Middle East Technical University | Department of Computer Engineering

CENG 424

Logic For Computer Science

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Assignment 2

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Answer 1

1. My proof:

1	$p \Rightarrow q$	Premise
2	$q \Rightarrow r$	Premise
3	$(\neg\neg p \Rightarrow r) \Rightarrow ((\neg\neg p \Rightarrow \neg r) \Rightarrow \neg p)$	CR
4	$(p \Rightarrow r) \Rightarrow ((p \Rightarrow \neg r) \Rightarrow \neg p)$	Double Negation Elimination: 3
5	$(q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$	II
6	$p \Rightarrow (q \Rightarrow r)$	MP: 5,2
7	$(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$	ID
8	$(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$	MP: 7,6
9	$p \Rightarrow r$	MP: 8,1
10	$((p \Rightarrow \neg r) \Rightarrow \neg p)$	MP: 4,9

Firstly, I realized that the final answer resembles the form of the consequent of the CR rule, so I wrote the rule and worked towards proving the antecedent true so I could apply Modus Ponens. Proving the antecedent was simple, as we have studied this exact case in our lectures.

2. My proof:

1	$\neg s \vee \neg r \Rightarrow \neg t$	Premise
2	$s \Rightarrow p \vee \neg r$	Premise
3	$p \wedge s \Rightarrow q$	Premise
4	t	Assumption
5	$\neg(\neg s \vee \neg r) \vee \neg t$	Implication Elimination: 1
6	$\neg(\neg s \vee \neg r)$	Disjunction Elimination: 5,4
7	$(s \wedge r) \Leftrightarrow \neg(\neg s \vee \neg r)$	OQ
8	$s \wedge r$	Equivalence: 7,6
9	s	Conjunction Elimination: 8
10	r	Conjunction Elimination: 8
11	$p \vee \neg r$	MP: 2,9
12	p	Disjunction Elimination: 11,10
13	$p \wedge s$	Conjunction Introduction: 12,9
14	q	MP: 3,13

Since I assumed t as a premise and derived q, from the deduction theorem, I proved $t \Rightarrow q$. Firstly, since I could not use Modus Tollens, I had to use standard rules of inference to prove the negation of the antecedent of the first term. Then, using OQ/De Morgan's law I changed it to a conjunction of s and r, and from there to derive q we just had to follow the implications in terms 2 and 3 using Modus Ponens.

Answer 2

To use propositional resolution, we need to first convert the sentence into clausal form. Converting the premises:

$$\text{Premises: } (r \Rightarrow s) \wedge ((t \wedge q) \Rightarrow \neg s)$$

Implications out:

$$(\neg r \vee s) \wedge (\neg(t \wedge q) \vee \neg s)$$

Negations in:

$$(\neg r \vee s) \wedge (\neg t \vee \neg q \vee \neg s)$$

Distribution: -
Operators out:

$$\{\neg r, s\}, \{\neg t, \neg q, \neg s\}$$

Now, we convert the goal:

$$\text{Goal: } (t \wedge q) \Rightarrow \neg r$$

Implications out:

$$\neg(t \wedge q) \vee \neg r$$

Negations in:

$$\neg t \vee \neg q \vee \neg r$$

Distribution: -
Operators out:

$$\{\neg t, \neg q, \neg r\}$$

First, we will try to disprove the sentence. We will write the premises and goal, and try to derive the empty clause:

1	$\{\neg r, s\}$	Premise
2	$\{\neg t, \neg q, \neg s\}$	Premise
3	$\{\neg t, \neg q, \neg r\}$	Goal
4	$\{\neg t, \neg q, \neg r\}$	2,1

Since we are stuck and could not derive the empty clause, the statement is probably true. We will verify by now writing the premises with the negated goal and trying to derive the empty clause.

Negated goal: $\neg(\neg t \vee \neg q \vee \neg r)$

Negations in:

$$t \wedge q \wedge r$$

Operators out:

$$\{t\}, \{q\}, \{r\}$$

Now for the proof:

1	$\{\neg r, s\}$	Premise
2	$\{\neg t, \neg q, \neg s\}$	Premise
3	$\{t\}$	Negated Goal
4	$\{q\}$	Negated Goal
5	$\{r\}$	Negated Goal
6	$\{\neg q, \neg s\}$	3,2
7	$\{\neg s\}$	6,4
8	$\{s\}$	5,1
9	$\{\}$	8,7

Since we derived the empty clause for the negated goal, the sentence is **true**.

Answer 3

1. We can write each sentence as a propositional statement:

i) $(HOL \wedge SNO) \Rightarrow GSK$
ii) $(GFR \vee GFL) \wedge (\neg GFR \vee \neg GFL)$
iii) $\neg SFL$
iv) SFR
v) $HOL \wedge GFL$
vi) $(SFL \wedge GSK) \Rightarrow GFL$
vii) $(SFR \wedge GSK) \Rightarrow GFR$

We assume there is conjunction in between the statements. Note: I changed the second statement (added the second bracket after $\wedge$ to imply that he cannot be in both places at the same time) and added statements vi) and vii) after the correction from ODTUClass.

2.
First, we have to convert the above propositional statements into CNF clauses. First statement:

$$\text{i) } (HOL \wedge SNO) \Rightarrow GSK$$

$$\neg(HOL \wedge SNO) \vee GSK$$

$$\neg HOL \vee \neg SNO \vee GSK$$

$$\{\neg HOL, \neg SNO, GSK\}$$

Second statement:

$$\text{ii) } (GFR \vee GFL) \wedge (\neg GFR \vee \neg GFL)$$

$$\{GFR, GFL\},\ \{\neg GFR, \neg GFL\}$$

Third statement:

$$\text{iii) } \neg SFL$$

$$\{\neg SFL\}$$

Fourth statement:

$$\text{iv) } SFR$$

$$\{SFR\}$$

Fifth statement:

$$\text{v) } HOL \wedge GFL$$

$$\{HOL\},\ \{GFL\}$$

Sixth statement:

$$\text{vi) } (SFL \land GSK) \Rightarrow GFL$$
$$\neg(SFL \land GSK) \lor GFL$$
$$\neg SFL \lor \neg GSK \lor GFL$$
$$\{\neg SFL, \neg GSK, GFL\}$$

Seventh statement:

$$\text{vii) } (SFR \land GSK) \Rightarrow GFR$$
$$\neg(SFR \land GSK) \lor GFR$$
$$\neg SFR \lor \neg GSK \lor GFR$$
$$\{\neg SFR, \neg GSK, GFR\}$$

Now, since we need to prove it is not snowing, our goal is: $\neg SNO$. The negated goal will be: SNO.

Now, we can finally construct the proof:

1	$\{\neg HOL, \neg SNO, GSK\}$	Premise
2	$\{GFR, GFL\}$	Premise
3	$\{\neg GFR, \neg GFL\}$	Premise
4	$\{\neg SFL\}$	Premise
5	$\{SFR\}$	Premise
6	$\{HOL\}$	Premise
7	$\{GFL\}$	Premise
8	$\{\neg SFL, \neg GSK, GFL\}$	Premise
9	$\{\neg SFR, \neg GSK, GFR\}$	Premise
10	$\{SNO\}$	Negated Goal
11	$\{\neg HOL, GSK\}$	10,1
12	$\{GSK\}$	11,6
13	$\{\neg SFR, GFR\}$	12,9
14	$\{GFR\}$	13,5
15	$\{\neg GFR\}$	7,3
16	$\{\}$	15,14

Since we derived the empty clause for the negated goal, we proved it is **not snowing**.