Recall:

Deterministic $M = (Q, \Sigma, \delta, s, F)$

- $\delta : Q \times \Sigma \to Q$
- $\hat{\delta} : Q \times \Sigma^* \to Q$

Acceptance of $x \in \Sigma^*$: $\hat{\delta}(s, x) \in F$

For $a \in \Sigma$, $x \in \Sigma^*$

- Base Case: $\hat{\delta}(q, \epsilon) = q$
- 2nd BC: $\hat{\delta}(q, a) = \delta(q, a)$
- Rec. Def: $\hat{\delta}(q, xa) = \delta(\underbrace{\hat{\delta}(q, x)}_{\text{evals to a state}}, a)$

Nondeterministic $N = (Q, \Sigma, \Delta, S, F)$

- $\Delta : Q \times \Sigma \to 2^Q$
- $\hat{\Delta} : Q \times \Sigma^* \to 2^Q$

"maps to any combination of states in Q"

Acceptance of $x \in \Sigma^*$: $\hat{\Delta}(S, x) \cap F \neq \emptyset$

Def: For $A \subseteq Q$, $a \in \Sigma$, $x \in \Sigma^*$

- BC $\hat{\Delta}(A, \epsilon) = A$
- 2nd BC $\hat{\Delta}(A, a) = \bigcup_{q \in A} \Delta(q, a)$
- Rec Def $\hat{\Delta}(A, xa) = \bigcup_{q \in \hat{\Delta}(A, x)} \Delta(q, a)$

Transitioning from NFA to DFA

NFA: $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$

Construct DFA ~~$M = (Q_M, \Sigma, \Delta_M,$~~

$$M = (Q_M, \Sigma, \delta_M, s_M, F_M)$$

- $Q_M = 2^{Q_N} \overset{\text{def}}{=} \{A \mid A \subseteq Q_N\}$

- $\delta_M(A, a) \overset{\text{def}}{=} \hat{\Delta}_N(A, a)$

  "state" in M

  set of states in N

- $s_M \overset{\text{def}}{=} S_N$

- $F_M \overset{\text{def}}{=} \{A \subseteq Q_N \mid A \cap F_N \neq \emptyset\}$

Summary: "set of states" in N becomes a "state" in M

WTP: N and M accept the same set of strings.

Toolbox Addition:

Lemma 1: For any $x, y \in \Sigma^*$ and $A \subseteq Q$, $\hat{\Delta}(A, xy) = \hat{\Delta}(\hat{\Delta}(A, x), y)$

"Transitioning over xy is same as trans. over x <u>then</u> y"

$\hat{\Delta}(\underline{\hat{\Delta}(A,x)}, y)$

Toolbox Addition:

Lemma 2: For any $A \subseteq Q_N$ and $x \in \Sigma^*$

$$\hat{\delta}(A, x) = \hat{\Delta}_N(A, x)$$

Proof by induction on $|x|$ (length of x)

Base case: $x = \epsilon$

$$\hat{\delta}_M(A, \epsilon) = \hat{\Delta}_N(A, \epsilon)$$

$$\downarrow \qquad \downarrow$$

$$A = A \quad \square$$

Ind Step: Assume true for x of arbitrary length ← I.H.

WTS true for xa

$\hat{\delta}_M(A, x) = \hat{\Delta}_N(A, x)$ Induction Hypothesis

$\delta_M(\underline{\hat{\delta}_M(A, x)}, a) = \delta_M(\underline{\hat{\Delta}_N(A, x)}, a)$ Apply δ_M to both sides

$\quad '' \quad = \hat{\Delta}_N(\hat{\Delta}_N(A, x), a)$ Construction Def

$\delta_M(\hat{\delta}_M(A, x), a) = \hat{\Delta}_N(A, xa)$ Lemma 1

$\hat{\delta}_M(A, xa) = \hat{\Delta}(A, xa)$ Def of $\hat{\delta}$ $\square$

N and M accept the same strings.

$$x \in L(M) \iff x \in L(N)$$

$x \in L(M)$	Assume
$\iff \hat{\delta}_M(s_M, x) \in F_M$	Def of acceptance on DFA
$\iff \cancel{\hat{\Delta}_N(S_M, x) \in F_M}$	Lemma 2
$\hat{\Delta}_N(s_M, x) \in F_M$ ← technically incorrect bc $\hat{\Delta}$ is a set	
$\iff \hat{\Delta}_N(S_N, x) \cap F_N \neq \emptyset$	Plugged in def of S_N and F_N (construction)
$x \in L(N)$	Def of acceptance on NFA □