Homework 3

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5.1

The code chunk below is used to check my answers for question 5.1. There are 5 outputs from this code chunk. Each output corresponds to a part of question 5.1. Hand work is shown on the notebook paper section of this question.

```
a1 <- c(1,2,3)
a2 <- c(4,6,8)

a <- as.matrix(cbind(a1,a2))

b1 <- c(1,1,2)
b2 <- c(3,4,5)

b <- as.matrix(cbind(b1,b2))

c1 <- c(3,5)
c2 <- c(8,4)
c3 <- c(1,0)

c <- as.matrix(cbind(c1,c2,c3))

#part 1
print("part 1")
```

```
## [1] "part 1"
```

```
a + b
```

```
##      a1 a2
## [1,]  2  7
## [2,]  3 10
## [3,]  5 13
```

```
#part 2
print("part 2")
```

```
## [1] "part 2"
```

```
a - b
```

```
##      a1 a2
## [1,]  0  1
## [2,]  1  2
## [3,]  1  3
```

```r
#part 3
print("part 3")
```

```
## [1] "part 3"
```

```r
a %*% c
```

```
##      c1 c2 c3
## [1,] 23 24  1
## [2,] 36 40  2
## [3,] 49 56  3
```

```r
#part 4
print("part 4")
```

```
## [1] "part 4"
```

```r
a %*% t(b)
```

```
##      [,1] [,2] [,3]
## [1,]   13   17   22
## [2,]   20   26   34
## [3,]   27   35   46
```

```r
#part 5
print("part 5")
```

```
## [1] "part 5"
```

```r
t(b) %*% a
```

```
##    a1 a2
## b1  9 26
## b2 26 76
```

5.4

The code chunk below is used to check my answers for question 5.4. There are 3 outputs from this code chunk, corresponding to part 1 **Y'Y**, part 2 **X'X** and part 3 **X'Y** respectively. Hand work is shown on the notebook paper section of this question.

```r
x <- c(8,4,0,-4,-8)
y <- c(7.8,9,10.2,11.0,11.7)

ymat <- as.matrix(y)
ytrans <- t(ymat)

ytransy <- ytrans %*% ymat

#Answer to part 1

print("part 1")
```

```
## [1] "part 1"
```

```r
ytransy
```

```
##        [,1]
## [1,] 503.77
```

```r
xmat <- as.matrix(cbind(rep(1,5),x))


xtrans <- t(xmat)


xtransx <- xtrans %*% xmat

#Answer to part 2
print("part 2")
```

```
## [1] "part 2"
```

```r
xtransx
```

```
##       x
##   5   0
## x 0 160
```

```r
xtransy <- xtrans %*% ymat

#Answer to Question 5.12
invxtransx_5.12 <- solve(xtransx)
invxtransx_5.12
```

```
##             x
##   0.2 0.00000
## x 0.0 0.00625
```

```r
#Answer to part 3
print("part 3")
```

```
## [1] "part 3"
```

```r
xtransy
```

```
##    [,1]
##    49.7
## x -39.2
```

5.6

The code chunk below is used to check my answers for question 5.6. There are 3 outputs from this code chunk, corresponding to part 1 **Y'Y**, part 2 **X'X** and part 3 **X'Y** respectively. Hand work is shown on the notebook paper section of this question.

```r
x <- c(1,0,2,0,3,1,0,1,2,0)
y <- c(16,9,17,12,22,13,8,15,19,11)

ymat <- as.matrix(y)
xmat <- as.matrix(cbind(rep(1,10),y))

#part 1
print("part 1")
```

```
## [1] "part 1"
```

```
ybary <- t(ymat) %*% ymat
ybary
```

```
##      [,1]
## [1,] 2194
```

```
#part 2
print("part 2")
```

```
## [1] "part 2"
```

```
xbarx <- t(xmat) %*% xmat
xbarx
```

```
##          y
##    10  142
## y 142 2194
```

```
#part 3
print("part 3")
```

```
## [1] "part 3"
```

```
xbary <- t(xmat) %*% ymat
xbary
```

```
##   [,1]
##    142
## y 2194
```

5.10

The code chunk below showcases the work and answers for question 5.10. The inverse of a is shown with the first output below, ainv. The inverse of b is shown in the second output below, binv.

```
a1 <-  c(2,3)
a2 <- c(4, 1)

a <- as.matrix(cbind(a1, a2))
#a

print("Here is matrix a.")
```

```
## [1] "Here is matrix a."
```

```
a
```

```
##      a1 a2
## [1,]  2  4
## [2,]  3  1
```

```
ainv <- solve(a)


print("Here is the inverse of matrix a.")
```

```
## [1] "Here is the inverse of matrix a."
```

```
ainv
```

```
##    [,1] [,2]
## a1 -0.1  0.4
## a2  0.3 -0.2
```

```
b1 <- c(4,6,10)
b2 <- c(3,5,1)
b3 <- c(2,10,6)

b <- as.matrix(cbind(b1,b2,b3))

print("Here is matrix b.")
```

```
## [1] "Here is matrix b."
```

```
b
```

```
##      b1 b2 b3
## [1,]  4  3  2
## [2,]  6  5 10
## [3,] 10  1  6
```

```
binv <- solve(b)
print("Here is the inverse of matrix b.")
```

```
## [1] "Here is the inverse of matrix b."
```

```
binv
```

```
##          [,1]        [,2]        [,3]
## b1  0.1086957 -0.08695652  0.10869565
## b2  0.3478261  0.02173913 -0.15217391
## b3 -0.2391304  0.14130435  0.01086957
```

5.12

$(X'X)^{-1}$ is depicted in the output of the code chunk below.

```
x <- c(8,4,0,-4,-8)
y <- c(7.8,9,10.2,11.0,11.7)

ymat <- as.matrix(y)
ytrans <- t(ymat)
ytransy <- ytrans %*% ymat


xmat <- as.matrix(cbind(rep(1,5),x))
xtrans <- t(xmat)


xtransx <- xtrans %*% xmat
xtransy <- xtrans %*% ymat

#Answer to Question 5.12
invxtransx_5.12 <- solve(xtransx)
print("X'X inverse:")
```

```
## [1] "X'X inverse:"
```

```
invxtransx_5.12
```

```
##             x
##   0.2 0.00000
## x 0.0 0.00625
```

5.14

The solutions are $y_1 = 4.5$ and $y_2 = 1$. The work for this question is depicted in the outputs below. Using the R code provided by this class's website, I found the inverse of the coefficient matrix and multiplied it by the constand matrix to find the solution matrix.

```
#Part A
mat5.14 <- as.matrix(cbind(c(4,2),c(7,3)))

print("part a, coefficient matrix")
```

```
## [1] "part a, coefficient matrix"
```

```
mat5.14
```

```
##      [,1] [,2]
## [1,]    4    7
## [2,]    2    3
```

```
mat5.14end <- as.matrix(c(25,12))

print("part a, constant matrix")
```

```
## [1] "part a, constant matrix"
```

```
mat5.14end
```

```
##      [,1]
## [1,]   25
## [2,]   12
```

```
invX <- solve(mat5.14)
#invX

Y<- invX %*% mat5.14end
print("part b, matrix solutions")
```

```
## [1] "part b, matrix solutions"
```

```
Y
```

```
##      [,1]
## [1,]  4.5
## [2,]  1.0
```