PRE 572

RESOURCE MANAGEMENT

Materials Management

Materials management is concerned with decisions about purchasing materials and services, inventories, production levels, staffing, patterns, schedules and distribution. Such decision often affects the entire organisation, either directly, or indirectly.

Materials management is a function, which aims for integrated approach towards the management of materials in an industrial undertaking. Its main objective is cost reduction and efficient handling of materials at all stages and in all sections of the undertaking. Its function includes several important aspects connected with material, such as, purchasing, storage, inventory control, material handling, standardisation etc.

Materials management is defined as "*the function responsible for the coordination of planning, sourcing, purchasing, moving, storing and controlling materials in an optimum manner so as to provide a pre-decided service to the customer at a minimum cost*".

From the definition it is clear that the scope of materials management is vast. The functions of materials management can be categorized in the following ways: (as shown in Fig. 1.1.)

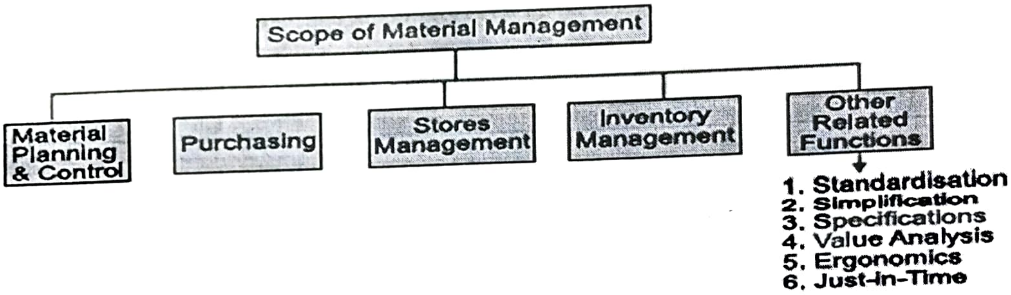


Fig 1.1: Scope of Materials Management

1. **Materials planning and control:** This involves estimating the individual requirements of parts, preparing materials budget, forecasting the levels of inventories, scheduling the orders and monitoring the performance in relation to production and sales.
2. **Purchasing:** This includes selection of sources of supply finalization in terms of purchase, placement of purchase orders, follow-up, maintenance of smooth relations with suppliers, approval of payments to suppliers, evaluating and rating suppliers.
3. **Stores control or management:** This involves physical control of materials, preservation of stores, minimization of obsolescence and damage through timely disposal and efficient handling, maintenance of stores records, proper location and stocking. A store is also responsible for the physical verification of stocks and reconciling them with book figures. A store plays a vital role in the operations of a company.
4. **Inventory control or management:** Inventory generally refers to the materials in stock. It is also called the idle resource of an enterprise. Inventories represent those items, which are either stocked for sale or they are in the process of manufacturing or they are in the form of materials, which are yet to be utilized. It is, necessary to hold

inventories of various kinds to act as a buffer between supply and demand for efficient operation of the system. Thus, an effective control on inventory is a must for smooth and efficient running of the production cycle with least interruptions.

5. **Other related activities**

(i) *Standardization*: Standardization means producing maximum variety of products from the minimum variety of materials, parts, tools and processes. It is the process of establishing standards or units of measure by which extent, quality, quantity, value, performance etc. may be compared and measured.

(ii) ***Simplification***: The concept of simplification is closely related to standardization. Simplification is the process of reducing the variety of products manufactured. Simplification is concerned with the reduction of product range, assemblies, parts, materials and design.

(iii) *Specifications*: It refers to a precise statement that formulizes the requirements of the customer. It may relate to a product, process or a service.

(iv) ***Value analysis***: Value analysis is concerned with the costs added due to inefficient or unnecessary specifications and features. It makes its contribution in the last stage of product cycle, namely, the maturity stage. At this stage research and development no longer make positive contributions in terms of improving the efficiency of the functions of the product or adding new functions to it.

(v) ***Ergonomics (Human Engineering)***: The human factors or human engineering is concerned with man-machine system. Ergonomics is "the design of human tasks, man-machine system, and effective accomplishment of the job, including displays for presenting information to human sensors, controls for human operations and complex man-machine systems." Each of the above functions are dealt in detail.

1.1 Material Planning and Control

Material planning is a scientific technique of determining in advance the requirements of raw materials, ancillary parts and components, spares etc. as directed by the production programme. It is a subsystem in the overall planning activity. There are many factors, which influence the activity of material planning. These factors can be classified as macro and micro systems.

1. *Macro factors*: Some of the micro factors which affect material planning, are price trends, business cycles, Govt. import policy etc.
2. *Micro factors*: Some of the micro factors that affect material planning are plant capacity utilization, rejection rates, lead times, inventory levels, working capital, delegation of powers and communication.

1.1.2 Techniques of Material Planning

One of the techniques of material planning is bill of material explosion. Material planning through bill of material explosion is shown below in Fig. 1.2.

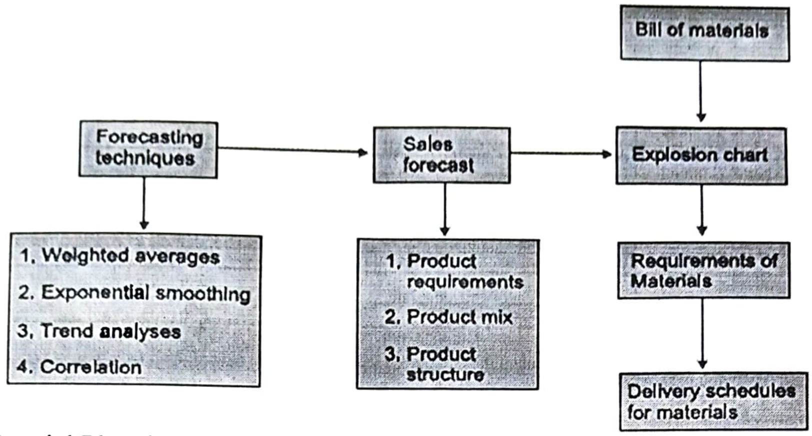


Fig. 1.2 Material Planning

The basis for material planning is the forecast demand for the end products. Forecasting techniques such as weighted average method, exponential smoothening and time series models are used for the same. Once the demand forecast is made, it is possible to go through the exercise of material planning. **Bill of materials** is a document which shows list of materials required, unit consumption location code for a given product. An **explosive chart** is a series of bill of material grouped in a matrix form so that combined requirements for different components can be done requirements of various materials are arrives at from the demand forecast, using bill of materials, through explosion charts. Thus material requirement plan will lead to the development of delivery schedule of the materials and purchasing of those material requirements.

1.2 Purchasing Methods and Supplier Selection

Purchasing is an important function of materials management. In any industry purchase means buying of equipment, materials, tools, parts etc. required for industry.

Traditionally, the main activities of a purchasing manager were to beat up potential suppliers on price and then buy products from the lowest cost supplier that could be found. That is still an important activity, but there are other activities that are becoming equally important. Because of this, the purchasing activity is now seen as part of a broader function called procurement. The procurement function can be broken into five main activity categories:

i. Purchasing
ii. Consumption Management
iii. Vendor Selection
iv. Contract Negotiation
v. Contract Management

1.2.1 Strategic Role of Purchasing

Purchasing function has a strategically indispensible role, it covers the sourcing end of supply with the delivery end of suppliers.

- **The classical definition** of purchasing is to obtain materials and/or services of the right quality in the right quantity, from the right source, deliver them to the right place at the right time.
- **The composite definition of purchasing** is the process undertaken by the organizational unit which, either as a function or as part of an integrated supply, is responsible for procuring supplies of materials and services of the right quality, quantity, time and price, and the management of the suppliers, thereby contributing to the competitive advantages of the achievement of the corporate strategy.

The operational processes of purchasing function can be represented by the diagram shown in Figure 1.3. It basically intermediates the company's internal operations with the suppliers, ensuring the right suppliers are found and engaged in a process of supply and delivering the required materials, components and services that best suit the internal operations. These are just the visible parts of purchasing function. Beyond these processes, there are more important high level strategic decisions to be made for the purchasing, through the purchasing and by the purchasing. Typically make-or-buy decisions, supply based rationalization, supplier development and etc..

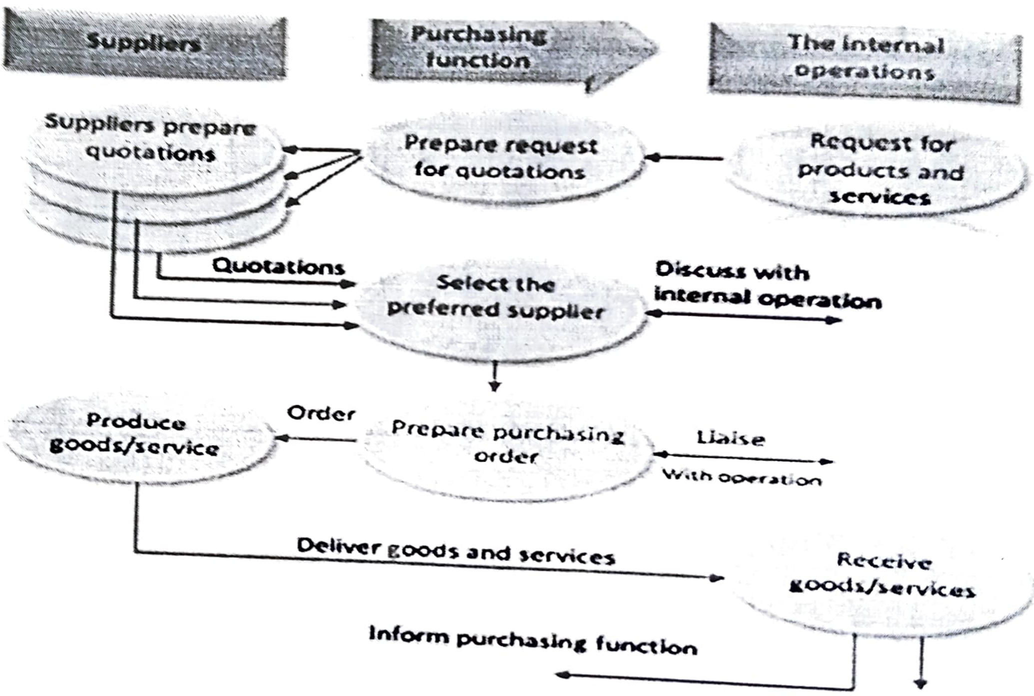


Figure 1.3: Operational Purchasing Activities (Slack, 1998)

1.2.2 Purchasing Portfolio

"What is the best way to managing purchasing?" "What is the best strategy for purchasing and supply?" Dr. Peter Kraljic from McKinsey claimed that there is no single best way existed for managing purchasing in all circumstances. His answer to the question can be paraphrased as that although there is no single best way, but if we know which product category we are purchasing, there would be a best way to do the purchasing for this product category. He looked in two dimensions to categorize all the purchased products. One is the supply risk of the product from the market; two is the financial impact of the purchased material. Thus a four categories matrix emerged presenting four different types of purchased product.

