lab4Solutions

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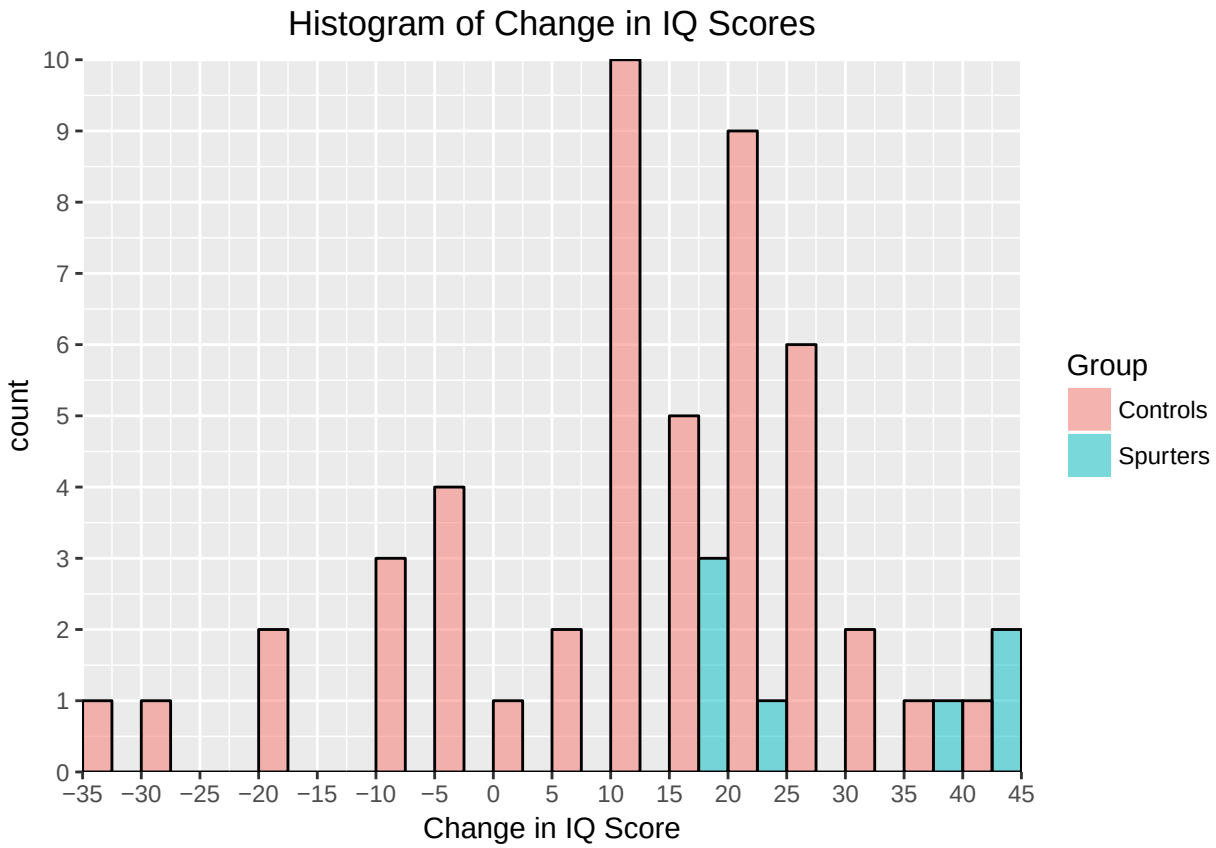
Problem 1

```
## Warning: package 'ggplot2' was built under R version 3.3.2
```

```
x = c(18, 40, 15, 17, 20, 44, 38)
y = c(-4, 0, -19, 24, 19, 10, 5, 10,
      29, 13, -9, -8, 20, -1, 12, 21,
      -7, 14, 13, 20, 11, 16, 15, 27,
      23, 36, -33, 34, 13, 11, -19, 21,
      6, 25, 30,22, -28, 15, 26, -1, -2,
      43, 23, 22, 25, 16, 10, 29)
iqData = data.frame(Treatment =
                      c(rep("Spurters", length(x)),
                        rep("Controls", length(y))),
                    Gain = c(x, y))
#iqData = read.csv("pygmalion.csv", header = TRUE)
```

