Set Notation Review

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Review: Set Notation and Operations

Sets - Definitions

A ***set*** is an unordered collection of objects.

The following are sets:

- $\{1, 2, 3\}$
- "all multiples of 7"
- {apples, 7, `True`}

Sets don't inherently have an order.

Sets - Terminology

A set is a ***finite set*** if it has a finite number of elements.

Any set that is not finite is an ***infinite set***.

Let A be a finite set. The number of different elements in A is called its ***cardinality***.

The cardinality of a finite set is denoted $|A|$.

Examples

$\{1, 2, 3\}$ is a finite set. Its cardinality is 3.

"all multiples of 7" is an infinite set.

Sets - Notation

$a \in A$ means a is an element of A.

$a \notin A$ means a is *not* an element of A.

Example

Let $A = \{apples, bananas, oranges\}$

"apples" $\in A$

"blueberries" $\notin A$

Sets - Notation

Sets are commonly expressed using *set notation*.

Within braces, we can write a rule consisting of a function, a vertical bar, and a set to which the function is applied.

Sets - Notation

Example

We can express the set $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$ as...

Subsets

A is a ***subset*** of B if and only if every element of A is an element of B.

Can also be written: $A \subseteq B$

Examples

Let $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5\}$.

$A \subseteq B$.

Let $A = \{1, 2, 3\}$ and $B = \{3, 2, 1\}$.

$A \subseteq B$ and $B \subseteq A$.

Equality

$A = B$ if and only if every element of A is an element of B and conversely every element of B is an element of A. That is, $A \subseteq B$ and $B \subseteq A$.

Example

Let $A = \{1, 2, 3\}$ and $B = \{3, 2, 1\}$. $A \subseteq B$ and $B \subseteq A$, so $A = B$

Common Sets

- There are some standard symbols that represent specific sets you will see:
- The set of **Natural Numbers** $\mathbb{N}$ is the set of all whole numbers ≥ 0, $\{0, 1, 2, 3, 4, ...\}$.*
- The set of **Integers** $\mathbb{Z}$ is the set of all whole numbers, $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
- The set of **Rational Numbers** $\mathbb{Q}$ are numbers that can be represented as a quotient of whole numbers, $\{\frac{p}{q} \mid p, q \in \mathbb{Z}\}$
- The set of **Real Numbers** $\mathbb{R}$ is all *real* numbers.

Tuples

A k-***tuple*** is an ordered sequence of k elements, which we write down in parentheses, $(a_1, a_2, \ldots, a_k)$.

> The most common tuple seen in math is the coordinate pair (x, y) on a graph.
>
> A 2-tuple is commonly called an *ordered pair*.

Two tuples are equal if and only if all of their corresponding elements are equal. $(a_1, a_2, \ldots, a_k)$ iff for all $i \in [1, \ldots, k]$ we have $a_i = b_i$.

Concatenation

Concatenation is used to join two strings or lists by putting all elements of the second string behind all elements of the first.

Example

> The concatenation of “hello” and “world” is “helloworld”.

Range

$[a, b]$ is the set of whole numbers $\geq a$ and $\leq b$.

(a, b) is the set of whole numbers $> a$ and $< b$.

Examples

> $[1, 5] = \{1, 2, 3, 4, 5\}$
>
> $(1, 5) = \{2, 3, 4\}$
>
> $[1, 5) = \{1, 2, 3, 4\}$

Set Operations

Set Operations

- $a \in B$ means a is an element of B.

Set Operations

- $a \notin B$ means a is *not* an element of B.
- Note that technically, $a \in B$ and $a \notin B$ are predicates! They take an element and a set as input and give True or False as an output.

Subset

Let A and B be sets. We say that A is a **subset** of B if and only if every element of A is an element of B.

We write $A \subseteq B$ to denote the fact that A is a subset of B.

Equality

Using Predicate Logic

- Remember this?
- For all sets A and B, $A = B$ if and only if every element of A is an element of B and every element of B is an element of A
- $\forall A, B, A = B \leftrightarrow (A \subseteq B \text{ and } B \subseteq A)$

Complement

The complement of a set A, denoted $A^{\mathcal{C}}$ is the set of all elements in the universe U that are *not* in A.

Using Set Notation

$$A^{\mathcal{C}} = \{x | x \notin A\}$$

Using Predicate Logic

$$\forall a, a \in A^{\mathcal{C}} \leftrightarrow a \notin A$$

or, equivalently, $\forall a \in U, a \notin A^{\mathcal{C}} \leftrightarrow a \in A$

Intersection

$A \cap B$ are the elements that are both in A and B.

Using Set Notation

$$A \cap B = \{x | x \in A \wedge x \in B\}$$

Using Predicate Logic

$$\forall A, B, x, x \in A \cap B \leftrightarrow (x \in A \wedge x \in B)$$

Union

$A \cup B$ are the elements that are either in A or B.

Using Set Notation

$$A \cup B = \{x | x \in A \vee x \in B\}$$

Using Predicate Logic

$$\forall A, B, x, x \in A \cup B \leftrightarrow (x \in A \vee x \in B)$$

Difference

The **difference** of sets A and B is the set that contains all elements in A that are not in B.

Using Set Notation

$$A - B = A \setminus B = \{\, x | x \in A \wedge x \notin B\}$$

Using Predicate Logic

$$\forall A, B, x, x \in A - B \leftrightarrow x \in A \wedge x \notin B$$

Difference Cont.

Example

Let $A = \{1, 3, 5, 7\}$ and $B = \{4, 5, 6, 7, 8\}$.

$A - B = \{1, 3\}$.

Example

Let $C = \{\bigcirc, \diamondsuit, \Box, \heartsuit\}$ and $e = \heartsuit$.

$C - \{e\} = \{\bigcirc, \diamondsuit, \Box\}$.

Xor

Using Set Notation

$$A \oplus B = \{x | x \in A \oplus x \in B\}$$

Using Predicate Logic

$$\forall A, B, x, x \in A \oplus B \leftrightarrow x \in A \oplus x \in B$$

Cartesian Product

The **cartesian product** of A and B,

Using Set Notation

$$A \times B = \{(a, b) | \forall a \in A, \forall b \in B\}$$

Using Predicate Logic

$$\forall A, B, a, b, ((a, b) \in A \times B) \leftrightarrow (a \in A \wedge b \in B)$$

Powerset

The powerset of a set A, denoted $\mathcal{P}(A)$ is the set of all subsets of A

Using Set Notation

$$\mathcal{P}(A) = \{S | S \subseteq A\}$$

- $|\mathcal{P}(A)| = 2^{|A|}$

Next

There's a lesson on Gradescope!