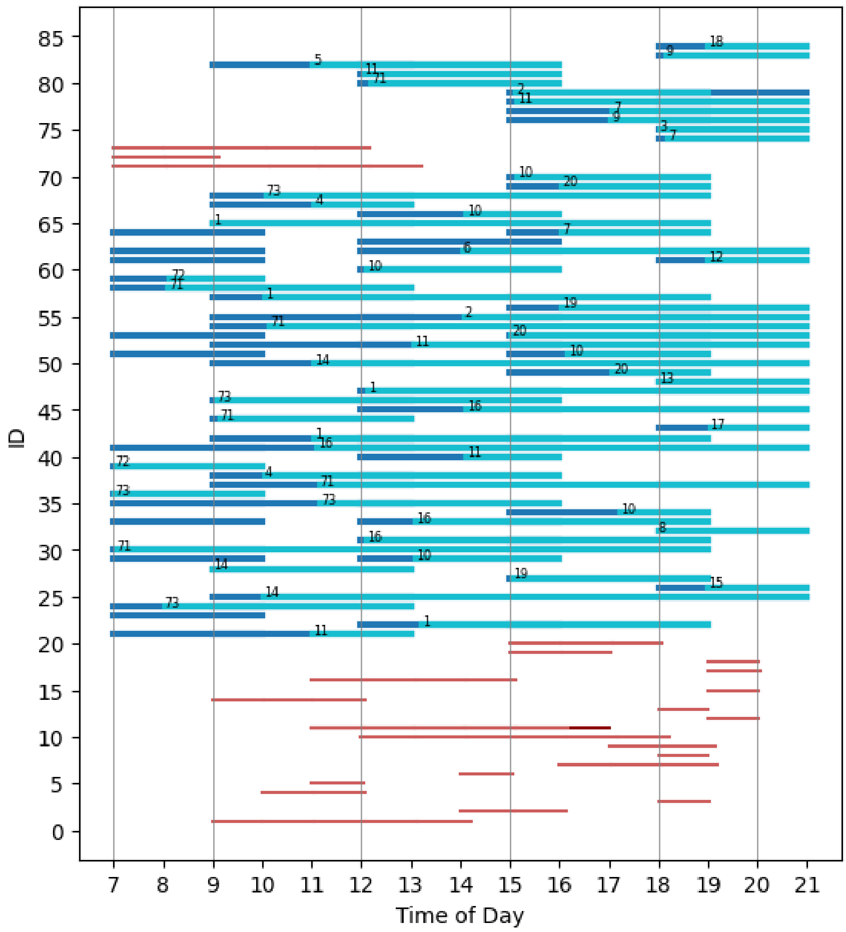


Figure 14: A graph showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20 and 71-73) or a client (21-70 and 74-84). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work and needs to meet working hour requirements. Vertical lines separate time windows.

3.4 Scenario 4: Removal of Carers

The fictional care company continues operating as it did in Scenario 3. However, carers may occasionally become unavailable due to illness or personal reasons. Two scenarios are considered to examine how such staff changes affect the daily care schedule.

3.4.1 Scenario 4a: Removal of Carer 10

In Scenario 3, carer 10 visited 6 clients (Figure 15), making their workday the longest among all carers. Their removal from the staff list may therefore have a big impact on the other carer's schedules.

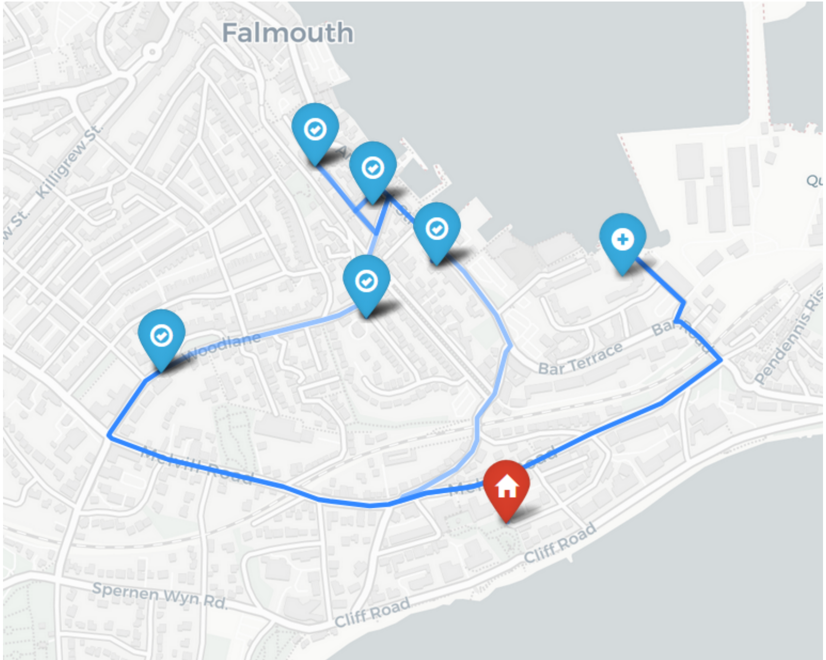
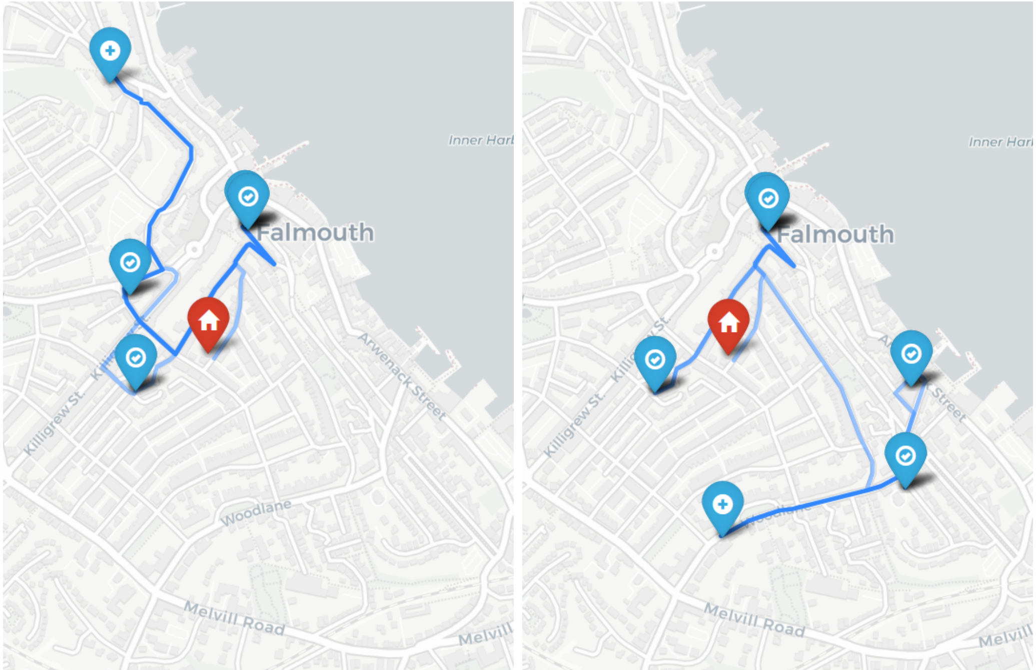