Part 1

```
xLimits = seq(min(iqData$Gain) - (min(iqData$Gain) %% 5),
              max(iqData$Gain) + (max(iqData$Gain) %% 5),
              by = 5)

ggplot(data = iqData, aes(x = Gain,
                          fill = Treatment,
                          colour = I("black"))) +
  geom_histogram(position = "dodge", alpha = 0.5,
                 breaks = xLimits, closed = "left")+
  scale_x_continuous(breaks = xLimits,
                     expand = c(0,0))+
  scale_y_continuous(expand = c(0,0),
                     breaks = seq(0, 10, by = 1))+
  ggtitle("Histogram of Change in IQ Scores") +
  labs(x = "Change in IQ Score", fill = "Group") +
  theme(plot.title = element_text(hjust = 0.5))
```



From the histograms, I know that the randomly selected "spurters" group has a different distribution than the "controls" group. This could indicate that teachers being told that a specific group of students is expected to perform particularly well will pay more attention and time on that group and resulting in more improvement over the year.

Part 2

$$p(X_1 ... X_{n_s}) = \prod_{i=1}^{n_s} \frac{1}{\sqrt{2{\sigma_s}^2\pi}} e^{-\frac{(x_i-\mu_s)^2}{2{\sigma_s}^2}} = \prod_{i=1}^{n_s} \frac{1}{\sqrt{2{\lambda_s}^{-1}\pi}} e^{-\frac{\lambda_s(x_i-\mu_s)^2}{2}}$$

$$p(X_1 ... X_{n_c}) = \prod_{i=1}^{n_c} \frac{1}{\sqrt{2{\sigma_c}^2\pi}} e^{-\frac{(x_i-\mu_c)^2}{2{\sigma_c}^2}} = \prod_{i=1}^{n_c} \frac{1}{\sqrt{2{\lambda_c}^{-1}\pi}} e^{-\frac{\lambda_c(x_i-\mu_c)^2}{2}}$$

$$p(\mu_s, \lambda_s | m, c, a, b) = \frac{b^a\sqrt{c}}{\Gamma(a)\sqrt{2\pi}} \lambda_s^{a-0.5} e^{-b\lambda_s} e^{-\frac{c\lambda_s(\mu_s-m)^2}{2}}$$

$$p(\mu_c, \lambda_c | m, c, a, b) = \frac{b^a\sqrt{c}}{\Gamma(a)\sqrt{2\pi}} \lambda_c^{a-0.5} e^{-b\lambda_c} e^{-\frac{c\lambda_c(\mu_c-m)^2}{2}}$$

$$(\mu_s, \lambda_s)|x_{1:n_s} \sim \text{NormalGamma}\left(m' = \frac{cm + n_s\bar{x}}{c+n_s}, c' = c + n_s, a' = a + \frac{n_s}{2}, b' = b + \frac{1}{2}\sum_{i=1}^{n_s}(x_i - \bar{x})^2 + \frac{n_s c}{c+n_s}\frac{(\bar{x}-m)^2}{2}\right)$$

$$= \text{NormalGamma}(24, 8, 4, 855)$$

$$(\mu_c, \lambda_c)|y_{1:n_c} \sim \text{NormalGamma}\left(m^* = \frac{cm + n_c\bar{y}}{c+n_c}, c^* = c + n_c, a^* = a + \frac{n_c}{2}, b^* = b + \frac{1}{2}\sum_{i=1}^{n_c}(y_i - \bar{y})^2 + \frac{n_c c}{c+n_c}\frac{(\bar{y}-m)^2}{2}\right)$$

$$= \text{NormalGamma}(11.8, 49, 24.5, 6344)$$

```r
prior = data.frame(m = 0, c = 1, a = 0.5, b = 50)
findParam = function(prior, data){
  postParam = NULL
  c = prior$c
  m = prior$m
  a = prior$a
  b = prior$b
  n = length(data)
  postParam = data.frame(m = (c*m + n*mean(data))/(c + n),
                c = c + n,
                a = a + n/2,
                b =  b + 0.5*(sum((data - mean(data))^2)) +
                  (n*c *(mean(data)- m)^2)/(2*(c+n)))
  return(postParam)
}
postS = findParam(prior, x)
postC = findParam(prior, y)
```

