Middle East Technical University Department of Computer Engineering

CENG 424

Logic for Computer Science

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Assignment 3

Name SURNAME: Aly Asad GILANI
Student ID: 2547875

1 Answer

1.1 The only thing I know is that I know nothing.

First, lets define our constants:

- $K(x, y)$: Relation constant, represents whether x knows y.
- i: Object constant, represents the speaker of the sentence.
- n: Object constant, represents the proposition "nothing".

Now, using these constants, we can construct the following relational logic sentence:

$$K(i, n) \land \forall_x (K(i, x) \Rightarrow (x \Leftrightarrow n))$$

The statement to the left of the AND ($\land$) means that the speaker "knows nothing", and the right statement means that the speaker knows nothing else.

1.2 A period is a punctuation mark that indicates an abbreviation or the end of a sentence.

First, lets define our constants:

- $P(x)$: Relation constant, represents whether x is a punctuation mark.
- $I(x, y)$: Relation constant, represents whether x "indicates" y.
- *period*: Object constant, represents the period.
- *abbr*: Object constant, represents abbreviation.
- *end*: Object constant, represents the end of a sentence.

Now, using these constants, we can construct the following relational logic sentence:

$$P(period) \land (I(period, abbr) \lor I(period, end))$$

The statement to the left of the AND ($\land$) means that the period is a punctuation mark, and the right statement means that the period indicates an abbreviation or the end of a sentence.

2 Answer

We have three premises, and we need to prove the conclusion. My steps:

1.	$\forall_x (P(x) \vee Q(x))$	Premise
2.	$\exists_x \neg Q(x)$	Premise
3.	$\forall_x (R(x) \Rightarrow \neg P(x))$	Premise
4.	$\neg Q(c)$	$\exists_x$ Instantiation: 2
5.	$P(c) \vee Q(c)$	$\forall_x$ Instantiation: 1
6.	$P(c)$	$\vee$ Elimination: 4, 5
7.	$R(c) \Rightarrow \neg P(c)$	$\forall_x$ Instantiation: 3
8.	$\neg R(c)$	Modus Tollens: 6,7
9.	$\exists_y \neg R(y)$	$\exists_y$ Generalization: 8

Since we have reached the conclusion, we have proven the validity of the question. All the rules I used have been taught in class and are present in the slides.

3 Answer

For this question, I will first create the set of premises, all in terms of the relation $P(x, y, p)$:

1. $P(a, b, 0.6)$
2. $P(a, c, 0.4)$
3. $P(b, d, 0.3)$
4. $P(b, e, 0.5)$
5. $P(b, f, 0.2)$
6. $P(c, g, 0.9)$
7. $P(c, h, 0.1)$
8. $P(e, i, 0.9)$
9. $P(e, j, 0.1)$

From the hint provided, we can formalize the property that given the premises 1) $P(x, y, p_1)$ and 2) $P(y, z, p_2)$, we can derive 3) $P(x, z, p_1 \times p_2)$ using (1) and (2). We will name this property "Transitivity".

Now, we can use the premises and the Transitivity property to find the probability ending at node j:

1.	$P(a, b, 0.6)$	Premise
2.	$P(a, c, 0.4)$	Premise
3.	$P(b, d, 0.3)$	Premise
4.	$P(b, e, 0.5)$	Premise
5.	$P(b, f, 0.2)$	Premise
6.	$P(c, g, 0.9)$	Premise
7.	$P(c, h, 0.1)$	Premise
8.	$P(e, i, 0.9)$	Premise
9.	$P(e, j, 0.1)$	Premise
10.	$P(a, e, 0.3)$	Transitivity: 1,4
11.	$P(a, j, 0.03)$	Transitivity: 10, 9

Since we have reached node j from the starting node a, we have found the probability of the computation as **0.03**.