Exercise sheet 03 – Tristan Koning

2a) Uniform Distr. has the same propability c for each value $a < x < b$

$$PDF = \int_a^b p(x \mid a, b)\, dx = 1$$

$$\Leftrightarrow \int_a^b c \; dx = 1 \Leftrightarrow c(b-a) = 1$$

$$\Leftrightarrow c = \frac{1}{b-a}$$

This ensures that the probability over the interval $[a, b]$ is equal to 1.

2b) Since the uniform distribution has the same probability for every point x, we can simply take the midpoint of the interval:

$$\mu = \frac{a+b}{2}$$

2c) $\cancel{Var(x) = \int_a^b (x - \frac{a-b}{\;}}$

$\cancel{Var(X) = E[(x-\mu)^2] = E[(X - \frac{a+b}{2})^2]}$

$\Rightarrow$

2c) $Var(X) = E(X^2) - (E(X))^2$

$$E(X) = \mu = \frac{a+b}{2}$$

$$E(X^2) = \int_a^b x^2 \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a} \int_a^b x^2\, dx$$

$$= \frac{1}{b-a} \cdot \left[\frac{x^3}{3} \right]_a^b = \frac{1}{b-a} \cdot \left(\frac{b^3}{3} - \frac{a^3}{3} \right) = \frac{b^3 - a^3}{3(b-a)}$$

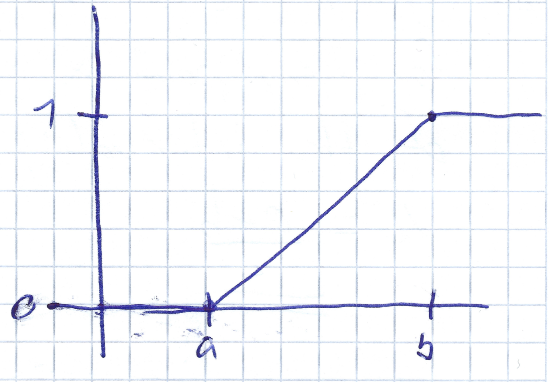
$$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} = \frac{b^2+ab+a^2}{3}$$

$$Var(X) = E(X^2) - \mu^2 = \frac{b^2+ab+a^2}{3} - \frac{b^2+2ab+a^2}{4}$$

$$= \frac{4(b^2+ab+a^2) - 3(b^2+2ab+a^2)}{12} = \frac{b^2-2ab+a^2}{12}$$

$$= \underline{\underline{\frac{(b-a)^2}{12}}}$$

2d) Looking at the graph of the CDF on the right, we can see that a is the starting point where the CDF starts rising. Therefore, we center our points x to a and apply the gradient (probability).



$F(x) = \underline{\frac{x-a}{b-a}}$ | if $a < x < b$

$F(x) = \underline{\underline{0}}$ | if $x < a$

$F(x) = \underline{\underline{1}}$ | if $x > b$