Universität Zürich
Institut für Informatik

HS 2023

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Numerical Methods in Informatics - Exercise 2

Hand out: 19.10.2023 - Due to: 01.11.2023

Please upload your solutions to the Olat system.

Theory

2.1 Matrix Factorization and Subspaces

a) (20 Min, 6 Points) LU Factorization

Please calculate the LU factorization ($A = LU$) without pivoting of the following matrix step by step:

$$A = \begin{pmatrix} 2 & 4 & 6 & 2 \\ 1 & 3 & 9 & 2 \\ 4 & 10 & 15 & 6 \\ 5 & 8 & 7 & 4 \end{pmatrix}$$

First, we transform A into an echelon form using only row replacement operations:

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 \\ 1 & 3 & 9 & 2 \\ 4 & 10 & 15 & 6 \\ 5 & 8 & 7 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 6 & 2 \\ 0 & 1 & 6 & 1 \\ 0 & 2 & 3 & 2 \\ 0 & -2 & -8 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 6 & 2 \\ 0 & 1 & 6 & 1 \\ 0 & 0 & -9 & 0 \\ 0 & 0 & 9 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 6 & 2 \\ 0 & 1 & 6 & 1 \\ 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U$$

To calculate L, we take the value of the top pivot of each column in A and divide all entries from pivot and below. Fill the other entries with 0.

To pivot columns: $\begin{bmatrix} 2 \\ 1 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \begin{bmatrix} -9 \\ 9 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 3/2 & -2 & -9/9 & 1 \end{bmatrix}$$

b) (30 Min, 4 Points) Subspaces

Given the matrix A, find a basis for $ColA$ and a basis for $NulA$

$$A = \begin{pmatrix} -6 & 18 & -4 & -14 \\ 4 & -12 & 8 & 16 \\ 3 & -9 & -2 & 2 \end{pmatrix}$$

Nul A:

$Ax = 0 \Rightarrow$ Reduce A

$$A = \begin{bmatrix} -6 & 18 & -4 & -14 \\ 4 & -12 & 8 & 16 \\ 3 & -9 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2/3 & 7/3 \\ 0 & 0 & 16/3 & 20/3 \\ 0 & 0 & -4 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2/3 & 7/3 \\ 0 & 0 & 1 & 5/4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 3/2 \\ 0 & 0 & 1 & 5/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Leftrightarrow$ $x_1 - 3x_2 + \frac{3}{2}x_4 = 0$, $x_3 + \frac{5}{4}x_4 = 0$, $0 = 0$

$\Leftrightarrow$ $x_1 = 3x_2 - \frac{3}{2}x_4$, $x_3 = -\frac{5}{4}x_4$

↳ x_2, x_4, x_5 are free variables

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3x_2 - \frac{3}{2}x_4 \\ x_2 \\ -\frac{5}{4}x_4 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3/2 \\ 0 \\ -5/4 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

($\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ = s, $\begin{bmatrix} -3/2 \\ 0 \\ -5/4 \\ 1 \\ 0 \end{bmatrix}$ = u, $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ = v)

↳ Basis for Nul A: $\{s, u, v\}$

Col A:

A in reduced echelon form:

$$\begin{bmatrix} 1 & -3 & 0 & 3/2 \\ 0 & 0 & 1 & 5/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↳ Basis of col A are all colums that contain a leading 1 from the original A.

↳ col $A = \begin{bmatrix} -6 & -4 \\ 4 & 8 \\ 3 & -2 \end{bmatrix}$