

Lab 5: Introduction to Rejection Sampling in R - STA 360/602

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1 Agenda

We can often end up with posterior distributions that we only know up to a normalizing constant. For example, in practice, we may derive

$$p(\theta | x) \propto p(x | \theta)p(\theta)$$

and find that the normalizing constant $p(x)$ is very difficult to evaluate. Such examples occur when we start building non-conjugate models in Bayesian statistics.

Given such a posterior, how can we approximate it's density? One way it's using importance sampling. As an example, let's suppose our resulting posterior distribution is

$$f(x) \propto \sin^2(\pi x), x \in [0, 1].$$

In order to understand how to approximate the density (normalized) of f , we will investigate the following tasks:

1. Plot the densities of $f(x)$ and the Unif(0,1) on the same plot.
2. According to the rejection sampling approach sample from $f(x)$ using the Unif(0,1) pdf as an enveloping function.
3. Plot a histogram of the points that fall in the acceptance region. Do this for a simulation size of 10^2 and 10^5 and report your acceptance ratio. Compare the ratios and histograms.
4. Repeat Tasks 1 - 3 for Beta(2,2) as an enveloping function.
5. Compare your results with the results in Task 3.