Teaching Bayes: A Binomial-Beta Lab

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Goal of the Lab and Tasks

Recall the Beta-Bernoulli model:

$$X \mid \theta \sim \text{Bernoulli}(\theta)$$

$$\theta \sim \text{Beta}(a, b)$$

where a, b are fixed parameters.

- Let's determine whether the probability that a worker will fake an illness is truly 1%. Your task is to assist me!
- Let's outline our tasks and then solve them.

Task 1

- Simulate some data using the `rbinom` function of size $n = 100$ and probability equal to 1%.
- In order to replicate results, let's set our seed using `set.seed(123)`.

Task 2

- Write a function that takes as its inputs the data you simulated above and a sequence of θ values of length 1000 and produces Likelihood values based on the Binomial Likelihood.
- Plot your sequence and its corresponding Likelihood function.

Task 3

- Write a function that takes as its inputs prior parameters `a` and `b` for the Beta-Bernoulli model and the observed data, and produces the posterior parameters you need for the model.
- Generate the posterior parameters for a non-informative prior i.e. `(a,b) = (1,1)` and for an informative case `(a,b) = (3,1)`.

Task 4

- Create two plots, one for the informative and one for the non-informative case to show the posterior distribution and superimpose the prior distributions on each along with the likelihood.
- What do you see? (Remember to turn the y-axis ticks off since superimposing may make the scale non-sense).

Task 5

- Based on the informative case, generate a 95% credible interval with 1000 posterior draws and a 95% confidence interval for your parameter of interest, and use xtable to output these. What is the problem?
- Based on the data you simulated, do you conclude that the true value higher or lower than 1%?