

# Cosmological Phase Transitions and Gravitational Wave Signatures from Topological Mass Generation in Quantum Cellular Automata

Haobo Ma<sup>1</sup> and Wenlin Zhang<sup>2</sup>

<sup>1</sup>*Independent Researcher*

<sup>2</sup>*National University of Singapore*

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We formulate a quantum cellular automaton (QCA) cosmology in which Dirac mass emerges as a topological impedance of the underlying discrete dynamics. Using a split-step QCA we show that the effective Dirac mass is controlled by a coin parameter that carries a chiral winding number, and that an information-scrambling phase transition dynamically freezes this parameter from a ballistic to a massive topological phase. Imposing a Micro-Parallelism Axiom on the local Hilbert space leads to a kinematic symmetry isomorphic to  $SU(3) \times SU(2) \times U(1)$ , reproducing the Standard Model gauge group upon gauging and quotienting by the common center. We further derive a holographic entropy constraint that relates the cosmological constant  $\Lambda$  to the QCA cell size and a holographic redundancy factor  $\xi$ , so that an impedance-freezing cosmological phase transition generically produces a stochastic gravitational-wave background whose ultraviolet cutoff frequency  $f_{\text{cut}} \sim c/(\pi\ell_{\text{cell}})$  and Bragg-like oscillations are fixed in terms of  $\Lambda$  and  $\xi$ . This provides a concrete observational handle on microscopic spacetime discreteness in this class of QCA cosmologies.

*Introduction.*—The observed acceleration of the universe and the tiny value of the cosmological constant  $\Lambda$  sit uneasily beside the apparently arbitrary pattern of Standard Model (SM) masses and charges. The Higgs mechanism explains how gauge bosons and fermions acquire mass, but it does not address why the SM gauge group is  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , nor why the vacuum energy is so small compared with microscopic scales. At the same time, the direct detection of gravitational waves has opened a new window on the early universe, in which stochastic gravitational-wave backgrounds (SGWBs) can probe physics far beyond the reach of colliders. Discrete and information-theoretic approaches suggest that continuum fields and particles may emerge from underlying quantum networks or cellular automata, with topology playing a central role in the effective excitations [1–3].

In this Letter we develop a cosmological scenario based on a quantum cellular automaton (QCA), in which Dirac mass is reinterpreted as a *topological impedance* of the discrete dynamics. Using a split-step QCA we show that: (i) the effective Dirac mass is controlled by a coin angle that carries a chiral winding number, and an information-scrambling phase transition dynamically freezes this angle from a ballistic to a massive topological phase; (ii) a Micro-Parallelism Axiom for the vacuum structure leads to a kinematic symmetry isomorphic to  $SU(3) \times SU(2) \times U(1)$ ; and (iii) enforcing a finite-entropy holographic bound introduces a large holographic redundancy factor  $\xi$  that relates the QCA cell size  $\ell_{\text{cell}}$  to  $\Lambda$ , so that an impedance-freezing transition generically produces an SGWB whose ultraviolet cutoff frequency obeys  $f_{\text{cut}} \propto \xi\sqrt{\Lambda}$ . This relation ties the macroscopic cosmological constant to a microscopic lattice scale and predicts a characteristic knee and Bragg-like oscillations in the GW spectrum, providing a concrete observational handle on microscopic spacetime discreteness within this QCA

framework.

*Mass as Topological Impedance.*—We model the universe, in a radially reduced description, as a split-step QCA on a cubic lattice  $\mathcal{L} \simeq \ell_{\text{cell}}\mathbb{Z}^3$ . The single-step evolution

$$U = S_- C(\theta_2) S_+ C(\theta_1)$$

combines conditional shifts  $S_{\pm}$  with coin rotations  $C(\theta) = e^{-i\theta\sigma_y}$  acting on a chiral spinor  $\Psi$  [3, 4]. In momentum space, the conditional shifts are well approximated, for long-wavelength modes, by a symmetric shift operator acting on the chiral basis,

$$S_{\pm}(k) \simeq \exp\left(-\frac{i}{2} k\ell_{\text{cell}} \sigma_z\right), \quad (1)$$

so that the Floquet operator can be written explicitly as

$$U(k) = e^{-ik\ell_{\text{cell}}\sigma_z/2} e^{-i\theta_2\sigma_y} e^{-ik\ell_{\text{cell}}\sigma_z/2} e^{-i\theta_1\sigma_y}. \quad (2)$$