Figure 15: Final scheduled route for carer 10 in Scenario 3. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

Scheduling of the remaining 22 carers takes 4 seconds. Of these carers, 20 work overtime, with the maximum length of overtime being 12 minutes. All carers meet their working hours requirement. Overall, these results are similar to those in Scenario 3 (see Section 3.3).
Removal of carer 10 impacts carer 11's schedule, allowing them to meet their working hours requirement in this scenario. They also travel a total of 3.9 kilometres in this scenario, which is 0.5 kilometres more than in Scenario 3. Only half of the clients that were seen by carer 11 in Scenario 3 are also part of their schedule in this scenario (Table 5). Those clients are closest to carer 11, indicating that the allocation of clients is more stable the closer they are to a carer (Figure 16).

Table 5: Clients visited by carer 11 in Scenario 3 compared to Scenario 3, from earliest to latest visit. Clients that feature in both scenarios are highlighted in bold.

Scenario 3	Scenario 4a
21	**21**
81	**81**
52	**40**
40	66
78	70
-	51

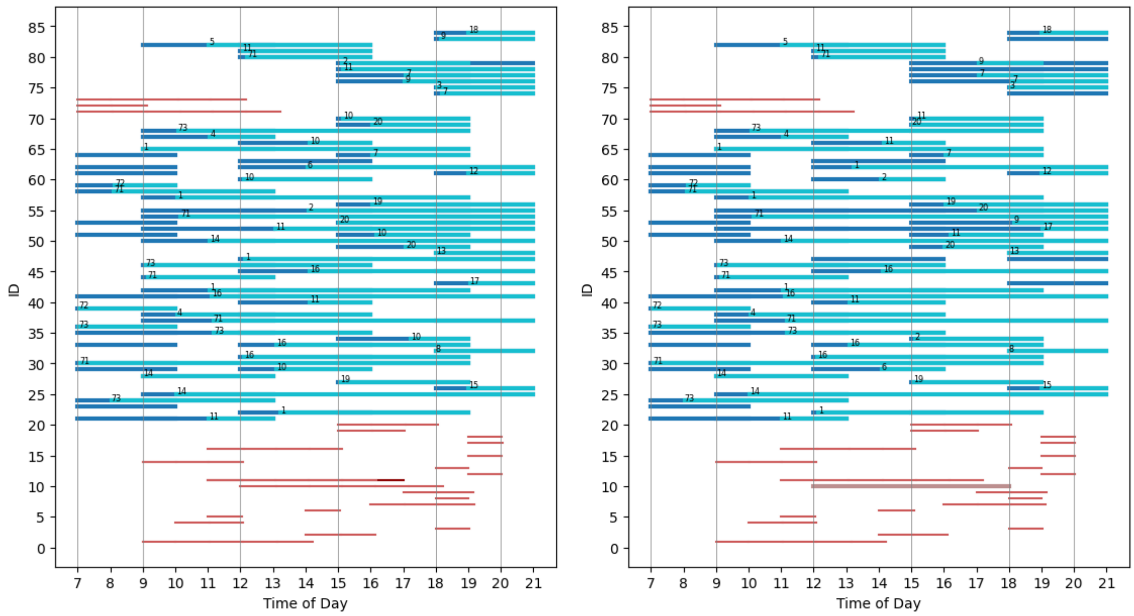


(a) Old route taken in Scenario 3. (b) New route taken in Scenario 4a.

Figure 16: Final scheduled routes for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

Out of all 61 clients, 7 are not seen by carer, which is an increase by 5 compared to Scenario 3. This loosely matches with the 6 hours of work previously done by carer 10, who is now unavailable. One might expect the clients that are not seen by a carer in this scenario to be identical to those that were previously visited by carer 10. This is not the case, however. In Scenario 3, clients 23 (see Remark 3.7) and 63 were not visited by any carer, and clients 29, 34, 51, 60, 66, 70 were visited by carer 10. In Scenario 4a however, the clients that are not visited by any carer are also 23 and 63, as well as 43, 47, 74, 78, and 83. This is because other carers are scheduled to care for the clients previously assigned to carer 10, leaving their own clients without a carer visit. Most of carer 10's clients are assigned to carers 2 and 11, which is due to the similar time of day during which these carers work (Figure 17b).

The removal of carer 10 causes many clients to be visited earlier in the day, leaving clients with evening time windows without a carer visit. While the assignment of clients to carers during the first two time windows is unchanged, the assignments in Scenarios 3 and 4a differ greatly for the 12 pm – 3 pm, 3 pm – 6 pm, and 6 pm – 8 pm time windows (see Figures 17b and 17a).



(a) Daily schedule in Scenario 3 when all 23 carers are available to work.

(b) Daily schedule in Scenario 4a. Carer 10 (light brown) is unavailable.

