



## MACHINE LEARNING FUNDAMENTALS



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Tillyard Auditorium

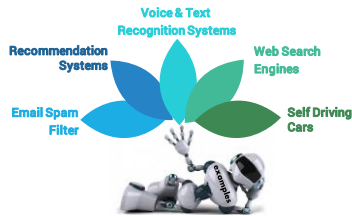
Session 1: Feb 19<sup>th</sup>

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### What is Machine Learning?

☞ In its simplest definition **Machine Learning** is the science (and art) of programming computers so they can learn from data ☞



*"Arthur Samuel, an American pioneer in the field of computer gaming and artificial intelligence, coined the term 'Machine Learning' in 1959 while at IBM"*

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### Any Application in Energy Sector? Let's listen to Uncle Rob



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## Three Different Type of Machine Learning

## 01 Supervised Learning

- Labeled data
- Direct feedback
- Predict outcome/future

## 02 Unsupervised Learning

- No label/targets
- No feedback
- Find hidden structure in data

## 03 Reinforcement Learning

- Decision process
- Reward system
- Learn series of action



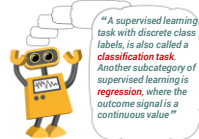
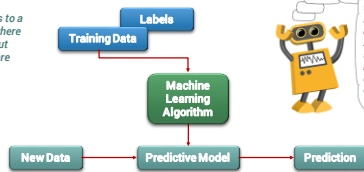
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## Supervised Learning

⊞ The main goal in supervised learning is to learn a model from **labeled training data** that allows us to make predictions about unseen or future data ⊞

"Here, the term **supervised** refers to a set of samples where the desired output signals (labels) are already **known**."



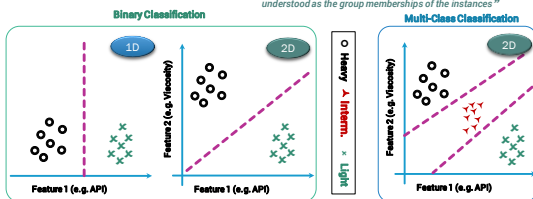
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## Classification for Predicting Class Labels

⊞ Classification is a subcategory of supervised learning where the goal is to predict the **categorical class labels** of new instances, based on past observations ⊞

"Those class labels are **discrete, unordered** values that can be understood as the group memberships of the instances"

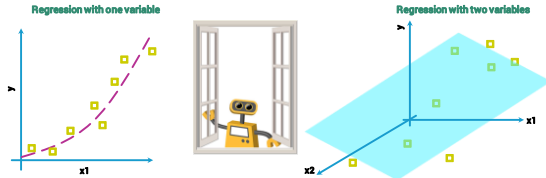


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### Regression for Predicting Continuous Outcomes

☞ In regression analysis, we are given a number of predictor (explanatory) variables and a **continuous response** variable (outcome or target), and we try to find a relationship between those variables that allows us to predict an outcome ☞

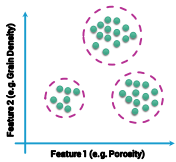


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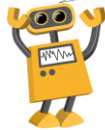
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### Discovering Hidden Structures with Unsupervised Learning

☞ In unsupervised learning, unlabeled data or data of unknown structure should be dealt with. Using unsupervised learning techniques, it is possible to explore the structure of the data to extract meaningful information without the guidance of a known outcome variable or reward function ☞



#### Sub-filed1: Finding Subgroups with Clustering

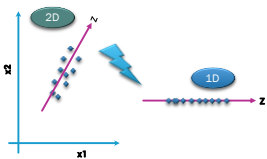


"**Clustering** is the task of dividing the population or data points into a number of groups such that data points in the same groups are more similar to other data points in the same group than those in other groups"

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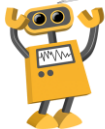
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### Discovering Hidden Structures with Unsupervised Learning



☞ **Isopach** mapping in petroleum geology is an example of dimensionality reduction for easier visualization purpose ☞

#### Sub-filed1: Dimensionality Reduction



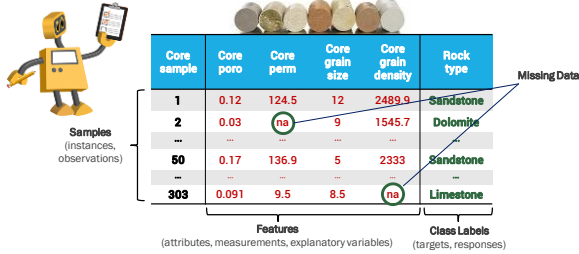
"**Dimensionality reduction** or dimension reduction is the process of reducing the number of random variables under consideration, by obtaining a set of principal variables. Approaches can be divided into feature selection and feature extraction."