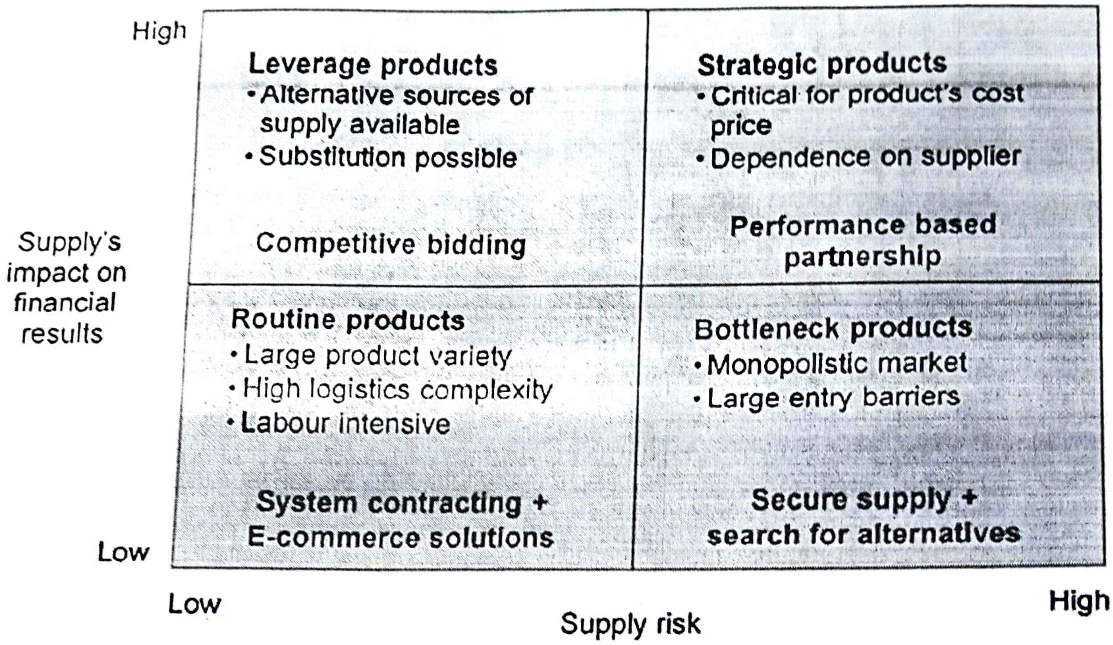


Figure 1.4: Kraljic Purchasing Portfolio Matrix

Leverage Products

Leverage products are those you buy from your supplier that will have significant impact on the finance of your own final product, but it is relatively easy to buy from the supply market, hence low market risk. For example, wood is a leverage product for a wood furniture manufacture to purchase. This is because most of the furniture they do are made of wood. A large portion of the cost come from wood price and the furniture prices are dependent on the wood cost in the market place. However, unless it is rare wood which they hardly use, the supply of wood is plentiful and there is very low risk of short supply.

For such leverage products, Kraljic proposes a purchasing strategy of competitive bedding. Understandably, competitive bidding will only work if there is more than one supplier. The low risk factor of leverage products supports this strategy. Alternative suppliers are in this case available and substitution of supplier is possible. The buyer can then benefits on the lower price and cost advantages. It is important to understand that the buyer need to do so, because the high financial impact the leveraged products.

Routine Products

Routine products are those materials that have very little impact on the buyer's own products and also there are plenty to choose from in the market place. Examples of these products or materials are small fixtures, pant and standard components like springs and nuts. They don't cost much in comparison to the total cost of your own products. The market availability for those materials is very high and there is no risk of supply.

For such routine products, Kraljic proposes a purchasing strategy of system contracting plus e-commerce solutions, because these products have large varieties and high logistics complexity and often labour intensive in handling. The sophisticated computer based system ordering suits well with the nature of the products. Although the alternative sources of supply are available, bidding is not recommended. This is because the low cost nature of the materials made the bidding unnecessary; and the variety complexity of the materials will make the bidding unaffordable.

Strategic Products

Strategic products are those components that have high level of financial impact to your final product. They are very expensive to develop and manufacture, and often involve high technology contents. They are not usually available in the market place, thus high supply risk. The buyer will need to contract for the manufacturing rather than pay for the delivery. The examples of these products are engines for automobile and compressor for the refrigerator.

For the strategic products, Kraljic proposed a purchasing strategy of performance based partnership. That is to create a partnership relation with the supplier and work together to develop and manufacture the components. The proposal is obviously for good reasons. This type of components usually is not make to stock it at lease needs to be ordered to the specific requirement, often the components will have to be designed and developed together with the supplier. Hence, strictly speaking, it is no longer the purchasing of products, but rather purchasing of the developing and manufacturing capabilities from the suppliers. That's why performance based is necessary, because it is about the capability.

Bottleneck Products

Bottleneck products are those components that may or may not cost too much in comparison with the total material cost, but they must have them and they are very difficult to get hold of. The supply risk for those components is high, and the availability of the components is not guaranteed. For example, the small amount of precious metal required for the exhaust purification system is a typical bottleneck material. Without it, the automobile will not pass the environmental standard and will not be allowed on the road. There are only very few supply in the world that have the blessing of the natural resource for those precious metal. Demand is much higher than supply.

For such bottleneck products, Kraljic proposed a purchasing strategy of securing supply plus searching for alternatives. You must secure your current supply because you have no choice, you must have it; you need to search for alternatives because the constant risk of cut-throat. The alternatives include radical new designs which may use different materials that are not short of supply. This is the relationship setting that the supplier has the up-hand; buyers will have to take some diversions.

1.2.3 Suppliers Selection

One of the mission critical tasks of purchasing functions is to identify and select the suppliers. This is particularly true when we talk about strategic components and bottleneck components. In both of these categories, suppliers are by no means ascertained. The quality of the suppliers and the righteousness of their selection will have direct implications on the supply chains long term competitiveness. The processes of going about selecting suppliers can be suggested as follows, but by no means comprehensive and universal. The processes to go about selecting the suppliers are as follows:

i. Set up selection criteria
ii. Initial contact
iii. Formal evaluation
iv. Price quotation
v. Financial data
vi. Reference checking
vii. Supplier visit
viii. Audits, assessments or surveys
ix. Initiation test

Apparently, these processes are mainly around setting and taking measures against the criteria. However, there are three significantly different approaches toward supplier selection. **The first** is based on the product that the supplier can deliver. This approach will normally check the product prototype to see if the quality and technical specifications can be met and the delivery terms are satisfactory. **The second** is based on the capability that the supplier displays. It typical checks whether the supplier has the design and development capability, strategic investment in technology and skills, and up to scratch management. This capability approach is often used for long-term supplier selection and can be done well before the idea of component is taking shape. **The third** is the combination of product and capability selection. It applies to when a strategically important new part is to be outsourced to a new supplier. Not only the supplier must comply with all the product specific requirements but also should have the capability of making future generation of the products in the long run, so as to sustain the supply chain development.

1.2.4 Tools for Supplier Selection

To facilitate the process of supplier selection, quantitative tools are beneficial. They can make the selection process more rationale; they serve as the platform for meaningful discussion or debate; they provide traceable documents; they form the factual contents for decision-making. Three basic quantitative tools are introduced here for managers to get started on creating their own tools for their own tools for their own industry and products.

The first one is called the **'categorical method'** as shown in Table 1.1. You define the selection criteria first, for example quality, delivery and service; then you make three category judgement: good (+), unsatisfactory (-), and neutral (0) against the criteria for each supplier; finally you sum up the judgement into a total score for each supplier; the highest one will be selected. This method is very simple to apply but is rather subjective. Hence, it is recommended to form a multi-functional team to make collective judgement in order to limit the bias from individuals.

Table 1.1: Categorical Method

Supplier	Performance Characteristics			
	Quality	Delivery	Service	Total
A	Good (+)	Unsatisfactory (-)	Neutral (0)	0
B	Neutral (0)	Good (+)	Good (+)	++
C	Neutral (0)	Unsatisfactory (-)	Neutral (0)	-

The second method is called the **"cost ratio method"** as shown in Figure 1.6. Similarly you set up the required criteria against the alternative suppliers. What's the different from the categorical method is that it is not entirely based on the people's subjective judgement. It makes use of some available data on the quality performance; service standard and delivery reliability for instance. With those historically collected data, you will be able to establish the corresponding cost ratios for each criterion in terms of how much the penalty cost needs to be added. The original quoted unit prices from different suppliers will then be adjusted to generate the net-adjusted costs. It the becomes clear that if the supplier selection is based on the original quoted unit price, then the supplier C should be selected because its cost is the lowest. However, if the supplier selection process takes into account of the suppliers historical performance in the three areas and use the net-adjusted cost, the lowest cost suppliers is A not C. the little home work of considering the cost ratio has made very different choice in the selection.

Table 1.2: Cost ratio method

Supplier	Quality Cost Ratio (%)	Delivery Cost Ratio (%)	Service Cost Ratio (%)	Total Penalty Ratio (%)	Quoted Price/Unit (N)	Net Adjusted Cost (N)
A	2	2	1	5	16.00	16.80
B	5	4	2	11	15.40	17.01
C	7	3	5	15	15.00	17.25

The problem with the two methods is that all the criteria are treated equally in the selection process. This is not really true in real-world business. People have preferences toward different criteria for various reasons. In some circumstances, quality is more important than delivery, and in other situations the delivery is more than quality. The third method takes care of these preferences by assigning a weight to each criterion. It is called the **"linear-average method"** as shown in Table 1.3. The weight is a score that reflects the relative importance of the criterion. The sum of all the weight scores is normally 100. Whatever judgement score multiplied by the weight becomes the adjusted judgement score; and then add all the adjusted scores together to generate the total selection score for the supplier. It should be noted that the weight can also be applied to the cost ratio method, or any other method. In the end, the most appropriate supplier selection tool perhaps is the combination of some of those methods.

Selection Criteria	Weight	Supplier A		Supplier B		Supplier C	
		Score	Total	Score	Total	Score	Total
Quality	52	8	415	5	260	6	312
Delivery	26	3	78	8	208	3	78
Service	22	5	110	8	176	5	110
Total			604		644		500

1.2.5 Towards Knowledge Based Sourcing

Purchasing practice and theory never stops developing. It really is a dynamic and evolving subject in both theory and practice. Looking back at the recent three decades of purchasing development, an evolution pattern starts to emerge. Depend on how one would like to take out of it; the pattern maybe presented in different ways. Here we frame the evolution pattern into two perspectives, each of which has four key stages. The first perspective is on the operational focuses and the second perspective is mainly on the characteristics changes.

The operational focus perspective classifies the purchasing function into four stages:

Stage one can be called **"Product Centered Purchasing".** The operation is basically concentrated exclusively upon the purchasing of the tangible and its outcomes on the overall businesses. It is usually measured in the five rights (right price, right time, right quantity, right quality and from the right sources).

Stage two can be called **"Process Centered Purchasing.** It is predominantly process focused operation. It moves beyond the direct outcomes of the purchasing activities and into the processes through which the outcomes are delivered. This means that managers realized that the processes are the enablers, and often the controllers of the purchasing outcomes.

Stage three can be called **"Relational Purchasing"**. The focus of the operation is not just on the process but also on the inter-organisational relationships. The relationship has been taken on as the key management instrument to enhance the product quality and technological advances; it also had massive positive impact on the suppliers' integration and development.

Stage four can be called **"performance Centered Purchasing"**. It focuses on the optimum business performance as a whole and managing the purchasing functions contributions to the overall business performance objectives and delivery. It is a system approach.

The characteristics focus perspective classifies the purchasing function into four stages:

Stage one is **"Passive"** in character. In this stage the purchasing can be defined as lack of strategic directions and is mainly reactive to operational requirements. High proportion of purchasing manager's time is on routine operations with low visibility to the supply chain. The supplier selection is based on price and availability only.

Stage two is **"independent"** in character. In this stage the purchasing may have adopted the latest technology and process, but may have not got the strategy that aligned with the competition. Links between purchasing and technical disciplines may have been established; performance based on cost reduction; top management recognizes the importance of professional development and the opportunities in purchasing contributing to profitability.

Stage three is **"Supportive"** in character. Purchasing starts to support firms competitive strategy by adopting purchasing techniques and products which strengthens the firms competitive position. Suppliers are considered as key competitive resource. The supply market, products evolution and capabilities are continuously monitored and analysed.

Stage four is **"Integrative"** in character. In this stage purchasing fully integrates with firm's purchasing function. Multifunctional teams and cross functional training of purchasing professional begin to take hold. Open and close communication with other functional departments is hard wired into the processes. Purchasing is measured in terms of its contribution to the overall success of the firm.

There is no doubt that purchasing function has now become a much more sophisticated process and has much wider and deeper impact to the business performance. It is moving away from the short-term towards long-term; from a function to processes; from transactional to relational; from cost saving to performance enhancing. The picture of purchasing in the future perhaps can be described as the knowledge-based purchasing, which is built on the knowledge about whole business objectives and stakeholders' interest, the knowledge about the suppliers and their capabilities and potential, the knowledge about people and their emotion towards relationship and culture; and the knowledge about technology up-taking.

