Limits in Calculus

Alyse Ammerman

Due Date: September 14, 2021

The Theorem that will be shown in this paper is used for showing how limits can be applied to polynomial functions($p(x)$. Since as a limit of X approaches a real number,a, the result of that limit will be the output of x. Because of that, we can apply that to a this theorem where, when a limit is applied to a function that is a polynomial, the limit is applied to every x-term, causing x to approach the value listed in the limit,a, in each individual x-term; and therefore the value of that limit will be p(a). The theorem as explained is illustrated below...

Theorem 0.1. *For any polynomial*

$$p(x) = c_0 + c_1 x + \cdots + c_n x^n$$

and for any real number

$$\lim_{x \to a} p(x) = c_0 + c_1 x + \cdots + c_n a^n = p(a)$$

Proof.

$$\begin{aligned}
\lim_{x \to a} p(x) &= \lim_{x \to a} (c_0 + c_1 x + \cdots + c_n x^n) && (1) \\
&= \lim_{x \to a} c_0 + \lim_{x \to a} c_1 x + \cdots + \lim_{x \to a} c_n x^n && (2) \\
&= \lim_{x \to a} c_0 + c_1 \lim_{x \to a} x + \cdots + c_n \lim_{x \to a} x^n && (3) \\
&= c_0 + c_1 a + \cdots + c_n a^n = p(a) && (4)
\end{aligned}$$

$\square$

As you see in step (3), the constants (c_n's) can also be pulled out into the front because it will not impact or change the limit of x in those terms. The limit can then be isolated to the x part of each term to get us to step (4) where the value a is approached for each x; finally getting us to the output, $p(a)$.

Lets try this out-

Example 0.1. *In this example lets consider the function,*

$$f(x) = 8 + 6x + 12x^2 + 2x^3.$$

Here we will using the theorem listed earlier in this section to show how $\lim_{x\to 8} f(x) = f(8)$.

$$\begin{aligned}
\lim_{x\to 8} f(x) &= \lim_{x\to 8}(8 + 6x + 12x^2 + 2x^3) \\
&= \lim_{x\to 8} 8 + \lim_{x\to 8} 6x + \lim_{x\to 8} 12x^2 + \lim_{x\to 8} 2x^3 \\
&= \lim_{x\to 8} 8 + 6\lim_{x\to 8} x + 12\lim_{x\to 8} x^2 + 2\lim_{x\to 8} x^3 \\
&= 8 + 6(8) + 12(8)^2 + 2(8)^3
\end{aligned}$$

$$\lim_{x\to 8} f(x) = f(8) = 1848$$