10/05/2021 TRANSFORMATION

Transformation means to convert or change an image. We have different types of transformation:

① Translation
② Rotation
③ Scaling
④ ~~Shearing~~ ~~Shirley~~ Shearing
⑤ Reflection.

Translation

Translation moves an object to a different position on a string. A point in 2D can be translated by adding translation co-ordinates t_x and t_y to the original co-ordinates x and y to get x' and y'

NB: Translation only changes the displacement, it does not change the size.

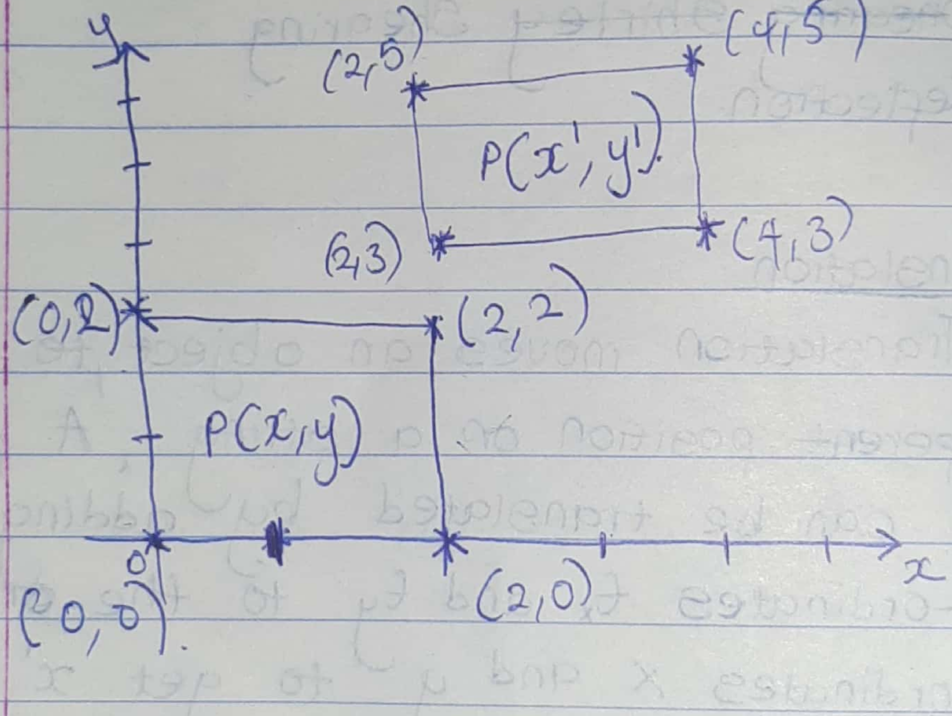
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \qquad \begin{aligned} x' &= t_x + x \\ y' &= t_y + y. \end{aligned}$$

Example

Given a square with position $\overset{x}{(0,}\overset{y}{0)}$, (2,0), (0,2), (2,2). Given $t_x = 2$ & $t_y = 3$

Solution

i) Draw the Co-ordinate



ii) input positions in the formula.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$\therefore (0,0) \Rightarrow (2,3)$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \quad \therefore (2,0) \Rightarrow (4,3)$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad \therefore (0,2) \Rightarrow (2,5)$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \quad \therefore (2,2) \Rightarrow (4,5)$$

Rotation

This involves the circular movement of an object around a centre/axis

$$\text{Rotation } (\phi) = R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\Downarrow$