1.3 Stores and Inventory Control

Production refers to the capacity to make and store products. The facilities of production are **factories and warehouses.** The fundamental decision that managers face when making production decisions is how to resolve the trade-off between responsiveness and efficiency. If factories and warehouses are built with a lot of excess capacity, they can be very flexible and respond quickly to wide swings in product demand. Facilities where all or almost all capacity is being used are not capable of responding easily to fluctuations in demand. On the other hand, capacity costs money and excess capacity is idle capacity not in use and not generating revenue. So the more excess capacity that exists, the less efficient the operation becomes.

Factories can be built to accommodate one of two approaches to manufacturing:

Product Focus: A factory that takes a product focus performs the range of different operations required to make a given product line from fabrication of different product parts to assembly of these parts.

Functional Focus: A functional approach concentrates on performing just a few operations such as only making a select group of parts or only doing assembly. These functions can be applied to making many different kinds of products.

A product approach tends to result in developing expertise about a given set of products at the expense of expertise about any particular function. A functional approach results in expertise about particular functions instead of expertise in a given product. Companies need to decide which approach or what mix of these two approaches will give them the capability and expertise they need to best respond to customer demands.

As with factories, warehouses too can be built to accommodate different approaches. There are three main approaches to use in warehousing:

Stock Keeping Unit (SKU) Storage: In this traditional approach, all of a given type of product is stored together. This is an efficient and easy to understand way to store products.

Job Lot Storage: In this approach, all the different products related to the needs of a certain type of customer or related to the needs of a particular job are stored together. This allows for an efficient picking and packing operation but usually requires more storage space than the traditional SKU storage approach.

Crossdocking: An approach that was pioneered by Wal-Mart in its drive to increase efficiencies in its supply chain. In this approach, product is not actually warehoused in the facility. Instead the facility is used to house a process where trucks from suppliers arrive and unload large quantities of different products. These large lots are then broken down into smaller lots. Smaller lots of different products are recombined according to the needs of the day and quickly loaded onto outbound trucks that deliver the products to their final destinations.

1.3.1 Stores Management

Stores play a vital role in the operations of company. It is in direct touch with the user departments in its day-to-day activities. The most important purpose served by the stores is to provide uninterrupted service to the manufacturing divisions. Further, stores are often equated directly with money, as money is locked up in the stores.

Functions of Stores

The functions of stores can be classified as follows:

1. To receive raw materials, components, tools, equipment's and other items and account for them.
2. To provide adequate and proper storage and preservation to the various items.
3. To meet the demands of the consuming departments by proper issues and account for the consumption.
4. To minimise obsolescence, surplus and scrap through proper codification, preservation and handling.
5. To highlight stock accumulation, discrepancies and abnormal consumption and effect control measures.
6. To ensure good housekeeping so that material handling, material preservation, stocking, receipt and issue can be done adequately.
7. To assist in verification and provide supporting information for effective purchase action.

1.3.2 Inventory Management

Inventory is spread throughout and includes everything from raw material to work in process to finished goods that are held by the manufacturers, distributors, and retailers in a supply chain. Again, managers must decide where they want to position themselves in the trade-off between responsiveness and efficiency. Holding large amounts of inventory allows a company to be very responsive to fluctuations in customer demand. However, the creation and storage of inventory is a cost and to achieve high levels of efficiency, the cost of inventory should be kept as low as possible.

There are three basic decisions to make regarding the creation and holding of inventory:

1. Cycle Inventory
2. Safety Inventory and
3. Seasonal Inventory

Inventory management is a set of techniques that are used to manage the inventory levels within different companies. The aim is to reduce the cost of inventory as much as possible while still maintaining the service levels that customers require. Inventory management takes its major inputs from the demand forecasts for products and the prices of products. With these two inputs, inventory management is an ongoing process of balancing product inventory levels to meet demand and exploiting economies of scale to get the best product prices.

As we discussed in Chapter 1, there are three kinds of inventory: (1) cycle inventory; (2) seasonal inventory; and (3) safety inventory. **Cycle inventory and seasonal inventory** are both influenced by economy of scale considerations. The cost structure of the companies in any supply chain will suggest certain levels of inventory based on production costs and inventory carrying cost. **Safety inventory** is influenced by the predictability of product demand. The less predictable product demand is, the higher the level of safety inventory is required to cover unexpected swings in demand.

The inventory management operation in a company or an entire supply chain is composed of a blend of activities related to managing the three different types of inventory. Each type of inventory has its own specific challenges and the mix of these challenges will vary from one company to another and from one supply chain to another.

Cycle Inventory

This is the amount of inventory needed to satisfy demand for the product in the period between purchases of the product. Companies tend to produce and to purchase in large lots in order to gain the advantages that economies of scale can bring. However, with large lots also come increased carrying costs. **Carrying costs** come from the cost to store, handle, and insure the inventory. Managers face the tradeoff between the reduced cost of ordering and better prices offered by purchasing product in large lots and the increased carrying cost of the cycle inventory that comes with purchasing in large lots.

Cycle inventory is the inventory required to meet product demand over the time period between placing orders for the product. Cycle inventory exists because economies of scale make it desirable to make fewer orders of large quantities of a product rather than continuous orders of small product quantity. The end-use customer of a product may actually use a product in continuous small amounts throughout the year. But the distributor and the manufacturer of that product may find it more cost efficient to produce and stock the product in large batches that do not match the usage pattern.

Cycle inventory is the buildup of inventory due to the fact that production and stocking of inventory is done in lot sizes that are larger than the ongoing demand for the product. For example, a distributor may experience an ongoing demand for Item A that is 100 units per week. The distributor finds, however, that it is most cost effective to order in batches of 650 units. Every six weeks or so the distributor places an order causing cycle inventory to build up in the distributor's warehouse at the beginning of the ordering period. The manufacturer of Item A that all the distributors order from may find that it is most efficient for them to manufacture in batches of 14,000 units at a time. This also results in the buildup of cycle inventory at the manufacturer's location.

Inventory Cost Structures

One of the most important prerequisites for effective inventory management is an understanding of the cost structure. Inventory cost structures incorporate the following four types of costs:

1. **Item cost:** This is the cost of buying or producing the individual inventory items. The item cost is usually expressed as a cost per unit multiplied by the quantity procured or produced. Sometimes item cost is discounted if enough units are purchased at one time.
2. **Ordering (or set up) costs:** These are costs of ordering and receiving inventory. They include typing purchase order, expediting the order, transportation costs, receiving costs, and so on. Ordering costs are generally expressed in fixed Naira per ordering regardless of order size. When a firm produces its own inventory instead of ordering it from a supplier, the costs of machine setup (e.g., preparing equipment for the job by adjusting the machine, changing cutting tools) are analogous to ordering costs; they are expressed as a fixed charge per run regardless of the size of the run.
3. **Carrying (or holding) cost:** This is associated with physically having items in storage for a period of time. Holding costs are stated in either of two ways: as a percentage of unit price, for example, a 15 percent annual holding cost means that it will cost 15 kobo to hold N1 of inventory for a year or in Naira per unit. The carrying cost usually consists of three components:
    i. **Cost of capital:** When items are carried in inventory, the capital invested is not available for other purposes. This represents a cost of foregone opportunities for other investments, which is assigned to inventory as an opportunity cost.
    ii. **Cost of storage:** This includes variable space cost, insurance, and taxes. In some cases, a part of the storage cost is fixed, for example, when a ware house is owned and cannot be used for other purpose. Such fixed costs should not be included in the cost of inventory storage. Similarly, taxes and insurance should be included only if they vary with inventory levels
    iii. **Costs of obsolescence, deterioration, and loss:** Obsolescence costs should be assigned to items which have a high risk of becoming obsolete; the higher the risk, the higher the costs. Perishable products such as fresh seafood, meat and poultry and blood should be charged with deterioration costs when the item deteriorates over time. The costs of loss include pilferage and breakage costs associated with holding items in inventory. For example, items that are easily concealed (e.g. pocket cameras, transistor radios, calculators) or fairly expensive (e.g. cars TVs) are prone to theft.
4. **Stock out or shortage costs** result when demand exceeds the supply of inventory on hand. These costs can include the sale lost because material is not on hand, loss of customer goodwill due to delay in delivery of order, late charges and similar costs. Also, if the shortage occurs in an item carried for internal use (e.g. to supply and assembly line), the cost of lost production or downtime is considered a shortage cost. Shortage costs are usually difficult to measure, and they are often subjectively estimated. Estimates can be based on the concept of foregone profits.

Economic Order Quantity (EOQ)

Given the cost structure of a company, there is an order quantity that is the most cost-effective amount to purchase at a time.

Inventory models deal with idle resources like men, machines, money and materials. These models are concerned with two decisions: how much to order (purchase or produce) and when to order so as to minimize the total cost.

For the first decision—how much to order, there are two basic costs are considered namely, inventory carrying costs and the ordering or acquisition costs. As the quantity ordered is increased, the inventory carrying cost increases while the ordering cost decreases. The 'order quantity' means the quantity produced or procured during one production cycle. Economic order quantity is calculated by balancing the two costs. Economic Order Quantity (EOQ) is that size of order which minimizes total costs of carrying and cost of ordering. *i.e.*, Minimum Total Cost occurs when Inventory Carrying Cost = Ordering Cost.

Determination of EOQ by analytical method

In order to derive an economic lot size formula following assumptions are made:

(*a*) Demand is known and uniform.
(*b*) Let D denotes the total number of units purchase/produced and Q denotes the lot size in each production run.
(*c*) Shortages are not permitted, *i.e.*, as soon as the level of the inventory reaches zero, the inventory is replenished.
(*d*) Production or supply of commodity is instantaneous.
(*e*) Lead-time is zero.
(*f*) Set-up cost per production run or procurement cost is C3.
(*g*) Inventory carrying cost is C1 = CI, where C is the unit cost and I is called inventory carrying cost expressed as a percentage of the value of the average inventory.

This fundamental situation can be shown on an inventory-time diagram, (Fig. 1.5) with Q on the vertical axis and the time on the horizontal axis. The total time period (one year) is divided into *n* parts.

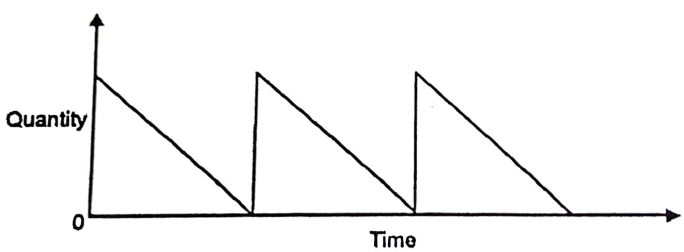


Fig. 1.5

The most economic point in terms of total inventory cost exists where,

$$Inventory\ carrying\ cost = Annual\ ordering\ cost\ (set-up\ cost)$$

$$Average\ inventory = \frac{1}{2}(maximum\ level + minimum\ level) = \frac{(Q+0)}{2} = \frac{Q}{2}$$

$$Total\ inventory\ carrying\ cost$$
$$= Average\ inventory \times Inventory\ carrying\ cost\ per\ unit$$

$$\text{Total inventory carrying cost} = \frac{Q}{2} \times C_1 = \frac{QC_1}{2} \qquad (1)$$

$$\text{Total annual ordering costs}$$
$$= \text{Number of orders per year} \times \text{ordering cost per order}$$

$$\text{Total annual ordering costs} = \left(\frac{D}{Q}\right) \times C_3 = \frac{DC_3}{Q}$$

Now, summing up the total inventory cost and the total ordering cost, we get the total inventory cost C(Q).

$$\text{Total cost of production run}$$
$$= \text{Total inventory carrying cost} + \text{Total annual annual ordering cost}$$

$$C(Q) = \frac{QC_1}{2} + \frac{DC_3}{Q} \qquad (3)$$

Equation (3) is the cost equation

But, the total cost is minimum when the inventory carrying costs becomes equal to the total annual ordering costs. Therefore,

$$\frac{QC_1}{2} = \frac{DC_3}{Q}$$

$$Q + \frac{2DC_3}{Q}$$

$$Q^2 = \frac{2DC_3}{C_1}$$

$$Q = \sqrt{\frac{2DC_3}{C_1}}$$

$$\text{Optimal quatity (EOQ)}, Q_0 = \sqrt{\frac{2DC_3}{C_1}} \qquad (4)$$

$$\text{Optimal number of orders}, N_0 = \frac{D}{Q_0} \qquad (5)$$

$$\text{Optimal order interval}, t_0 = \frac{365}{N_0} \text{in days} = \frac{1}{N_0} \text{in years or } t_0 = \frac{Q_0}{D}$$

$$\text{Average yearly cost } (TC) = \sqrt{2C_3DC_1} \qquad (7)$$

Example 4: An oil engine manufacturer purchases lubricants at the rate of #42 per piece from a vendor. The requirements of these lubricants are 1800 per year. What should be the ordering quantity per order, if the cost per placement of an order is #16 and inventory carrying charges per naira per year is 20.