Figure 17: Graphs showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20 and 71-73) or a client (21-70 and 74-84). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work and needs to meet working hour requirements. Light brown = an unavailable carer's previous availability. Vertical lines separate time windows.

In summary, the removal of carer 10 affects the schedule of all other carers that work during or after carer 10's shift. The overall workload for carers does not change, however. On the other hand, clients are greatly affected by just one carer becoming unavailable, either through a change in the carer they are allocated to, or by not being visited by a carer at all.

3.4.2 Scenario 4b: Removal of Carer 14

In Scenario 3, carer 14 was one of only two carers that was scheduled in the morning between 9 am and 12 pm. Unlike carer 10, they only visited 3 clients (Figure 18), but their removal from the staff list may affect the schedule of other clients due to the time of day they are normally scheduled to work.

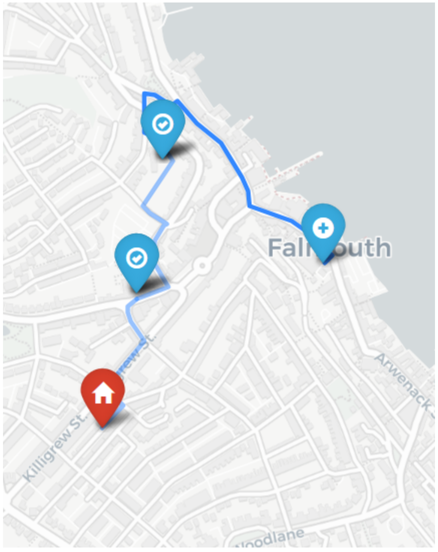
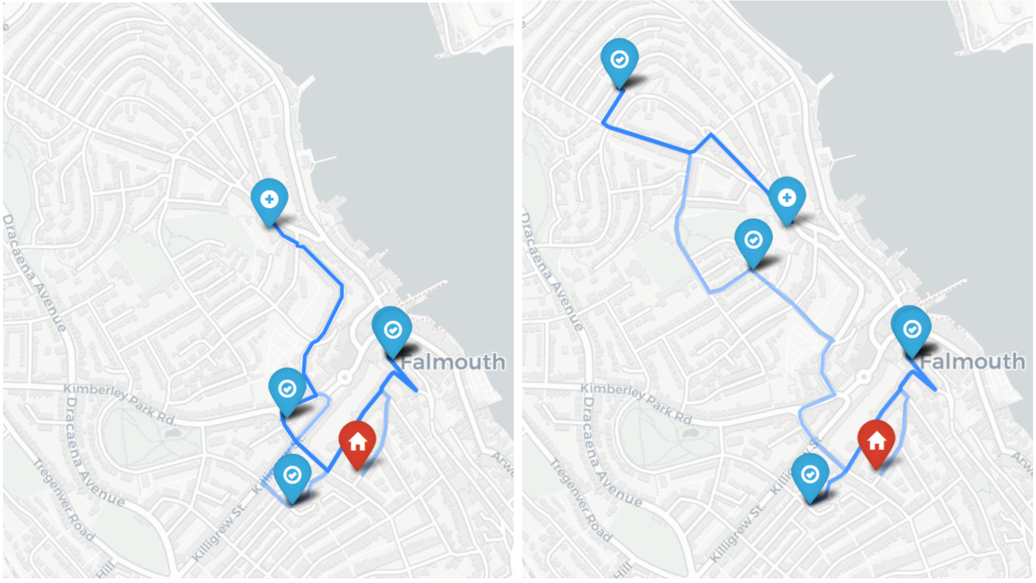


Figure 18: Final scheduled route for carer 14 in Scenario 3. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

After removing carer 14, scheduling of the other 22 carers takes 4 seconds. As before, 20 of these carers work some amount of overtime, with the maximum being 13 minutes. All carers meet their working hours requirement. Overall, these outcomes are similar to those in Scenarios 3 and 4a. Changes to carer 11's schedule are less drastic when compared to Scenario 4a. This time, the carer only travels 0.2 kilometres further than in Scenario 3. Most of the clients visited by carer 11 stay the same between Scenario 3 and 4b, although additional, more remote clients are visited in this scenario (Table 6). The clients closest to the carer feature in both scenarios' routes, once more indicating that the allocation of proximal clients is more stable than of distant ones (Figure 19).

Table 6: Clients visited by carer 11 in Scenario 3 compared to Scenario 4b and, from earliest to latest visit. Clients that feature in both scenarios are highlighted in bold.

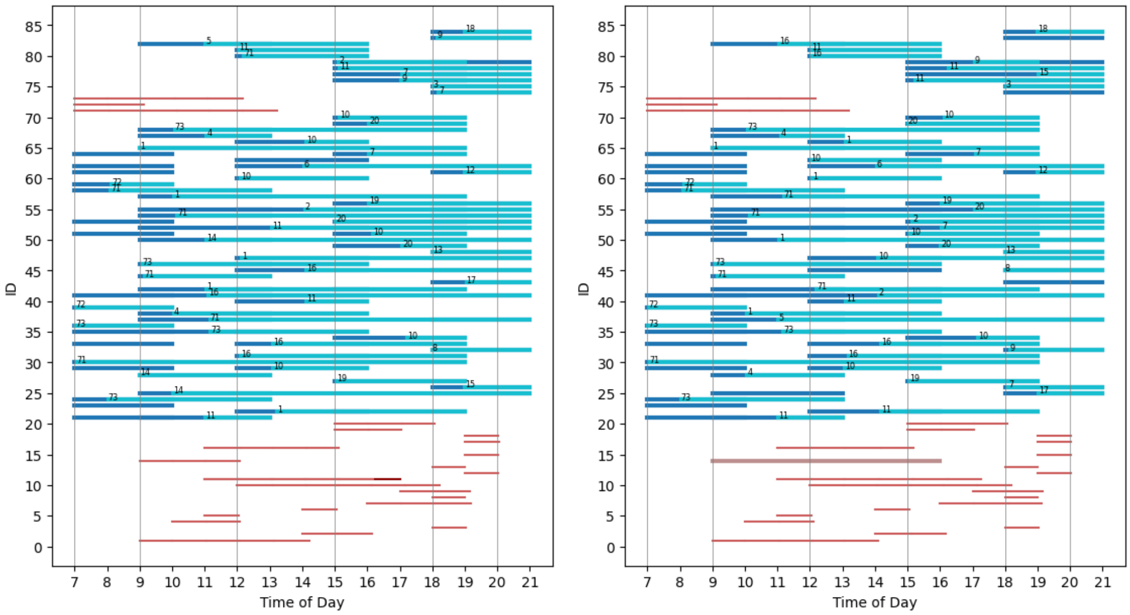
Scenario 3	Scenario 4b
21	**21**
81	**81**
52	**40**
40	22
78	76
-	**78**



(a) Old route taken in Scenario 3.