anticlockwise = -ve

clockwise = +ve

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\Downarrow$

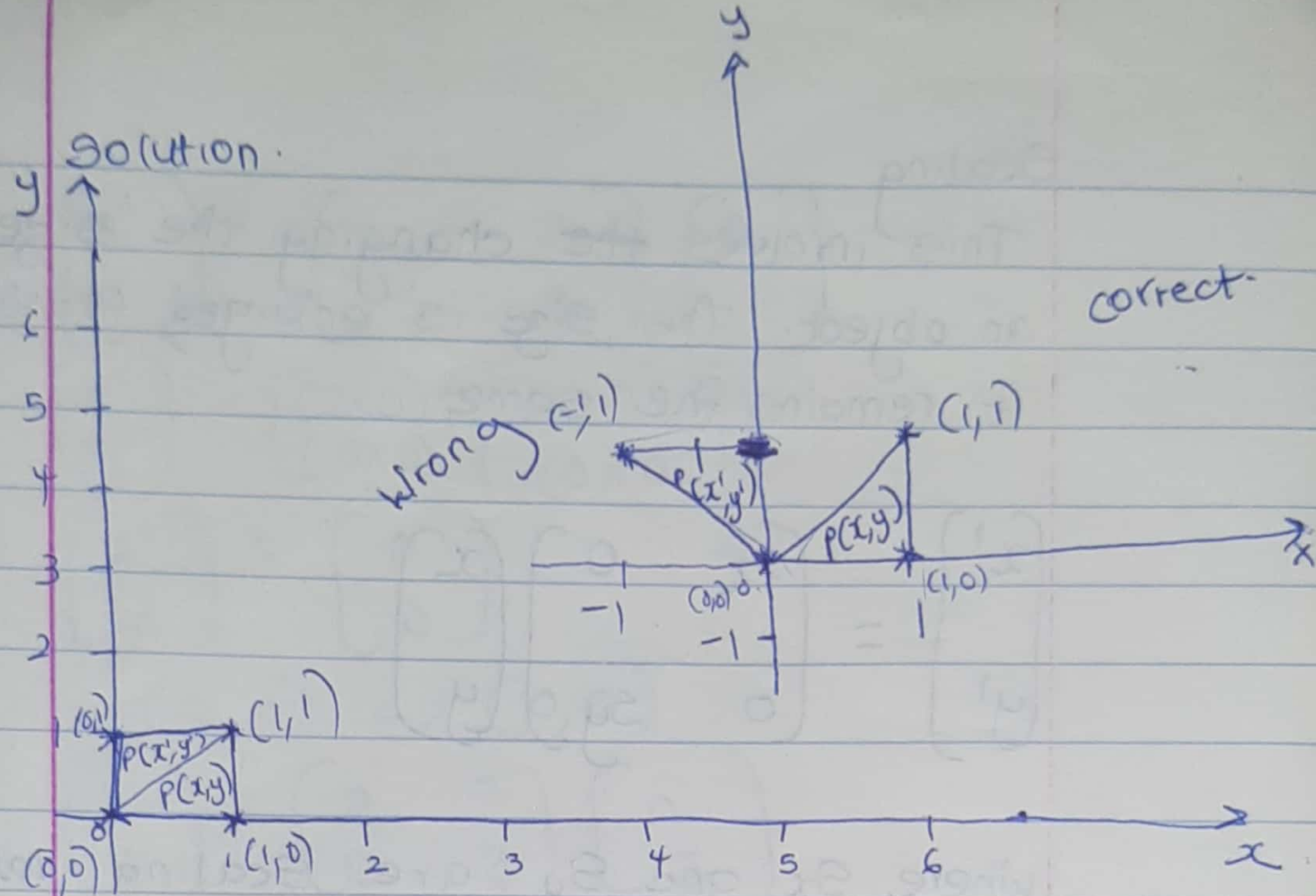
anticlockwise = +ve

clockwise = -ve

Example

Rotate the triangle ~~at~~ (0,0), (1,0), (1,1) at 90° anticlockwise.

Solution.



Input positions in the formula.

$$R = \begin{pmatrix} \cos(-90) & \sin(-90) \\ -\sin(-90) & \cos(-90) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(Solve the rest completely).

(0,0) => (0, 0)

(1,0) => (0, 1)

(1,1) => (-1, 1)

Scaling

This involves ~~the~~ changing the size of an object. The size is enlarged, reduced or it remains the same.

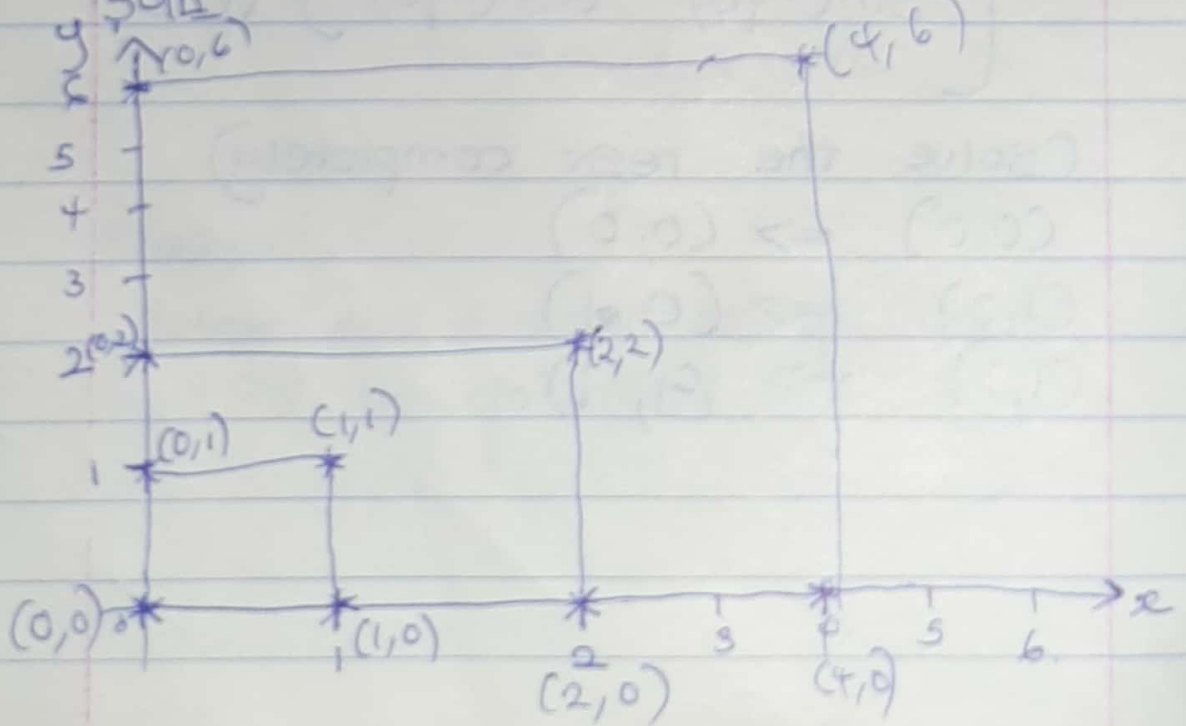
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} S_x & 0 \\ 0 & S_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Where, S_x and S_y are scaling values.

- **Example**

Scale the square (0,0) (2,0) (0,2) (2,2) where $S_x = 2$ and $S_y = 3$.

Soln



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} (2\times0)+(0\times0) \\ (0\times0)+(0\times0) \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) \Rightarrow (0,0)$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2\times2 + 0\times0 \\ 0\times2 + 3\times0 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$(2,0) \Rightarrow (4,0)$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 2\times0 + 0\times2 \\ 0\times0 + 3\times2 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$(0,2) \Rightarrow (0,6)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2\times2 + 0\times2 \\ 0\times2 + 3\times2 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$(2,2) \Rightarrow (4,6)$

if $S_x = 0.5$ & $S_y = 0.5$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.5\times0 + 0\times0 \\ 0\times0 + 0.5\times0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) \Rightarrow (0,0)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.5\times2 + 0\times0 \\ 0\times2 + 0.5\times0 \end{bmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$(2,0) \Rightarrow (1,0)$.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0.5\times 0 + 0\times 2 \\ 0\times 0 + 0.5\times 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$(0,2) \Rightarrow (0,1)$