Solution

Given data are:

Number of lubricants to be purchased, D = 1800 per year

Procurement cost, C_3 = #16 per order

Inventory carrying cost, CI = C_1 = #42 × #0.20 = #8.40 per year

$$Optimal\ quatity\ (EOQ), Q_0 = \sqrt{\frac{2DC_3}{C_1}} = \sqrt{\frac{2 \times 16 \times 1800}{8.4}} = 82.8 \cong 83 lubricants$$

Example 5: A manufacturing company purchase 9000 parts of a machine for its annual requirements ordering for month usage at a time, each part costs #20. The ordering cost per order is #15 and carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchase policy for the company. What advice would you offer and how much would it save the company per year?

Solution

Given data are:

Number of lubricants to be purchased, D = 9000 parts per year

Cost of part, C_s = #20

Procurement cost, C_3 = #15 per order

Inventory carrying cost, CI = C_1 = 15% of average inventory per year = #20 × 0.15 = #3 per each part per year

$$Optimal\ quatity\ (EOQ), Q_0 = \sqrt{\frac{2DC_3}{C_1}} = \sqrt{\frac{2 \times 9000 \times 15}{3}} = 300\ units$$

$$Optimal\ order\ interval, t_0 = \frac{Q_0}{D} = \frac{300}{9000} = \frac{1}{30} years = \frac{1}{30} \times 365\ days = 122\ days$$

$$minimum\ average\ cost\ (TC) \overset{o}{=} \sqrt{2C_3DC_1} = \sqrt{2 \times 15 \times 9000 \times 3} = \#900$$

If the company follows the policy of ordering every month, then the annual ordering cost is

$$= 12 \times \#15 = \#180$$

$$Lot\ size\ of\ inventory\ each\ month = \frac{9000}{12} = 750$$

$$Average\ inventory\ at\ anytime = \frac{Q}{2} = \frac{750}{2} = 375$$

$$Therefore, storage\ cost\ at\ anytime = Average\ inventory\ at\ anytime \times C_1$$
$$= 375 \times 3 = \#1125$$

$$\boldsymbol{Total\ annual\ cost = storage\ cost\ at\ anytime + annual\ ordering\ cost = 1125 + 180}$$
$$= \#1305$$

Hence, the company should purchase 300 parts at time interval of 1/30 year instead of ordering 750 parts each month.

$$\boldsymbol{The\ net\ saving\ of\ the\ company\ will\ be}$$
$$= Total\ annual\ cost - minimum\ average\ cost = 1305 - 900 = \#405$$

Good inventory management requires a company to know the EOQ for all the products it buys. The EOQ for different products changes over time so a company needs an ongoing measurement process to keep the numbers accurate and up to date.

Seasonal Inventory

This is inventory that is built up in anticipation of predictable increases in demand that occur at certain times of the year. For example, it is predictable that demand for antifreeze will increase in the winter. If a company that makes antifreeze has a fixed production rate that is expensive to change, then it will try to manufacture product at a steady rate all year long and build up inventory during periods of low demand to cover for periods of high demand that will exceed its production rate. The alternative to building up seasonal inventory is to invest in flexible manufacturing facilities that can quickly change their rates of production of different products to respond to increases in demand. In this case, the tradeoff is between the cost of carrying seasonal inventory and the cost of having more flexible production capabilities.

Seasonal inventory happens when a company with a fixed amount of productive capacity decides to produce and stockpile products in anticipation of future demand. If future demand is going to exceed productive capacity, then the answer is to produce product in times of low demand that can be put into inventory to meet the high demand in the future.

Decisions about seasonal inventory are driven by a desire to get the best economies of scale given the capacity and cost structure of each company in the supply chain. If it is expensive for a manufacturer to increase productive capacity, then capacity can be considered as fixed. Once the annual demand for the manufacturer's products is determined, the most efficient schedule to utilize that fixed capacity can be calculated.

This schedule will call for seasonal inventory. Managing seasonal inventory calls for demand forecasts to be accurate since large amounts of inventory can be built up this way and it can become obsolete, or holding costs can mount if the inventory is not sold off as anticipated. Managing seasonal inventory also calls for manufacturers to offer price incentives to persuade distributors to purchase the product and put it in their warehouses well before demand for it occurs.

Safety Inventory

Inventory that is held as a buffer against uncertainty. If demand forecasting could be done with perfect accuracy, then the only inventory that would be needed would be cycle inventory. But since every forecast has some degree of uncertainty in it, we cover that uncertainty to a greater or lesser degree by holding additional inventory in case demand is suddenly greater than anticipated. The tradeoff here is to weigh the costs of carrying extra inventory against the costs of losing sales due to insufficient inventory.

Safety inventory is necessary to compensate for the uncertainty that exists in a supply chain. Retailers and distributors do not want to run out of inventory in the face of unexpected customer demand or unexpected delay in receiving replenishment orders, so they keep safety stock on hand. As a rule, the higher the level of uncertainty, the higher the level of safety stock that is required.

Safety inventory for an item can be defined as the amount of inventory on hand for an item when the next replenishment EOQ lot arrives. This means that the safety stock is inventory that does not turn over. In effect, it becomes a fixed asset and it drives up the cost of carrying inventory. Companies need to find a balance between their desire to carry a wide range of products and offer high availability on all of them, and their conflicting desire to keep the cost of inventory as low as possible. That balance is reflected quite literally in the amount of safety stock that a company carries.

Four Ways to Reduce Safety Inventory

i. Reduce demand uncertainty: learn to do better product demand forecast
ii. Reduce order leads times: shorter lead times mean less safety inventory needed for coverage
iii. Reduce lead time variability: further reduces need for safety inventory
iv. Reduce availability uncertainty: ensure product availability when demand occurs'

1.4 Standardization

Standardization means producing maximum variety of products from the minimum variety of materials, parts, tools and processes. It is the process of establishing standards or units of measure by which extent, quality, quantity, value, performance etc., may be compared and measured.

Advantages of Standardization

All the sections of company will be benefited from standardization as mentioned below.

1. **Benefits to Design Department**
   (a) Fewer specifications, drawings and part list have to prepared and issued.
   (b) More time is available to develop new design or to improve established design.

(c) Better resource allocation.
(d) Less qualified personnel can handle routine design work.

2. **Benefits to Manufacturing Department**
   (a) Lower unit cost.
   (b) Better quality products.
   (c) Better methods and tooling.
   (d) Increased interchangeability of parts.
   (e) Better utilization of manpower and equipment.
   (f) Accurate delivery dates.
   (g) Better services of production control, stock control, purchasing, etc.
   (h) More effective training.
3. **Benefits to Marketing Department**
   (a) Better quality products of proven design at reasonable cost leads to greater sales volume.
   (b) Increased margin of profit.
   (c) Better product delivery.
   (d) Easy availability of sales part.
   (e) Less sales pressure of after-sales services.
4. **Benefits to Production Planning Department**
   (a) Scope for improved methods, processes and layouts.
   (b) Opportunities for more efficient tool design.
   (c) Better resource allocation.
   (d) Reduction in pre-production activities.
5. **Benefits to Production Control Department**
   (a) Well proven design and methods improve planning and control.
   (b) Accurate delivery promises.
   (c) Fewer delays arise from waiting for materials, tools, etc.
   (d) Follow-up of small batches consumes less time.
6. **Benefits to Purchase and Stock Control Department**
   (a) Holding of stock of standard items leads to less paper work and fewer requisitions and orders.
   (b) Storage and part location can be improved.
   (c) Newer techniques can be used for better control of stocks.
   (d) Because of large purchase quantities involved, favourable purchase contracts can be made.
7. **Benefits to Quality Control Department**
   (a) Better inspection and quality control is possible.
   (b) Quality standards can be defined more clearly.
   (c) Operators become familiar with the work and produce jobs of consistent quality.
8. **Other Benefits**
   (a) Work study section is benefited with efficient break down of operations and effective work measurement.
   (b) Costing can obtain better control by installing standard costing.
   (c) More time is available to the supervisors to make useful records and preserve statistics.
   (d) Reduced reductions and scrap.
   (e) Helps supervisors to run his department efficiently and effectively.

Disadvantages of Standardization

Following are the disadvantages of standardization:

i. Reduction in choice because of reduced variety and consequently loss of business or customer.
ii. Standard once set, resist change and thus standardization may become an obstacle to progress.
iii. It tends to favour only large companies.
iv. It becomes very difficult to introduce new models because of less flexible production facilities and due to high cost of specialised production equipment.

1.5 Simplification

The concept of simplification is closely related to standardization. Simplification is the process of reducing the variety of products manufactured. Simplification is concerned with the reduction of product range, assemblies, parts, materials and design.

1.7.1 Advantages of Simplification

Following are the advantages of simplification:

i. Simplification involves fewer, parts, varieties and changes in products; this reduces manufacturing operations and risk of obsolescence.
ii. Simplification reduces variety, volume of remaining products may be increased.
iii. Simplification provides quick delivery and better after-sales services.
iv. Simplification reduces inventory and thus results in better inventory control.
v. Simplification lowers the production costs.
vi. Simplification reduces price of a product.
vii. Simplification improves product quality.

Interest rate is the rental value of money. It represents the growth of capital per unit period. The period may be a month, a quarter, semiannual or a year. An interest rate 15% compounded annually means that for every hundred naira invested now, an amount of #15 will be added to the account at the end of the first year. So, the total amount at the end of the first year will be #115. At the end of the second year, again 15% of #115, i.e. #17.25 will be added to the account. Hence the total amount at the end of the second year will be #132.25. The process will continue thus till the specified number of years.

If an investor invests a sum of #100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 2.1.

Table 2.1 Compound Amounts (amount of deposit = #100.00)

Year End	Interest (#)	Compound Amount (#)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

The formula to find the future worth in the third column is

$$F = P \times (1 + i)^n$$

Where;
P = principal amount invested at time 0,
F = future amount,
i = interest rate compounded annually,
n = period of deposit.

The maturity value at the end of the fifth year is #201.14. This means that the amount #201.14 at the end of the fifth year is equivalent to #100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 2.1. This explanation assumes that the inflation is at zero percentage.

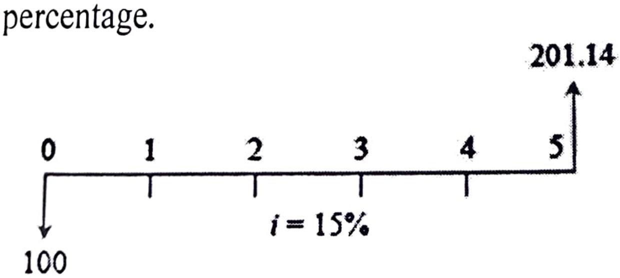


Fig. 2.1: Time value of money

Alternatively, the above concept may be discussed as follows: If we want #100.00 at the end of the nth year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in Table 2.2.

Table 2.2 Present Worth Amounts (rate of interest = 15%)

Year End	Present Worth	Compound Amount (#)
0		100.00
1	86.96	100.00
2	75.61	100.00
3	65.75	100.00
4	57.18	100.00
5	49.72	100.00
6	43.29	100.00
7	37.59	100.00
8	32.69	100.00
9	28.43	100.00
10	24.72	100.00

The formula to find the present worth in the second column is

$$P = \frac{F}{(1+i)^n}$$

From Table 2.2, it is clear that if we want #100 at the end of the fifth year, we should now deposit an amount of #49.72. Similarly, if we want #100.00 at the end of the 10th year, we should now deposit an amount of #24.72.