(b) New route taken in Scenario 4b.

Figure 19: Final scheduled routes for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.



(a) Daily schedule in Scenario 3 when all 23 carers are available to work.

(b) Daily schedule in Scenario 4b. Carer 14 (light brown) is unavailable.

Figure 20: Graphs showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20 and 71-73) or a client (21-70 and 74-84). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work and needs to meet working hour requirements. Light brown = an unavailable carer's previous availability. Vertical lines separate time windows.

The four clients not visited by a carer in this scenario were also part of the list of clients not seen in Scenario 4a. Similarly to Scenario 4a, carer 14's clients are reallocated to other carers, which causes clients that need to be seen later in the day to not be visited as carers fulfill their working hours requirements earlier in the day. Other than in Scenario 4a however, the schedule of client visits changes less drastically for all time windows after 9 am (see Figures 20b and 20a).
As in Scenario 4a, the removal of a carer affects the schedule of all other carers. However, despite carer 14 working earlier in the day, there are fewer changes caused by their unavailability compared to carer 10. This is because there are fewer clients originally allocated to carer 14, which in turn causes fewer changes to other carers' schedules.

3.4.3 General Remarks

A care company can handle carer absence in two ways. For one, the schedule of the remaining carers can be adjusted as was done in Scenario 4. Alternatively, a back-up carer can be scheduled to cover just the absent carer's route, allowing the rest of the care staff's schedules to remain unchanged.

Remark 3.8. *In case of a temporary absence due to illness or vacations, a real care company may also simply schedule more hours for their existing carers without needing to rely on back-up carers. This algorithm however does not allow for the purposeful scheduling of additional hours of overtime. Instead, regular carers that agree to cover a colleagues' shift are simply treated as back-up carers for these purposes.*

Should a carer's absence only be temporary, one back-up carer covering their shifts may be a simpler solution. However, in cases when a carer is permanently removed from the staff list, the care company may consider both options and choose the one that ensures higher client and staff satisfaction.
Regarding the two scenarios outlined above, the unavailability of carer 10 may more easily be covered by one back-up carer to allow the remaining staff to keep their existing schedules. The absence of carer 14 on the other hand may more easily be compensated by rescheduling the remaining carers.

3.5 Scenario 5: Removal of Clients

In addition to carers, clients may also stop being a part of the care company's clientele. Based on the results from Scenario 3, two scenarios for how the removal of clients may impact the carers' schedules are considered.

3.5.1 Scenario 5a: Removal of Client 55

Client 55 is flexible regarding their visit times and may be visited at any point between 9 am and 8 pm. Their removal may therefore have a large impact on carer schedules. However, they live in an area of Falmouth that is not close to many carers compared to other clients (Figure 21).

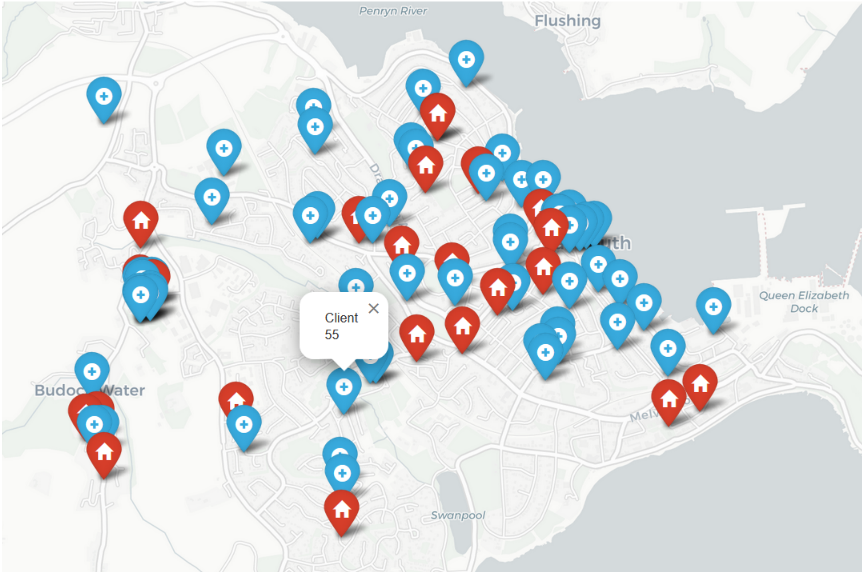
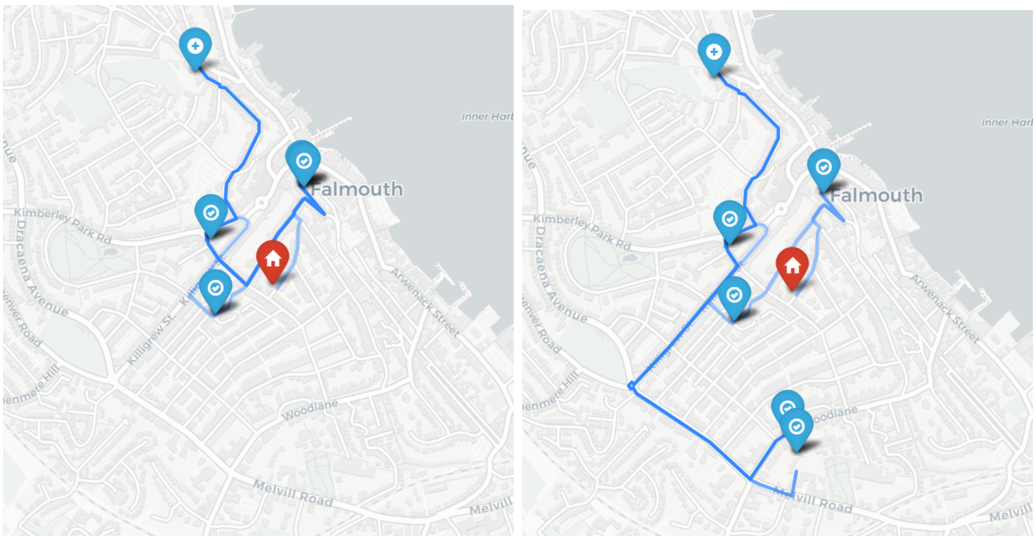


