Question 6

Implementation of the first case is similar to the code developed in question 5, with modified boundary condition as given in the question:

case 1 : $u(x,0) = (1-x)^4 (1+x)$

the code for this boundary condition implementation is given in the file

"gulana_q6_case1.py"

In this implementation $\mu = \Delta t / \Delta x^2 = 1$, $\theta = 0$, and $\alpha(t) = 1$. The result is stored in

"gulana_q6-case1.jpeg"

To reproduce the figure, simply run the code as it is.

To verify this, the code "gulana_q4_advection.py" is modified to have case 1 boundary conditions, all other terms constant. The output figure and code are found in, respectively

"gulana_q6_verification.jpeg"

"gulana_q6_verification.py"

Comparison of these graphs show that the same numerical values are obtained for $\theta = 0$ in the implicit theta scheme.

<u>case 2</u> $u(x,0) = (1-x)^4(1+x)\left(\sum_{k=0}^{3} b_k \phi_k(x) + C\right)$

The code implementing this boundary condition can be found in:

"gulana_q6_case2.py"

to display θ-μ parameter plots, value of b_k were fixed to be

$$b_k = [0.7703829, 0.57331959, 0.86436473, 0.82221663]$$

and value of θ-μ were varied as follows by changing the input to the function of $\underline{N}$. (where N = no. of timesteps in my code)

$N = 16.67$ by varying $N = 6$

case 1 : $N = 16.67$, $\theta = 0$ saved as "case2_96_mu_16.67_theta_0.jpeg"

case 2 : $N = 16.67$ $\theta = 0.5$ saved as "case2_96_mu_16.67_theta_0.5.jpeg"

case 3 : $N = 16.67$ $\theta = 1$ saved as "case2_96_mu_16.67_theta_0.jpeg"

$\mu = 10$ by varying $N = 10$

case 4 : $\mu = 10$, $\theta = 0$ saved as
"case2_q6_mu_10_theta_0.jpeg"

case 5 : $\mu = 10$, $\theta = 0.5$ saved as
"case2_q6_mu_10_theta_0.5.jpeg"

case 6 : $\mu = 10$, $\theta = 1$ saved as
"case2_q6_mu_10_theta_1.jpeg"

$\mu = 1$ by varying $N = 100$

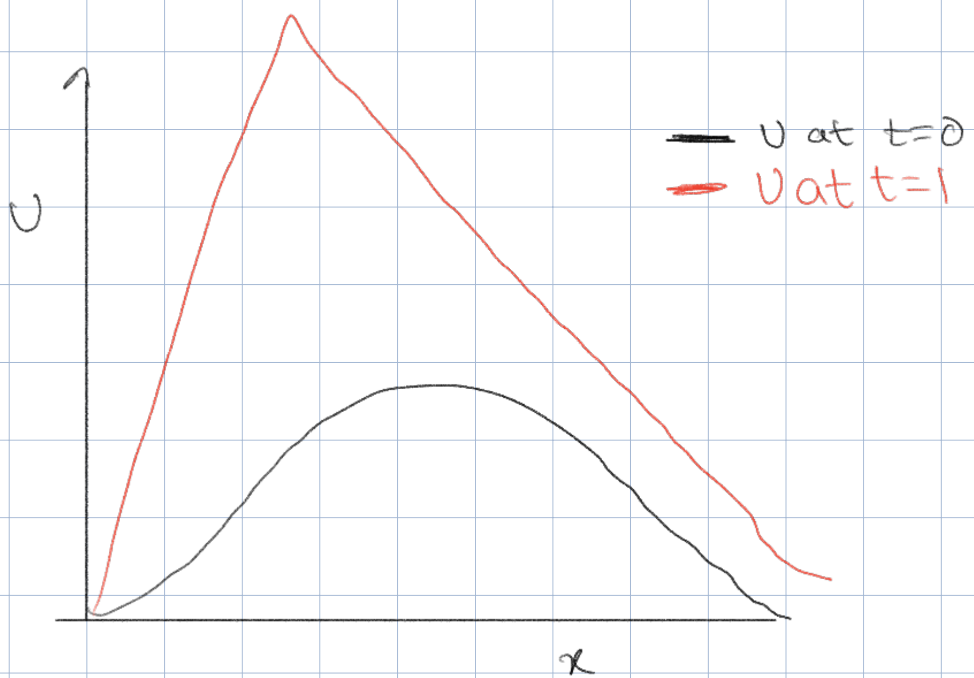
case 7 : $\mu = 100$ $\theta = 0$ saved as
"case2_q6_mu_100_theta_0.jpeg"

case 8 : $\mu = 100$ $\theta = 0.5$ saved as
"case2_q6_mu_100_theta_0.5.jpeg"

case 9 : $\mu = 100$ $\theta = 1$ saved as
"case2_q6_mu_100_theta_1.jpeg"

NOTE: to reproduce graphs, simply run
"gulana_q6_case2.py"

⇒ Upon analysis, at smaller values of dt, the scheme for $\theta = 0$ is unstable and violates the maximum principle as U at t=0 is less than U at t=1. It follows that instability rises in the explicit scheme. At theta=1, the scheme is always stable, proving it is unconditionally stable for any values of dt. This is expected. Below shows a figure of how the maximum principle is violated.



Sketch above: Violation of max. principle.