**Problem 1**

Answer each question about this context-free grammar $G$.
(You must answer all parts correct for homework credit.)

$$R \to XRX \mid S$$
$$S \to aTb \mid bTa$$
$$T \to XTX \mid X \mid \epsilon$$
$$X \to a \mid b$$

a. What are the variables of $G$?

b. What are the terminals of $G$?

c. Which is the start variable of $G$?

d. Give three strings in $L(G)$.

e. Give three strings not in $L(G)$.

f. In english, give a general description of what language is $L(G)$ describing.

**Problem 2**

For each part, let the language be $\Sigma = \{0, 1\}$. Define a CFG that generates the language. Specify all elements of the tuple definition $P = (Q, \Sigma, \Gamma, \delta, s, F)$.
(You must answer at least two parts correctly for homework credit.)

a. $\{w \mid w$ contains at least three 1s$\}$

b. $\{w \mid w$ starts and ends with the same symbol$\}$

c. {The set of strings with more 0's than 1's}

d. $\{w \mid w$ starts and ends with the same symbol$\}$

e. $\{w \mid$ the length of w is odd and its middle symbol is a 1$\}$

f. The empty set

**Problem 3**

Define a PDA that accepts the language $\{ww^{\mathcal{R}} | w \in \{a, b\}^*$.
Provide a 6-tuple definition. (For $\delta$ you can use a transition table or draw a state diagram.)
*Hint: This is going to be very similar to our in-class example!*

**Problem 4**

The following CFG generates an arithmetic expression. Let $\Sigma = \{a, +, \times, (, )\}$ and your start variable is $E$.

Convert this into a PDA using the procedure we defined in class.

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid a$$

**Problem 5**

Let $C = \{a^i b^j c^k | 0 \leq i \leq j \leq k\}$. Use the pumping lemma to prove that $C$ is not a CFL.

**Problem 6**

In class, we showed $A = \{a^n b^n\}$ is context-free and $B = \{a^n b^n c^n | n \geq 0\}$ is *not* context-free.

Say I wanted to (incorrectly) prove that $A$ is *not* context free by choosing $s = a^p b^p$ and trying to show that there is no way to divide $s = uvxyz$ such that for each $i \geq 0$, $uv^i xy^i z \in A$, $|vy| > 0$, $|vxy| \leq p$.

Choose a $u$, $v$, $x$, $y$, and $z$ that contradicts this claim. In other words, choose them such that for each $i \geq 0$, $uv^i xy^i z \in A$.

*(Note that this isn't sufficient for proving that $A$ is context-free, but rather we're just showing a counter example to an incorrect claim.)*

**Problem 7**

Let $B$ be the language of all palindromes over $\{0, 1\}$ containing equal numbers of $0$s and $1$s. Show that $B$ is not context free.

**Problem 8**

Let $G$ be the following grammar. Use the DK-test to show that $G$ is not a DCFG.

$$R \to S \mid T$$
$$S \to aSb \mid ab$$
$$T \to aTbb \mid abb$$