Also, this concept can be stated as follows:

A person has received a prize from a finance company during the recent festival contest. But the prize will be given in either of the following two modes:

1. Spot payment of #24.72 or
2. #100 after 10 years from now (this is based on 15% interest rate compounded annually).

If the prize winner has no better choice that can yield more than 15% interest rate compounded annually, and if 15% compounded annually is the common interest rate paid in all the finance companies, then it makes no difference whether he receives #24.72 now or #100 after 10 years.

On the other hand, let us assume that the prize winner has his own business wherein he can get a yield of 24% interest rate (more than 15%) compounded annually, it is better for him to receive the prize money of #24.72 at present and utilize it in his business. If this option is followed, the equivalent amount for #24.72 at the end of the 10th year is #212.45. This example clearly demonstrates the time value of money.

2.1 Interest Formulas and their Applications

While making investment decisions, computations will be done in many ways. To simplify all these computations, it is extremely important to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment-decision making, the various interest formulas are presented first.

Interest rate can be classified into *simple interest rate* and *compound interest rate*.

In **simple interest**, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In **compound interest**, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period.

The notations which are used in various interest formulae are as follows:

P = principal amount
n = No. of interest periods
i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)
F = future amount at the end of year n
A = equal amount deposited at the end of every interest period
G = uniform amount which will be added/subtracted period after period to/ from the amount of deposit A1 at the end of period 1

2.1.1 Single-Payment Compound Amount

Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram of this situation is shown in Fig. 2.2

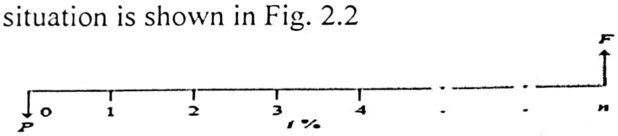


Fig. 2.2 Cash flow diagram of single-payment compound amount

The formula to obtain the single-payment compound amount is

$$F = P(1+i)^n = P(F/P, i, n)$$

Where: **(F/P, i, n)** is called as single-payment compound amount factor.

Example 1: A person deposits a sum of # 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

P = #20,000
i = 18% compounded annually
n = 10 years

$$F = P(1+i)^n = P(F/P, i, n) = 20{,}000P(F/P, 18\%, 10) = 20{,}000 \times 5.234 = \#104{,}680$$

The maturity value of #20,000 invested now at 18% compounded yearly is equal to #104,680 after 10 years.

2.1.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 2.3.

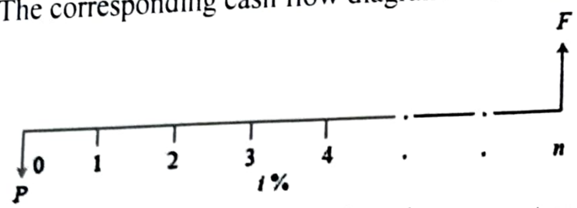


Fig. 3.3 Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$F = \frac{P}{(1+i)^n} = F(P/F, i, n)$$

Where: **(*P*/*F*, *i*, *n*)** is termed as *single-payment present worth factor.*

Example 2: A person wishes to have a future sum of #1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

F = #100,000
i = 15%, compounded annually
n = 10 years

$$F = \frac{P}{(1+i)^n} = F(P/F, i, n) = 100{,}000(P/F, 15\%, 10) = 100{,}000 \times 0.2472 = \#24720$$

The person has to invest #24,720 now so that he will get a sum of #100,000 after 10 years at 15% interest rate compounded annually.

2.1.3 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 2.4.

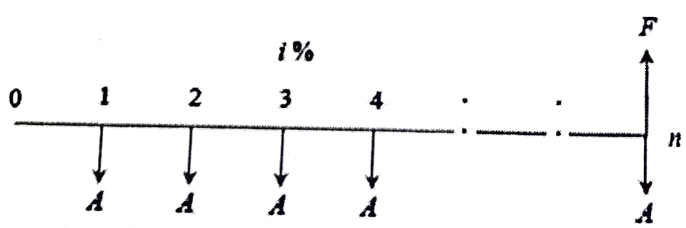


Fig. 2.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 2.4,

A = equal amount deposited at the end of each interest period

n = No. of interest periods
i = rate of interest
F = single future amount

The formula to get F is

$$F = A\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

Where: **(*F*/*A*, *i*, *n*)** is termed as *equal-payment series compound amount factor.*

Example 3 A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of #10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution

A = #10,000
n = 25 years
i = 20%
F = ?

The corresponding cash flow diagram is shown in Fig. 2.5.

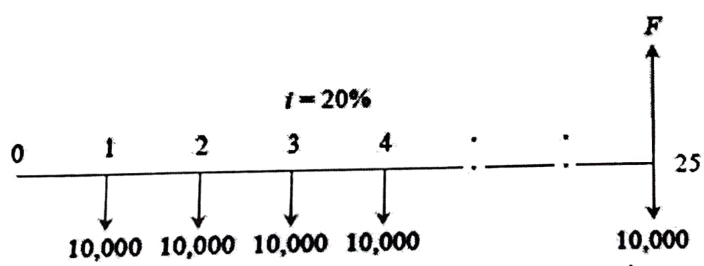


Fig. 2.5 Cash flow diagram of equal-payment series compound amount.

$$F = A\frac{(1+i)^n - 1}{i} = A(F/A.i,n) = 10{,}000(F/A, 20\%, 25) = 10{,}000 \times 471.981$$
$$= \#4{,}719{,}810$$

The future sum of the annual equal payments after 25 years is equal to #4,719,810.

2.1.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 2.6.

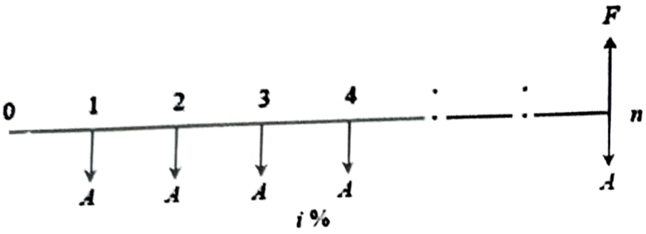


Fig. 2.6 Cash flow diagram of equal-payment series sinking fund

In Fig. 2.6,

A = equal amount to be deposited at the end of each interest period
n = No. of interest periods
i = rate of interest
F = single future amount at the end of the nth period

The formula to get F is

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

Where: $(\boldsymbol{A/F, i, n})$ is called as *equal-payment series sinking fund factor.*

Example 4: A company has to replace a present facility after 15 years at an outlay of #500,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

Solution

F = #500,000
n = 15 years
i = 18%
A = ?

The corresponding cash flow diagram is shown in Fig. 2.7.

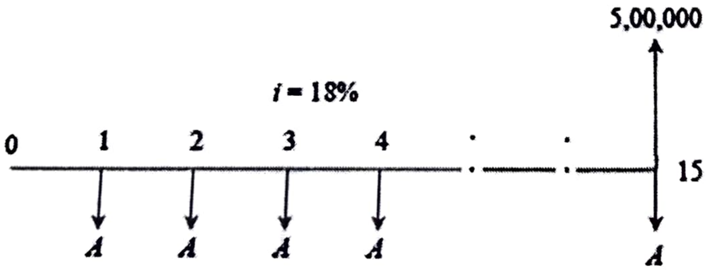


Fig. 7 Cash flow diagram of equal-payment series sinking fund

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F, i, n) = 500{,}000(A/F, 18\%, 15) = 500{,}000 \times 0.0164$$
$$= \#8{,}200$$

The annual equal amount which must be deposited for 15 years is #8,200

2.1.5 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 2.8. Here,

P = present worth
A = annual equivalent payment
i = interest rate
n = No. of interest periods

The formula to compute P is

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

Where: $(P/A, i, n)$ is called *equal-payment series present worth factor.*

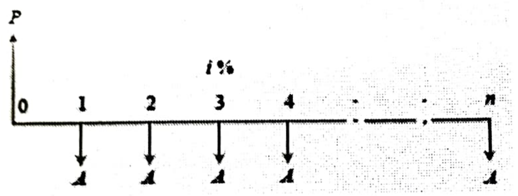


Fig. 2.8 Cash flow diagram of equal-payment series present worth amount

Example 5: A company wants to set up a reserve which will help the company to have an annual equivalent amount of #1,000,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

Solution

A = #1,000,000
i = 15%
n = 20 years
P = ?

The corresponding cash flow diagram is illustrated in Fig. 2.9.

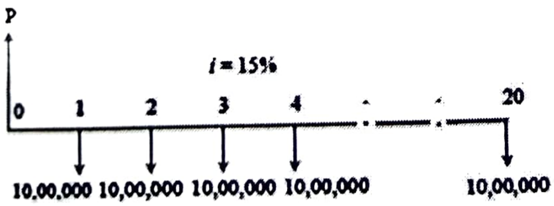


Fig. 2.9: Cash flow diagram of equal-payment series present worth amount.

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n) = 1{,}000{,}000 \times (P/A, 15\%, 20) = 1{,}000{,}000 \times 6.2593$$
$$= 6{,}259{,}300$$

The amount of reserve which must be set-up now is equal to #6,259,300.

2.1.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period (Fig. 2.10).

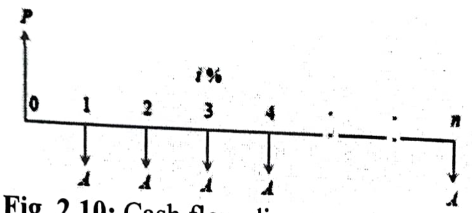


Fig. 2.10: Cash flow diagram of equal-payment series capital recovery amount.

P = present worth (loan amount)
A = annual equivalent payment (recovery amount)
i = interest rate
n = No. of interest periods

The formula to compute P is as follows:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

Where, **(A/P, i, n)** is called *equal-payment series capital recovery factor.*

Example 6: A bank gives a loan to a company to purchase an equipment worth #1,000,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution

P = #1,000,000
i = 18%
n = 15 years
A = ?

The corresponding cash flow diagram is shown in Fig. 2.11.

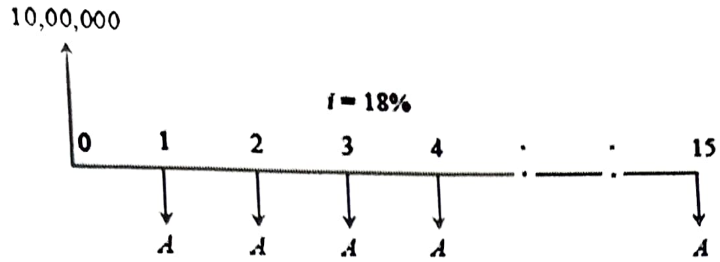


Fig. 2.11: Cash flow diagram of equal-payment series capital recovery amount.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n) = 1{,}000{,}000 \times (A/P, 18\%, 15) = 1{,}000{,}000 \times 0.1964$$
$$= \#196{,}400$$

The annual equivalent installment to be paid by the company to the bank is #196,400.

2.1.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 2.12.

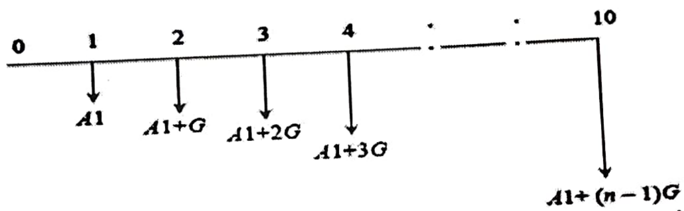


Fig. 2.12: Cash flow diagram of uniform gradient series annual equivalent amount

The formula to compute A under this situation is

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A1 + G(A/G, i, n)$$

Where, $(A/G, i, n)$ is called *uniform gradient series factor.*

Example 7 A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is #4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of #500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution,

$A1 = \#4{,}000$
$G = \#500$
$i = 15\%$
$n = 10$ years

$A = ?$ & $F = ?$

The cash flow diagram is shown in Fig. 2.13.

