

# Set Notation Review

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## Review: Set Notation and Operations

### Sets - Definitions

A *set* is an unordered collection of objects.

The following are sets:

- $\{1, 2, 3\}$
- “all multiples of 7”
- $\{\text{apples}, 7, \text{True}\}$

Sets don't inherently have an order.

### Sets - Terminology

A set is a *finite set* if it has a finite number of elements.

Any set that is not finite is an *infinite set*.

Let  $A$  be a finite set. The number of different elements in  $A$  is called its *cardinality*.

The cardinality of a finite set is denoted  $|A|$ .

### Examples

$\{1, 2, 3\}$  is a finite set. Its cardinality is 3.

“all multiples of 7” is an infinite set.

### Sets - Notation

$a \in A$  means  $a$  is an element of  $A$ .

$a \notin A$  means  $a$  is *not* an element of  $A$ .

### Example

Let  $A = \{apples, bananas, oranges\}$

“apples”  $\in A$

“blueberries”  $\notin A$

### Sets - Notation

Sets are commonly expressed using *set notation*.

Within braces, we can write a rule consisting of a function, a vertical bar, and a set to which the function is applied.

### Sets - Notation

#### Example

We can express the set  $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$  as...

### Subsets

$A$  is a **subset** of  $B$  if and only if every element of  $A$  is an element of  $B$ .

Can also be written:  $A \subseteq B$

#### Examples

Let  $A = \{1, 3, 5\}$  and  $B = \{1, 2, 3, 4, 5\}$ .

$$A \subseteq B.$$

Let  $A = \{1, 2, 3\}$  and  $B = \{3, 2, 1\}$ .

$$A \subseteq B \text{ and } B \subseteq A.$$

### Equality

$A = B$  if and only if every element of  $A$  is an element of  $B$  and conversely every element of  $B$  is an element of  $A$ . That is,  $A \subseteq B$  and  $B \subseteq A$ .

#### Example

Let  $A = \{1, 2, 3\}$  and  $B = \{3, 2, 1\}$ .  $A \subseteq B$  and  $B \subseteq A$ , so  $A = B$

## Common Sets

- There are some standard symbols that represent specific sets you will see:
- The set of **Natural Numbers**  $\mathbb{N}$  is the set of all whole numbers  $\geq 0$ ,  $\{0, 1, 2, 3, 4, \dots\}$ .\*
- The set of **Integers**  $\mathbb{Z}$  is the set of all whole numbers,  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ .
- The set of **Rational Numbers**  $\mathbb{Q}$  are numbers that can be represented as a quotient of whole numbers,  $\{\frac{p}{q} \mid p, q \in \mathbb{Z}\}$
- The set of **Real Numbers**  $\mathbb{R}$  is all *real* numbers.

## Tuples

A *k-tuple* is an ordered sequence of  $k$  elements, which we write down in parentheses,  $(a_1, a_2, \dots, a_k)$ .

The most common tuple seen in math is the coordinate pair  $(x, y)$  on a graph.

A 2-tuple is commonly called an *ordered pair*.

Two tuples are equal if and only if all of their corresponding elements are equal.  $(a_1, a_2, \dots, a_k)$  iff for all  $i \in [1, \dots, k]$  we have  $a_i = b_i$ .

## Concatenation

**Concatenation** is used to join two strings or lists by putting all elements of the second string behind all elements of the first.

### Example

The concatenation of “hello” and “world” is “helloworld”.

## Range

$[a, b]$  is the set of whole numbers  $\geq a$  and  $\leq b$ .

$(a, b)$  is the set of whole numbers  $> a$  and  $< b$ .

### Examples

$$[1, 5] = \{1, 2, 3, 4, 5\}$$

$$(1, 5) = \{2, 3, 4\}$$

$$[1, 5) = \{1, 2, 3, 4\}$$

## Set Operations

### Set Operations

- $a \in B$  means  $a$  is an element of  $B$ .

### Set Operations

- $a \notin B$  means  $a$  is *not* an element of  $B$ .
- Note that technically,  $a \in B$  and  $a \notin B$  are predicates! They take an element and a set as input and give True or False as an output.

### Subset

Let  $A$  and  $B$  be sets. We say that  $A$  is a **subset** of  $B$  if and only if every element of  $A$  is an element of  $B$ .

We write  $A \subseteq B$  to denote the fact that  $A$  is a subset of  $B$ .

### Equality

#### Using Predicate Logic

- Remember this?
- For all sets  $A$  and  $B$ ,  $A = B$  if and only if every element of  $A$  is an element of  $B$  and every element of  $B$  is an element of  $A$
- $\forall A, B, A = B \leftrightarrow (A \subseteq B \text{ and } B \subseteq A)$

### Complement

The complement of a set  $A$ , denoted  $A^c$  is the set of all elements in the universe  $U$  that are *not* in  $A$ .

#### Using Set Notation

$$A^c = \{x | x \notin A\}$$

#### Using Predicate Logic

$$\forall a, a \in A^c \leftrightarrow a \notin A$$

or, equivalently,  $\forall a \in U, a \notin A^c \leftrightarrow a \in A$

### Intersection

$A \cap B$  are the elements that are both in  $A$  and  $B$ .

### Using Set Notation

$$A \cap B = \{x | x \in A \wedge x \in B\}$$

### Using Predicate Logic

$$\forall A, B, x, x \in A \cap B \leftrightarrow (x \in A \wedge x \in B)$$

### Union

$A \cup B$  are the elements that are either in  $A$  or  $B$ .

### Using Set Notation

$$A \cup B = \{x | x \in A \vee x \in B\}$$

### Using Predicate Logic

$$\forall A, B, x, x \in A \cup B \leftrightarrow (x \in A \vee x \in B)$$

### Difference

The **difference** of sets  $A$  and  $B$  is the set that contains all elements in  $A$  that are not in  $B$ .

### Using Set Notation

$$A - B = A \setminus B = \{x | x \in A \wedge x \notin B\}$$

### Using Predicate Logic

$$\forall A, B, x, x \in A - B \leftrightarrow x \in A \wedge x \notin B$$

### Difference Cont.

#### Example

Let  $A = \{1, 3, 5, 7\}$  and  $B = \{4, 5, 6, 7, 8\}$ .

$$A - B = \{1, 3\}.$$

#### Example

Let  $C = \{\bigcirc, \diamond, \square, \heartsuit\}$  and  $e = \heartsuit$ .

$$C - \{e\} = \{\bigcirc, \diamond, \square\}.$$

## Xor

### Using Set Notation

$$A \oplus B = \{x | x \in A \oplus x \in B\}$$

### Using Predicate Logic

$$\forall A, B, x, x \in A \oplus B \leftrightarrow x \in A \oplus x \in B$$

## Cartesian Product

The **cartesian product** of  $A$  and  $B$ ,

### Using Set Notation

$$A \times B = \{(a, b) | \forall a \in A, \forall b \in B\}$$

### Using Predicate Logic

$$\forall A, B, a, b, ((a, b) \in A \times B) \leftrightarrow (a \in A \wedge b \in B)$$

## Powerset

The powerset of a set  $A$ , denoted  $\mathcal{P}(A)$  is the set of all subsets of  $A$

### Using Set Notation

$$\mathcal{P}(A) = \{S | S \subseteq A\}$$

- $|\mathcal{P}(A)| = 2^{|A|}$

## Next

There's a lesson on Gradescope!