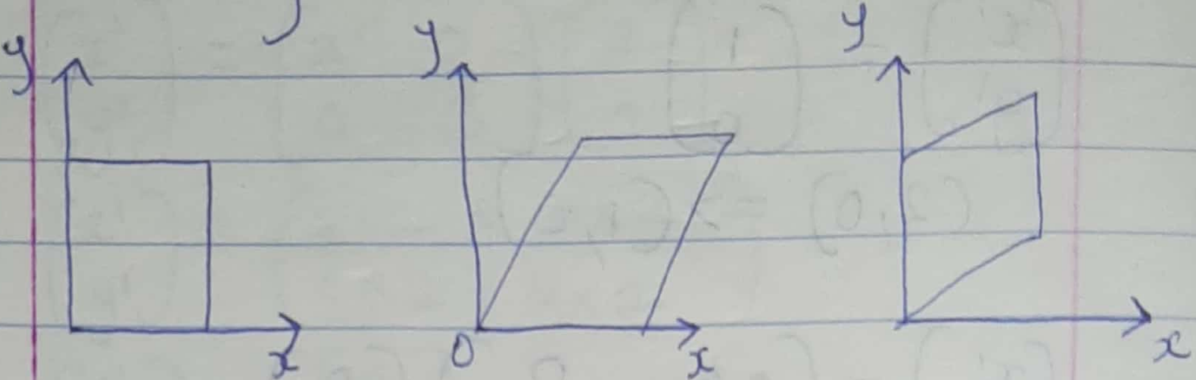
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0.5\times 2 + 0\times 2 \\ 0\times 2 + 0.5\times 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$(2,2) \Rightarrow (1,1)$

Shearing



a) Original Object

b) Object after x-shear

c) Object after y-shear

x-shear, $\begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

y-shear, $\begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

Example

Given a square (0,0), (0,2), (2,0), (2,2) compute the shear of the square

Given $sh_x = 2$ and $sh_y = 2$

Soln

x-shear $= \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Scanned with CamScanner

$$x\text{-shear} = \begin{bmatrix} 1\times 0 + 2\times 0 \\ 0\times 0 + 1\times 0 \end{bmatrix}$$

$$x\text{-shear} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) \Rightarrow (0,0)$

$$x\text{-shear} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 0 + 2\times 2 \\ 0\times 0 + 1\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$(0,2) \Rightarrow (4,2)$

$$x\text{-shear} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 2 + 2\times 0 \\ 0\times 2 + 1\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$(2,0) \Rightarrow (2,0)$

$$x\text{-shear} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times2 + 2\times2 \\ 0\times2 + 1\times2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$(2,2) => (6,2).$

y-shear:

$$y\text{-shear} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times0 + 0\times0 \\ 2\times0 + 1\times0 \end{pmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) => (0,0)$

$$y\text{-shear} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times0 + 0\times2 \\ 2\times0 + 1\times2 \end{pmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$(0,2) \Rightarrow (0,2)$

$$y\text{-shear} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 2 + 0\times 0 \\ 2\times 2 + 1\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

