

## ⑤ Tabular representation

$$\begin{array}{c|ccccccccc} n & \dots & -2 & -1 & 0 & 1 & 2 & 3 & \dots \\ \hline x(n) & & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \dots \end{array}$$



UNIVERSITY OF BENIN

Question .....

Write on both sides of the paper

Do not write  
in this  
margin

Do not write  
in this  
margin

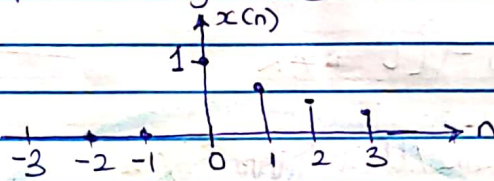
There are several alternative ways of describing the sample value of a DT Signal. The most common are.

### ① Sequence Notation

$$x = \{ \dots 0, 0, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots \}$$

\* The bar on the top of the symbol 1 indicates origin ( $n=0$ ) or an arrow

### ② Graphical



### ③ Explicit mathematical expression $x(n) = \begin{cases} 0, & n < 0, \\ 2^{-n} & n \geq 0 \end{cases}$

### ④ Recursive approach $x(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}x(n-1) & n > 0, \end{cases}$

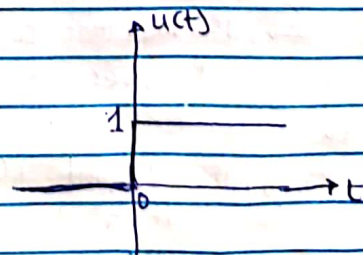
Depending on the specific sequence  $x(n)$ , some approach may lead to more compact representation than others

## ELEMENTARY SIGNALS (BASIC)

In Signal processing a proper reference frame is needed in order to place the signals to be analyzed. Since we deal with signals, any reference frame of interest has to be made of signals with well known properties, which reflect the kind of information we want to extract. There are several elementary signals that feature prominently in the study of digital signals and DSP. These basic signals will be the basis of our discussion.

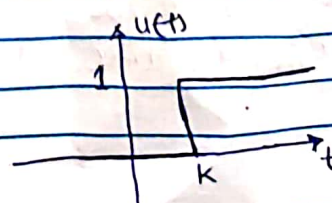
### ① Unit STEP FUNCTION.

Also called the Dirac Delta function is defined by  $u(t) = \begin{cases} 1, & t \geq 0 \\ 0 & t < 0 \end{cases}$



This function can also be shifted by a factor an argument

$$u(t-k) = \begin{cases} 1, & t \geq k \\ 0, & t < k \end{cases}$$





Do not write  
in this  
margin



UNIVERSITY OF BENIN

Question .....

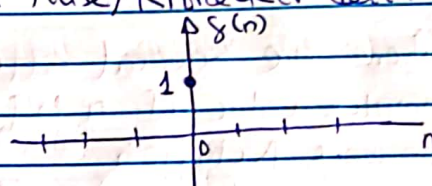
Write on both sides of the paper

Do not write  
in this  
margin

## DT SIGNAL

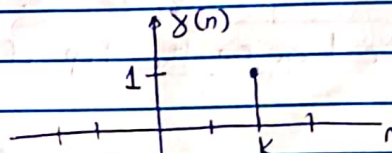
→ Unit Sample Sequence/Unit Impulse/Unit Pulse/Kronecker delta  
Denoted as  $\delta(n)$  and is defined as

$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$



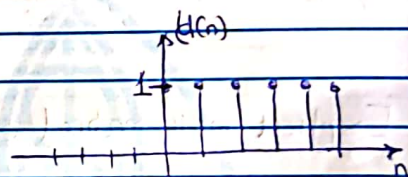
The USS can also be shifted by an argument

$$\delta(n-k) = \begin{cases} 1, & n=k \\ 0, & n \neq k \end{cases}$$



→ Unit Step Sequence/Unit Step Signal  
Denoted as  $u(n)$  and is defined as

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



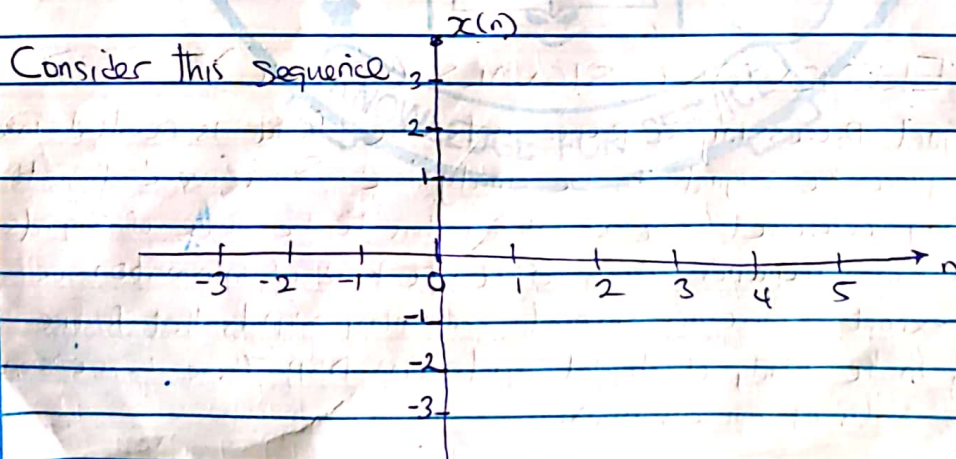
It is the discrete time counterpart for the Unit Step function.