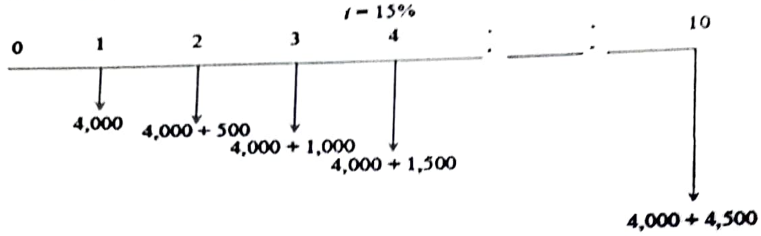


Fig. 2.13: Cash flow diagram of uniform gradient series annual equivalent amount

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A1 + G(A/G, i, n) = 4{,}000 + 500(A/G, 15\%, 10)$$
$$= 4{,}000 + (500 \times 3.3832) = \#5{,}691.60$$

This is equivalent to paying an equivalent amount of #5,691.60 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A, i, n) = A(F/A, 15\%, 10) = 5{,}691.60(20.304) = \#115{,}562.25$$

Where $A = A1 + G(A/G, i, n)$

At the end of the 10th year, the compound amount of all his payments will be #115,562.25.

Example 8 **A person is planning for his retired life. He has 10 more years of service. He would like to deposit #8,500 at the end of the first year and thereafter he wishes to deposit the amount with an annual decrease of #500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.**

Solution

$A1 = \#8{,}500$
$G = -\#500$
$i = 15\%$
$n = 10$ years
$A = ?$ & $F = ?$

The cash flow diagram is shown in Fig. 2.14.

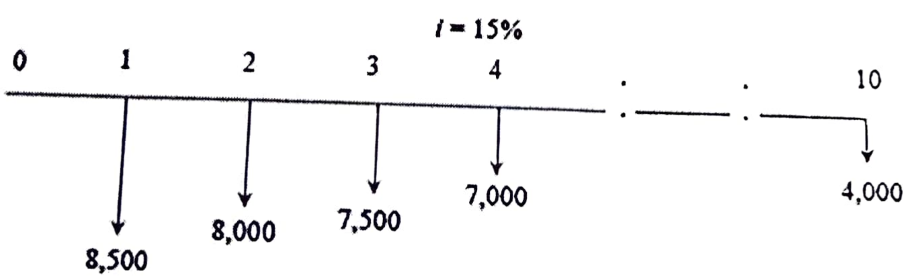


Fig. 2.14: Cash flow diagram of uniform gradient series annual equivalent amount

$$A = A1 - G\frac{(1+i)^n - in - 1}{i(1+i)^n - i} = A1 - G(A/G, i, n) = 8{,}500 - 500(A/G, 15\%, 10)$$
$$= 8{,}500 - (500 \times 3.3832) = \#6{,}808.40$$

This is equivalent to paying an equivalent amount of #6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A, i, n) = A(F/A, 15\%, 10) = 6{,}808.40(20.304) = \#138{,}237.75$$

At the end of the 10th year, the compound amount of all his payments is #138,237.75.

2.2 Bases for Comparison of Alternatives

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has a huge sum of money for potential investment and there are three different projects whose initial outlay and annual revenues during their lives are known. The executive has to select the best alternative among these three competing projects.

There are several bases for comparing the worthiness of the projects. These bases are:

1. Present worth method
2. Future worth method
3. Annual equivalent method
4. Rate of return method

2.2.1 Present Worth Method of Comparison

In this method of comparison, the cash flows of each alternative will be reduced to time zero by assuming an interest rate *i*. Then, depending on the type of decision, the best alternative will be selected by comparing the present worth amounts of the alternatives.

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

In a cost dominated cash flow diagram, the costs (outflows) will be assigned with positive sign and the profit, revenue, salvage value (all inflows), etc. will be assigned with negative sign.

In a revenue/profit-dominated cash flow diagram, the profit, revenue, salvage value (all inflows to an organization) will be assigned with positive sign. The costs (outflows) will be assigned with negative sign.

In case the decision is to select the alternative with the minimum cost, then the alternative with the least present worth amount will be selected. On the other hand, if the decision is to select the alternative with the maximum profit, then the alternative with the maximum present worth will be selected.

Revenue-Dominated Cash Flow Diagram

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 2.15.

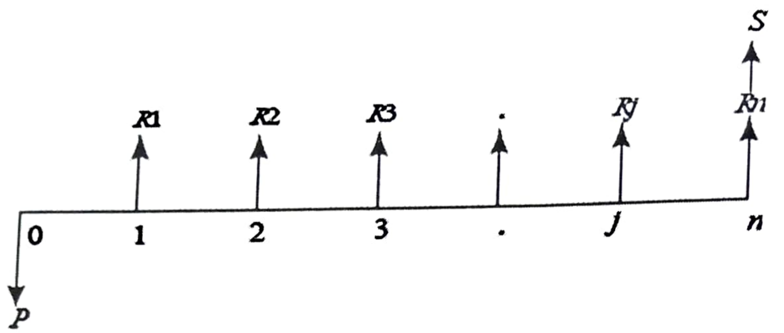


Fig. 2.15: Revenue-dominated cash flow diagram.

In Fig. 2.15, *P* represents an initial investment and *Rj* the net revenue at the end of the *j*th year. The interest rate is *i*, compounded annually. *S* is the salvage value at the end of the *n*th year.

To find the present worth of the above cash flow diagram for a given interest rate, the formula is

$$PW(i) = -P + R1\left[\frac{1}{(1+i)^1}\right] + R2\left[\frac{1}{(1+i)^2}\right] + \cdots + Rj\left[\frac{1}{(1+i)^j}\right] + Rn\left[\frac{1}{(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]$$

In this formula, expenditure is assigned a negative sign and revenues are assigned a positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. **Finally, the alternative with the maximum present worth amount should be selected as the best alternative.**

Cost-Dominated Cash Flow Diagram

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 2.16

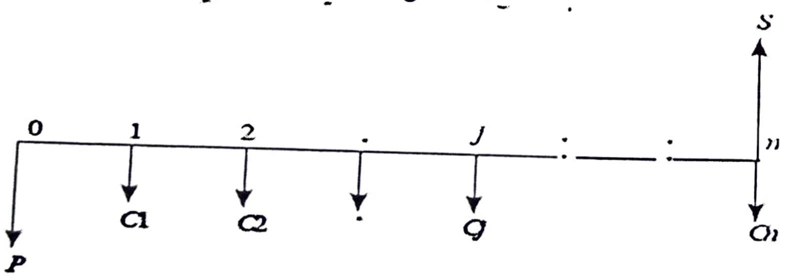


Fig. 2.16: Cost-dominated cash flow diagram.

In Fig. 2.16, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

To compute the present worth amount of the above cash flow diagram for a given interest rate i, we have the formula

$$PW(i) = -P + C1\left[\frac{1}{(1+i)^1}\right] + C2\left[\frac{1}{(1+i)^2}\right] + \cdots + Cj\left[\frac{1}{(1+i)^j}\right] + Cn\left[\frac{1}{(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]$$

In the above formula, the expenditure is assigned a positive sign and the revenue a negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. **Finally, the alternative with the minimum present worth amount should be selected as the best alternative.**

Example 9: Alpha Industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table 2.3. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

Table 2.3

	Initial outlay (#)	**Annual Revenue (#)**	**Life (years)**
Technology 1	1,200,000	400,000	10
Technology 2	2,000,000	600,000	10
Technology 3	1,800,000	500,000	10

Solution In all the technologies, the initial outlay is assigned a negative sign and the annual revenues are assigned a positive sign.

Technology 1

Initial outlay, P = #1,200,000
Annual revenue, A = #400,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is as shown in Fig. 2.17

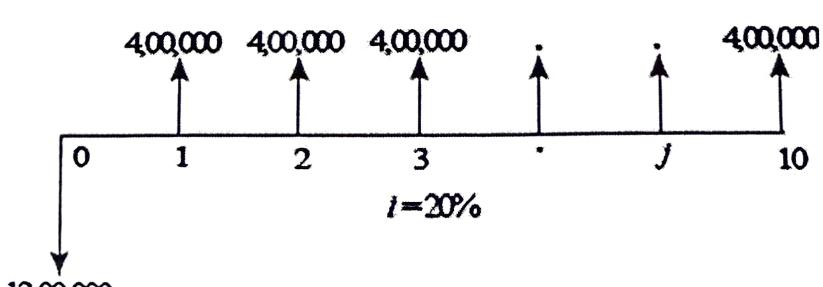


Fig. 2.17: Cash flow diagram for technology 1

The present worth expression for technology 1

$$PW(i)_1 = -P + A(P/A, i, n)$$

Where *A(P/A, i ,n)*is the equal payment series present worth amount

$$PW(20\%)_1 = -1{,}200{,}000 + 400{,}000 \times (P/A, 20\%, 10)$$
$$= -1{,}200{,}000 + 400{,}000(4.1925) = \#477{,}000$$

Technology 2

Initial outlay, P = #2,000,000
Annual revenue, A = #600,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 2.17:

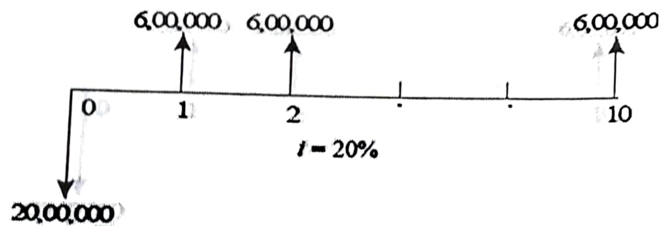


Fig. 2.18: Cash flow diagram for technology 2

The present worth expression for technology 2

$$PW(i)_2 = -P + A(P/A, i, n)$$

Where *A(P/A, i ,n)*is the equal payment series present worth amount

$$PW(20\%)_2 = -2{,}000{,}000 + 600{,}000 \times (P/A, 20\%, 10)$$
$$= -2{,}000{,}000 + 600{,}000(4.1925) = \#515{,}500$$

Technology 3

Initial outlay, P = #1,800,000
Annual revenue, A = #500,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 2.18

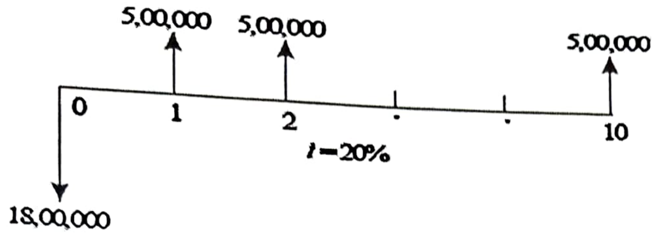


Fig. 2.19: Cash flow diagram for technology 3

The present worth expression for technology 3

$$PW(i)_3 = -P + A(P/A, i, n)$$

Where $A(P/A,\ i\ ,n)$is the equal payment series present worth amount

$$PW(20\%)_3 = -1{,}800{,}000 + 500{,}000 \times (P/A, 20\%, 10)$$
$$= -1{,}800{,}000 + 500{,}000(4.1925) = \#296{,}250$$

From the above calculations, it is clear that the present worth of technology 2 is the highest among all the technologies. Therefore, technology 2 is suggested for implementation to expand the production.