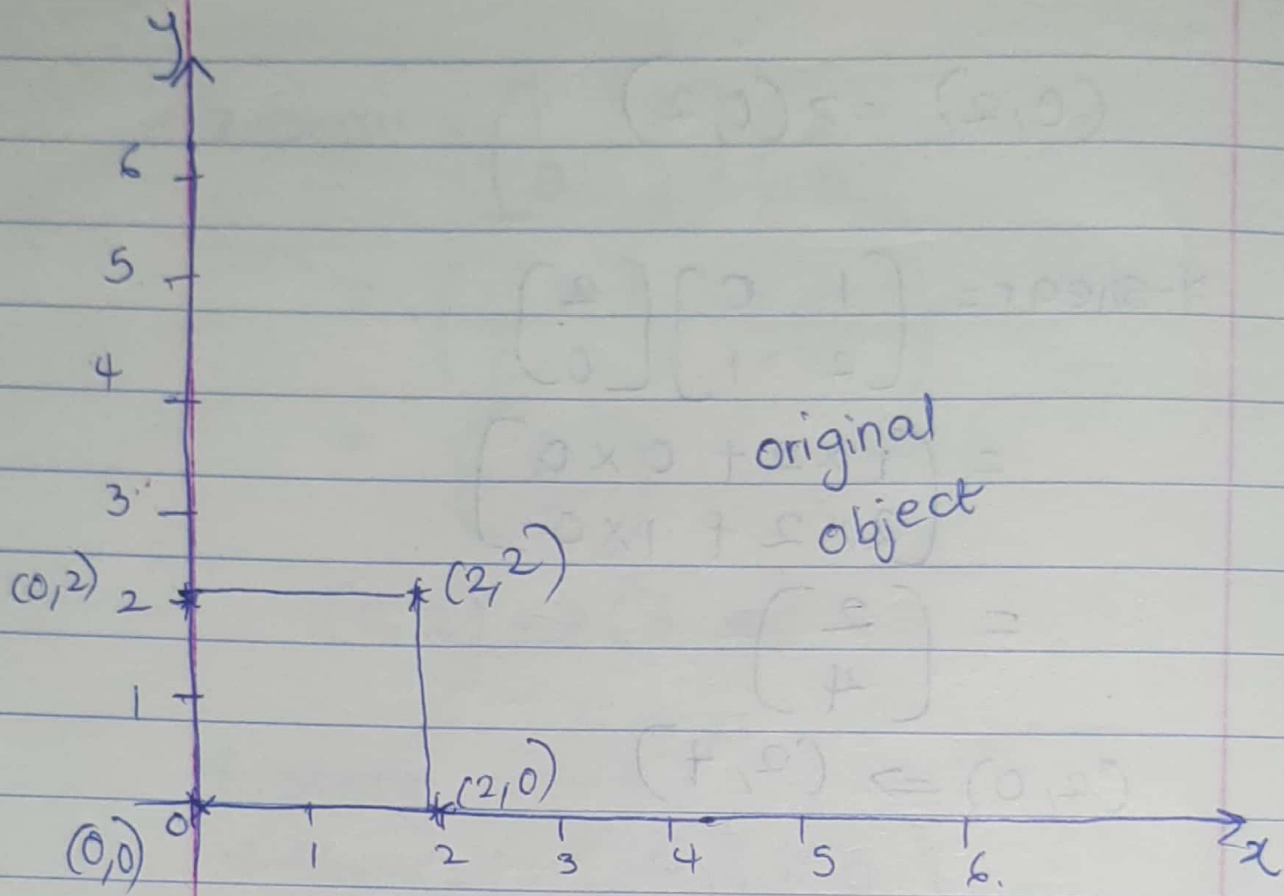
$(2,0) \Rightarrow (2,4)$

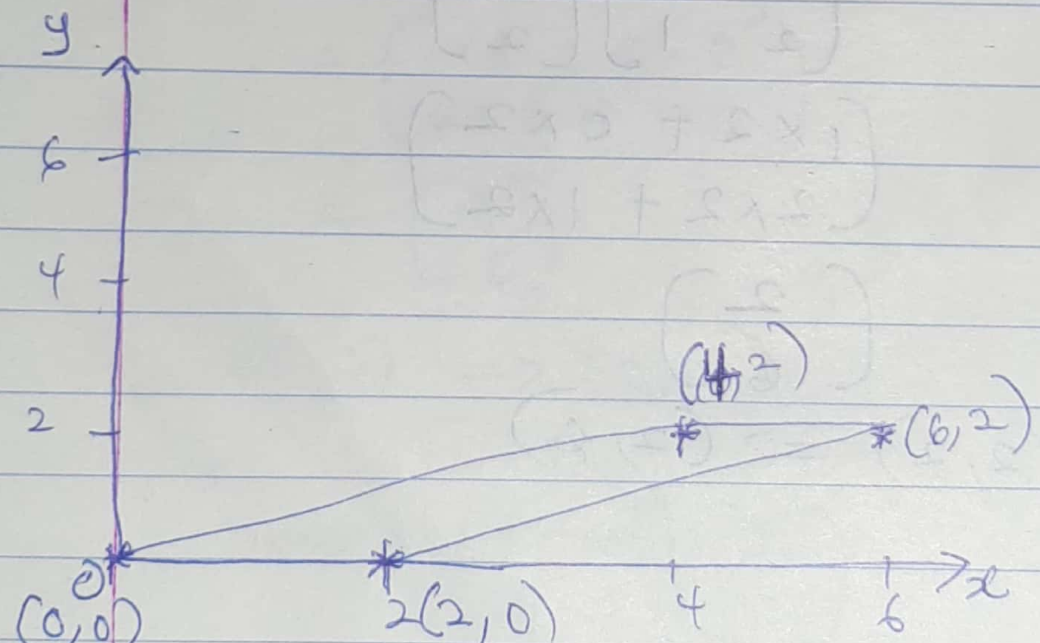
$$y\text{-shear} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1\times 2 + 0\times 2 \\ 2\times 2 + 1\times 2 \end{bmatrix}$$

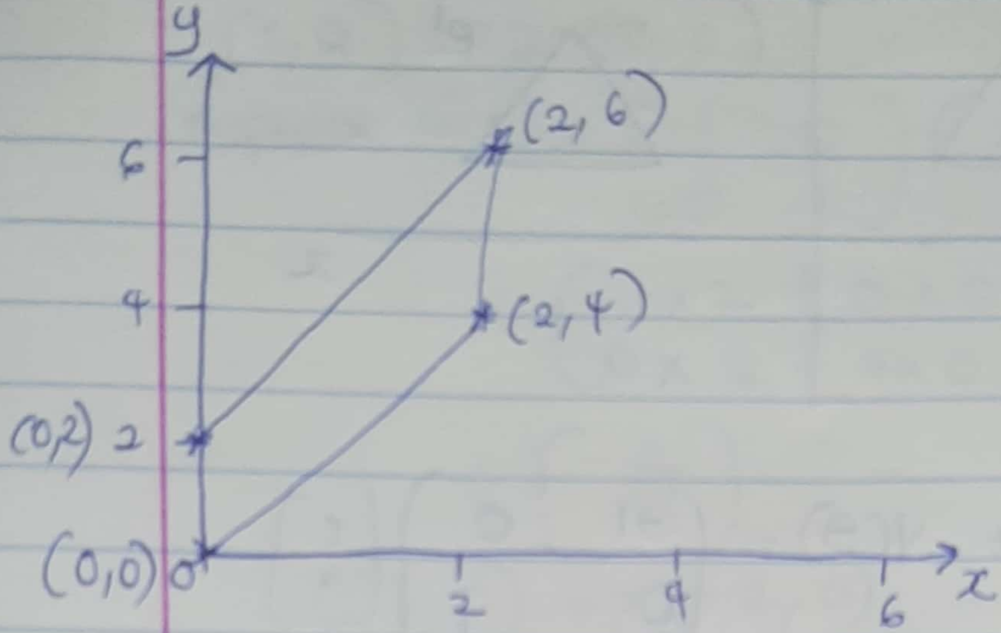
$$\begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$(2,2) \Rightarrow (2,6)$