Figure 21: Map of Falmouth showing the location of 23 carers' (red) and 61 clients' (blue) homes. A pop-up shows the location of client 55.

Scheduling of carers takes 4 seconds. Out of all 23 carers, 20 work some amount of overtime, with the maximum being 16 minutes. Carer 7 is the only carer not to meet their working hours requirement. Compared to Scenario 3, only client 23 is not visited by a carer (see Remark 3.7). Overall, these results are similar to Scenario 3.
The schedule of carer 11 is also different compared to Scenario 3, despite client 55 not being a part of that carer's schedule (Table 7). While carer 11 visits one additional client in this scenario, their clients are more spread out, resulting in the carer back-tracking to travel to their final client (Figure 22). As a result of this, they travel a total of 4.6 kilometres, compared to 3.4 kilometres in Scenario 3.



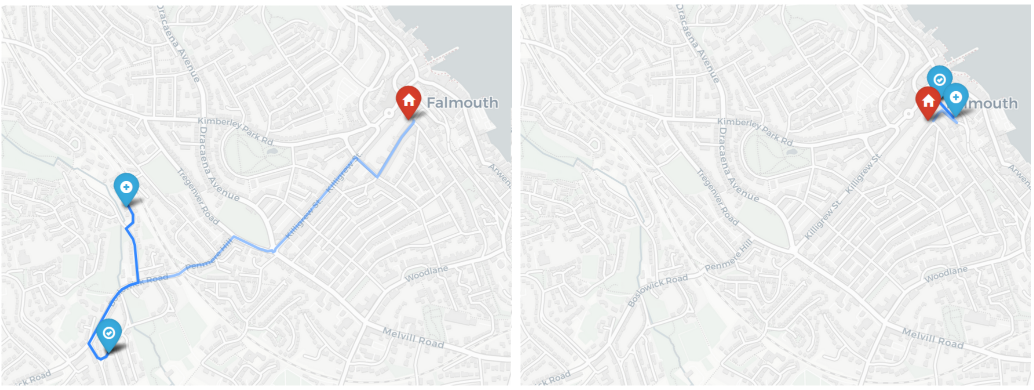
(a) Old route taken in Scenario 3. (b) New route taken in Scenario 5a.

Figure 22: Final scheduled routes for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

Table 7: Clients visited by carer 11 in Scenario 3 compared to Scenario 5a, from earliest to latest visit. Clients that feature in both scenarios are highlighted in bold.

Scenario 3	Scenario 5a
21	**21**
81	**81**
52	**52**
40	47
78	51
-	**78**

In Scenario 3, client 55 was the first of two clients visited by carer 2, who travelled a total of 2.4 kilometres. The removal of this client therefore drastically changes carer 2's entire route and schedule, resulting in them only travelling 0.3 kilometres (Figure 23).



(a) Old route taken in Scenario 3.

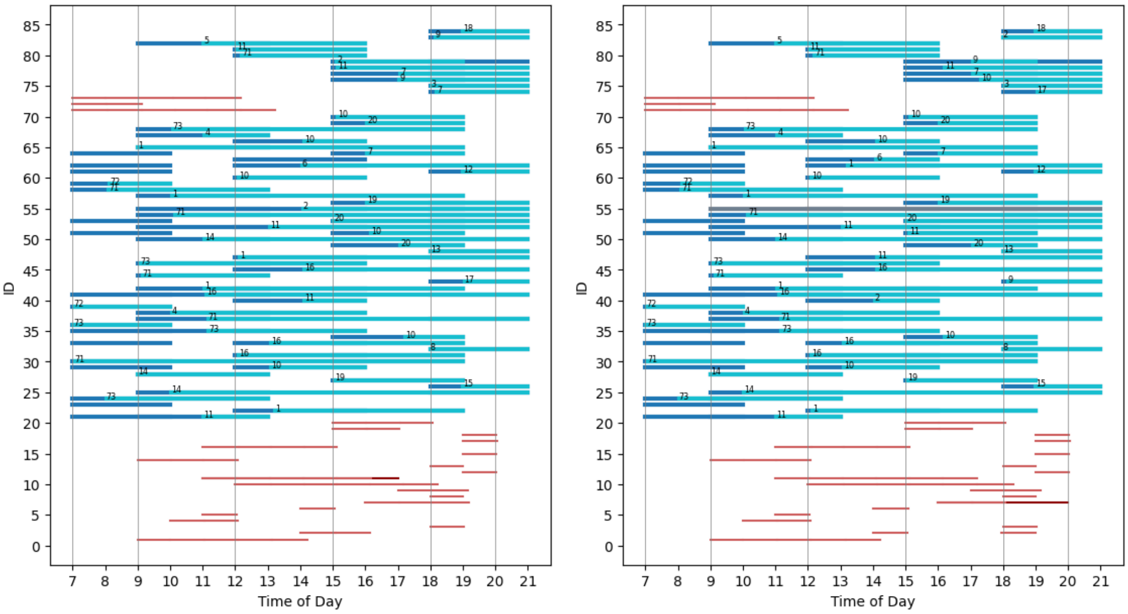
(b) New route taken in Scenario 5a.

