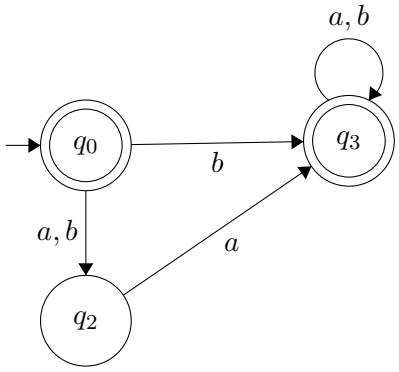
Problem 1

Design a DFA for the following languages over the alphabet $\Sigma = \{a, b\}$:

- $L_1 = \{w :$ every a is immediately followed by a $b\}$
- $L_2 = \{w : w$ has an even number of $a's$ and an odd number of $b's\}$
- $L_3 = \{w : w$ contains the substring $baa\}$
- $L_4 = \{w : w$ every odd position of w is an $a\}$
- $L_5 = \{\epsilon, a\}$

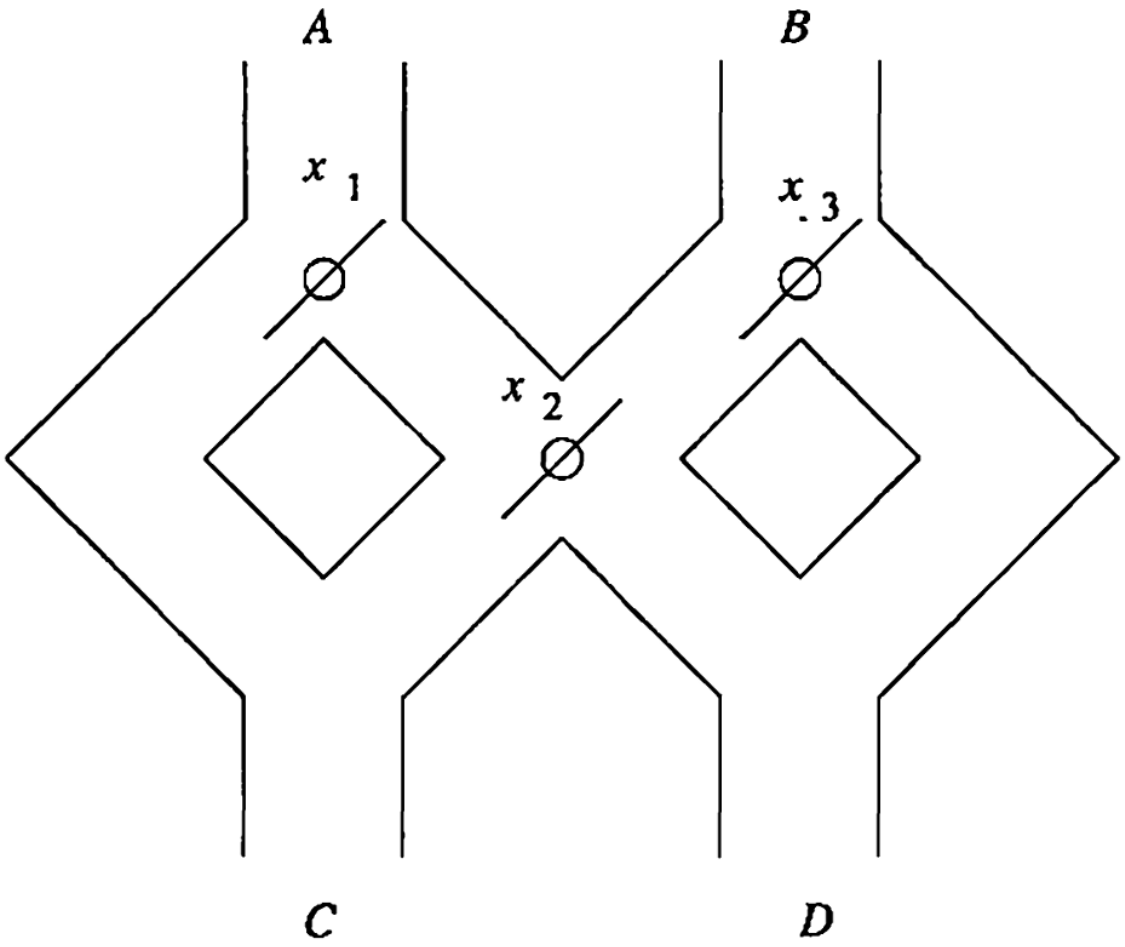
[Tug22]

Problem 2



Provide a 5-tuple definition for the NFA above. [Tug22]

Problem 3



In the figure is a marble-rolling toy. A marble is dropped at A or B. Levers x_1, x_2, x_3 cause the marble to fall either to the left or to the right. Whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

Model this toy by a finite automaton. Let the inputs A and B represent the input into which the marble is dropped. Let acceptance correspond to the marble exiting at D; nonacceptance represents a marble exiting at C.

[HMU01]

**Hint: think of your "states" in terms of lever positions.*

Problem 4

$L_{\text{float}} = \{w : w \text{ is the string representation of a floating point number}\}$ Assume the following syntax for floating point numbers:

- A floating point number is an optional sign, followed by a decimal number, followed by an optional exponent.
- A decimal number may be of the form x or $x.y$, where x and y are nonempty strings of decimal digits.
- An exponent beings with `E` and is followed by an optional sign and then an integer.
- An integer is a nonempty string of decimal digits.

The following strings are examples of floating point numbers:

$$+3.0, 3.0, 0.3\text{E}1, 0.3\text{E}+1, -0.3\text{E}+1, -3\text{E}8, 7$$

Show that L_{float} is regular by constructing a DFA that recognizes L_{float}. [Tug22]

Problem 5

In our lecture, we defined three automata:

- $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$
- $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$
- $M_3 = (Q_3, \Sigma, \delta_3, s_3, F_3)$

with

- $Q_3 = Q_1 \times Q_2$
- $\delta_3((p, q), a) = \big(\delta_1(p, a), \delta_2(q, a)\big)$
- $s_3 = (s_1, s_2)$
- $F_3 = F_1 \times F_2$

Prove $\hat{\delta_3}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$.
Hint: Prove by induction on the length of x.

References

[HMU01] John E Hopcroft, Rajeev Motwani, and Jeffrey D Ullman. Introduction to automata theory, languages, and computation. *Acm Sigact News*, 32(1):60–65, 2001.

[Tug22] Randal Tuggle. Homework problem for comp 455. HW2, 2022.