Problem 1

Let $w^{\mathcal{R}}$ represent the string w in reverse order. Prove by construction that, for any language A, with $A^{\mathcal{R}} = \{w^{\mathcal{R}} | w \in A\}$, if A is regular, then so is $A^{\mathcal{R}}$.

Hint: You want to use the the automaton M that accepts A to construct a new automaton $M_{\mathcal{R}}$ and prove that $x \in L(M) \iff x^{\mathcal{R}} \in M_{\mathcal{R}}$.

[Sip96]

Problem 2

Give a regular expression for the following subsets of $\{a, b\}^*$.

(a) $\{x \mid x$ contains an even number of a's$\}$

(b) $\{x \mid x$ contains an odd number of b's$\}$

(c) $\{x \mid x$ contains an even number of a's or an odd number of b's$\}$

(d) $\{x \mid x$ contains an even number of a's and an odd number of b's$\}$

[Koz07]

Problem 3

Define an NFA accepting the the set of strings matching the regular expression

$$(01 \cup 011 \cup 0111)^*$$

Now, design a DFA that accepts the same set of strings.

[Koz07]

	0	1	ε
q_0	$\{q_1\}$	$\varnothing$	$\varnothing$
q_1	$\varnothing$	$\{q_2\}$	$\varnothing$
q_2	$\varnothing$	$\{q_3\}$	$\{q_0\}$
q_3	$\varnothing$	$\{q_0\}$	$\{q_0\}$

Table 1: NFA transition table for $(01 \cup 011 \cup 0111)^*$

Problem 4

Define a DFA accepting the the set of strings matching the regular expression $\big((000)^* \cup (111)^*\big)^*$. [Koz07]

Problem 5

For each of the following languages, give:

- a language definition w using set notation,
- two strings that are members of the language, and

- two strings that are *not* members

Assume the alphabet $\Sigma = \{a, b\}$.

1. a^*b^*
2. $a^* \cup b^*$
3. $aba \cup bab$
4. $(\Sigma\Sigma)^*$
5. $(a \cup \epsilon)b^*$

Problem 6

Use the pumping lemma to show the following languages aren't regular.

1. $A_1 = \{a^{2^n} | n \geq 0\}$
2. $A_2 = \{w | w \in \{0,1\}^*$ is a palindrome$\}$
3. $A_3 = \{0^{2i}1^i | i \geq 0\}$

Problem 7

Describe error in the following "proof" that a^*b^* is not a regular language.

We will start by assuming it is regular and finding a contradiction.

$B = \{a^*b^*\}$ is a regular language (Assumption) (1)

$s = a^p b^p \in A$ (Plugged p into line 1) (2)

s can be split into $s = xyz$, where for any $i \geq 0$ the string $xy^iz \in B$ (Pumping lemma) (3)

$\nexists s = xyz$, where for any $i \geq 0$ the string $xy^iz \in B$ (Demonstrated in proof that a^nb^n is not regular) (4)

Lines 3 and 4 contradict. $\rightarrow\leftarrow$ (5)

References

[Koz07] Dexter C Kozen. *Automata and computability*. Springer Science & Business Media, 2007.

[Sip96] Michael Sipser. Introduction to the theory of computation. *ACM Sigact News*, 27(1):27–29, 1996.