Toolbox

You can use the fact that these problems are decidable/undecidable for your proofs.

Decidable Problems The following are decidable problems:

- **Finite Automata**
  - $A_{\text{DFA}} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts input string } w\}$
  - $A_{\text{NFA}} = \{\langle B, w\rangle \mid B \text{ is an NFA that accepts input string } w\}$
  - $A_{\text{REX}} = \{\langle R, w\rangle \mid R \text{ is a regular expression that generates string } w\}$
  - $E_{\text{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$
  - $EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$
- **Context Free Grammars**
  - $A_{\text{CFG}} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}$
  - $E_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$
- **Turing Machines**
  - $A_{\text{LBA}} = \{\langle M, w\rangle \mid M \text{ is an LBA that accepts string } w\}$

Undecidable Problems The following are undecidable problems:

- **Context Free Grammars**
  - $ALL_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$
- **Turing Machines**
  - $A_{\text{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$
  - $HALT_{\text{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ halts on input } w\}$
  - $E_{\text{TM}} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$
  - $REGULAR_{\text{TM}} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is a regular language.}\}$
  - $EQ_{\text{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$
  - $E_{\text{LBA}} = \{\langle M\rangle \mid M \text{ is an LBA where } L(M) = \emptyset\}$

Problem 1

Let $ALL_{\text{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

Problem 2

Let $INFINITE_{\text{PDA}} = \{\langle M\rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$.
Show that $INFINITE_{\text{PDA}}$ is decidable.
Hint: there's a similar proof that $INFINITE_{\text{DFA}}$ is decidable in the book. [**?**]

Problem 3

Let $S = \{\langle M \rangle \mid M$ is a DFA that accepts $w^{\mathcal{R}}$ whenever it accepts $w\}$. Show that S is decidable.

Problem 4

Is the following problem decidable or undecidable? Justify your answer.
$EV_{\text{LBA}} = \{\langle M, w \rangle \mid \ M$ is an LBA that accepts string w and $|w|$ is even$\}$

Problem 5

Let $EQ_{\text{CFG}} = \{G$ and H are CFGs and $L(G) = L(H)\}$. Show that EQ_{CFG} is undecidable. You can use informal reducibility or mapping reducibility.

Problem 6

Let $T = \{\langle M \rangle \mid M$ is a TM that accepts $w^{\mathcal{R}}$ whenever it accepts $w\}$. Show that T is undecidable. You can use informal reducibility or mapping reducibility.

Problem 7

Let $SUB_{\text{TM}} = \{\langle M_1, M_2 \rangle \mid M_1$ and M_2 are TMs and $L(M_1) \subseteq L(M_2)\}$ Show that SUB_{TM} is undecidable. You can use informal reducibility or mapping reducibility.