% latex table generated in R 3.3.1 by xtable 1.8-2 package % Mon Feb 13 09:01:32 2017

	m	c	a	b
prior	0.00	1.00	0.50	50.00
Spurters Posterior	24.00	8.00	4.00	855.00
Controls Posterior	11.80	49.00	24.50	6343.98

Table 1: Parameters

```r
# sampling from two posteriors

sim = 1000
# initialize vectors to store samples
#mus = NULL
#lambdas = NULL
#muc = NULL
#lambdac = NULL


# for(i in 1:sim){
#   lambdas[i] = rgamma(1, postS$a, postS$b)
#   lambdac[i] = rgamma(1, postC$a, postC$b)
#   mus[i] = rnorm(1, postS$m, sqrt(1/(postS$c*lambdas[i])))
#   muc[i] = rnorm(1, postC$m, sqrt(1/(postC$c*lambdac[i])))
# }
lambdas = rgamma(sim, postS$a, postS$b)
lambdac = rgamma(sim, postC$a, postS$b)
mus = sapply(sqrt(1/(postS$c*lambdas)),rnorm, n = 1, mean = postS$m)
muc = sapply(sqrt(1/(postC$c*lambdac)),rnorm, n = 1, mean = postC$m)

simDF = data.frame(lambda = c(lambdas, lambdac),
                   mu = c(mus, muc),
                   Treatment = rep(c("Spurters", "Controls"),
                                   each = sim))
simDF$lambda = simDF$lambda^{-0.5}
```

```
ggplot(data = simDF, aes(x = mu, y = lambda, colour = Treatment, shape = Treatment)) +
  geom_point(alpha = 0.2) +
  labs(x = expression(paste(mu, " (Mean Change in IQ Score)")),
       y = expression(paste(lambda^{-1/2}, " (Std. Dev. of Change)")))  +
  ggtitle("Posterior Samples")+
  theme(plot.title = element_text(hjust = 0.5))
```

The simulated scatterplot does look similar to Figure one in that the controls groups is more concentrated with a smaller average change in IQ Score and a smaller variance while the spurters group has a larger spread and a higher average change in IQ score. ### Part 3

```
# approximate posterior probabilty
(apprx = mean(mus > muc))
```

```
## [1] 0.989
```