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## Basic Terminology and Notations

Rock Typing Data Set



Core sample	Core poro	Core perm	Core grain size	Core grain density	Rock type
1	0.12	124.5	12	2489.9	Sandstone
2	0.03	na	9	1545.7	Dolomite
...	...	...	...	...	...
50	0.17	136.9	5	2333	Sandstone
...	...	...	...	...	...
303	0.091	9.5	8.5	na	Limestone

Missing Data

Samples (instances, observations)

Features (attributes, measurements, explanatory variables)

Class Labels (targets, responses)

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## Basic Terminology and Notations

The "core samples" dataset consisting of 303 samples and four features can then be written as a  $X \in \mathbb{R}^{303 \times 4}$

$$X = \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & x_1^{(3)} & x_1^{(4)} \\ x_2^{(1)} & x_2^{(2)} & x_2^{(3)} & x_2^{(4)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{303}^{(1)} & x_{303}^{(2)} & x_{303}^{(3)} & x_{303}^{(4)} \end{bmatrix}$$

Superscript  $i$  to refer to the  $i$ th training sample, and the subscript  $j$  to refer to the  $j$ th dimension of the training dataset

Each row in this feature matrix represents one rock instance and can be written as a four-dimensional row vector  $x^{(i)} \in \mathbb{R}^{1 \times 4}$

$$x^{(i)} = [x_1^{(i)} \quad x_2^{(i)} \quad x_3^{(i)} \quad x_4^{(i)}]$$

Each feature is a 150-dimensional column vector  $x_j \in \mathbb{R}^{150 \times 1}$

$$x_j = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_{303}^{(1)} \end{bmatrix}$$

Similarly, we store the target variables (here, class labels) as a 303-dimensional column vector:

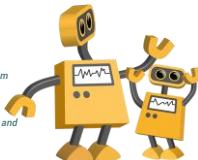
$$y = \begin{bmatrix} y^{(1)} \\ \vdots \\ y^{(303)} \end{bmatrix} \quad (y \in \{ss, dl, lm\})$$

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## Training Data - Test Data

"The observations in the **training set** comprise the experience that the algorithm uses to learn. In supervised learning problems, each observation consists of an observed response variable and one or more observed explanatory variables"



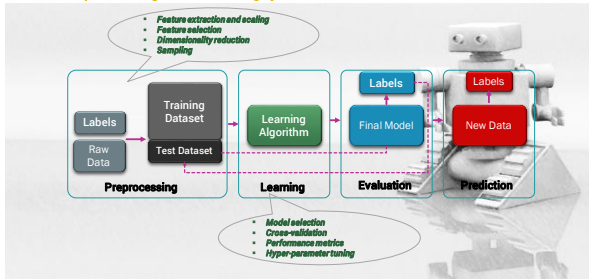
"The **test set** is a similar collection of observations that is used to evaluate the performance of the model using some performance metric. It is important that no observations from the training set are included in the test set."



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## A Roadmap for Building Machine Learning Systems

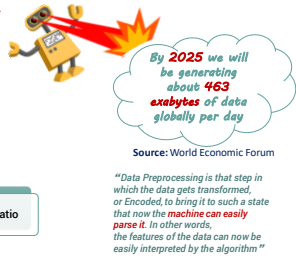
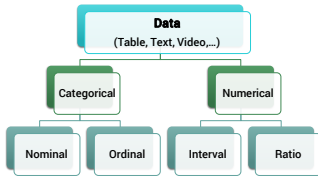


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## Data Preprocessing

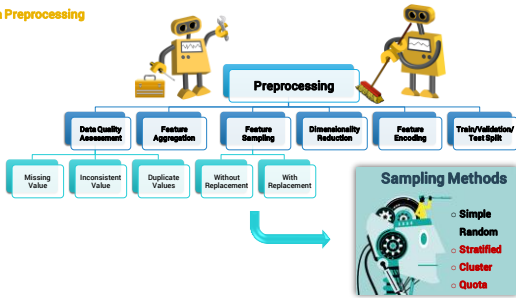
⊗ Data preprocessing is one of the most (if not the most) crucial steps in any machine learning application ⊗



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## Data Preprocessing



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## Simple Linear Regression

Simple linear regression can be used to model a **linear relationship between one response variable and one explanatory variable (feature)**