For  $|k\ell_{\text{cell}}| \ll 1$  and  $|\theta_i| \ll 1$ , expanding Eq. (2) to first order in these small parameters yields

$$U(k) \simeq \mathbb{I} - i[(k\ell_{\text{cell}})\sigma_z + (\theta_1 + \theta_2)\sigma_y] + \mathcal{O}(k^2, \theta_i^2, k\theta_i). \quad (3)$$

Comparing this with  $U(k) = \exp[-iH_{\text{eff}}(k)\Delta t/\hbar]$ , we identify the effective Hamiltonian  $H_{\text{eff}}(k) \simeq (\hbar/\Delta t)[(k\ell_{\text{cell}})\sigma_z + (\theta_1 + \theta_2)\sigma_y]$ . Writing  $p = \hbar k$ , we recover the Dirac form

$$H_{\text{eff}} \simeq cp\sigma_z + mc^2\sigma_y, \quad (4)$$

with emergent light speed  $c \equiv \ell_{\text{cell}}/\Delta t$  and Dirac mass

$$mc^2 = \hbar(\theta_1 + \theta_2)/\Delta t. \quad (5)$$

Consequently, we interpret mass not as an intrinsic constant, but as a *topological impedance*: a local scattering amplitude obstructing ballistic information flow

along the QCA. For the split-step construction with time-reversal and particle-hole symmetries, the quasi-momentum Floquet operator  $\hat{U}(k)$  falls into the BDI symmetry class and admits a chiral winding number  $\mathcal{W}$  [5]. Along the parameter line relevant for our cosmological construction,  $\theta_{1,2} = 0$  corresponds to a topologically trivial, massless phase with  $\mathcal{W} = 0$ , while turning on a nonzero coin angle opens a gap with  $|\mathcal{W}| = 1$ , i.e., a massive topological phase protected by topology.

*Vacuum structure and gauge groups.*—The manifold supporting this transition is defined by the *Micro-Parallelism Axiom* [7]: each macroscopic spacetime point is resolved into a finite stack of microscopic branches carrying parallel information streams. The corresponding local Hilbert space factorizes as

$$\mathcal{H}_{\text{loc}} \cong \mathcal{H}_{\text{matter}} \otimes \mathbb{C}_{\text{space}}^3 \otimes \mathbb{C}_{\text{time}}^2, \quad (6)$$

together with a local phase register on which  $U(1)_{\text{phase}}$  acts. The axiom further postulates that the three spatial branches and two temporal branches are individually indistinguishable, so that any orthonormal basis on each factor is physically equivalent. The group of local basis changes that preserves transition amplitudes and the inner product on  $\mathcal{H}_{\text{loc}}$  is therefore  $U(3)_{\text{space}} \times U(2)_{\text{time}} \times U(1)_{\text{phase}}$ . Factoring out the overall unobservable phase of the full local Hilbert space and imposing that the remaining symmetry acts transitively on the internal unit spheres, while being compact, connected and non-Abelian, singles out the minimal choice

$$G_{\text{kin}} \simeq SU(3)_{\text{space}} \times SU(2)_{\text{time}} \times U(1)_{\text{phase}}. \quad (7)$$

Gauging these local symmetries and identifying transformations that differ only by a common center element yields the physical gauge group

$$G_{\Sigma} \cong \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}, \quad (8)$$

since the centers  $\mathbb{Z}_3 \subset SU(3)$ ,  $\mathbb{Z}_2 \subset SU(2)$  and the integer-charged subgroup of  $U(1)$  share a common  $\mathbb{Z}_6$  that acts trivially on all matter multiplets. This matches the Standard Model gauge group. A detailed classification is presented in Ref. [7].

*The Cosmological Phase Transition.*—We define an effective temperature  $T$  of the QCA network in terms of the information scrambling rate,  $\lambda_s(T)$ , which quantifies the density of non-trivial unitary updates per causal four-volume. We treat  $T$  as a monotonic reparametrization of  $\lambda_s$  without assuming full thermal equilibrium. In the early universe ( $T \gg T_c$ ) the scrambling is maximal and the local coin operator explores the  $SU(2)$  manifold ergodically, leading to a chirally symmetric phase with vanishing order parameter  $\langle \theta \rangle = 0$  (massless).

As the universe expands and the scrambling rate drops below a critical threshold, the system undergoes a Kibble-Zurek freeze-out of the coin degree of freedom.

