1. Brenda says: $A \rightarrow C \equiv \neg A \vee C$

Anthony says: $B \wedge \neg C$

Charles says: $\neg C \wedge (A \vee (A \wedge B) \vee B \vee (A \wedge B)) \equiv \neg C \wedge (A \vee B)$

(a) $(\neg A \vee C) \wedge (B \wedge \neg C) \wedge (\neg C \wedge (A \vee B))$

(b) No, because the formula is satisfiable due to $(\neg A \wedge B \wedge \neg C)$ is a model.

(c) If all were guilty Anthony and Charles would be lying. This is because if they weren't guilty they would be telling the truth.

(d) $\neg A \vee C = T$, A is False & C is False

$B \wedge \neg C = T$, B is True & C is False

$\neg C \wedge (A \vee B) = T$, C is False & A is False so B is True

Brenda is Guilty and Anthony and Charles are innocent.

2. $F = (P \vee Q) \rightarrow ((P \vee Q \vee \neg R) \wedge (R \vee P \vee Q))$

$\equiv (P \vee Q) \rightarrow ((P \vee Q) \vee (\neg R \wedge R))$

$\equiv (P \vee Q) \rightarrow ((P \vee Q) \vee \text{False})$

$\equiv (P \vee Q) \rightarrow (P \vee Q)$

$\equiv \neg(P \vee Q) \vee (P \vee Q)$

$\equiv T$

F is a Tautology thus, $\neg F$ is a contradiction.

3.

(3.1) $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

$\equiv \neg(P \rightarrow Q) \vee ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

$\equiv \neg(P \rightarrow Q) \vee (\neg(Q \rightarrow R) \vee (P \rightarrow R))$

$\equiv \neg(\neg P \vee Q) \vee (\neg(\neg Q \vee R) \vee (\neg P \vee R))$

$\equiv (P \wedge \neg Q) \vee (Q \wedge \neg R) \vee (\neg P \vee R)$

$\equiv P \wedge (\neg Q \vee Q) \wedge \neg R \vee (\neg P \vee R)$

$\equiv (P \wedge \neg R) \vee (\neg P \vee R)$

$\equiv (P \vee \neg P \vee R) \wedge (\neg R \vee \neg P \vee R)$

$\equiv (\neg P \vee R \vee \neg R) \vee (P \wedge \neg P)$

$\equiv (\neg P \vee R \vee \neg R) \vee \text{False}$

$\equiv \neg P \vee (R \vee \neg R)$

$\equiv \neg P \vee T$

$\equiv T$

CNF, Tautology

(3.2) $(P \rightarrow Q) \leftrightarrow (P \rightarrow R)$

$\equiv ((P \rightarrow Q) \rightarrow (P \rightarrow R)) \wedge ((P \rightarrow R) \rightarrow (P \rightarrow Q))$

$\equiv (\neg(P \rightarrow Q) \vee (P \rightarrow R)) \wedge (\neg(P \rightarrow R) \vee (P \rightarrow Q))$

$\equiv (\neg(\neg P \vee Q) \vee (\neg P \vee R)) \wedge (\neg(\neg P \vee R) \vee (\neg P \vee Q))$

$\equiv ((P \wedge \neg Q) \vee (\neg P \vee R)) \wedge ((P \wedge \neg R) \vee (\neg P \vee Q))$

$\equiv ((P \vee \neg P \vee R) \wedge (\neg Q \vee \neg P \vee R)) \wedge ((P \vee \neg P \vee Q) \wedge (\neg R \vee \neg P \vee Q))$

$\equiv ((\neg P \vee R \vee \neg Q) \vee (P \wedge \neg P)) \wedge ((\neg P \vee Q \vee \neg R) \vee (P \wedge \neg P))$

$\equiv ((\neg P \vee R \vee \neg Q) \vee \text{False}) \wedge ((\neg P \vee Q \vee \neg R) \vee \text{False})$

$\equiv (\neg P \vee R \vee \neg Q) \wedge (\neg P \vee Q \vee \neg R)$

CNF

(3.3) $(P \wedge Q) \rightarrow (\neg P \leftrightarrow Q)$

$\equiv \neg(P \wedge Q) \vee (\neg P \leftrightarrow Q)$

$\equiv \neg(P \wedge Q) \vee ((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P))$

