

# Lab 5: Introduction to Rejection Sampling in R - STA 360/602

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## 1 Agenda

We can often end up with posterior distributions that we only know up to a normalizing constant. For example, in practice, we may derive

$$p(\theta | x) \propto p(x | \theta)p(\theta)$$

and find that the normalizing constant  $p(x)$  is very difficult to evaluate. Such examples occur when we start building non-conjugate models in Bayesian statistics.

Given such a posterior, how can we approximate it's density? One way is using rejection sampling. As an example, let's suppose our resulting posterior distribution is

$$f(x) \propto \sin^2(\pi x), x \in [0, 1].$$

In order to understand how to approximate the density (normalized) of  $f$ , we will investigate the following tasks:

1. Plot the densities of  $f(x)$  and the Unif(0,1) on the same plot.
2. According to the rejection sampling approach sample from  $f(x)$  using the Unif(0,1) pdf as an enveloping function.
3. Plot a histogram of the points that fall in the acceptance region. Do this for a simulation size of  $10^2$  and  $10^5$  and report your acceptance ratio. Compare the ratios and histograms.
4. Repeat Tasks 1 - 3 for Beta(2,2) as an enveloping function.
5. Compare your results with the results in Task 3.