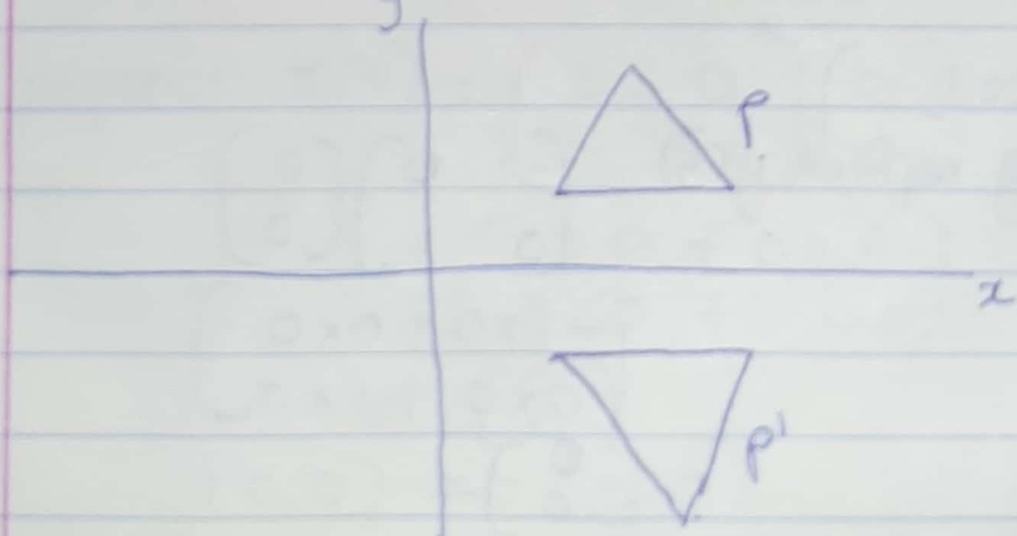
x-shear

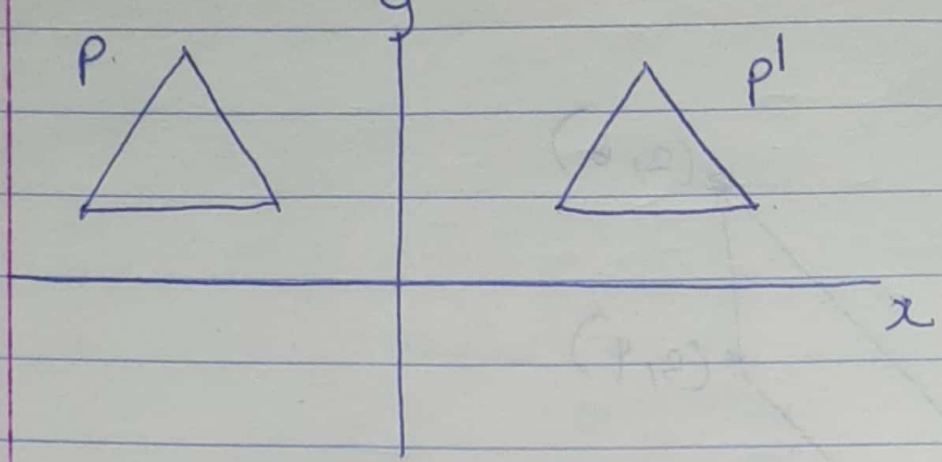


y-shear

Reflection

This is a mirror image of the original object. Reflection does not change the size of an original object.





Reflect $Y(S) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Reflect $x(S) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

<u>Example</u>

Compute the reflection of the triangle (0, 0) (2, 0) (1, 2) in the x and y axis

<u>Soln</u>

reflect $y(S) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -1 \times 0 + 0 \times 0 \\ 0 \times 0 + 1 \times 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(0,0) \Rightarrow (0,0)$.

reflect $y(s) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

$= \begin{bmatrix} -1 \times 2 + 0 \times 0 \\ 0 \times 2 + 1 \times 0 \end{bmatrix}$.

$= \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$(2,0) \Rightarrow (-2,0)$

reflect $y(s) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} -1 \times 1 + 0 \times 2 \\ 0 \times 1 + 1 \times 2 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$(1,2) \Rightarrow (-1,2)$.

reflect $x(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 0 + 0 \times 0 \\ 0 \times 0 + -1 \times 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(0,0) \Rightarrow (0,0)$

reflect $x(S) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$= \begin{pmatrix} 1\times 2 + 0\times 0 \\ 0\times 2 + (-1\times 0 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$(2,0) \Rightarrow (2,0)$

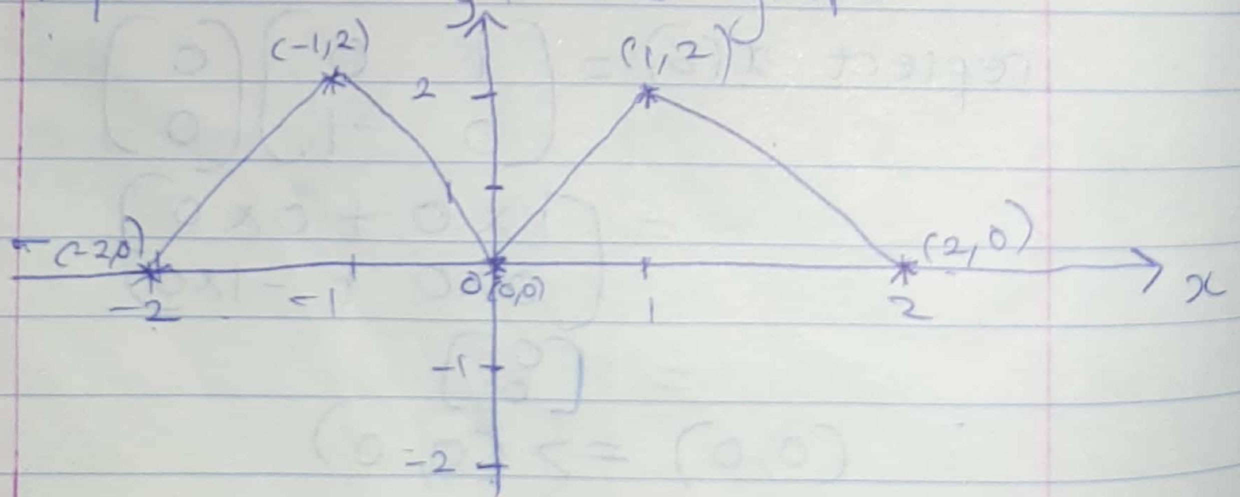
reflect $x(s) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{pmatrix} 1\times 1 + 0\times 2 \\ 0\times 1 + (-1\times 2 \end{pmatrix}$

