Problem 1

A set being closed over an operation means that the operation can be applied to elements of the set and still produce an element of the set.

Show that P is closed under:

1. union: $\forall A, B \in P, A \cup B \in P$
2. concatenation, $\forall A, B \in P, A \circ B \in P$
3. complement, $\forall A \in P, \bar{A} \in P$

Hint: This is a good candidate for a proof by construction. Think about what it means for A and B to be in P, and see if you can construct something with that information to show that, for example, $A \cup B \in P$.

Problem 2

A *triangle* in a graph is a 3-clique. Show that $TRIANGLE \in P$, where
$TRIANGLE = \{\langle G \rangle \mid G \text{ contains a triangle}\}$. [Sip96]

Problem 3

Show that $EQ_{\text{DFA}} \in P$.

Problem 4

Show that if P = NP, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete. [Sip96]

Problem 5

Let G be an *undirected* graph.

Show that $SPATH \in P$, where
$SPATH = \{\langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at most } k \text{ from } a \text{ to } b\}$

References

[Sip96] Michael Sipser. Introduction to the theory of computation. *ACM Sigact News*, 27(1):27–29, 1996.