Lab 4: Do a teacher's expectations influence student achievement?

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February 8, 2017

Let's first load packages that we'll need in this assignment and also load the data.

```
## Warning: package 'dplyr' was built under R version 3.4.4
```

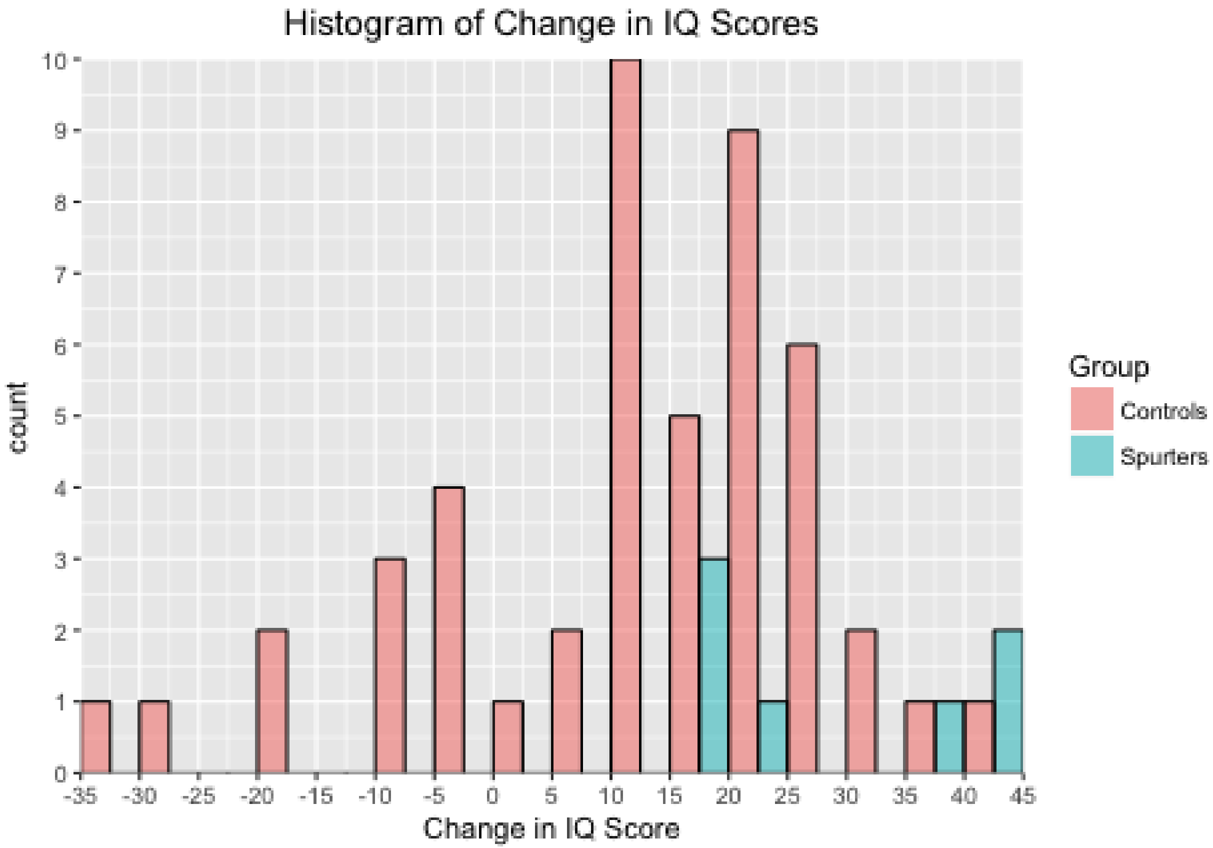
```
## Warning: package 'xtable' was built under R version 3.4.4
```

```r
# input data
x = c(18, 40, 15, 17, 20, 44, 38)
y = c(-4, 0, -19, 24, 19, 10, 5, 10,
      29, 13, -9, -8, 20, -1, 12, 21,
      -7, 14, 13, 20, 11, 16, 15, 27,
      23, 36, -33, 34, 13, 11, -19, 21,
      6, 25, 30,22, -28, 15, 26, -1, -2,
      43, 23, 22, 25, 16, 10, 29)
# store data in data frame
iqData = data.frame(Treatment = c(rep("Spurters", length(x)), rep("Controls", length(y))),
Gain = c(x, y))
```

