Lab 3: Introduction to Decision Theory in R - STA 360/602

Rebecca C. Steorts

1 Agenda

In class, you saw the resource allocation example. We will now go through how to reproduce parts of the lecture using R in Tasks 1-2 and Tasks 3–5 should be completed in your weekly homework assignment. Let's briefly recall the problem statement and set up.

Suppose public health officials in a small city need to decide how much resources to devote toward prevention and treatment of a certain disease, but the fraction θ of infected individuals in the city is unknown.

Suppose they allocate enough resources to accomodate a fraction c of the population. If c is too large, there will be wasted resources, while if it is too small, preventable cases may occur and some individuals may go untreated. After deliberation, they tentatively adopt the following loss function:

$$\ell(\theta, c) = \begin{cases} |\theta - c| & \text{if } c \geq \theta \\ 10|\theta - c| & \text{if } c < \theta. \end{cases}$$

By considering data from other similar cities, they determine a prior $p(\theta)$. For simplicity, suppose $\boldsymbol{\theta} \sim \text{Beta}(a, b)$ (i.e., $p(\theta) = \text{Beta}(\theta|a, b)$), with $a = 0.05$ and $b = 1$. They conduct a survey assessing the disease status of $n = 30$ individuals, $x_1, \ldots, x_n$. This is modeled as $X_1, \ldots, X_n \overset{iid}{\sim} \text{Bernoulli}(\theta)$, which is reasonable if the individuals are uniformly sampled and the population is large. Suppose all but one are disease-free, i.e., $\sum_{i=1}^{n} x_i = 1$.

1. We know $p(\theta|x)$ as an updated Beta, so we can numerically compute this integral for each c. Reproduce Figure 1 from lecture, illustrating $\rho(c, x)$ for our example. Also, work through where the minimum occurs numerically ($c \approx 0.08$).

2. Now perform a sensitivity analysis for the prior assumption (Beta(a,b)). What do you find?

3. Consider the Bayes procedure ($c \approx 0.08$), $c = \bar{x}, c = 0.1$. Reproduce Figure 2. Explain your findings.

4. Plot the frequentist risk $R(\theta, \delta)$ as a function of θ for the three procedures in the previous task. Report your findings.

5. Based on your plot of the frequentist risk, consider the three estimators—the constant, the mean, and the Bayes estimators. Which estimators are admissible? Be sure to explain why or why they are not admissible.