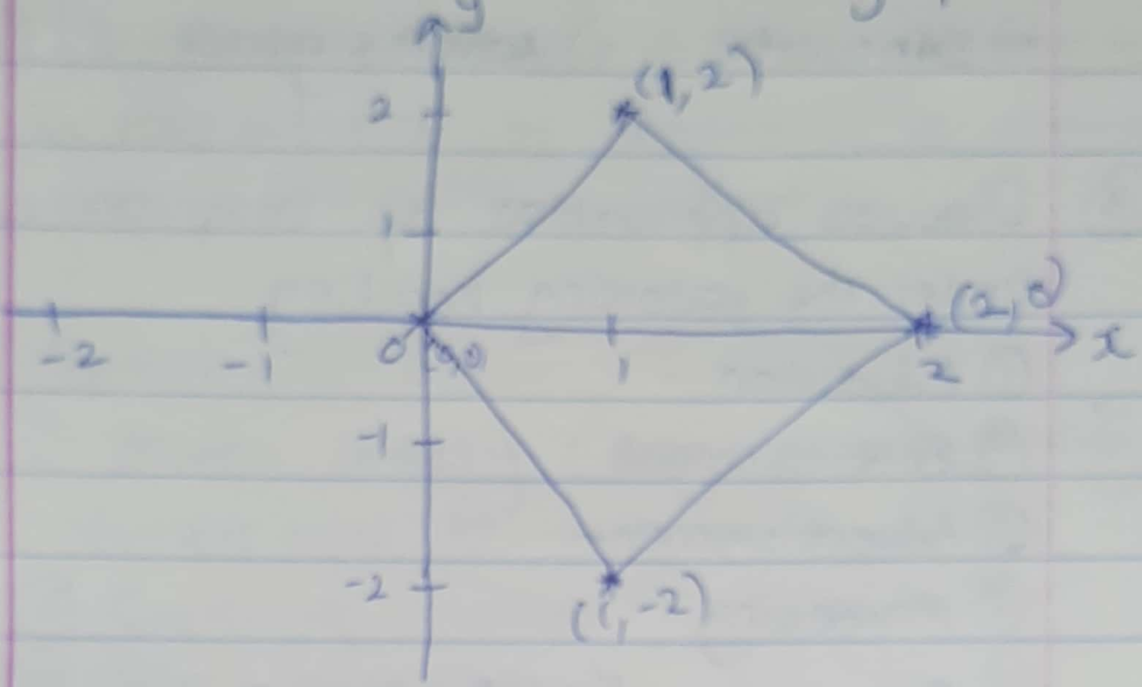
$= \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$(1,2) \Rightarrow (1, -2)$

Reflection in y axis graph.



Reflection in x axis graph.



Assignment

① What is Animation?

② Explain the different types of Animation

① Traditional Animation.

② Computer Animation

i) Digital 2D

ii) Digital 3D

iii) Motion capture

③ Motion Graphics

④ Stop motion

i claymation ii) Puppet iii) cutout

iv) Silhouette v) Action figures vi) Pixelation

③ Discuss application of computer animation under the following heading
① Education
② Entertainment
③ Advertisement
④ Marketing
⑤ Scientific visualization or medical
⑥ Creative Art
⑦ Gaming
⑧ Simulation
⑨ Architecture and Engineering
10) Manufacturing.

④ Describe and illustrate with sketches the 12 principles of Animation.

⑤ Why is animation described as an illusion?

CPE 556 - ENGR. 191.

Computer graphics is the generation or manipulation of images in a computer using a computer software.

Applications

1. Mixed reality (Augmented reality / Virtual reality)
2. Gaming
3. Film Industry
4. Image Editing / Processing
5. Engineering (CAD)
6. Marketing
7. Medical Imaging

Softwares

1. Maya
2. Unity
3. Photoshop
4. After Effects
5. Blender
6. 3DS Max
7. AutoCAD
8. Proteus

A pixel is the smallest indivisible element of a display.

Raster is another name for an arrangement of pixels.

Raster is a representation of a 3D image on a 2D plane.

OpenGL stands between the hardware & software

Scan conversion is the process of converting an image from its 2D representation to its pixelated or rasterized form.

More pixels require more hardware and memory resources

CURVES

i) Tangent-to-the-curve

ii) Normal-to-the-curve

* study the general eqn of the lines and curves

CPE 556 - EKAR IRUKWU

HOMOGENEOUS CO-ORDINATES

<u>Translation</u>

$$\cancel{S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ t_x & t_y & 1 \end{bmatrix}}$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

<u>Rotation</u>

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<u>Scaling</u>

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 1

Perform the following transformation independently and find the resultant transformation matrix

i) Scale the object two times in the x direction and three times in y direction

ii) Rotate the object by 90° anticlockwise

iii) Translate the object one (1) unit in x direction and ~~One(1)~~ Two(2) unit in y direction

Soln

i) Scale

$S_x = 2$

$S_y = 3$

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

ii) Rotate.

$\theta = 90$, anticlockwise

$$R = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

iii) Translate

$t_x = 1$

$t_y = 2$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

∴ The resultant transformation is

$T_R = SRT$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 0 \\ 3 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Question 2

A triangle defined by the vertices A (0, 2), B (0, 3), C (1, 2) Perform the following transformation and find the final position of the triangle.

i. Scale the triangle by factor 1.5

ii. Rotate the triangle by 270^{0} anticlockwise

iii. Translate the triangle 2 unit in x-direction.

NB: Final position of Triangle = Initial position X Resultant Transformation

Sol$\underline{\underline{n}}$

i) Scale

$S_x = 1.5$

$S_y = 1.5$

$$S = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ii) Rotate

$\theta = 270^{0}$

anticlockwise

$$R = \begin{bmatrix} \cos 270 & -\sin 270 & 0 \\ \sin 270 & \cos 270 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scanned with CamScanner

$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) Translate

$t_x = 2$

$t_y = 0$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

∴ The resultantant transformation is

$T_R = S\ R\ T$

$$T_R = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1.5\times0 + (0\times -1) + (0\times0) & 1.5\times1 + 0\times0 + 0\times0 & 1.5\times0 + 0 + 0 \\ :0 & 1.5\times1 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

∴ The final position is

Final position = Initial Position × Resultant Transformation

To change Triangle to Homogenous system.

