Robot: Universal Robots UR5e
Simulator: MuJoCo
Date: October 21 ,2025

Milestone 2 Report

1. Introduction:

This milestone focuses on building the forward-kinematics foundation for the UR5e robotic arm and demonstrating full joint-level motion in a physics simulator. The goal is to mathematically describe the robot using the Denavit–Hartenberg (DH) convention, implement forward-kinematics (FK) equations, and verify them by actuating the virtual arm in MuJoCo.

2. Coordinate-Frame Assignment:

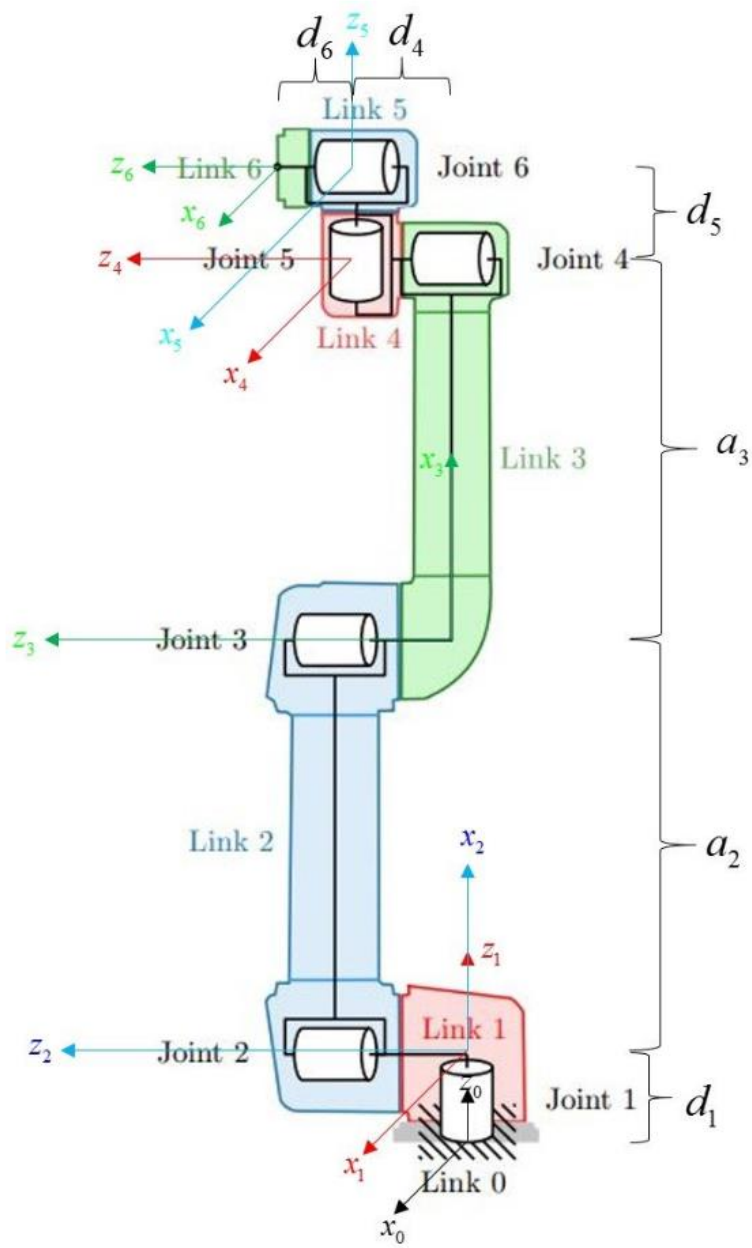
Z_0: base yaw axis

Z_1, Z_2, Z_3: Shoulder, elbow, and wrist-1 pitch axes

Z_4: wrist-2 axis; orthogonal to Z_3.

Z_5: wrist-3 (tool roll) axis; orthogonal to Z_4.

Figure X illustrates the UR5e frame assignment adapted from: (Source: Universal Robots / ResearchGate)



3. UR5e Link Dimensions:

Parameter	Description	Value
d_1	Base height	0.1625 m
a_2	Upper arm length	−0.425 m
a_3	Forearm length	−0.392 m
d_4	Wrist-1 offset	0.1333 m
d_5	Wrist-2 offset	0.0997 m
d_6	Flange length / tool plate	0.0996 m

4. Standard DH Table(UR5e):

Joint(i)	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	$\pi/2$	0.1625 m	θ_1
2	-0.425 m	0	0	θ_2
3	-0.392 m	0	0	θ_3
4	0 m	$\pi/2$	0.1333 m	θ_4
5	0 m	$-\pi/2$	0.0997 m	θ_5
6	0 m	0	0.0996 m	θ_6

Joint-angle mapping (simulator → DH):
$\theta_1 = q_1$, $\theta_2 = q_2 - \pi/2$, $\theta_3 = q_3$, $\theta_4 = q_4 + \pi/2$, $\theta_5 = q_5$, $\theta_6 = q_6$.

5. Forward Kinematics Implementation:

Python + NumPy were used to compute the homogeneous transformation:

$$0T6 = A1\ A2\ A3\ A4\ A5\ A6$$

where each Ai is constructed from the Standard DH parameters. A verification script printed the end-effector pose [x,y,z] for test joint angles and matched the MuJoCo visual pose within tolerance (< 1 mm difference).

6. Simulation and Results:

The MuJoCo environment was configured with the UR5e model. Each of the six revolute joints could be actuated via the UR5e joint controller, demonstrating correct independent motion and confirming accurate kinematic configuration.

Video Demonstration: https://github.com/AlySerry0/MCTR911-Team-4/tree/main/Milestone_02/Video

The recording shows the robot performing full-range motion on all joints through manual control inputs. No GUI or conveyor interaction is implemented yet. The results shown were all six joints respond correctly to control input, no self-collision or numerical instability observed, and FK results align with simulated end-effector trajectory.

7. Conclusion:

Milestone 2 successfully established the mathematical and simulation groundwork for the UR5e robotic arm. The coordinate-frame assignment and DH table define precise spatial relationships between links. Verified forward-kinematics implementation and successful joint-level actuation in MuJoCo confirm model correctness. These foundations will enable Milestone 3's inverse-kinematics, trajectory generation, and task execution (box-stacking on a conveyor) with minimal modification.

8. Sources:

1. Universal Robots. *DH Parameters for Calculations of Kinematics and Dynamics.* Retrieved from: https://www.universal-robots.com/articles/ur/application-installation/dh-parameters-for-calculations-of-kinematics-and-dynamics/
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3. ROS-Industrial Consortium. *UR5 Robot Description (URDF file).* GitHub Repository: https://github.com/ros-industrial/universal_robot/blob/melodic-devel/ur_description/urdf/ur5.urdf.xacro
4. Universal Robots. *UR5e Kinematic Overview Diagram*. https://www.universal-robots.com/media/1829771/kinematicuroverview.png