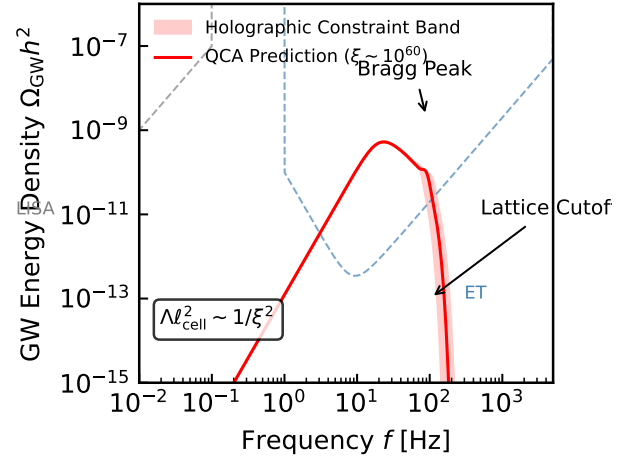


FIG. 1. Stochastic gravitational-wave spectrum from the QCA topological phase transition. The shaded band illustrates the range of cutoff positions and corresponding amplitudes obtained when the redundancy parameter  $\xi$  is varied under the holographic constraint (10). The solid curve shows a representative QCA prediction with a pronounced knee and Bragg-like oscillations near the lattice cutoff frequency  $f_{\text{cut}} \sim c/(\pi \ell_{\text{cell}})$ . The dashed lines indicate the projected sensitivities of LISA and the Einstein Telescope. The position of the cutoff is fixed by  $\Lambda$  and  $\xi$  through Eqs. (10) and (12), linking the microscopic QCA scale to an observable feature in the GW spectrum.

We model this via an effective free energy density  $\mathcal{F}[\theta] = \varepsilon(\theta) - Ts(\theta) \approx \alpha(T)\theta^2 + \beta\theta^4$ , where the quadratic coefficient  $\alpha(T)$  flips sign at  $T_c$  while  $\beta > 0$  remains approximately constant. This continuous symmetry-breaking transition freezes the topological impedance to one of the degenerate minima at  $\theta = \pm\theta_0 \neq 0$ , dynamically generating mass from the underlying information flow.

*Unified constraints and holographic redundancy.*—To predict the characteristic scales of the SGWB we must fix the microscopic lattice spacing  $\ell_{\text{cell}}$ . Our starting point is a discrete de Sitter patch of radius  $R_{\Lambda} \sim 1/\sqrt{\Lambda}$  tiled by QCA cells of linear size  $\ell_{\text{cell}}$ . The number of bulk cells and surface “pixels” are then  $N_{\text{bulk}} \simeq 4\pi R_{\Lambda}^3/(3\ell_{\text{cell}}^3)$  and  $N_{\text{surf}} \simeq 4\pi R_{\Lambda}^2/\ell_{\text{cell}}^2$ . In Planck units, the de Sitter horizon entropy is  $S_{\Lambda} \sim \pi R_{\Lambda}^2$ , so the holographic principle requires that the number of independent horizon degrees of freedom scale as  $N_{\text{dof}} \propto R_{\Lambda}^2$ . Identifying  $N_{\text{surf}}$  (up to an  $\mathcal{O}(1)$  factor) with  $N_{\text{dof}}$  implements this bound for the QCA discretization.

We then introduce the *holographic redundancy*

$$\xi \equiv \frac{N_{\text{bulk}}}{N_{\text{surf}}} \simeq \frac{R_{\Lambda}}{3\ell_{\text{cell}}}, \quad (9)$$

which counts the number of bulk cells per independent horizon degree of freedom. Eliminating  $R_{\Lambda}$  in favor of  $\Lambda$  gives the geometric scaling  $\Lambda \ell_{\text{cell}}^2 \simeq 1/(9\xi^2)$ . A more careful accounting of geometric and entropic factors, pre-

sented in the Supplemental Material [8], refines this to

$$\Lambda \ell_{\text{cell}}^2 \approx \frac{9}{64} \frac{1}{\xi^2}. \quad (10)$$

The observed smallness of  $\Lambda$  ( $\sim 10^{-122}$  in Planck units) then corresponds to an immense redundancy  $\xi \sim 10^{61}$ , assuming  $\ell_{\text{cell}}$  is of order the Planck length. More generally, Eq. (10) defines a one-parameter family of QCA discretizations labeled by  $\xi$ , which we use below to explore scenarios where the effective lattice cutoff is redshifted into the observational GW window. In this picture the small vacuum energy density reflects the fact that most bulk QCA microstates are highly redundant: the vast majority of them are entangled and do not contribute independently to the horizon surface area.