$A(0,2) \Rightarrow (0, 2, 1)$

$B(0,3) \Rightarrow (0, 3, 1)$

$C(1,2) \Rightarrow (1, 2, 1)$

$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$P_F = P_I \times T_R$

$$P_F = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$P_f = \begin{bmatrix} 0+-3+2 & 0+0+0 & 0+0+1 \\ 0+-4.5+2 & 0+0+0 & 0+0+1 \\ 0+-3+2 & 1.5+0+0 & 0+0+1 \end{bmatrix}$$

$$P_f = \begin{array}{c} A' \\ B' \\ C' \end{array} \begin{bmatrix} -1 & 0 & 1 \\ -2.5 & 0 & 1 \\ -1 & 1.5 & 1 \end{bmatrix}$$

$\therefore$ Convert P_f to 2D coordinate

$A' = (-1, 0)$

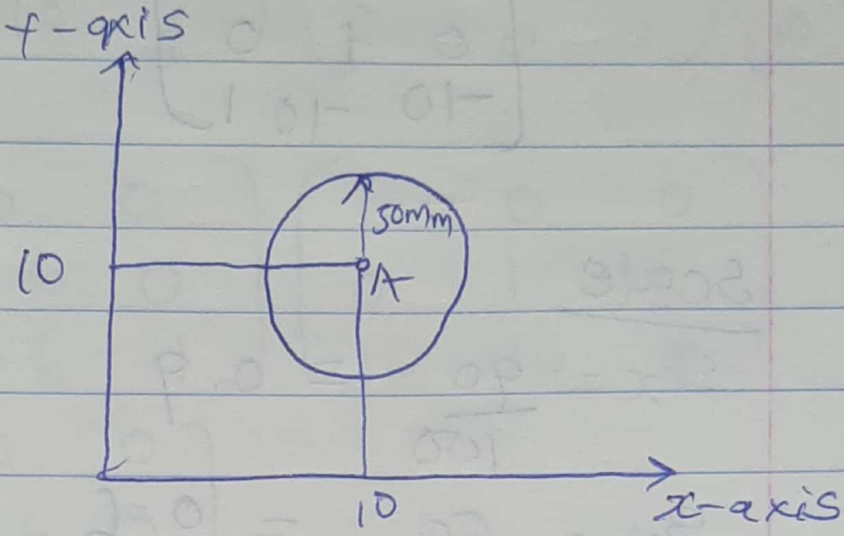
$B' = (-2.5, 0)$

$C' = (-1, 1.5)$

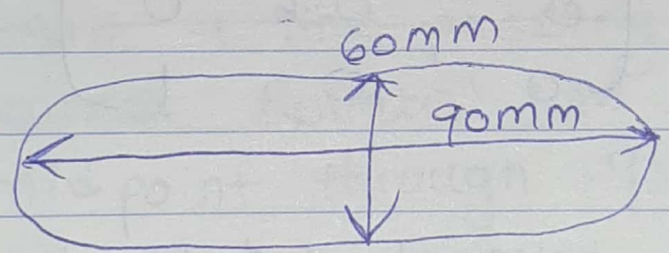
Then, plot the graph for the Initial 2D co-ordinate points and the final 2D co-ordinate points, on the same graph.

Question 3

The figure shows a circle with radius 50mm centered at A(10, 10)



To be converted into an ellipse with major axis 90mm and minor axis 60mm. Find the total transformation



Soln

$S_x = 0.9$, $S_y = 0.6$ $\left(\frac{\text{new position}}{\text{initial position}}\right)$

$t_x = -10$ $t_y = -10$:

(its negative in other to move A(10,10) to the origin.)

Translate

$$T1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -10 & -10 & 1 \end{bmatrix}$$

Scale

$$S_x = \frac{90}{100} = 0.9$$

$$S_y = \frac{60}{100} = 0.6$$

$$S = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, take it back to normal position

$$T2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 10 & 1 \end{bmatrix}$$

$$TR = T1 \cdot S \cdot T2$$

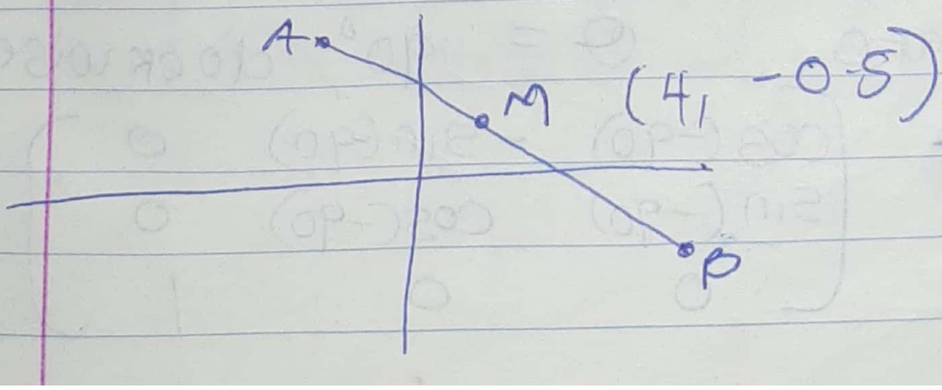
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -10 & -10 & 1 \end{bmatrix} \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 10 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.6 & 0 \\ -9 & -6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 10 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.6 & 0 \\ 1 & 4 & 1 \end{bmatrix}$$

Question (Take-Home)

~~Translate a line passing through two points~~ Rotate line AB about its midpoint through 90° clockwise. Find the final position of the line if initial position is A $(-3, 8)$; B$(11, -9)$



To get M, find the midpoint of a line

$M_x = \frac{x_1 + x_2}{2}$, $M_y = \frac{y_1 + y_2}{2}$

$M_x = \frac{-3+11}{2}$, $M_y = \frac{8-9}{2}$

$M_x = 4$ $\quad$ $M_y = -0.5$.

i) Translate & move to origin
ii) ~~move to origin~~ Apply rotation formu
iii) ~~conve~~ move back to initial position

i) Translate

$t_x = -4$

$t_y = 0.5$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0.5 & 1 \end{bmatrix}$$

ii) Rotate $\quad \theta = 90^\circ$, clockwise

$$R = \begin{bmatrix} \cos(-90) & -\sin(-90) & 0 \\ \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ii) Translate

$t_x = 4 \quad t_y = -0.5$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -0.5 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -0.5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -0.5 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -0.5 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3.5 & -4.5 & 1 \end{bmatrix}$$

Question 5

A triangle having vertices A(3, 3); B(8, 5); C(5, 8) is first translate 2 units in x direction. Then it is scaled by two(2) units at point (5, 6) and finally rotated 90° anticlockwise at point (2, 5). Depict the initial and final position of the triangle.