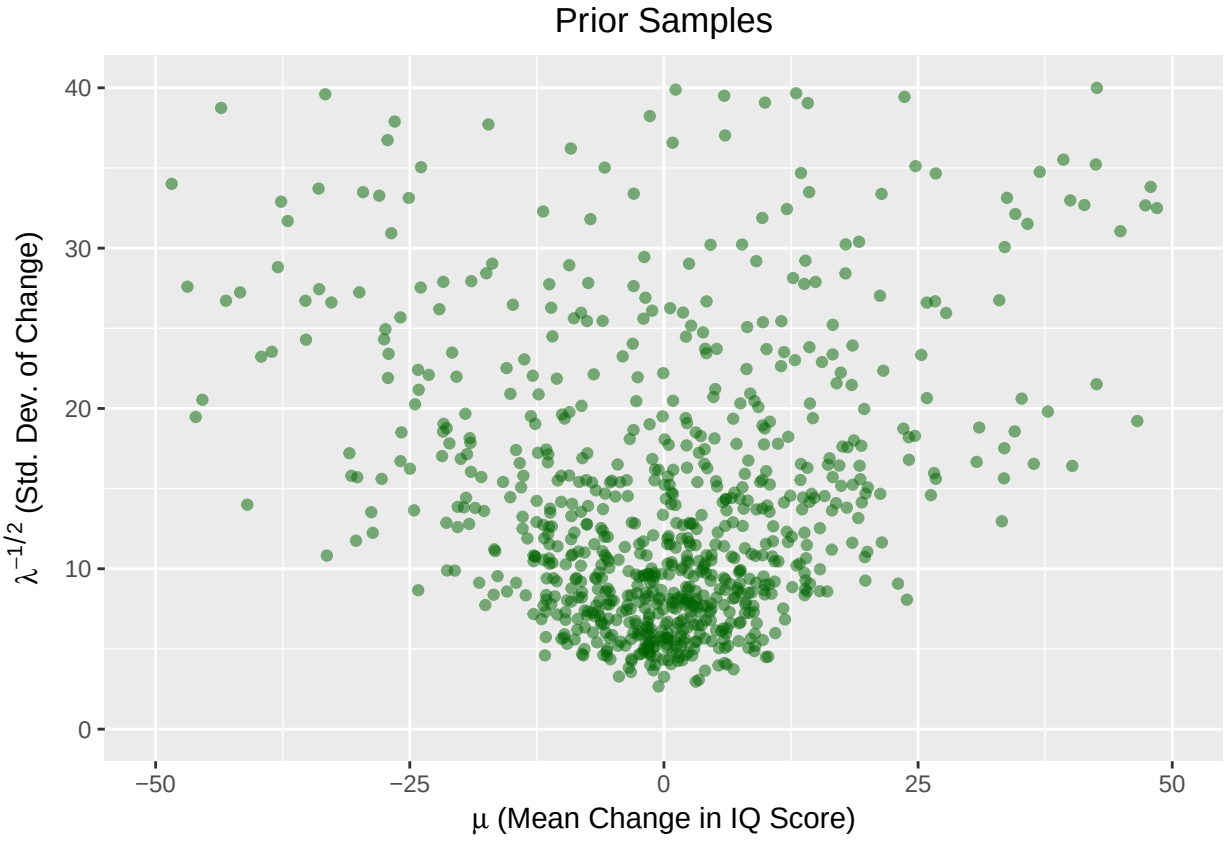
The posterior probabilty that $\mu_s > \mu_c$ is approximately 98.9%.

Part 4

```
# sample mu and lambda from prior
sim = 1000
# initialize vectors to store samples
#mu = NULL
#lambda = NULL

# for(i in 1:sim){
#   lambda[i] = rgamma(1, prior$a, prior$b)
```

```
#   mu[i] = rnorm(1, prior$m, sqrt(1/(prior$c*lambda[i])))
# }
lambda = apply(as.matrix(sim), 1,rgamma, prior$a, prior$b)
mu = sapply(sqrt(1/(prior$c*lambda)), rnorm, n = 1, mean = prior$m)

simPrior = data.frame(lambda, mu)
simPrior$lambda = simPrior$lambda^{-0.5}

ggplot(data = simPrior, aes(x = mu, y = lambda)) +
  geom_point(alpha = 0.5, colour = "darkgreen") +
  labs(x = expression(paste(mu, " (Mean Change in IQ Score)")),
       y = expression(paste(lambda^{-1/2}, " (Std. Dev. of Change)"))) +
  ggtitle("Prior Samples")+
  theme(plot.title = element_text(hjust = 0.5))+
  xlim(-50,50) + ylim(0, 40)
```

```
## Warning: Removed 212 rows containing missing values (geom_point).
```

