Problem 1

Answer each question about this context-free grammar G.
(You must answer all parts correct for homework credit.)

$$
\begin{aligned}
R &\to XRX \mid S \\
S &\to aTb \mid bTa \\
T &\to XTX \mid X \mid \epsilon \\
X &\to a \mid b
\end{aligned}
$$

a. What are the variables of G?

b. What are the terminals of G?

c. Which is the start variable of G?

d. Give three strings in $L(G)$.

e. Give three strings not in $L(G)$.

f. In english, give a general description of what language is $L(G)$ describing.

Problem 2

For each part, let the language be $\Sigma = \{0, 1\}$. Define a CFG that generates the language. Specify all elements of the quadruple definition.
(You must answer at least two parts correctly for homework credit.)

a. $\{w \mid w$ contains at least three 1s$\}$

b. $\{w \mid w$ starts and ends with the same symbol$\}$

c. {The set of strings with more 0's than 1's}

d. $\{0^n 1^{2n} | n \geq 0\}$

e. $\{w \mid$ the length of w is odd and its middle symbol is a $1\}$

f. The empty set

Problem 3

Define a PDA that accepts the language $\{ww^{\mathcal{R}} | w \in \{a, b\}^*$.
Provide a 6-tuple definition. (For δ you can use a transition table or draw a state diagram.)
Hint: This is going to be very similar to our in-class example!

Problem 4

The following CFG generates an arithmetic expression. Let $\Sigma = \{a, +, \times, (,)\}$ and your start variable is E.

Convert this into a PDA using the procedure we defined in class.

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid a$$

Problem 5

Let $C = \{a^i b^j c^k | 0 \leq i \leq j \leq k\}$. Use the pumping lemma to prove that C is not a CFL.

Problem 6

In class, we showed $A = \{a^n b^n\}$ is context-free and $B = \{a^n b^n c^n | n \geq 0\}$ is *not* context-free.

Say I wanted to (incorrectly) prove that A is *not* context free by choosing $s = a^p b^p$ and trying to show that there is no way to divide $s = uvxyz$ such that for each $i \geq 0$, $uv^i xy^i z \in A$, $|vy| > 0$, $|vxy| \leq p$.

Choose a u, v, x, y, and z that contradicts this claim. In other words, choose them such that for each $i \geq 0$, $uv^i xy^i z \in A$.

(Note that this isn't sufficient for proving that A is context-free, but rather we're just showing a counter example to an incorrect claim.)

Problem 7

Let B be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.