Example 10: An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

Bid	Engineer's Estimates		
	Initial Cost (#)	Service life (years)	Annual operations & maintenance cost (#)
Alpha Elevator Inc.	450,000	15	27,000
Beta Elevator Inc.	540,000	15	28,500

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

Solution

Bid 1: Alpha Elevator Inc

Initial cost, P = #450,000
Annual operation and maintenance cost, A = #27,000
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 1 is shown in Fig. 2.18

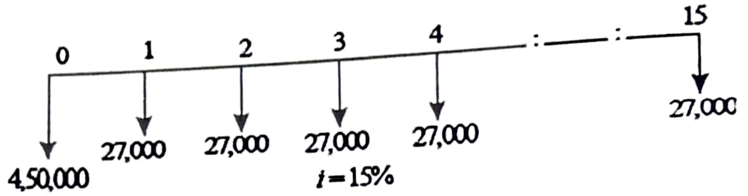


Fig. 2.20: Cash flow diagram for bid 1

The present worth of the above cash flow diagram is computed as follows:

$$PW(i)_1 = P + A(P/A, i, n)$$

Where *A(P/A, i ,n)*is the equal payment series present worth amount

$$PW(15\%) = 450{,}000 + 27{,}000 \times (P/A, 15\%, 15) = 450{,}000 + 27{,}000(5.8474)$$
$$= \#607{,}879.80$$

Bid 2: Beta Elevator Inc
Initial cost, P = #540,000
Annual operation and maintenance cost, A = #28,500
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 2 is shown in Fig. 2.19

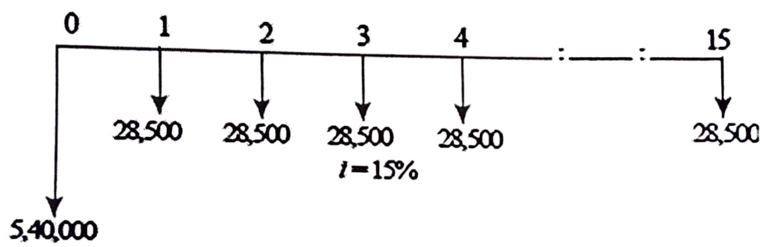


Fig. 2.21: Cash flow diagram for bid 2

The present worth of the above cash flow diagram is computed as follows:

$$PW(i)_2 = P + A(P/A, i, n)$$

Where *A(P/A, i ,n)*is the equal payment series present worth amount

$$PW(15\%) = 540{,}000 + 28{,}500 \times (P/A, 15\%, 15) = 540{,}000 + 28{,}500(5.8474)$$
$$= \#706{,}650.90$$

The total present worth cost of bid 1 is less than that of bid 2. Hence, bid 1 is to be selected for implementation. That is, the elevator from Alpha Elevator Inc. is to be purchased and installed in the new building.

Example 11: A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is #1,600,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of #200,000 each. Suggest the best alternative for the company using the present worth basis at i = 18%, compounded annually.

Solution There are two alternatives available for the company:

1. Down payment of #1,600,000
2. Down payment of #400,000 and 10 annual equal installments of #200,000 each

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 2.20.

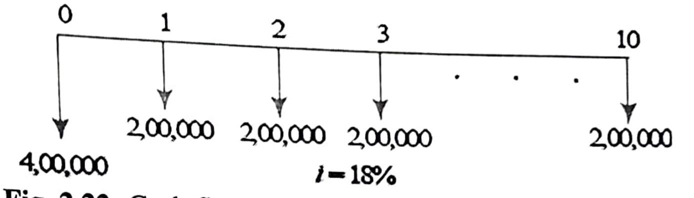


Fig. 2.22: Cash flow diagram for the second alternative.

The present worth of the above cash flow diagram is computed as

$$PW(i)_1 = -P + A(P/A, i, n)$$

Where *A(P/A, i ,n)*is the equal payment series present worth amount

$$PW(18\%) = 400{,}000 + 200{,}000 \times (P/A, 18\%, 10) = 400{,}000 + 200{,}000(4.4941)$$
$$= \#1{,}298{,}820$$

The present worth of this option is #1,298,820, which is less than the first option of complete down payment of #1,600,000. Hence, the company should select the second alternative to buy the fully automated granite cutting machine.

Example 13: Novel Investment Ltd. accepts #10,000 at the end of every year for 20 years and pays the investor #8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts #10,000 at the end of every year for 20 years and pays the investor #15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with i = 12%.

Solution Novel Investment Ltd's plan. The cash flow diagram of Novel Investment Ltd's plan is shown in Fig. 2.23

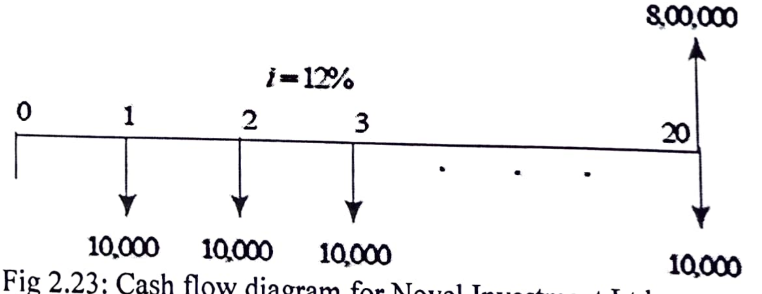


Fig 2.23: Cash flow diagram for Novel Investment Ltd.

The present worth of the above cash flow diagram is computed as

$$PW(i) = A(P/A, i, n) + F(P/F, i, n)$$

Where A(P/A, i, n) is the equal payment series present worth and F(P/F, i, n) is the single payment present worth.

$$PW(12\%) = -10{,}000(P/A, 12\%, 20) + 800{,}000(P/F, 12\%, 20)$$
$$= -10{,}000(7.4694) + 800{,}000(0.1037) = \#8{,}266$$

Innovative Investment Ltd plan: The cash flow diagram of the Innovative Investment Ltd's plan is illustrated in Fig. 2.24

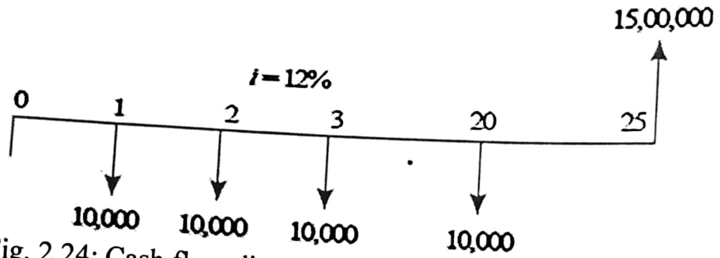


Fig. 2.24: Cash flow diagram for Innovative Investment Ltd.

The present worth of the above cash flow diagram is calculated as

The present worth of the above cash flow diagram is computed as

$$PW(i) = A(P/A, i, n) + F(P/F, i, n)$$

Where A(P/A, i, n) is the equal payment series present worth and F(P/F, i, n) is the single payment present worth.

$$PW(12\%) = -10{,}000(P/A, 12\%, 20) + 1{,}500{,}000(P/F, 12\%, 25)$$
$$= -10{,}000(7.4694) + 1{,}500{,}000(0.0588) = \#13{,}506$$

The present worth of Innovative Investment Ltd plan is more than that of Novel Investment Ltd plan. Therefore, Innovative Investment Ltd plan is the best from investor's point of view.

Example 14: A small business with an initial outlay of #12,000 yields #10,000 during the first year of its operation and the yield increases by #1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

Solution

Initial investment, $P = \#12{,}000$
Income during the first year, $A = \#10{,}000$
Annual increase in income, $G = \#1{,}000$
$n = 10$ years

$i = 18\%$, compounded annually

The cash flow diagram for the small business is depicted in Fig. 2.24

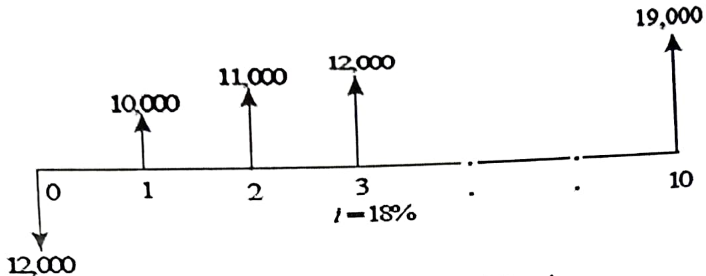


Fig. 2.25: Cash flow diagram for the small business.

The equation for the present worth is

$$PW(i) = P + (A1 + G(A/G, i, n))(P/A, i, n)$$

Where, $P + (A1+G(A/G,\ i,\ n)$ is uniform gradient annual equivalent factor $(P/A,\ i,\ n)$ is the equal payment series present worth factor

$$\begin{aligned} PW(18\%) &= -12{,}000 + (10{,}000 + 1{,}000 \times (A/G, 18\%, 10)) \times (P/A, 18\%, 10) \\ &= -12{,}000 + (10{,}000 + 1{,}000 \times 3.1936) \times 4.4941 = \#47{,}293.36 \end{aligned}$$

The present worth of the small business is #47,293.36.

2.2.2 Future Worth Method

In the future worth method of comparison of alternatives, the future worth of various alternatives will be computed. Then, the alternative with the maximum future worth of net revenue or with the minimum future worth of net cost will be selected as the best alternative for implementation.

Revenue-Dominated Cash Flow Diagram

A generalized revenue-dominated cash flow diagram to demonstrate the future worth method of comparison is presented in Fig. 2.26.

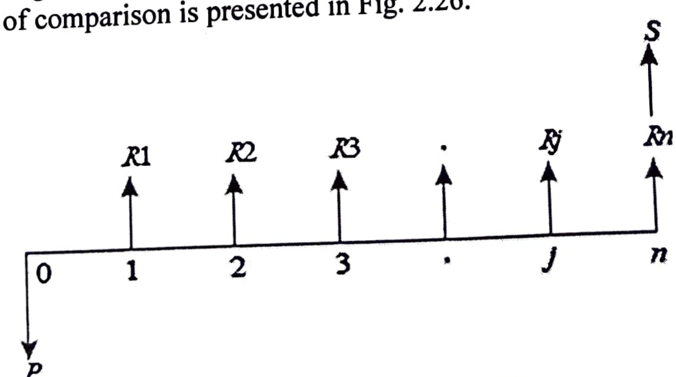


Fig 2.26: Revenue-dominated cash flow diagram.

In Fig. 2.26, P represents an initial investment, Rj the net-revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = -P(1+i)^n + R1(1+i)^{n-1} + R2(1+i)^{n-2} + \dots + Rj(1+i)^{n-j} + \dots + Rn + S$$

In the above formula, the expenditure is assigned with negative sign and the revenues are assigned with positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, **the alternative with the maximum future worth amount should be selected as the best alternative.**

Cost-Dominated Cash Flow Diagram

A generalized cost-dominated cash flow diagram to demonstrate the future worth method of comparison is given in Fig. 2.27.

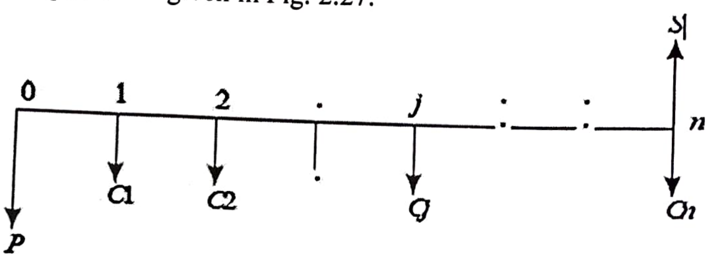


Fig. 2 27: Cost-dominated cash flow diagram.

In Fig. 2.27, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = -P(1+i)^n + C1(1+i)^{n-1} + C2(1+i)^{n-2} + \dots + Cj(1+i)^{n-j} + \dots + Cn + S$$

In this formula, the expenditures are assigned with positive sign and revenues with negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, **the alternative with the minimum future worth amount should be selected as the best alternative.**

Example 15: A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as given in the following table:

	Build Gas Station	Build Soft Ice-Cream Stand
First cost (#)	2,000,000	3,600,000
Annual property taxes (#)	80,000	150,000
Annual income (#)	800,000	980,000

Life of building (years)	20	20
Salvage value (#)	0	0

Evaluate the alternatives based on the future worth method at $i = 12\%$.

Alternative 1—**Build gas station**

First cost = #2,000,000
Net annual income = Annual income − Annual property tax = #800,000 − #80,000 = #720,000
Life = 20 years
Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is depicted in Fig.2.28

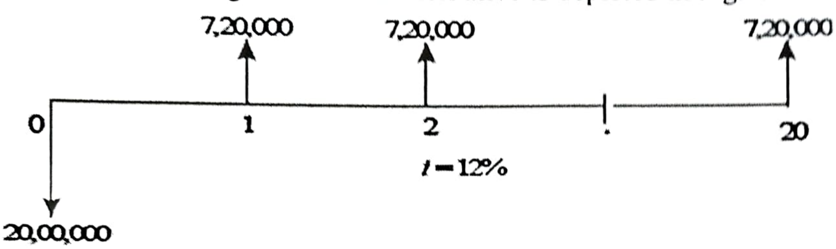


Fig 2.28: Cash flow diagram for alternative 1.