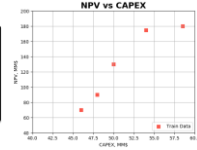
```
# code 1
import numpy as np
import matplotlib.pyplot as plt

actual_capex = np.array([46, 48, 50, 54, 58.5])
actual_npv = np.array([70, 90.2, 130, 175.1, 180])
fig, ax = plt.subplots()
ax.scatter(actual_capex, actual_npv,
           s=100, c='red', label='Train Data')
ax.set_xlabel('CAPEX (MM$)')
ax.set_ylabel('NPV (MM$)')
ax.grid(True)
plt.show()
```

Raw Input Data

Training Instance (scenario)	CAPEX (MM\$)	NPV (MM\$)
1	46	70
2	48	90.2
3	50	130
4	54	175.1
5	58.5	180

Matplotlib Output



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## Simple Linear Regression: Building Predictive Model Using Scikit-Learn Library

```
# code 2
from sklearn.linear_model import LinearRegression
sl_model = LinearRegression()
sl_model.fit(actual_capex, actual_npv)
predicted_npv = sl_model.predict(np.array([[52]]))
print(f"CAPEX of $52 should bring about an NPV of $%s (predicted_npv[0]:.1f)"
```

- LinearRegression** class is an **estimator**. We create one object of this estimator class and name it model
- The **fit()** method of **LinearRegression** learns the parameters of the following model for simple linear regression

$$y = \beta_0 + \beta_1 x$$

Intercept      coefficient(s)

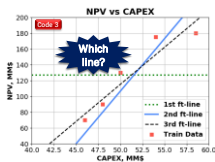
- After **LinearRegression** learned the parameters via **fit** method, **predict()** method can be used to predict the value of response for any explanatory variable

All estimators in scikit-learn implement **fit()** (to learn parameters of the model) and **predict()** (to predict the value of response) methods.

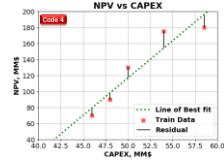
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## Simple Linear Regression: Evaluating the fitness of a model with a cost function



- A **cost function**, also called a **loss function**, is used to define and measure the error of a model.
- The differences between the model predicted by the model and the observed response in the **training set** are called **residuals** or **training errors**.
- The differences between the predicted and observed values in the **test data** are called **prediction errors** or **test errors**.

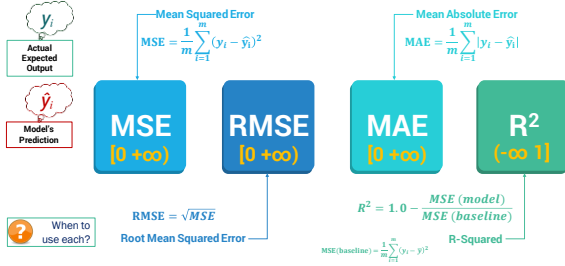


- Variety of cost functions can be defined to evaluate of the model's fitness, i.e., **MAG**, **MSE**
- A typical cost function for regression problem is **residual sum of squares**, which upon minimization will generate optimized values for fit function

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Select a Performance Measure (Metric): MSE, RMSE, MAE, SSR,  $R^2$ , Adj.  $R^2$ , ...



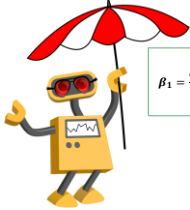
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Simple Linear Regression: Solving ordinary least squares for simple linear regression

$$y = \beta_0 + \beta_1 x$$

Goal: to solve the values of  $\beta_0$  and  $\beta_1$  that minimize the cost function



$$\beta_1 = \frac{\text{cov}(x, y)}{\text{var}(x)} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

- Goal 1: Calculate  $\beta_0$  and  $\beta_1$  for training data set
- Goal 2: Compare the result from task 1 with those of SKL model

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Simple Linear Regression: Evaluating the model

So far: We have used a learning algorithm to estimate a model's parameters from the training data

How can we assess whether our model is a good representation of the real relationship?

Test Instance	CAPEX (MM\$)	NPV (MM\$)	Predicted NPV (MM\$)
1	48	110	
2	49	85	
3	51	150	
4	56	180	
5	52	110	

- Goal 1: Use fitted model to fill the column in

- Goal 2: Several measures can be used to assess our model's predictive capabilities. For example,  $r$ -squared, measures how well the observed values of the response variables are predicted by the model.

$$R^2 = 1.0 - \frac{SS_{res}}{SS_{tot}} = 1.0 - \frac{\sum_{i=1}^n (y_i - \hat{h}(x_i))^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

- Goal 3: Calculate  $R^2$  for test data and compare with SKL results

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## Multiple Linear Regression: Upgrade of the Simple Linear Model

You might have some intuitions about the effect of time on the projects' NPV

Why  $R^2$  is poor?