Figure 23: Final scheduled routes for carer 2. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

This change in carer 2's route highlights how the algorithm handles the clustering of clients around carers throughout the day. With each visited client, the carer's current location is updated. This updated location is then used for the next clustering step to ensure carers are allocated to the clients closest to their location, not necessarily closest to their homes. The removal of client 55 causes carer 2 to visit a different client instead, which in turn affects the location of carer 2 for the next clustering iteration. As a result, the route that carer 2 takes is vastly different in this scenario compared to Scenario 3.

Unsurprisingly, the removal of one client does not affect the other carers' schedules as much as the removal of a carer did in Scenario 4. While there are small differences between the overall schedules (Figure 24a and Figure 24b), the general order and timing of carers visiting their clients is similar.

In summary, the removal of one client can drastically affect the carer's schedule that they were originally allocated to. This in turn can affect the remaining carers' schedules, although the impact is less significant when compared to the removal of carers (Section 3.4).



(a) Daily schedule in Scenario 3. All 61 clients are available.

(b) Daily schedule in Scenario 5a. Client 55 (grey) can no longer be visited.

Figure 24: Graphs showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20 and 71-73) or a client (21-70 and 74-84). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work and needs to meet working hour requirements. Grey = an unavailable client's previous availability. Vertical lines separate time windows.

3.5.2 Scenario 5b: Removal of Client 28

Unlike client 55, client 28 lives in the middle of Falmouth and in close proximity to several carers (Figure 25). The time window during which they can be visited is only between 9 am and 12 pm, however. Their removal from the client list may therefore impact the schedules of carers differently than the removal of client 55.

In this scenario, scheduling of 23 carers takes 5 seconds. Of these, 20 carers are required to work overtime. The maximum amount of overtime is 18 minutes, which is an increase by 6 minutes compared to Scenario 3. Only carer 10 does not meet their working hours requirement by about an hour. In Scenario 3, clients 23 and 63 were not visited by a carer. In this scenario, only client 23 is not seen by a carer (see Remark 3.7). Just like in Scenario 5a, removal of one client allows a different client to be visited instead. Overall, the total number of clients visited is unchanged, but the amount of overtime is higher compared to Scenario 3.

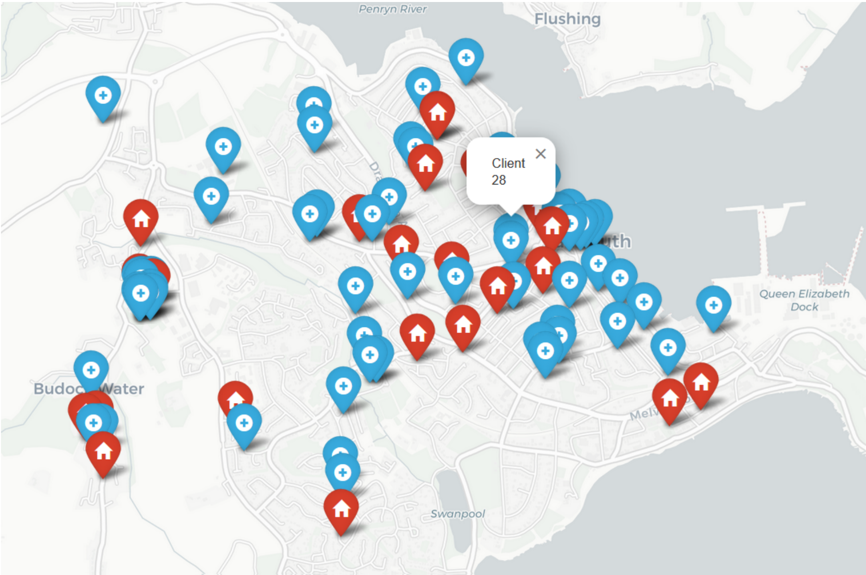
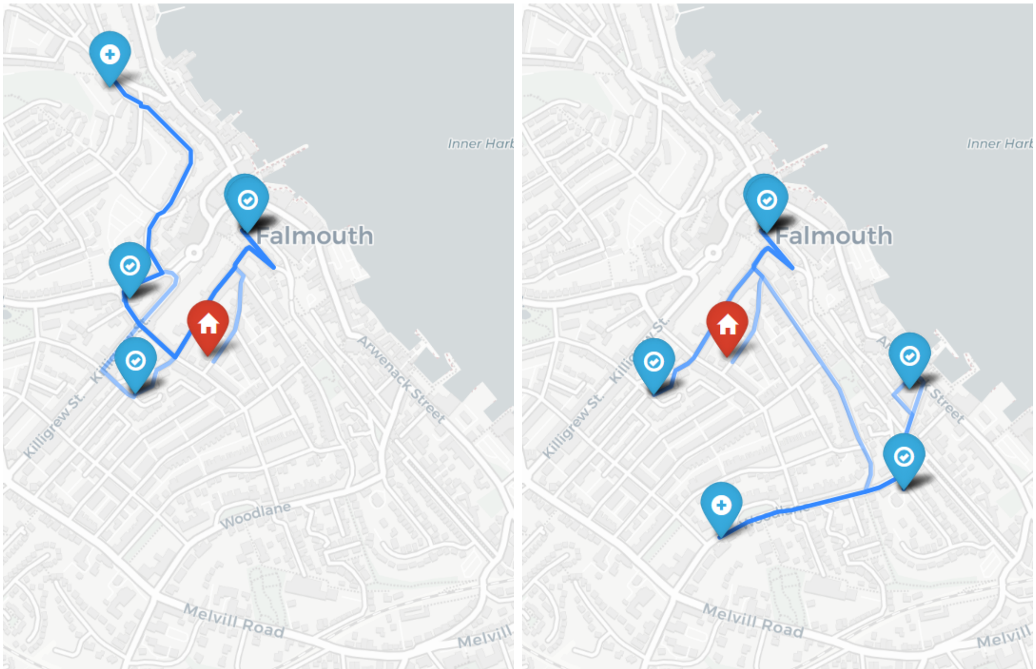


Figure 25: Map of Falmouth showing the location of 23 carers' (red) and 61 clients' (blue) homes. A pop-up shows the location of client 28.