It can also be shifted.

### ASSIGNMENT

There are other elementary Signals (Talk about them)

- \* Exponential Signal/sequence Real & Complex
- \* Sinusoidal Signal
- \* (Unit) ramp signal



It is important to note that a DT signal is not defined at instants between two successive samples. Additionally, it is wrong to think that  $x(n)$  is equal to zero if  $n$  is not an integer. Simply, the signal  $x(n)$  is not defined for non integer values of  $n$ .

Again if a discrete Time Signal is written as a sequence of numbers inside braces, the location of the sample value associated with the time index  $n=0$  is indicated by an arrow under it ( $\uparrow$ )





The sample values to its right are for positive values of  $n$ , and the sample values to its left are for negative values of  $n$ .  
So from the diagram,  $x(0) = 1$ ,  $x(-1) = 2$ ,  $x(2) = 0$

Therefore, for a sequence  $x(n)$ , which is zero for  $n < 0$ , can be represented as.  $x(n) = \{0, 1, 2, 4, 0, 0, 5, 6\}$

The time origin for a sequence  $x(n)$  which is zero for  $n < 0$ , is understood to be the ~~left~~ first (leftmost) point in the sequence. A finite-duration sequence can be represented as  $x(n) = \{3, -1, -2, 5, 0, 4, -1\}$

Whereas a finite-duration sequence that satisfies the condition  $x(n) = 0$  for  $n < 0$  can be represented as

$$x(n) = \{0, 1, 4, 1\}$$

Thus the first signal has seven samples or points (in time), thus it is called or identified as a seven-point sequence, and the other a four point sequence. The DT signal may be a finite-length or an infinite-length sequence. A finite-length (also called finite-duration or finite-extent) sequence is defined only for a finite time interval:

$N_1 \leq n \leq N_2$ , where  $-\infty < N_1$  and  $N_2 < \infty$  with  $N_2 \geq N_1$ . The length or duration of the above finite length sequence is  $N = N_2 - N_1 + 1$ .

Thus as said before, a length  $-N$  discrete-time sequence consists of  $N$  samples and is called an  $N$ -point sequence.

A finite length sequence can also be considered an infinite length sequence by assigning zero values to samples whose arguments are outside the above range. The process of lengthening a sequence by adding zero-valued samples is called appending with zeros or zero-padding.

ASS: Three types of Infinite-length sequence ① A Right-sided sequence ② A left sided sequence ③ A Two-sided sequence



Do not write  
in this  
margin



UNIVERSITY OF BENIN

Question .....

Write on both sides of the paper

112 D SP  
162 165  
163 512  
164 161

Do not write  
in this  
margin

## OPERATION OF SEQUENCE

### ① TIME REVERSAL / folding / Reflection

It involves replacing the independent variable  $n$  by  $-n$   
note that the origin remains the same

$$x(n) = \{1, 2, 3, 4, 5, 6, 7\}$$

$$x(-n) = \{7, 6, 5, 4, 3, 2, 1\}$$

### ② Shifting (Delay & Advance) $y(n) = x(n-N)$ if $N > 0$ = delay if $N < 0$ = advance

Shifting can be done to the left or to the right

for  $x(n)$ ;  $x(n-k)$

Shift the origin of the sequence to  $k$

set  $n-k=0$  (set the argument equal to zero)

Then move the origin to  $k$ .

$$y = x(n+k)$$

Therefore move the origin to the left

ASS: What if it is reversed? How do you shift.

$$y(n) = x(-n+2)$$

$$x(-n-2)$$

### ③ Amplitude Scaling

### ④ Addition / Subtraction — Zero padding is needed here.

ASS: Time Scaling for continuous signals and discrete signals.

Conjugate Symmetric Sequence

Conjugate asymmetric Sequence

Classification of Sequence based on Symmetry

There is a general misconception among student about commutative properties of shifting and folding. They are not commutative in nature. i.e. Folding delayed signal is not the same as delaying a folded signal. To illustrate.

①  $x(n)$  is first folded and then delayed by  $k$  units. So this means

$$x \& x\{-n-k\} = x(-n+k)$$

②  $x(n)$  is first delayed by  $k$  units and then folded.

$$x(-n-k)$$