Middle East Technical University Department of Computer Engineering

CENG 424

Logic for Computer Science

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Assignment 4

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Answer 1

First, we convert the premises into relational logic:

1. $\forall_x(LOVES(Jane, x) \Rightarrow TRAVELER(x))$
2. $\forall_y((PERSON(y) \wedge \neg EARN(y)) \Rightarrow \neg TRAVEL(y))$
3. $DOCTOR(Jim)$
4. $\forall_z(DOCTOR(z) \Rightarrow PERSON(z))$
5. $\forall_v((DOCTOR(v) \wedge \neg WORK(v)) \Rightarrow \neg EARN(v))$
6. $\forall_w(\neg TRAVEL(w) \Rightarrow \neg TRAVELER(w))$

Now, we convert the goal into relational logic: $\neg WORK(Jim) \Rightarrow \neg LOVES(Jane, Jim)$

Negated goal: $\neg(\neg WORK(Jim) \Rightarrow \neg LOVES(Jane, Jim))$

Now, we convert the premises and negated goal to clausal normal form using the INSEADO method:

1. $\forall_x(LOVES(Jane, x) \Rightarrow TRAVELER(x))$
   $\forall_x(\neg LOVES(Jane, x) \vee TRAVELER(x))$
   $\neg LOVES(Jane, x) \vee TRAVELER(x)$
   $\{\neg LOVES(Jane, x), TRAVELER(x)\}$

2. $\forall_y((PERSON(y) \wedge \neg EARN(y)) \Rightarrow \neg TRAVEL(y))$
   $\forall_y(\neg(PERSON(y) \wedge \neg EARN(y)) \vee \neg TRAVEL(y))$
   $\forall_y(\neg PERSON(y) \vee \neg\neg EARN(y) \vee \neg TRAVEL(y))$
   $\forall_y(\neg PERSON(y) \vee EARN(y) \vee \neg TRAVEL(y))$
   $\neg PERSON(y) \vee EARN(y) \vee \neg TRAVEL(y)$
   $\{\neg PERSON(y), EARN(y), \neg TRAVEL(y)\}$

3. $DOCTOR(Jim)$
   $\{DOCTOR(Jim)\}$

4. $\forall_z(DOCTOR(z) \Rightarrow PERSON(z))$
   $\forall_z(\neg DOCTOR(z) \vee PERSON(z))$
   $\neg DOCTOR(z) \vee PERSON(z)$
   $\{\neg DOCTOR(z), PERSON(z)\}$

5. $\forall_v((DOCTOR(v) \wedge \neg WORK(v)) \Rightarrow \neg EARN(v))$
   $\forall_v(\neg(DOCTOR(v) \wedge \neg WORK(v)) \vee \neg EARN(v))$
   $\forall_v(\neg DOCTOR(v) \vee \neg\neg WORK(v) \vee \neg EARN(v))$
   $\forall_v(\neg DOCTOR(v) \vee WORK(v) \vee \neg EARN(v))$
   $\neg DOCTOR(v) \vee WORK(v) \vee \neg EARN(v)$
   $\{\neg DOCTOR(v), WORK(v), \neg EARN(v)\}$

6. $\forall_w(\neg TRAVEL(w) \Rightarrow \neg TRAVELER(w))$
   $\forall_w(\neg\neg TRAVEL(w) \vee \neg TRAVELER(w))$
   $\forall_w(TRAVEL(w) \vee \neg TRAVELER(w))$
   $TRAVEL(w) \vee \neg TRAVELER(w)$
   $\{TRAVEL(w), \neg TRAVELER(w)\}$

Now, using INSEADO on the negated goal:

$\neg(\neg WORK(Jim) \Rightarrow \neg LOVES(Jane, Jim))$
$\neg(\neg\neg WORK(Jim) \vee \neg LOVES(Jane, Jim))$
$\neg WORK(Jim) \wedge \neg\neg LOVES(Jane, Jim)$
$\neg WORK(Jim) \wedge LOVES(Jane, Jim)$
$\{\neg WORK(Jim)\},\ \{LOVES(Jane, Jim)\}$

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Finally, we can use relational resolution using the CNF of premises and negated goal:

1.	$\{\neg LOVES(Jane, x), TRAVELER(x)\}$	Premise
2.	$\{\neg PERSON(y), EARN(y), \neg TRAVEL(y)\}$	Premise
3.	$\{DOCTOR(Jim)\}$	Premise
4.	$\{\neg DOCTOR(z), PERSON(z)\}$	Premise
5.	$\{\neg DOCTOR(v), WORK(v), \neg EARN(v)\}$	Premise
6.	$\{TRAVEL(w), \neg TRAVELER(w)\}$	Premise
7.	$\{\neg WORK(Jim)\}$	Negated Goal
8.	$\{LOVES(Jane, Jim)\}$	Negated Goal
9.	$\{TRAVELER(Jim)\}$	1, 8
10.	$\{TRAVEL(Jim)\}$	6, 9
11.	$\{\neg PERSON(Jim), EARN(Jim)\}$	2, 10
12.	$\{\neg DOCTOR(Jim), WORK(Jim), \neg PERSON(Jim)\}$	5, 11
13.	$\{\neg DOCTOR(Jim), WORK(Jim)\}$	4, 12
14.	$\{WORK(Jim)\}$	3, 13
15.	$\{\}$	7, 14

Since we arrived at an empty clause (contradiction) with the negated goal, we have proven our original goal is true.

Answer 2

Since we know that it should make a correct operation, each run should start at q_0, output the exact string "aba", and end at q_4. From inspection, I have found there to be a total of 6 correct operations outputting aba:

1. $q_0 \xrightarrow{0.4/a} q_2 \xrightarrow{0.9/b} q_3 \xrightarrow{0.3/a} q_4$
2. $q_0 \xrightarrow{0.4/\epsilon} q_3 \xrightarrow{0.3/a} q_1 \xrightarrow{0.9/\epsilon} q_3 \xrightarrow{0.4/b} q_4 \xrightarrow{0.1/\epsilon} q_3 \xrightarrow{0.3/a} q_4$
3. $q_0 \xrightarrow{0.4/\epsilon} q_3 \xrightarrow{0.3/a} q_1 \xrightarrow{0.9/\epsilon} q_3 \xrightarrow{0.4/b} q_4 \xrightarrow{0.1/a} q_4$
4. $q_0 \xrightarrow{0.4/\epsilon} q_3 \xrightarrow{0.3/a} q_4 \xrightarrow{0.2/b} q_3 \xrightarrow{0.3/a} q_4$
5. $q_0 \xrightarrow{0.4/\epsilon} q_3 \xrightarrow{0.3/a} q_4 \xrightarrow{0.1/\epsilon} q_3 \xrightarrow{0.4/b} q_4 \xrightarrow{0.1/\epsilon} q_3 \xrightarrow{0.3/a} q_4$
6. $q_0 \xrightarrow{0.4/\epsilon} q_3 \xrightarrow{0.3/a} q_4 \xrightarrow{0.1/\epsilon} q_3 \xrightarrow{0.4/b} q_4 \xrightarrow{0.1/a} q_4$

The above doesn't answer the sub-questions, but they will be useful in formulating their answers.

a.