As in Scenario 5a, the schedule for carer 11 is different compared to Scenario 3, despite the removed client not being a part of that carer's schedule (Table 8). This new route involves visiting an additional client and is 0.6 kilometres shorter than the one taken in Scenario 3 (Figure 26).



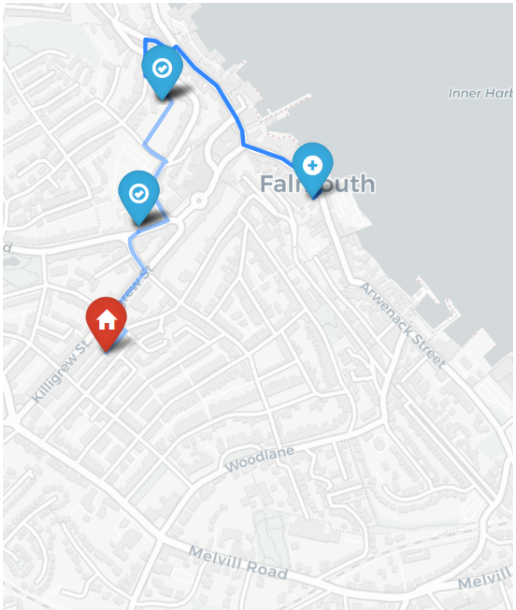
(a) Old route taken in Scenario 3. (b) New route taken in Scenario 5b.

Figure 26: Final scheduled routes for carer 11. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

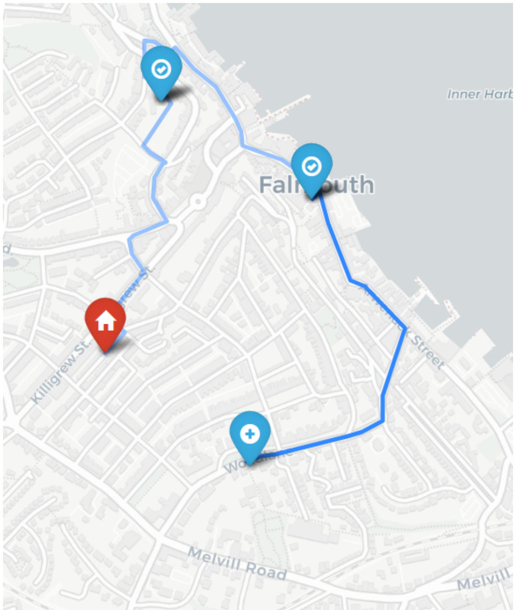
Table 8: Clients visited by carer 11 in Scenario 3 compared to Scenario 5b, from earliest to latest visit. Clients that feature in both scenarios are highlighted in bold.

Scenario 3	Scenario 5b
21	**21**
81	**81**
52	**40**
40	66
78	70
-	51

In Scenario 3, client 28 was visited by carer 14. The removal of that client does not drastically change carer 14's schedule: they still visit 3 clients in total, two of which are the same in both scenarios. However, carer 14 travels a total of 2.4 kilometres, which is 1 kilometre longer in this scenario than in Scenario 3. This is because their final client lives further away (Figure 27).



(a) Old route taken in Scenario 3.

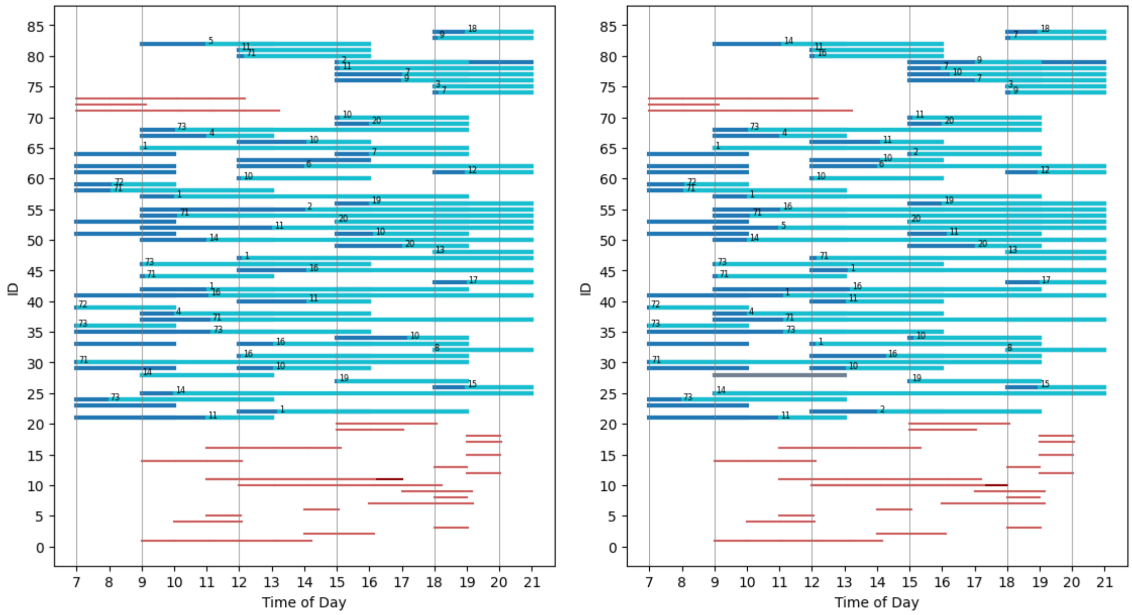


(b) New route taken in Scenario 5a.

Figure 27: Final scheduled routes for carer 14. The red icon represents the carer's home, the blue icons a client's home. A check-mark represents a client that has been visited by the carer. Faded routes have been travelled prior to the final step.

Similarly to Scenario 5a, the removal of client 28 only has a minor effect on the other carers' schedules. Clients are mostly visited by the same carer, often even still at the same time as they were in Scenario 3 (Figure 28).

The removal of client 28 causes a smaller change in the overall schedule of the carers compared to Scenario 5a. The impact of a client's removal is therefore mostly dependent on the time windows during which they can be visited, with a client with more time windows having a larger effect.



(a) Daily schedule in Scenario 3. All 61 clients are available.

