Middle East Technical University Department of Computer Engineering

CENG 424

Logic For Computer Science

Fall 2024-2025

Assignment 1

Name SURNAME: Aly Asad GILANI
Student ID: 2547875

Answer for Q1

1. From the slides, it says that for a given formula F to be satisfiable, its terminated tableau for $F = 1$ should have at least one open branch. I equate the given formula to 1 and find the semantic tableau:

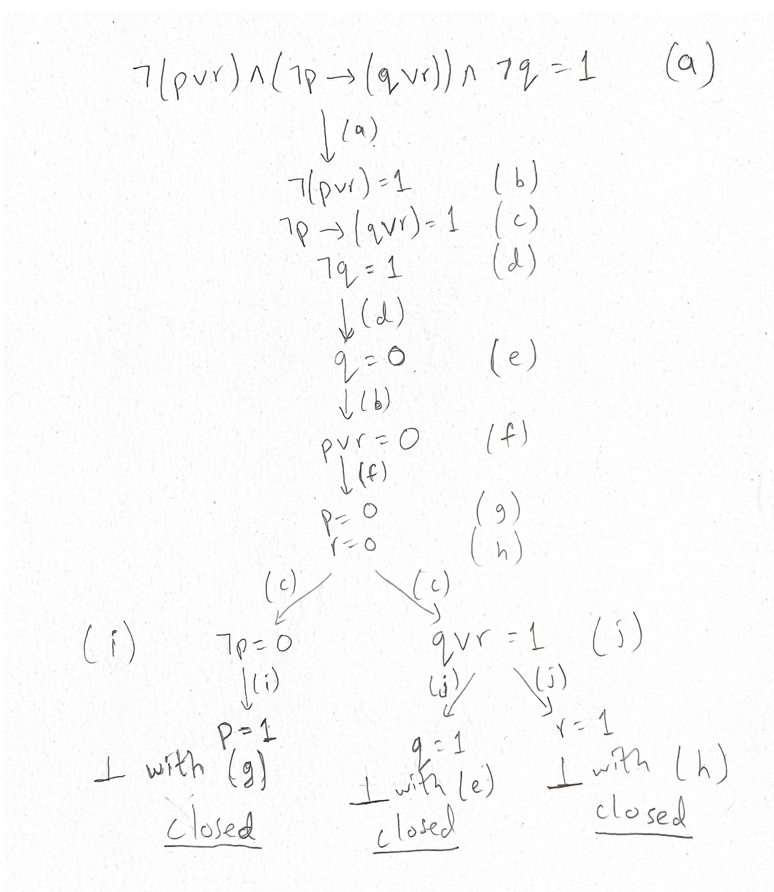

Figure 1: Semantic tableau of Q1.1

As we can see, there are no open branches for the given formula, therefore it is **unsatisfiable**.

2. Similar to part 1, we can see if the given formula is satisfiable or not by equating it to 1 and looking for an open branch in the terminated semantic tableau:

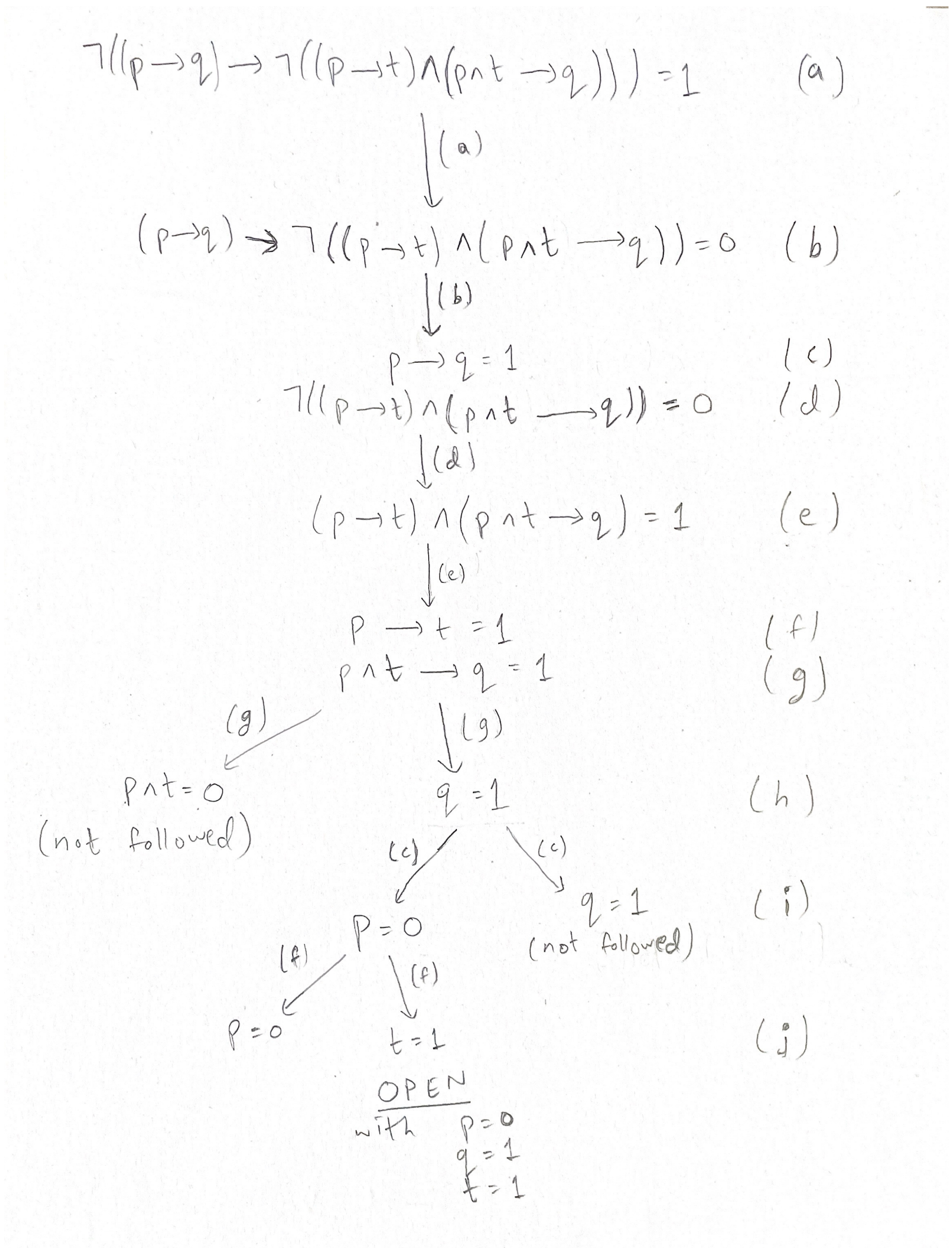

Figure 2: Semantic tableau of Q1.2

Since we found an open branch, the given formula is **satisfiable** with a possible interpretation of $p = 0,\ q = 1,\ t = 1$.

Answer for Q2

1. For DLL method, we need the formula in Conjunctive Normal Form (CNF). We will first convert the given formula F to CNF. Take:

$$F = A \wedge B \wedge C$$

Where

$$A = \neg(p \vee r) \quad | \quad B = \neg p \Rightarrow (q \vee r) \quad | \quad C = \neg q$$

Now, we simplify A using De Morgan's Law:

$$A = \neg(p \vee r) = \neg p \wedge \neg r$$

We simplify B using the implication property $a \Rightarrow b \equiv \neg a \vee b$:

$$B = \neg p \Rightarrow (q \vee r) = \neg(\neg p) \vee (q \vee r) = p \vee q \vee r$$

Now we can write the given formula in CNF:

$$F = A \wedge B \wedge C$$

$$F = (\neg p) \wedge (\neg r) \wedge (p \vee q \vee r) \wedge (\neg q)$$

Now, we can apply the DLL method to find satisfiability:

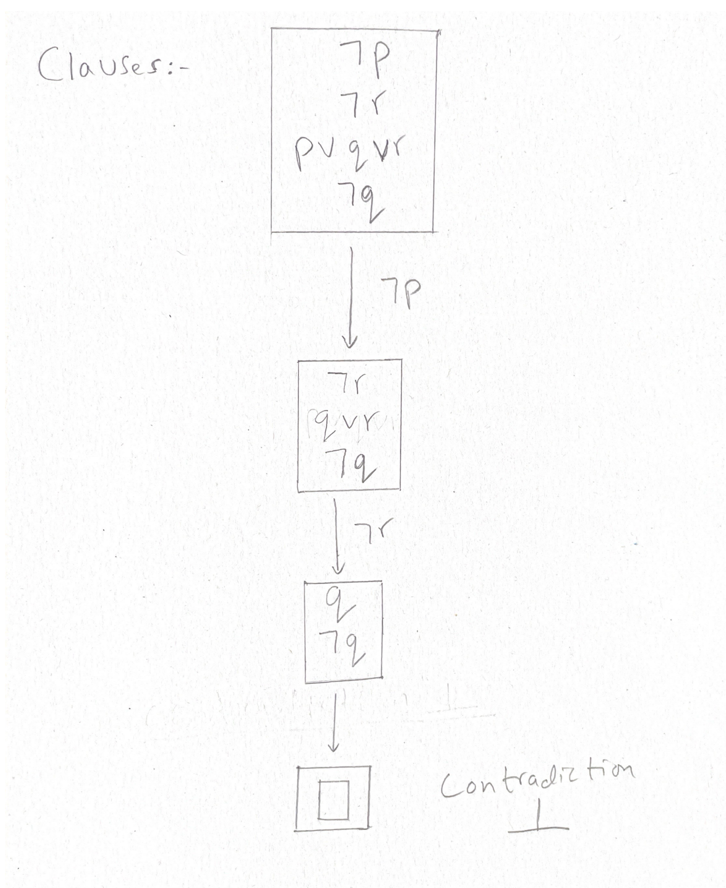

Figure 3: DLL method for Q2.1

Since there are no empty boxes and only a contradiction, the given formula is **unsatisfiable**.

b. Similar to Q2.1, the given formula F is not in CNF, so we must first convert it to CNF. Take:

$$A = (p \land q \land \neg w) \Rightarrow r$$

$$B = (p \land r) \Rightarrow (q \lor w)$$

$$C = q \Rightarrow \neg(r \land w)$$

$$D = q \lor \neg r \lor \neg w$$

Now, we can simplify our formula:

$$F = \neg(A \Rightarrow \neg(B \land p \land C)) \land D$$

Using the implication property $a \Rightarrow b \equiv \neg a \lor b$:

$$F = \neg(\neg A \lor \neg(B \land p \land C)) \land D$$

Using De Morgan's Law:

$$F = \neg\neg A \land \neg\neg(B \land p \land C) \land D$$

$$F = A \land B \land p \land C \land D$$

Now, we simplify A:

$$A = (p \land q \land \neg w) \Rightarrow r$$

Using implication property:

$$A = \neg(p \land q \land \neg w) \lor r$$

Using De Morgan's Law:

$$A = \neg p \lor \neg q \lor w \lor r$$

$$A = \neg p \lor \neg q \lor r \lor w$$

Now, we simplify B:

$$B = (p \land r) \Rightarrow (q \lor w)$$

Using implication property:

$$B = \neg(p \land r) \lor (q \lor w)$$

Using De Morgan's Law:

$$B = \neg p \lor \neg r \lor q \lor w$$

$$B = \neg p \lor q \lor \neg r \lor w$$

Now, we simplify C:

$$C = q \Rightarrow \neg(r \land w)$$

Using implication property:

$$C = \neg q \lor \neg(r \land w)$$

Using De Morgan's Law:

$$C = \neg q \lor \neg r \lor \neg w$$

Finally, we can write the given formula F in CNF:

$$F = A \wedge B \wedge p \wedge C \wedge D$$

$$F = (\neg p \vee \neg q \vee r \vee w) \wedge (\neg p \vee q \vee \neg r \vee w) \wedge (p) \wedge (\neg q \vee \neg r \vee \neg w) \wedge (q \vee \neg r \vee \neg w)$$

Now, we apply the DLL method to find satisfiability:

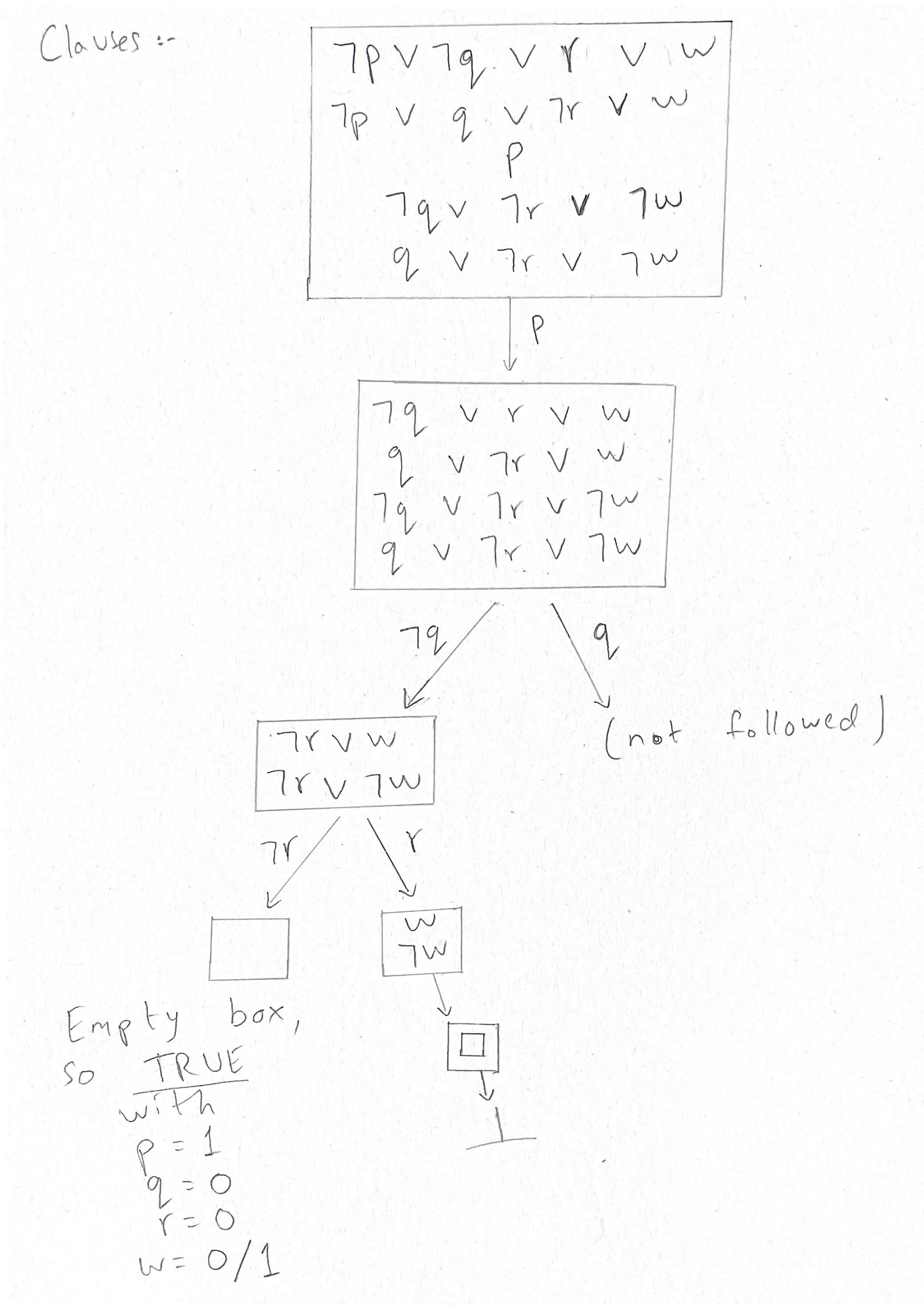

Figure 4: DLL method for Q2.2

Since we reached an empty box, the given formula is **satisfiable** with two possible interpretations of $p = 1,\ q = 0,\ r = 0,\ w =$either 0 or 1.