Problem 1

Design an NFA for the following languages over their respective alphabets:

- $L = \{w | w$ has $3m$ or $4m$ $a's$ for some $m \in \mathbb{N}\}$. L is over the alphabet $\Sigma = \{a, b\}$. [Tug22]
- $L = \{lab, cab, dab\}$. L is over the alphabet $\Sigma = \{a, b, c, d, l\}$ [HMU01]
- $L = \{1101, 101, 111\}$. L is over the alphabet $\Sigma = \{0, 1\}$. [HMU01]
- $L = \{w | w$ ends with $00\}$. L is over the alphabet $\Sigma = \{0, 1\}$. (For this one, your NFA can only have 3 states.) [Sip96]

Problem 2

In the previous problem, you defined an N such that $L(N) = \{w | w$ ends with $00\}$, Convert this NFA into a DFA.

Δ	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
q	$\emptyset$	$\{r\}$
$*r$	$\{p, r\}$	$\{q\}$

Table 1: NFA

Problem 3

Convert the NFA defined in Table 1 to a DFA. [HMU01]

Problem 4

In class we used the following lemma based on our definition of $\hat{\Delta}$. Prove it by induction on $|y|$.
For any $x, y \in \Sigma^*$ and $A \subseteq Q$,

$$\hat{\Delta}(A, xy) = \hat{\Delta}(\hat{\Delta}(A, x), y)$$

References

[HMU01] John E Hopcroft, Rajeev Motwani, and Jeffrey D Ullman. Introduction to automata theory, languages, and computation. *Acm Sigact News*, 32(1):60–65, 2001.

[Sip96] Michael Sipser. Introduction to the theory of computation. *ACM Sigact News*, 27(1):27–29, 1996.

[Tug22] Randal Tuggle. Homework problem for comp 455. HW2, 2022.