Task 1

Plot histograms for the change in IQ score for the two groups. Report your findings.

```r
xLimits = seq(min(iqData$Gain) - (min(iqData$Gain) %% 5),
              max(iqData$Gain) + (max(iqData$Gain) %% 5),
              by = 5)

ggplot(data = iqData, aes(x = Gain, fill = Treatment, colour = I("black"))) +
  geom_histogram(position = "dodge", alpha = 0.5, breaks = xLimits, closed = "left")+
  scale_x_continuous(breaks = xLimits,
                     expand = c(0,0))+
  scale_y_continuous(expand = c(0,0),
breaks = seq(0, 10, by = 1))+
  ggtitle("Histogram of Change in IQ Scores") + labs(x = "Change in IQ Score", fill = "Group") + theme(
```



From the histograms, I know that the randomly selected "spurters" group has a different distribution than the "controls" group. This could indicate that teachers being told that a specific group of students is expected to perform particularly well will pay more attention and time on that group and resulting in more improvement over the year.

Task 2

$$p(X_1 ... X_{n_s}|\mu_s, \lambda_s^{-1}) = \prod_{i=1}^{n_s} \frac{1}{\sqrt{2{\sigma_s}^2\pi}} e^{-\frac{(x_i-\mu_s)^2}{2{\sigma_s}^2}} = \prod_{i=1}^{n_s} \frac{1}{\sqrt{2{\lambda_s}^{-1}\pi}} e^{-\frac{\lambda_s(x_i-\mu_s)^2}{2}}$$

$$p(Y_1 ... Y_{n_c}|\mu_c, \lambda_c^{-1}) = \prod_{i=1}^{n_c} \frac{1}{\sqrt{2{\sigma_c}^2\pi}} e^{-\frac{(y_i-\mu_c)^2}{2{\sigma_c}^2}} = \prod_{i=1}^{n_c} \frac{1}{\sqrt{2{\lambda_c}^{-1}\pi}} e^{-\frac{\lambda_c(y_i-\mu_c)^2}{2}}$$

$$p(\mu_s, \lambda_s|m, c, a, b) = \frac{b^a\sqrt{c}}{\Gamma(a)\sqrt{2\pi}} \lambda_s^{a-0.5} e^{-b\lambda_s} e^{-\frac{c\lambda_s(\mu_s-m)^2}{2}}$$

$$p(\mu_c, \lambda_c|m, c, a, b) = \frac{b^a\sqrt{c}}{\Gamma(a)\sqrt{2\pi}} \lambda_c^{a-0.5} e^{-b\lambda_c} e^{-\frac{c\lambda_c(\mu_c-m)^2}{2}}$$

$$\begin{aligned}(\mu_s, \lambda_s)|x_{1:n_s} &\sim \text{NormalGamma}\left(m' = \frac{cm + n_s\bar{x}}{c + n_s}, c' = c + n_s, a' = a + \frac{n_s}{2}, b' = b + \frac{1}{2}\sum_{i=1}^{n_s}(x_i - \bar{x})^2 + \frac{n_s c}{c + n_s}\frac{(\bar{x} - m)^2}{2}\right) \\ &= \text{NormalGamma}(24, 8, 4, 855)\end{aligned}$$

$$\begin{aligned}(\mu_c, \lambda_c)|y_{1:n_c} &\sim \text{NormalGamma}\left(m^* = \frac{cm + n_c\bar{y}}{c + n_c}, c^* = c + n_c, a^* = a + \frac{n_c}{2}, b^* = b + \frac{1}{2}\sum_{i=1}^{n_c}(y_i - \bar{y})^2 + \frac{n_c c}{c + n_c}\frac{(\bar{y} - m)^2}{2}\right) \\ &= \text{NormalGamma}(11.8, 49, 24.5, 6344)\end{aligned}$$

```r
prior = data.frame(m = 0, c = 1, a = 0.5, b = 50)
findParam = function(prior, data){
  postParam = NULL
  c = prior$c
  m = prior$m
  a = prior$a
  b = prior$b
  n = length(data)
  postParam = data.frame(m = (c*m + n*mean(data))/(c + n),
                c = c + n,
                a = a + n/2,
                b =  b + 0.5*(sum((data - mean(data))^2)) +
                  (n*c *(mean(data)- m)^2)/(2*(c+n)))
  return(postParam)
}
postS = findParam(prior, x)
postC = findParam(prior, y)
```

% latex table generated in R 3.4.1 by xtable 1.8-3 package % Tue Feb 5 13:40:56 2019

