

STA 360/602

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1. Let

$$\begin{aligned} Y|\theta &\sim \text{Exp}(\theta) \\ \theta &\sim \text{Gamma}(a, b). \end{aligned}$$

Suppose we have a new observation $\tilde{Y}|\theta \sim \text{Exp}(\theta)$, where conditional on θ , Y and \tilde{Y} are independent. Show that

$$p(\tilde{y}|y) = \frac{b(a+1)(by+1)^{a+1}}{(b\tilde{y}+by+1)^{a+2}},$$

where a is an integer. (Note that this is a valid density function that integrates to 1).

2. Suppose

$$X_1, \dots, X_n | \theta \stackrel{iid}{\sim} \text{Poisson}(\theta).$$

- (a) Find Jeffreys' prior. Is it proper or improper?
 - (b) Find $p(\theta|x_1, \dots, x_n)$ under Jeffreys' prior.
3. Consider dose response models. The setup is the following: animals are tested for development of drugs or other chemical compounds. Someone administers various levels of doses to k batches of animals. The response variable is a dichotomous (binary) outcome. So, it might be alive or dead or maybe tumor or no tumor. Let x_i represent the data, n_i represent the number of animals receiving the i th dose, and y_i the number of positive outcomes for n_i animals.
 - (a) Suppose that $y_i \stackrel{iid}{\sim} \text{Binomial}(n_i, \theta_i)$, where θ_i is the probability of death (or tumor) for the i th animal that receives dose x_i . The typical modeling the prior on θ_i is a logistic regression. That is, we suppose that $\text{logit}(\theta_i) = \alpha + \beta x_i$. Write out the likelihood in a simple form (it will contain a product).
 - (b) Find Jeffreys' prior for (α, β) . Also, write down the equations you need to solve for finding the posterior modes α and β under the uniform prior for α and β .