* Understand application of computer graphics

Soln (Q.5)

1) Translate

$tx = 2 \qquad ty = 0$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

ii) scale

Ta

$t_x = -5 \quad , \quad t_y = -6$

$$T_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -6 & 1 \end{bmatrix}$$

Sb

$s_x = 2 \quad , \quad s_y = 2$

$$S_b = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Tb

$t_x = 5, \quad t_y = 6$

$$T_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 6 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -6 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 6 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -10 & -12 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 6 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -6 & 1 \end{bmatrix}$$

$$\therefore S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -6 & 1 \end{bmatrix}$$

(iii) Rotate

<u>Ta</u>

$t_x = -2, \quad t_y = -5$

$$T_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -5 & 1 \end{bmatrix}$$

<u>R_θ</u>

$\theta = 90°$, anticlockwise.

$$R_b = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_b = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<u>T_b</u>

$t_x = 2$, $t_y = 5$

$$T_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -5 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ -3 & 7 & 1 \end{bmatrix}$$

$$\therefore R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ -3 & 7 & 1 \end{bmatrix}$$

$\therefore$ The resultant transformation is

$T_R = T1 \times S \times R$

$$T_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -6 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ -3 & 7 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & -6 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ -3 & 7 & 1 \end{bmatrix}$$

$$T_R = \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ -9 & 8 & 1 \end{bmatrix}$$

$\therefore$ final Position $= \begin{bmatrix} 3 & 3 & 1 \\ 8 & 5 & 1 \\ 5 & 8 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ -9 & 8 & 1 \end{bmatrix}$

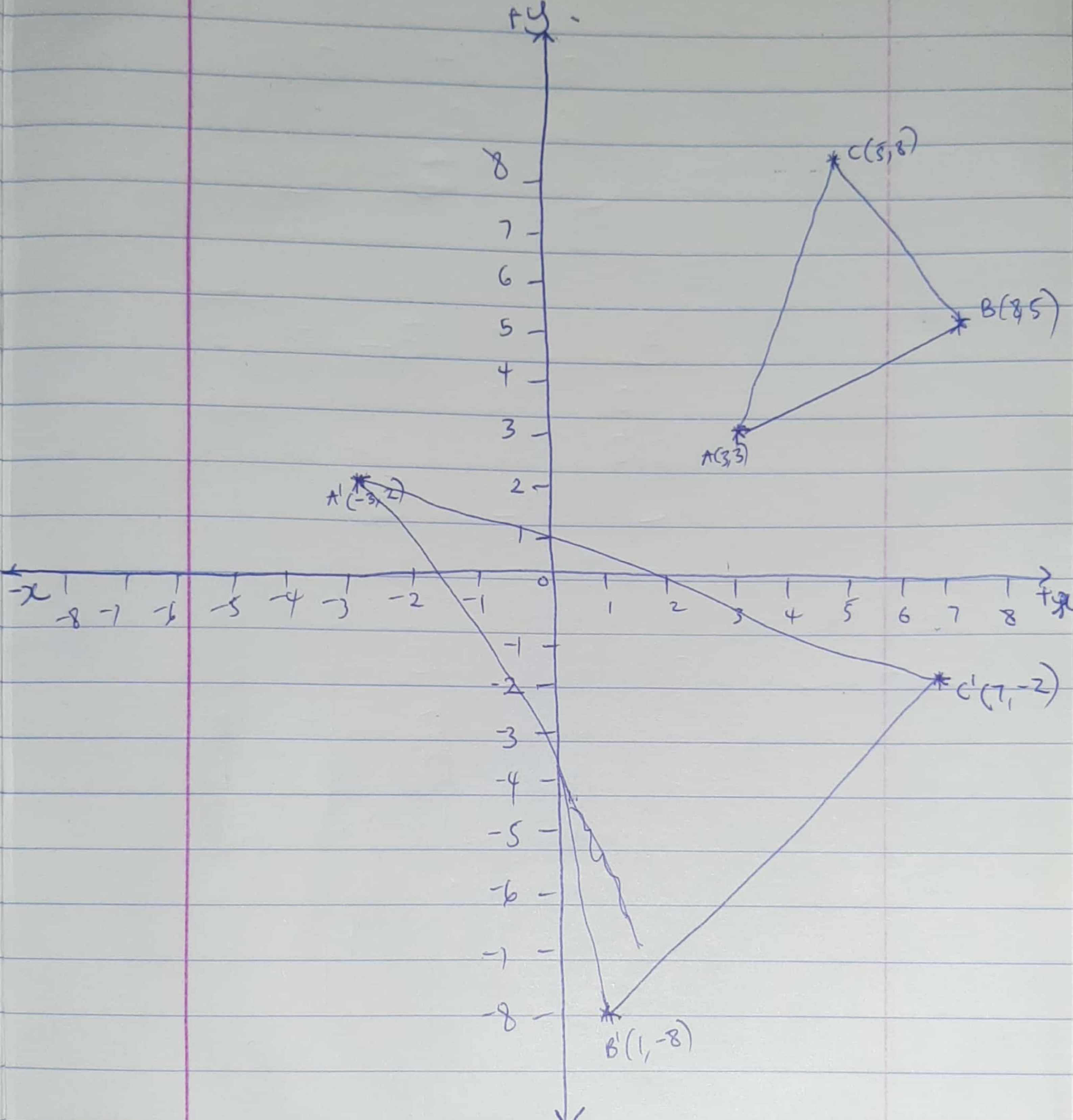
$$\begin{array}{c} A' \\ B' \\ C' \end{array} \begin{bmatrix} -3 & 2 & 1 \\ 1 & -8 & 1 \\ 7 & -2 & 1 \end{bmatrix}$$

converting to 2D coordinate

$A' = (-3, 2)$

$B' = (1, -8)$

$C' = (7, -2)$



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