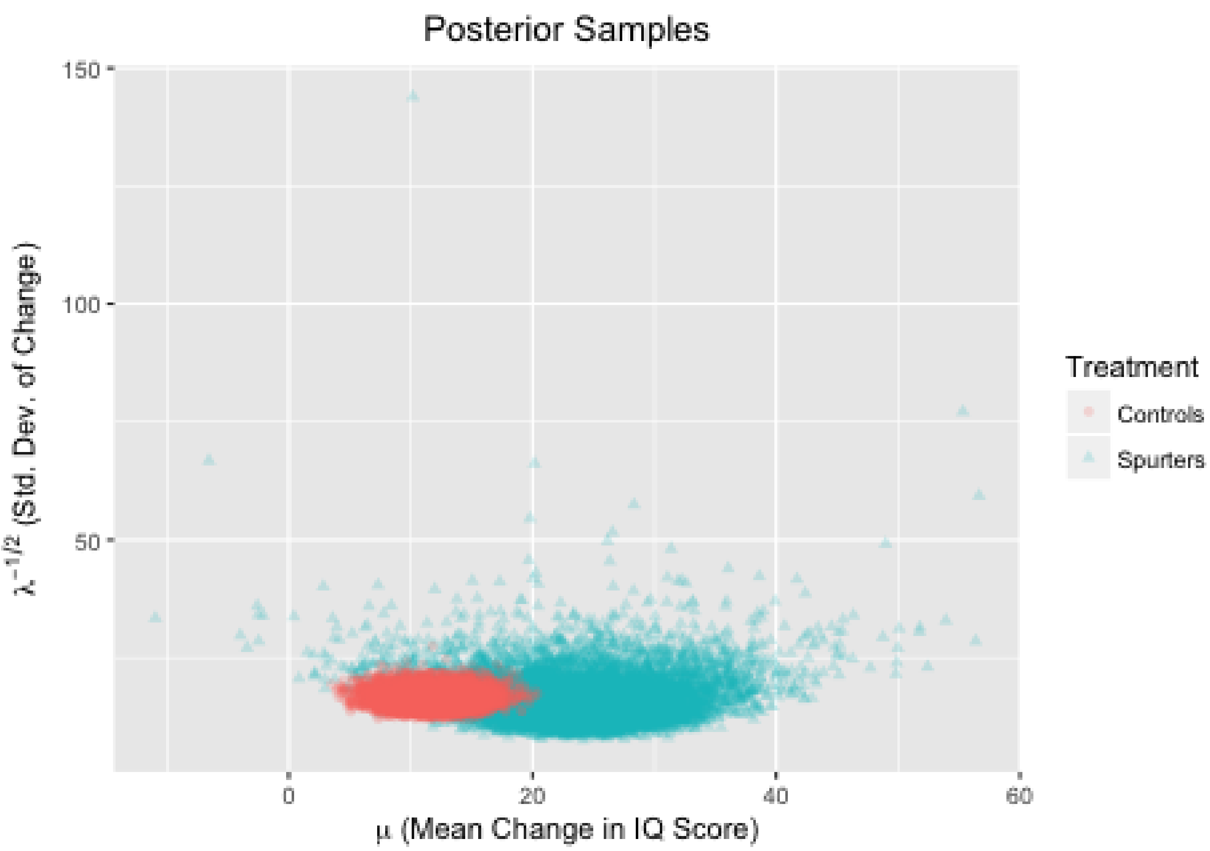
	m	c	a	b
prior	0.00	1.00	0.50	50.00
Spurters Posterior	24.00	8.00	4.00	855.00
Controls Posterior	11.80	49.00	24.50	6343.98

Table 1: Parameters

Task 3

```r
# sampling from two posteriors

sim = 1e+4
# initialize vectors to store samples
mus = NULL
lambdas = NULL
muc = NULL
lambdac = NULL

lambdas = rgamma(sim, shape = postS$a, rate = postS$b)
lambdac = rgamma(sim, shape = postC$a, rate = postC$b)
mus = sapply(sqrt(1/(postS$c*lambdas)),rnorm, n = 1, mean = postS$m)
muc = sapply(sqrt(1/(postC$c*lambdac)),rnorm, n = 1, mean = postC$m)

simDF = data.frame(lambda = c(lambdas, lambdac),
                   mu = c(mus, muc),
                   Treatment = rep(c("Spurters", "Controls"),
                                   each = sim))
simDF$lambda = simDF$lambda^{-0.5}

ggplot(data = simDF, aes(x = mu, y = lambda, colour = Treatment, shape = Treatment)) +
  geom_point(alpha = 0.2) +
  labs(x = expression(paste(mu, " (Mean Change in IQ Score)")),
       y = expression(paste(lambda^{-1/2}, " (Std. Dev. of Change)")))  +
```

```r
ggtitle("Posterior Samples")+
theme(plot.title = element_text(hjust = 0.5))
```



The simulated scatterplot does look similar to Figure 1 in that the control group is more concentrated with a smaller average mean change in IQ score, while the spurters group has a larger average mean change in IQ score.