(b) Daily schedule in Scenario 5b. Client 28 (grey) can no longer be visited.

Figure 28: Graphs showing when carers are scheduled to work as well as when and by whom clients are visited. X-axis is time of day, starting from 7 am and ending at 9 pm. Y-axis is the ID of either a carer (1-20 and 71-73) or a client (21-70 and 74-84). Light blue = a client is or was visited by a carer; the visiting carer is identified by their ID. Dark blue = a client has not been visited by a carer at that moment in time. Red = a carer is scheduled to either do care work or travel. Dark red = a carer is not scheduled to work, but is available to work and needs to meet working hour requirements. Grey = an unavailable client's previous availability. Vertical lines separate time windows.

3.5.3 General Remarks

In summary, the removal of a client causes many carers' schedules to remain largely unaffected. When they do change, they involve visiting clients that live further away than in Scenario 3, resulting in the slight increase in overall overtime.

While the removal of a client from a care company's clientele is less in the company's control, the examination of the effects of such removals are still valuable for considering the stability of the carers' schedules. A client no longer being visited does not cause a similar level of disruption as an unavailable carer and therefore does not need to be compensated by taking on additional clients as soon as possible.

3.6 Summary of Results

In total, the algorithm took between 4 and 5 seconds to schedule all carers. Scheduled overtime ranged from 12 to 18 minutes, which was caused by the travel time between locations.

An ideal outcome means that every carer works between 0 and 60 minutes of overtime and that every client is cared for. Mismatches in the availability of carers and clients throughout the day are the primary reason for non-ideal solutions. This is also why the travel time between points can be longer in certain scenarios; if during a given time window the only available clients live far away, the carers need to travel farther to ensure they are still cared for.

Changes to one carer's schedule also impact other carers. The impact is greater the more clients

were originally part of that carer's schedule. Similarly, changes in the clientele affect carers' schedules more if the time window during which a client could be visited is bigger. As a result of this, both the hiring of new staff and the addition of clients to the clientele needs to be considered based on a hypothetical schedule that includes these changes, instead of simply trying to identify and fill gaps in the schedule.
Allocation of clients to carers is more stable the closer clients are to the carer. Furthermore, if the overall route for a carer changes between two scenarios, the order in which the carer visits the same clients does not change. Continuity of care is therefore more likely for clients living close to their carers.

4 Future Work

There are a number of aspects common in the HHC problem that the CCC does not include. Future work may expand the algorithm and make it more applicable to the problems faced by real care companies.
One important aspect not included in this dissertation is acknowledging the difference in skill level among carers. The care company that the CCC is based on differentiates its carers based on their qualification level regarding PEG feeding. Clients that require this specialised care are rare but need to be considered nonetheless. This aspect can be included by using a binary variable that reflects the status of a client and carer in regards to PEG feeding (see Remark 2.12), as well as through a small modification of the clustering step. For every time window, the clients that require PEG feeding are the first to be allocated to the nearest qualified carer. Allocation of the remaining clients to carers then follows as before.
Dwell time at every client's location is assumed to be one hour. In reality, dwell time differs based on the number and complexity of the care tasks associated with each client. Differences in dwell times can be implemented by using weighted clustering so that every point's weight corresponds to the client's dwell time. In a fuzzy clustering algorithm, this can be achieved by replication of each point a number of times equal to the associated weight [21].
This version of the CCC assumes that every carer drives to their clients. In reality, there may be a number of carers that reach their clients on foot. The inclusion of such carers can be facilitated thanks to the OpenRouteService package, which can calculate travel time based on a number of different modes of transportation. However, the clustering step that allocates clients to carers requires the mode of transportation to the identical for all distances. Instead, every clustering step would first allocate clients to walking carers, and then allocate any remaining (and potentially more remote) clients to driving carers.
Synchronised visits are arguably the most complex aspect of the HHC problem. Clients that require a visit from two carers can be allocated to two carer clusters through the use of fuzzy clustering. The TSP can then be solved separately for both carers. The best combination of solutions that allows both carers to reach the client at similar times is then selected, as mentioned in Remark 2.16.

Remark 4.1. *Due to both carers' schedules prior to the synchronised visit, it is unlikely that the carers arrive at a client's location at the same time. The care company may therefore set an acceptable time window during which both carers may arrive.*

In this version of the CCC, clients are only visited once a day, often within a wide time window. In reality however, clients require several visits throughout a day, often within narrow time windows. Repeated visits throughout the day can be included by treating each visit to the same client as separate instances of once-a-day visits to different dummy-clients.

Remark 4.2. *As an example, client 85 needs to be visited three times during the day. The algorithm may then treat each visit as one visit to separate dummy-clients (clients 86-88) with different time windows.*

Similarly, carers may work non-consecutive hours throughout a workday. This can also easily be included in the algorithm by treating each shift as belonging to different dummy-carers.
The scheduling of carers can also be expanded to include a working week instead of just a day. To do this, only the subset of carers and clients that are available during each workday would be considered. The algorithm can then be run normally to produce the schedule for each day in the working week.
Finally, an interesting quirk of the algorithm became apparent in Section 3.2. Most of the time, the algorithm allocates clients to carers based on their flexibility. In certain situations however, the allocation instead prioritises proximity of clients to carers. Depending on the care company's priorities, this aspect can be tweaked to always prioritise the allocation of less flexible clients at the expense of travel time. This would ensure that the highest number of clients possible are cared for, but might reduce carer satisfaction.

5 Conclusion

Instead of using Linear Programming, the Commuting Carer Conundrum can be solved using a combination of graph theory and fuzzy clustering. By limiting the number of combinations to be explored, this algorithm solves the problem in a shorter time span than most solutions using Linear Programming. The resulting schedules strike a balance between workload fairness and coverage of clients. Changes to the initial conditions often have unexpected effects on the schedules of several carers. Thanks to the short run time however, these changes can quickly be explored to inform the recruitment of additional carers or expansion of the clientele. While the algorithm solving the CCC does not include every aspect common in the HHC problem, it allows for the straightforward implementation of these further aspects in the future.

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