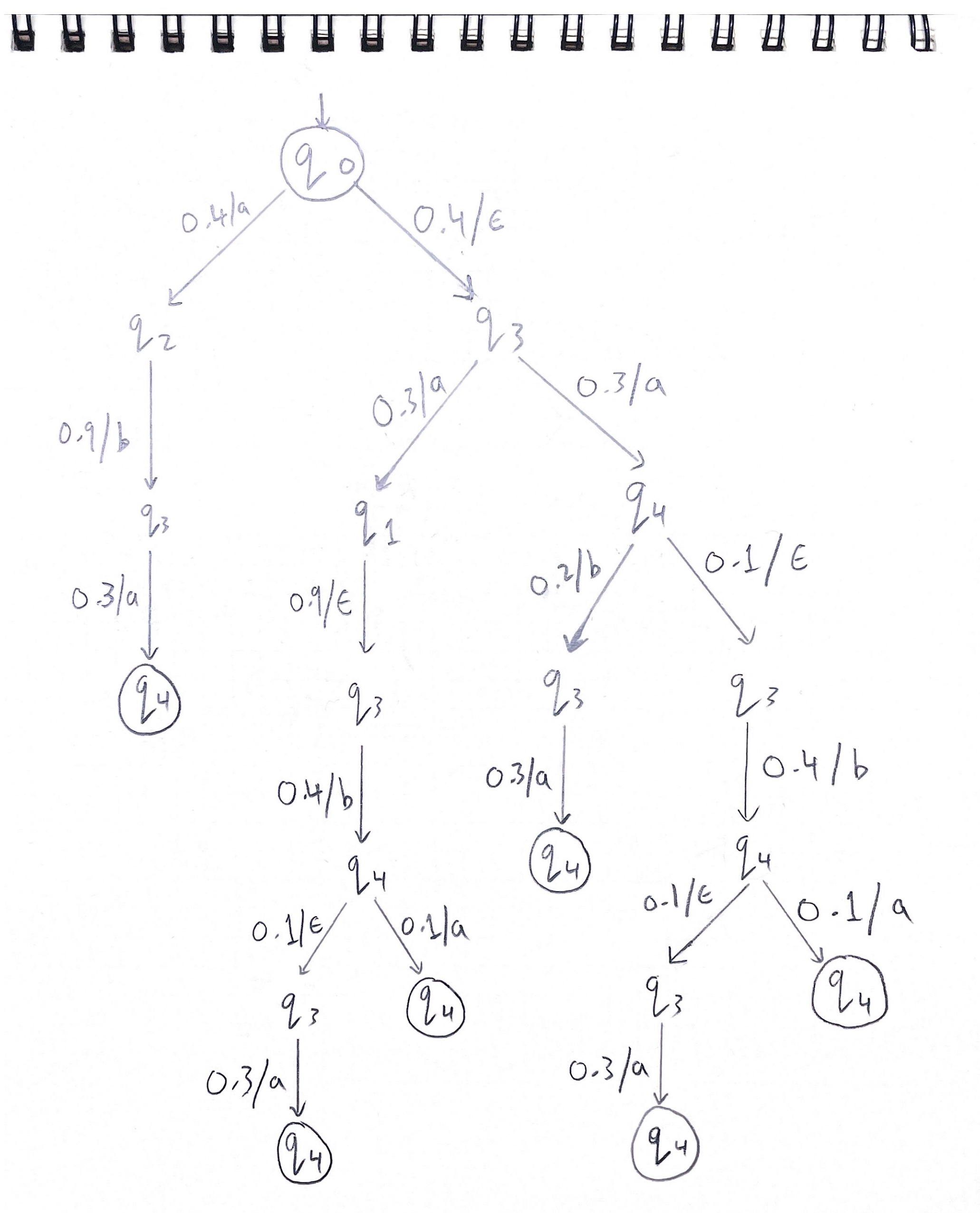
For this part, we can create a relation named T:

$$T(q, q', p, o)$$

Here, T is the relation for the transition, q is the initial state, q' is the final state, p is the probability of transition, and o is the output symbol, which can be either a, b, or ϵ (denoting no output).

b.

The partial probabilistic computation tree is given below. There are 6 total pathways ending at q_4. For each pathway, the probability should be multiplied by 0.6 to account for the probability of it terminating at state q_4.



c.

All the transitions are relevant, so the premises are:

1. $T(q_0, q_2, 0.4, a)$
2. $T(q_0, q_3, 0.4, \epsilon)$
3. $T(q_1, q_3, 0.9, \epsilon)$
4. $T(q_2, q_3, 0.9, b)$
5. $T(q_3, q_1, 0.3, a)$
6. $T(q_3, q_4, 0.3, a)$
7. $T(q_3, q_4, 0.4, b)$
8. $T(q_4, q_3, 0.1, \epsilon)$
9. $T(q_4, q_3, 0.2, b)$
10. $T(q_4, q_4, 0.1, a)$

d.

