IIT MANDI

4TH INTER IIT TECH MEET

2016

LET THE BRAINS COMPETE AGAIN

Portfolio Defender Problem Statement

PROBLEM:

Given a set of n assets $A = \{a_1, a_2, \ldots, a_n\}$. Each asset a_i is associated with an expected return (per period) r_i, and each pair of assets $< a_i, a_j >$ has a covariance σ_{ij}. The covariance matrix $[\sigma_{ij}]$ is symmetric and each diagonal element σ_{ii} represents the variance of asset a_i. A positive value R represents the expected return.

A portfolio can be represented by a set $W = \{w_1, w_2, \ldots, w_n\}$, where w_i represents the percentage wealth invested on asset a_i. The formulation of the basic problem can thus be defined as the following.

$$\begin{gathered} \text{Minimize } f(x) = \lambda \sum \sigma_{ij} w_i w_j - (1-\lambda) \sum r_i w_i \\ \textit{Subject to:} \\ \sum w_i = 1 \\ 0 \le w_i \le 1, i = 0, 1, \ldots, n \end{gathered} \quad \textbf{(A)}$$

A set of extended portfolio optimization problem with following additional constraints is given by

1. Total number of assets (out of n assets) in which investment has been done.
2. Upper and lower bounds on the investments (if any) in the assets.

$$\begin{gathered} \text{Minimize } f(x) = \lambda \sum \sigma_{ij} w_i w_j - (1-\lambda) \sum r_i w_i \\ \textit{Subject to:} \\ \sum w_i = 1 \\ \epsilon_i y_i \le w_i \le \delta_i y_i \\ \sum y_i = K \\ \epsilon_i, \delta_i > 0 \\ y_i \in \{0, 1\} \\ i = 0, 1, \ldots, n \\ 0 \le \lambda \le 1 \end{gathered} \quad \textbf{(B)}$$

Where the binary variable y_i is 1 if investment has been done in i^{th} asset and 0 if no investment has been done in i_{th}. Total number of assets in which the investment has been done is restricted to $K \leq n$ (i.e. $w_i > 0$ if $y_i = 1$ and $w_i = 0$ if $y_i = 0 \forall i$).

Write a generic code to handle any given number of assets n to solve the above models. The five data sets for different values of n are given. The data set contains the historical returns r_i and covariance matrix $[\sigma_{ij}]$. Using the code developed, solve the formulation **(A)** and **(B)** for $K = 10$, by taking $\lambda = \left\{0, \frac{1}{2000}, \frac{2}{2000}, \ldots, 1\right\}$ for the data sets provided. Plot the results of both the models and *benchmark unconstrained frontier* (provided in attached files) for all values of λ in a two dimensional graph with $x-$ axis as $\sum \sigma_{ij} w_i w_j$ and $y-$ axis as $\sum r_i w_i$ for each of the data set provided. Save the weights w_i for both the models and all the data sets and for all values of λ in separate files.

The test problems are the files: $port1.txt, port2.txt, \ldots, port5.txt$

The format of these data files is: number of assets (n) for each asset i $(i = 1, 2, \ldots, n)$: mean return, standard deviation of return for all possible pairs of assets: i, j, correlation between asset i and asset j

The unconstrained efficient frontiers for each of these data sets are available in the files: $portef1.txt, portef2.txt, \ldots, portef5.txt$

The format of these files is: for each of the calculated points (λ) on the unconstrained frontier: mean return $(\sum r_i w_i)$, variance $(\sum \sigma_{ij} w_i w_j)$ of return

FIND THE DATA SETS HERE :

https://github.com/interiittech/portfolio_defender_16/