$3 \to 2$

$A = L(\alpha)$ for reg exp α

$\to A = L(\alpha)$ for pattern α

Trivial.

All reg. exp. are patterns. $\square$

$2 \to 1$ $A = L(\alpha)$ for pattern α

$\to A$ is a regular language

Proof by:

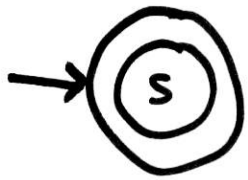
- induction
- cases
- construction

Base case:

- $a \in \Sigma$

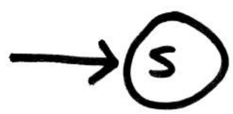
a is a reg language $\square$

- $\epsilon = \{\epsilon\}$



ϵ is a reg language $\square$

- $\emptyset = \{\}$



$\emptyset$ is a reg language. $\square$

Inductive Step: (Case 1)

1. $L(\beta)$ and $L(\gamma)$ are reg. ~~expressions~~ languages	I. H.
2. $L(\beta) \cup L(\gamma) = L(\beta \cup \gamma)$	Compound pattern def.
3. $L(\beta) \cup L(\gamma)$ is reg. language	lemma 1
4. $L(\beta \cup \gamma)$ is reg language	combined lines 2+3

WTP

Proving lemma 1:

Proof by construction where we combine automata for $L(\beta)$ and $L(\gamma)$ to make automaton for $L(\beta) \cup L(\gamma)$

← HW problem?

WTP

* Also have to prove cases for $\beta \circ \gamma$ and β^*

$1 \to 3$

A is regular language $\to$

$A = L(\alpha)$ for a reg. exp. α

We know:

$\exists$ an automaton N that accepts A

Strategically convert N to a regular expression

(Proof by construction)

What if a language isn't regular?

* Nonregular language *

Proving a language is nonregular.

$$B = \{a^n b^n \mid n \geq 0\}$$

ϵ

ab

$aabb$

$aaabbb$

Proof by contradiction

Assume B is regular.

There exist automaton M s.t. $B = L(M)$

- We have finite Q, $|Q| = K$ (K states)
- B should accept $a^n b^n$ $\quad n > K$

$$\underset{s}{\uparrow}\underbrace{aaaa\ldots aa}_{n}\;\underbrace{bbb\ldots b}_{n}\underset{r}{\uparrow}$$

r is an accept state! $r \in F$

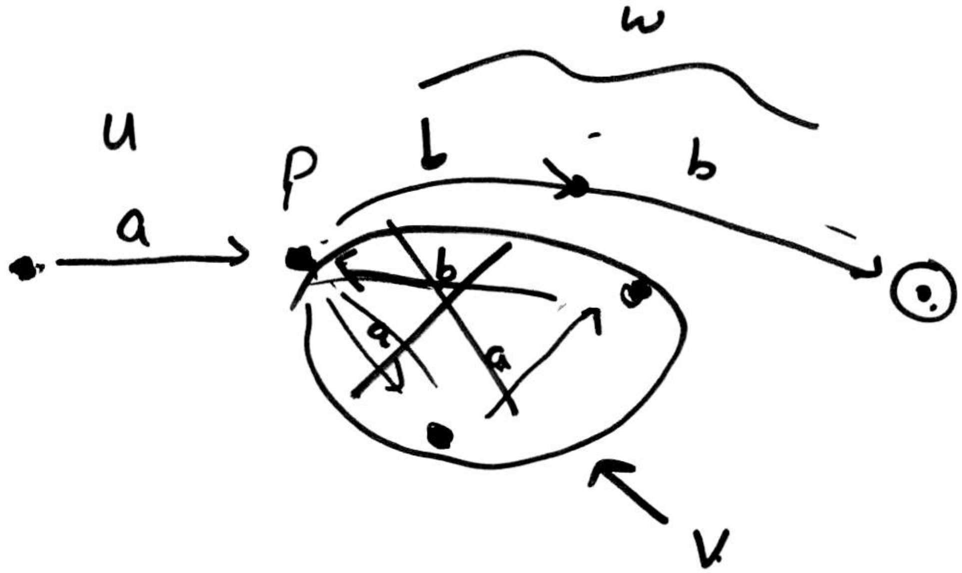
- Visiting $2n$ states, only K <u>unique</u> states
- visit at least one state twice

$\overbrace{aaa}^{u}\overbrace{\cancel{a\ldots a}}^{v}\overbrace{aa\,bbb\ldots b}^{w}$

s p p r

$a^n b^n = uvw$

uw should also end in accept state



we define v to be all a's so removing v would result in more bs than as.

uw

uvw

uvvw

The Pumping Lemma

If A is a reguelar language, then there exists a "pumping length" p, where is s ~~$=xyz$~~ is a string in A of length p, then $\exists$ xyz s.t. $s = xyz$ and

1. For each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$
3. $|xy| \leq p$

Using the Pumping Lemma