I will add a few extra premises to enforce the rules, and I will explain each of them:

1.

$$(T(x, y, p_1, o_1) \wedge T(y, z, p_2, o_2)) \rightarrow T(x, z, p_1 * p_2, o_1o_2)$$
$$\neg(T(x, y, p_1, o_1) \wedge T(y, z, p_2, o_2)) \vee T(x, z, p_1 * p_2, o_1o_2)$$
$$\neg T(x, y, p_1, o_1) \vee \neg T(y, z, p_2, o_2)) \vee T(x, z, p_1 * p_2, o_1o_2)$$
$$\{\neg T(x, y, p_1, o_1), \neg T(y, z, p_2, o_2)), T(x, z, p_1 * p_2, o_1o_2)\}$$

   This is the rule of transitivity, if there is a transition from x to y with probability p_1 and output o_1, as well as a transition from y to z with probability p_2 and output o_2, then through transitivity, there is a transition from x to z with probability $p_1 * p_2$ and output o_1o_2 concatenated with each other.

2.

$$T(q_0, q_4, p, aba) \rightarrow P(p * 0.6)$$
$$\neg T(q_0, q_4, p, aba) \vee P(p * 0.6)$$
$$\{\neg T(q_0, q_4, p, aba), P(p * 0.6)\}$$

   This rule represents the completion of a followed path. Here, the relation P represents *path*. A path is only valid if it started at q_0 and ended at q_4 and outputted the string aba, and all these conditions are being checked. We are multiplying the final probability of the path taken by 0.6 to account for the probability of the path ending at state q_4.

3.

$$(P(p_1) \wedge P(p_2)) \rightarrow goal(p_1 + p_2)$$
$$\neg(P(p_1) \wedge P(p_2)) \vee goal(p_1 + p_2)$$
$$\neg P(p_1) \vee \neg P(p_2) \vee goal(p_1 + p_2)$$
$$\{\neg P(p_1), \neg P(p_2), goal(p_1 + p_2)\}$$

This rule states that the probabilities of two paths can be added together to get the combined probability of outputting *aba* by traversing those paths. This rule will only be used once, to initiate the summation procedure, because of the existence of the next rule.

4.

$$(P(p_1) \wedge goal(p_2)) \rightarrow goal(p_1 + p_2)$$
$$\neg(P(p_1) \wedge goal(p_2)) \vee goal(p_1 + p_2)$$
$$\neg P(p_1) \vee \neg goal(p_2) \vee goal(p_1 + p_2)$$
$$\{\neg P(p_1), \neg goal(p_2), goal(p_1 + p_2)\}$$

This rule is similar to the previous one, it allows chaining of the *goal* relation so we can keep summing the probabilities of each distinct path one by one. The final value in *goal* will be the final probability for the question.

e.

With all the premises above, we are ready to use answer extraction method in relational resolution to find the final probability of PTS_1 making a correct operation and outputting *aba*:

1.	$\{T(q_0, q_2, 0.4, a)\}$	Premise from (c)
2.	$\{T(q_0, q_3, 0.4, \epsilon)\}$	Premise from (c)
3.	$\{T(q_1, q_3, 0.9, \epsilon)\}$	Premise from (c)
4.	$\{T(q_2, q_3, 0.9, b)\}$	Premise from (c)
5.	$\{T(q_3, q_1, 0.3, a)\}$	Premise from (c)
6.	$\{T(q_3, q_4, 0.3, a)\}$	Premise from (c)
7.	$\{T(q_3, q_4, 0.4, b)\}$	Premise from (c)
8.	$\{T(q_4, q_3, 0.1, \epsilon)\}$	Premise from (c)
9.	$\{T(q_4, q_3, 0.2, b)\}$	Premise from (c)
10.	$\{T(q_4, q_4, 0.1, a)\}$	Premise from (c)
11.	$\{\neg T(x, y, p_1, o_1), \neg T(y, z, p_2, o_2), T(x, z, p_1 * p_2, o_1 o_2)\}$	Premise from (d)
12.	$\{\neg T(q_0, q_4, p, aba), P(p * 0.6)\}$	Premise from (d)