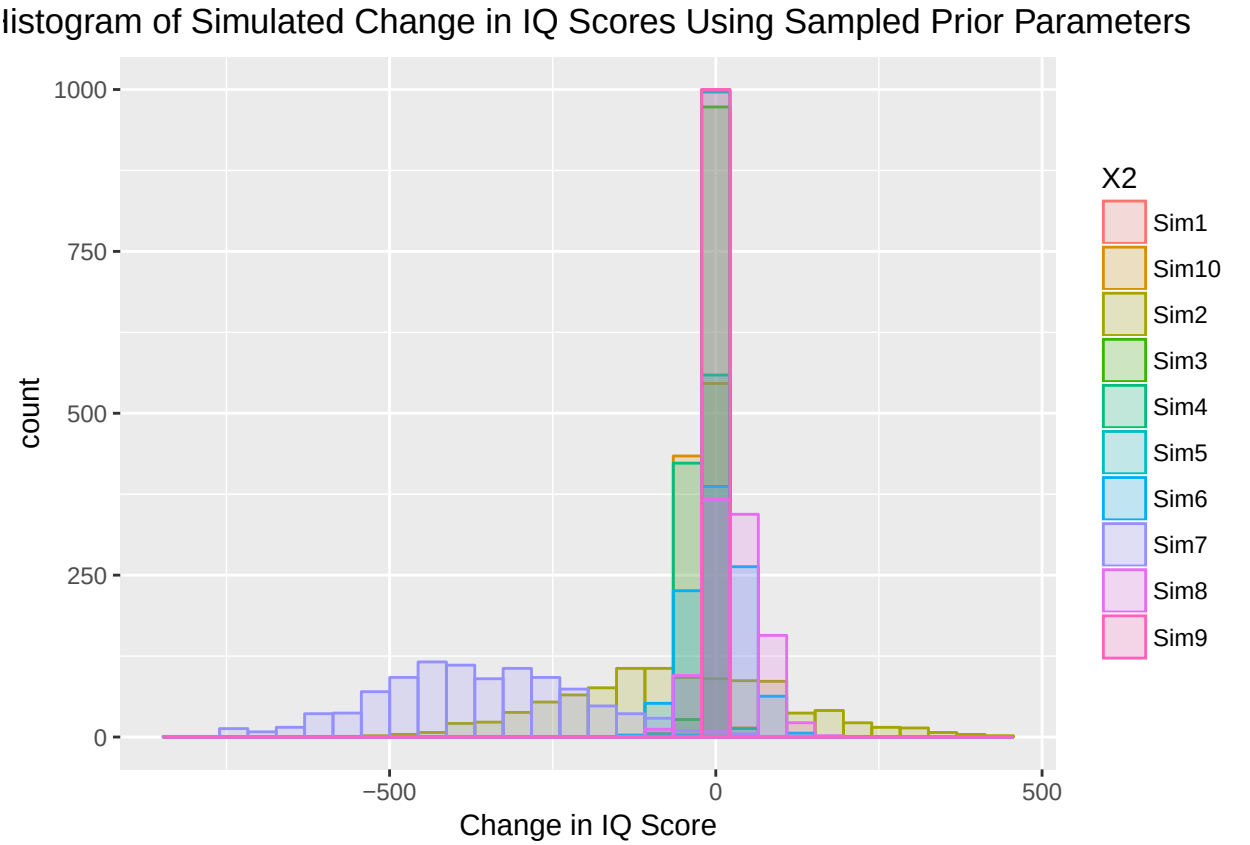


The plot simulated from the prior distribution is similar to Figure 2 in that it does seem to have a bottom-heavy convex spread centered at 0. The major difference is obviously the much larger range for both the mean change in IQ score and variance of change. It is very varied and seems to not add a lot of information to the data.

```
# sample 1000 data points from the latter 10 set of parameters

simData = mapply(rnorm, n = 1000,
                 mean = tail(simPrior$mu, n = 10),
                 sd = tail(simPrior$lambda, n = 10))
```

```r
colnames(simData) = paste0("Sim", 1:10)
simData = melt(simData)[,2:3]

ggplot(data = simData, aes(x = value, fill = X2, colour = X2)) +
  geom_histogram(alpha = 0.2, position = "identity") +
  ggtitle("Histogram of Simulated Change in IQ Scores Using Sampled Prior Parameters") +
  labs(x = "Change in IQ Score", lab = "Simulation") +
  theme(plot.title = element_text(hjust = 0.5))
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Based on the prior distribution of the likelihood parameters, the simulated data is slightly right skewed and approximately dentered around 0 with small tails, meaning that it is more likely to see little to none changes in IQ scores.