*Gravitational-wave signatures and lattice cutoff.*—The non-linear relaxation of the impedance field after the transition, together with the acoustic turbulence it sources in the surrounding medium, generates a stochastic GW background. We describe the continuous part of the spectrum by a broken power-law template  $\Omega_{\text{cont}}(f)$  that was originally derived for acoustic turbulence in first-order phase transitions [6], but which we use here as a phenomenological fit to the QCA-generated spectrum. We encode the effect of the QCA lattice through a universal window function

$$\Omega_{\text{QCA}}(f) = \Omega_{\text{cont}}(f) W(f). \quad (11)$$

Linear perturbations on a cubic QCA obey a discrete wave equation  $\ddot{h}_n = (c/\ell_{\text{cell}})^2(h_{n+1} - 2h_n + h_{n-1})$ , leading to a lattice dispersion relation  $\omega(k) = (2c/\ell_{\text{cell}})|\sin(k\ell_{\text{cell}}/2)|$ . This implies a strict Nyquist cutoff for propagating modes at

$$f_{\text{cut}} \simeq \frac{c}{\pi \ell_{\text{cell}}}. \quad (12)$$

Combining Eqs. (10) and (12) shows that, up to an  $\mathcal{O}(1)$  numerical factor,  $f_{\text{cut}} \propto \xi \sqrt{\Lambda}$ , so the ultraviolet knee of the spectrum is a derived quantity rather than a free parameter. For example, a redundancy parameter  $\xi \sim 10^{20}$  places the cutoff frequency  $f_{\text{cut}} \sim 100$  Hz, within the sensitivity range of ground-based detectors, whereas  $\xi \sim 10^{61}$  pushes it to the Planck scale. Assuming a latent heat parameter  $\alpha \sim 0.1$  and transition duration  $\beta/H_* \sim 100$ , the acoustic-turbulence fit predicts a peak amplitude  $\Omega_{\text{GW}} h^2 \sim 10^{-9}$ , accessible to next-generation detectors. This allows GW experiments to directly constrain the degree of holographic redundancy. Note that  $f_{\text{cut}}$  in Eq. (12) refers to the comoving frequency at the time of generation; the observed frequency today is redshifted by  $f_{\text{obs}} = f_{\text{cut}}(a_*/a_0)$ , where  $a_*$  is the scale factor at the transition. We have verified numerically, for a range of localized sources on the discrete QCA, that the resulting GW spectra are well fitted by Eq. (??); see

Supplemental Material for details. Motivated by the discrete sampling of the source on the lattice, we model the window function as

$$W(f) = \left[ \frac{\sin(\pi f/f_{\text{cut}})}{\pi f/f_{\text{cut}}} \right]^2. \quad (13)$$

For  $f \ll f_{\text{cut}}$  this window tends to unity, so the low-frequency spectrum coincides with the continuum prediction. For  $f \gtrsim f_{\text{cut}}$  it induces a sharp power-law suppression and oscillatory Bragg-like features characteristic of waves on a lattice. Continuous field theories have no analogue of the hard cutoff set by Eq. (12). Observing a knee and Bragg-like oscillations at a frequency fixed in this way would provide strong evidence for microscopic spacetime discreteness in this class of QCA cosmologies.

*Conclusion.*—We have presented a QCA-based cosmology in which Dirac mass arises as a topological impedance of a discrete quantum information network, the vacuum supports a kinematic symmetry isomorphic to the Standard Model gauge group, and holographic entropy bounds tie the microscopic cell size  $\ell_{\text{cell}}$  to the cosmological constant  $\Lambda$  through a large redundancy factor  $\xi$ . Together these ingredients imply that an impedance-freezing cosmological phase transition generically sources a stochastic gravitational-wave background whose ultraviolet cutoff frequency and Bragg-like oscillations are fixed in terms of  $\Lambda$  and  $\xi$ , as encoded in Eqs. (10) and (12). The simple scalings  $\Lambda \ell_{\text{cell}}^2 \sim 1/\xi^2$  and  $f_{\text{cut}} \sim c/(\pi \ell_{\text{cell}})$  make the stochastic GW spectrum a direct probe of microscopic spacetime discreteness and of the degree of holographic redundancy. A detection of a non-standard SGWB with a sharp knee and lattice-induced oscillations at the predicted frequency would strongly support this QCA picture, while the absence of such features in the relevant bands would constrain or falsify it. More broadly, the framework outlined here illustrates how quantum-information-theoretic architectures for spacetime can lead to sharp, testable predictions for upcoming GW experiments.

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  - [8] See Supplemental Material for derivations of the holographic entropy bound, the split-step QCA effective Hamiltonian, and the lattice dispersion relation.