$\equiv \neg(P \wedge Q) \vee ((\neg(\neg P) \vee Q) \wedge (\neg Q \vee \neg P))$

$\equiv (\neg P \vee \neg Q) \vee (P \vee Q) \wedge (\neg Q \vee \neg P)$

$\equiv (\neg P \vee P) \vee (\neg Q \vee Q) \wedge (\neg Q \vee \neg P)$

$\equiv T \vee T \wedge (\neg Q \vee \neg P)$

$\equiv (\neg Q \vee \neg P)$

CNF

(3.4) $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$

$\equiv \neg((P \rightarrow Q) \wedge \neg Q) \vee \neg P$

$\equiv \neg((\neg P \vee Q) \wedge \neg Q) \vee \neg P$

$\equiv ((P \wedge \neg Q) \vee Q) \vee \neg P$

$\equiv ((P \vee Q) \wedge (\neg Q \vee Q)) \vee \neg P$

$\equiv (P \vee Q) \vee \neg P$

$\equiv (P \vee \neg P) \vee Q$

$\equiv T \vee Q$

$\equiv T$

CNF, Tautology

4.

(4.1) $Q \wedge S \wedge \neg W \wedge (\neg P \vee \neg Q \vee V \vee \neg S) \wedge (S \vee \neg V) \wedge R$

$\equiv (1 \rightarrow Q) \wedge (1 \rightarrow S) \wedge (W \rightarrow 0) \wedge (P \wedge Q \wedge S \rightarrow V) \wedge (V \rightarrow S) \wedge (1 \rightarrow R)$

$\mathcal{A}(Q) = 1$

$\mathcal{A}(S) = 1$

$\mathcal{A}(W) = 0$

$\mathcal{A}(P) = 0$

$\mathcal{A}(V) = 0$

$\mathcal{A}(R) = 1$

This formula is Satisfiable, $\mathcal{A}(Q) = 1$, $\mathcal{A}(S) = 1$, $\mathcal{A}(W) = 0$, $\mathcal{A}(P) = 0$, $\mathcal{A}(V) = 0$, $\mathcal{A}(R) = 1 \vDash F$

4.2 $(\neg A \vee E) \wedge (\neg B \wedge (\neg C \vee (A \rightarrow B)) \wedge A \wedge (\neg E \vee C \vee \neg D) \wedge (D \wedge (D \vee F))$

$\equiv (\neg A \vee E) \wedge (\neg B \wedge (\neg C \vee (A \rightarrow B)) \wedge A \wedge (\neg E \vee C \vee \neg D) \wedge D$

$\equiv (A \rightarrow E) \wedge (B \rightarrow 0) \wedge (C \wedge A \rightarrow B) \wedge (1 \rightarrow A) \wedge (D \wedge E \rightarrow C) \wedge (1 \rightarrow D)$

$\mathcal{A}(A) = 1$

$\mathcal{A}(E) = 1$

$\mathcal{A}(B) = 0$

$\mathcal{A}(C) = 1/0? \leftarrow$

$\mathcal{A}(D) = 1$

$\mathcal{A}(F) = 0$

This formula is unsatisfiable because there is a contradiction regarding the proposition C where it can be neither 1 nor 0 because either answer would give an unsatisfiable formula. Thus, there exists no model for this formula.

4.3 $(P \wedge Q) \rightarrow (\neg P \leftrightarrow Q)$

$\equiv \neg(P \wedge Q) \vee (\neg P \leftrightarrow Q)$

$\equiv \neg(P \wedge Q) \vee ((\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P))$

$\equiv \neg(P \wedge Q) \vee ((\neg(\neg P) \vee Q) \wedge (\neg Q \vee \neg P))$

$\equiv (\neg P \vee \neg Q) \vee (P \vee Q) \wedge (\neg Q \vee \neg P)$

$\equiv (\neg P \vee P) \vee (\neg Q \vee Q) \wedge (\neg Q \vee \neg P)$

$\equiv T \vee T \wedge (\neg Q \vee \neg P)$

$\equiv (\neg Q \vee \neg P)$

CNF

$\equiv (Q \wedge P \rightarrow 0)$

$\mathcal{A}(Q) = 1$

$\mathcal{A}(P) = 0$

This formula is Satisfiable, $\mathcal{A}(Q) = 1$& $\mathcal{A}(P) = 0 \vDash F$