13.	$\{\neg P(p_1), \neg P(p_2), goal(p_1 + p_2)\}$	Premise from (d)
14.	$\{\neg P(p_1), \neg goal(p_2), goal(p_1 + p_2)\}$	Premise from (d)
15.	$\{\neg T(q_2, z, p_2, o_2), T(q_0, z, 0.4 * p_2, ao_2)\}$	1, 11
16.	$\{T(q_0, q_3, 0.36, ab)\}$	4, 15
17.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.36 * p_2, abo_2)\}$	11, 16
18.	$\{T(q_0, q_4, 0.108, aba)\}$	6, 17
19.	$\{P(0.0648)\}$	12, 18 (Path #1)
20.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.4 * p_2, o_2)\}$	2, 11
21.	$\{T(q_0, q_1, 0.12, a)\}$	5, 20
22.	$\{\neg T(q_1, z, p_2, o_2), T(q_0, z, 0.12 * p_2, ao_2)\}$	11, 21
23.	$\{T(q_0, q_3, 0.108, a)\}$	3, 22
24.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.108 * p_2, ao_2)\}$	11, 23
25.	$\{T(q_0, q_4, 0.0432, ab)\}$	7, 24
26.	$\{\neg T(q_4, z, p_2, o_2), T(q_0, z, 0.0432 * p_2, abo_2)\}$	11, 25
27.	$\{T(q_0, q_3, 0.00432, ab)\}$	8, 26
28.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.00432 * p_2, abo_2)\}$	11, 27
29.	$\{T(q_0, q_4, 0.001296, aba)\}$	6, 28
30.	$\{P(0.0007776)\}$	12, 29 (Path #2)
31.	$\{\neg T(q_4, z, p_2, o_2), T(q_0, z, 0.0432 * p_2, abo_2)\}$	11, 25
32.	$\{T(q_0, q_4, 0.00432, aba)\}$	10, 31
33.	$\{P(0.002592)\}$	12, 32 (Path #3)
34.	$\{T(q_0, q_4, 0.12, a)\}$	6, 20
35.	$\{\neg T(q_4, z, p_2, o_2), T(q_0, z, 0.12 * p_2, ao_2)\}$	11, 34
36.	$\{T(q_0, q_3, 0.024, ab)\}$	9, 35
37.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.024 * p_2, abo_2)\}$	11, 36
38.	$\{T(q_0, q_4, 0.0072, aba)\}$	6, 37
39.	$\{P(0.00432)\}$	12, 38 (Path #4)
40.	$\{\neg T(q_4, z, p_2, o_2), T(q_0, z, 0.12 * p_2, ao_2)\}$	11, 34
41.	$\{T(q_0, q_3, 0.012, a)\}$	8, 40
42.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.012 * p_2, ao_2)\}$	11, 41
43.	$\{T(q_0, q_4, 0.0048, ab)\}$	7, 42
44.	$\{\neg T(q_4, z, p_2, o_2), T(q_0, z, 0.0048 * p_2, abo_2)\}$	11, 43

45.	$\{T(q_0, q_3, 0.00048, ab)\}$	8, 44
46.	$\{\neg T(q_3, z, p_2, o_2), T(q_0, z, 0.00048 * p_2, abo_2)\}$	11, 45
47.	$\{T(q_0, q_4, 0.000144, aba)\}$	6, 46
48.	$\{P(0.0000864)\}$	12, 47 (Path #5)
49.	$\{T(q_0, q_4, 0.00048, aba)\}$	10, 44
50.	$\{P(0.000288)\}$	12, 49 (Path #6)
51.	$\{\neg P(p_2), goal(0.0648 + p_2)\}$	13, 19
52.	$\{goal(0.0655776)\}$	30, 51 (Paths 1, 2)
53.	$\{\neg P(p_1), goal(p_1 + 0.0655776)\}$	14, 52
54.	$\{goal(0.0681696)\}$	33, 53 (Paths 1, 2, 3)
55.	$\{\neg P(p_1), goal(p_1 + 0.0681696)\}$	14, 54
56.	$\{goal(0.0724896)\}$	39, 55 (Paths 1, 2, 3, 4)
57.	$\{\neg P(p_1), goal(p_1 + 0.0724896)\}$	14, 56
58.	$\{goal(0.072576)\}$	48, 57 (Paths 1, 2, 3, 4, 5)
59.	$\{\neg P(p_1), goal(p_1 + 0.072576)\}$	14, 58
60.	$\{goal(0.072864)\}$	50, 59 (All paths 1-6)

We finally arrived at our goal using only a single resolution instance, by computing the probabilities of going down each path, and then summing them. The final answer to the question is $\boldsymbol{p = 0.072864}$.