Training Instance	CAPEX (MM\$)	CAPEX Spent Time (Year)	NPV (MM\$)
1	46	5	70
2	48	4	90.2
3	50	3	130
4	54	5	175.1
5	58.5	3	180

We cannot proceed with simple linear regression, but we can use a generalization of simple linear regression that can use multiple explanatory variables called **multiple linear regression**.



$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$

multiple linear regression uses one coefficient per each explanatory variables (an arbitrary number of variables can be used)

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## Multiple Linear Regression: Solve for coefficient using normal equation

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_1^{(1)} + \beta_2 x_2^{(1)} \\ \beta_0 + \beta_1 x_1^{(2)} + \beta_2 x_2^{(2)} \\ \vdots \\ \beta_0 + \beta_1 x_1^{(n)} + \beta_2 x_2^{(n)} \end{bmatrix} = ?$$

Decompose the last matrix with summation terms into two matrices multiplication

X is called design matrix

$$Y = X\beta$$

$$\beta = (X^T X)^{-1} X^T Y$$

Closed-form solution for normal equation

When obtaining  $\beta$  values might be a challenge using this approach?

Write a code to calculate  $\beta$  values for the train data (table in previous slide)

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## Multiple Linear Regression: SKL implementation of MLR

```

import numpy as np
from sklearn.linear_model import LinearRegression

# From this point on we use the Official variables names for machine learning
x_train = np.array([[46, 5], [48, 4], [50, 3], [54, 5], [58.5, 3]])
y_train = np.array([70, 90.2, 130, 175.1, 180])

x_test = np.array([[46, 5], [48, 4], [54, 5], [58.5, 3], [50, 3]])
y_test = np.array([70, 90.2, 130, 175.1, 180])

# Create the model
ml_model = LinearRegression()
# Fit the model
ml_model.fit(x_train, y_train)
# Predict
beta0, beta1 = ml_model.coef_, ml_model.intercept_
print(f"beta0 = {beta0:.4f}, beta1 = {beta1:.4f}")
# Predict
y2 = ml_model.predict(x_test)
print(f"y2 = {y2}")

```



It appears that adding the spent time as an explanatory variable has improved the performance of our model



**Caution:** Evaluating a model on a single test set can provide inaccurate estimates of the model's performance. We can estimate its performance more accurately by training and testing on many partitions of the data.

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### Polynomial Regression: Special form of MLR



So far, we assumed that the real relationship between the explanatory variables and the response variable is linear. This assumption is not always true.

**Polynomial regression** (a special case of multiple linear regression) can add terms with degrees greater than one to the model

**Quadratic regression**, or regression with a second order polynomial, is given by the following formula (one explanatory variable):  $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$

What is the form of this equation in case of two explanatory variables?

What is the general form of this equation?



Note that the equation for polynomial regression is the same as the equation for multiple linear regression in vector notation



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### Polynomial Regression: SKL implementation of polynomial regression

```
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import PolynomialFeatures

x_train = np.array([[6], [8], [10], [14], [18]])
y_train = np.array([7, 9, 13, 17.5, 18])
x_test = np.array([[6], [8], [11], [16]])
y_test = np.array([8, 12, 15, 18])

quadratic_featurizer = PolynomialFeatures(degree=2)
x_train_quadratic = quadratic_featurizer.fit_transform(x_train)
x_test_quadratic = quadratic_featurizer.transform(x_test)

quadratic_reggressor = LinearRegression()
quadratic_reggressor.fit(x_train_quadratic, y_train)
r_sq = quadratic_reggressor.score(x_test_quadratic, y_test)
print(f"Quadratic regression r-square: {r_sq:.4f}")
```

**PolynomialFeatures** class is a transformer. We create one object of this transformer (here, `quadratic_featurizer`) to transform our explanatory variables.

**fit\_transform()** method of the transformer is used to tune the transformer and transform the train data into proper shape

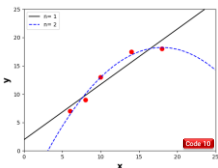
**transform()** method of the transformer is used to transform the test data into proper shape



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### Polynomial Regression: Be watchful for over-fitting

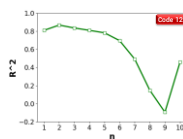
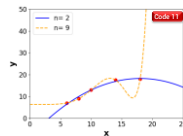


Try to regenerate this plot

$n = 1$   $R^2 = 81\%$   
 $n = 2$   $R^2 = 87\%$



- While quadratic and cubic regression models are the most common, we can add polynomials of any degree.
- The 9<sup>th</sup> polynomial regression model fits the training data almost exactly! The model's r-squared score, however, is 0.09. We created an extremely complex model that fits the training data exactly but fails to approximate the real relationship.
- This problem is called **over-fitting**.



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