The future worth of alternative 1 is computed as

$$FW_1(i) = P(F/P, i, n) + A(F/A, i, n)$$

P(F/P, i, n) is the future worth of a single payment and *A(F/A, i, n)* is the future worth equal payment series.

$$\begin{aligned} FW_1(12\%) &= -2{,}000{,}000(F/P, 12\%, 20) + 720{,}000(F/A, 12\%, 20) \\ &= -2{,}000{,}000(9.646) + 720{,}000(72.052) = \#32{,}585{,}440 \end{aligned}$$

Alternative 2—**Build soft ice-cream stand**

First cost = #3,600,000
Net annual income = Annual income − Annual property tax = #980,000 − #150,000 = #830,000
Life = 20 years
Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is shown in Fig. 2.29

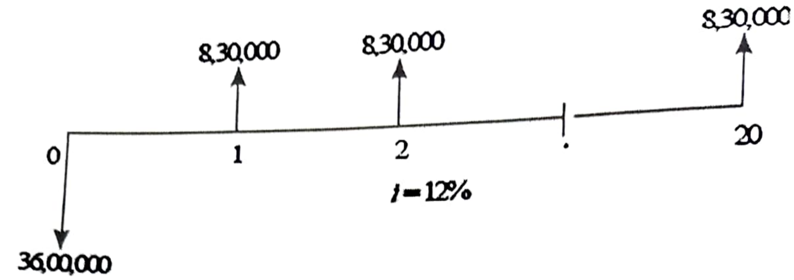


Fig 2.29: Cash flow diagram for alternative 2.

The future worth of alternative 2 is calculated as

$$FW_1(i) = P(F/P, i, n) + A(F/A, i, n)$$

Where *P(F/P, i, n)* is the future worth of a single payment and *A(F/A, i, n)* is the future worth equal payment series.

$$FW_1(12\%) = -3{,}600{,}000(F/P, 12\%, 20) + 830{,}000(F/A, 12\%, 20)$$
$$= -3{,}600{,}000(9.646) + 830{,}000(72.052) = \#25{,}077{,}560$$

The future worth of alternative 1 is greater than that of alternative 2. Thus, building the gas station is the best alternative.

Example 16: The cash flow diagram of two mutually exclusive alternatives are given in Figs. 2.30 and 2.31

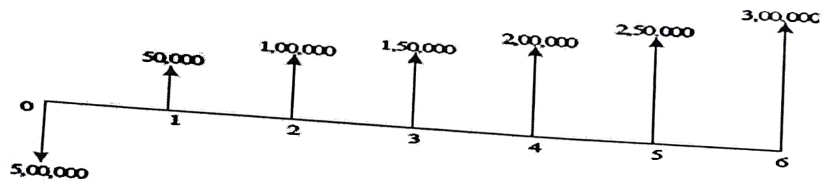


Fig 2.30: Cash flow diagram for alternative 1

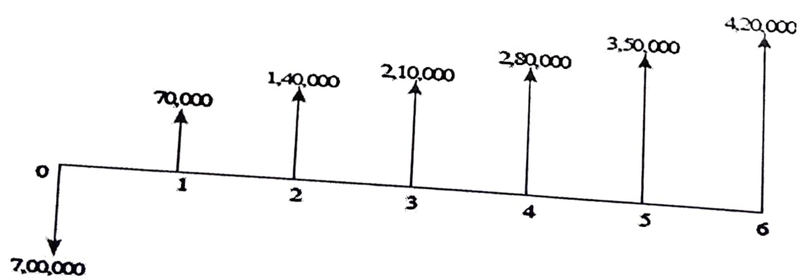


Fig 2.31: Cash flow diagram for alternative 2.

(a) Select the best alternative based on future worth method at $i = 8\%$.
(b) Rework part (a) with $i = 9\%$ and 20%

(a) Evaluation at $i = 8\%$

Alternative 1—**This comes under equal payment gradient series.**

$P = \#500{,}000$
$A1 = \#50{,}000$
$G = \#50{,}000$
$i = 8\%$
$n = 6$ years

The formula for the future worth of alternative 1 is

$$FW(i) = P(F/P, i, n) + [A1 + G(A/G, i, n)] \times (F/A, i, n)$$

Where P(F/P, i, n) is the single payment future worth sum, $A1 + G(A/G, i, n)$ is the uniform gradient series annual equivalent amount and $(F/A, i, n)$ is the future sum of annual equal payments.

$$\begin{aligned} FW_1(8\%) &= -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6) \\ &= -500{,}000(1.587) + [50{,}000 + 50{,}000(2.2764)] \times 7.336 \\ &= \#408{,}283.52 \end{aligned}$$

Alternative 2—**This comes under equal payment gradient series.**

$P = \#700{,}000$
$A1 = \#70{,}000$
$G = \#70{,}000$
$i = 8\%$
$n = 6$ years

The formula for the future worth of alternative 2 is

$$FW(i) = P(F/P, i, n) + [A1 + G(A/G, i, n)] \times (F/A, i, n)$$

Where P(F/P, i, n) is the single payment future worth sum, $A1 + G(A/G, i, n)$ is the uniform gradient series annual equivalent amount and $(F/A, i, n)$ is the future sum of annual equal payments.

$$\begin{aligned} FW_2(8\%) &= -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6) \\ &= -700{,}000(1.587) + [70{,}000 + 70{,}000(2.2764)] \times 7.336 \\ &= \#571{,}596.93 \end{aligned}$$

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(b) (i) Evaluation at i = 9%: Alternative 1

$P = \#500{,}000$
$A1 = \#50{,}000$
$G = \#50{,}000$
$n = 6$ years

The formula for the future worth of alternative 1 is as follows:

$$FW(i) = P(F/P, i, n) + [A1 + G(A/G, i, n)] \times (F/A, i, n)$$

Where P(F/P, i, n) is the single payment future worth sum, $A1 + G(A/G, i, n)$ is the uniform gradient series annual equivalent amount and $(F/A, i, n)$ is the future sum of annual equal payments.

$$\begin{aligned} FW_1(9\%) &= -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \times (F/A, 9\%, 6) \\ &= -500{,}000(1.677) + [50{,}000 + 50{,}000(2.2498)] \times 7.523 \\ &= \#383{,}912.27 \end{aligned}$$

Alternative 2

$P = \#700{,}000$
$A1 = \#70{,}000$
$G = \#70{,}000$
$n = 6$ years

The formula for the future worth of the alternative 2 is

$$FW(i) = P(F/P, i, n) + [A1 + G(A/G, i, n)] \times (F/A, i, n)$$

Where P(F/P, i, n) is the single payment future worth sum, $A1 + G(A/G, i, n)$ is the uniform gradient series annual equivalent amount and $(F/A, i, n)$ is the future sum of annual equal payments.

$$\begin{aligned} FW_2(9\%) &= -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \times (F/A, 9\%, 6) \\ &= -700{,}000(1.677) + [70{,}000 + 70{,}000(2.2498)] \times 7.523 \\ &= \#537{,}477.18 \end{aligned}$$

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(ii) Evaluation at i = 20%: Alternative 1

$P = \#500{,}000$
$A1 = \#50{,}000$
$G = \#50{,}000$
$n = 6$ years

The formula for the future worth of alternative 1 is

$$FW(i) = P(F/P, i, n) + [A1 + G(A/G, i, n)] \times (F/A, i, n)$$

Where P(F/P, i, n) is the single payment future worth sum, $A1 + G(A/G, i, n)$ is the uniform gradient series annual equivalent amount and $(F/A, i, n)$ is the future sum of annual equal payments.

$$FW_1(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6)$$
$$= -500{,}000(2.986) + [50{,}000 + 50{,}000(1.9788)] \times 9.93$$
$$= \# - 14{,}025.80$$

The negative sign of the future worth amount indicates that alternative 1 incurs loss.

Alternative 2

P = #700,000
$A1$ = #70,000
G = #70,000
n = 6 years

The formula for the future worth of alternative 2 is

$$FW(i) = P(F/P, i, n) + [A1 + G(A/G, i, n)] \times (F/A, i, n)$$

Where P(F/P, i, n) is the single payment future worth sum, $A1 + G(A/G, i, n)$ is the uniform gradient series annual equivalent amount and $(F/A, i, n)$ is the future sum of annual equal payments.

$$FW_2(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6)$$
$$= -700{,}000(2.986) + [70{,}000 + 70{,}000(1.9788)] \times 9.93 = \#19{,}636.12$$

The negative sign of the above future worth amount indicates that alternative 2 incurs loss. Thus, none of the two alternatives should be selected.

Example 2.17 M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

	Manufacturer		
	1	2	3
Initial cost (#)	8,000,000	7,000,000	9,000,000
Life (years)	12	12	12
Annual operation and maintenance cost(#)	800,000	900,000	850,000
Salvage value afetr 12 years	500,000	400,000	700,000

Which is the best alternative based on future worth method at i = 20%?

Solution Alternative 1—Manufacturer 1

First cost, P = #8,000,000
Life, n = 12 years
Annual operating and maintenance cost, A = #800,000
Salvage value at the end of furnace life, S = #500,000

The cash flow diagram for this alternative is shown in Fig. 2.32.

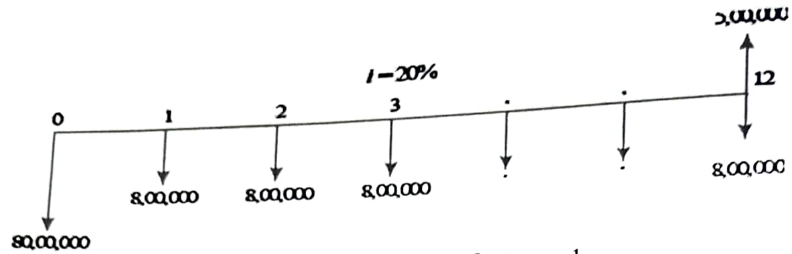


Fig. 2.32: Cash flow diagram for manufacturer 1

The future worth amount of alternative 1 is computed as

$$FW(i) = P(F/P, i, n) + P(F/A, i, n) - 500{,}000$$

Where $P(F/P, i, n)$ is the single payment future worth sum, $P(F/A, i, n)$ is future sum of annual equal payments and S is salvage value.

$$FW_1(20\%) = 8{,}000{,}000(F/P, 20\%, 12) + 800{,}000(F/A, 20\%, 12) - 500{,}000$$
$$= 8{,}000{,}000(8.916) + 800{,}000(39.581) - 500{,}000 = \#10{,}2492{,}800$$

Alternative 2— Manufacturer 2

First cost, P = #7,000,000
Life, n = 12 years
Annual operating and maintenance cost, A = #900,000
Salvage value at the end of furnace life = #400,000

The cash flow diagram for this alternative is given in Fig. 2.33

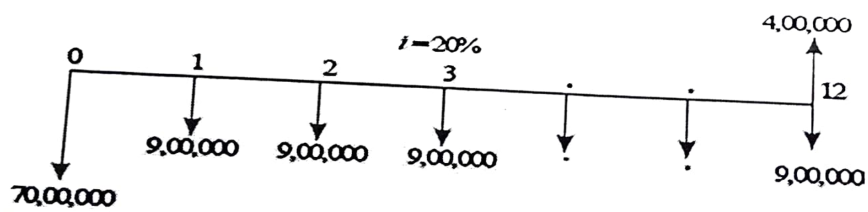


Fig. 2.33: Cash flow diagram for manufacturer 2

The future worth amount of alternative 2 is computed as

$$FW(i) = P(F/P, i, n) + P(F/A, i, n) - 500{,}000$$

Where $P(F/P, i, n)$ is the single payment future worth sum, $P(F/A, i, n)$ is future sum of annual equal payments and S is salvage value.

$$FW_2(20\%) = 7{,}000{,}000(F/P, 20\%, 12) + 900{,}000(F/A, 20\%, 12) - 400{,}000$$
$$= 7{,}000{,}000(8.916) + 900{,}000(39.581) - 400{,}000 = \#97{,}634{,}900$$

Alternative 3—Manufacturer 3

First cost, P = #9,000,000
Life, n = 12 years

Annual operating and maintenance cost, A = #850,000
Salvage value at the end of furnace life = #700,000

The cash flow diagram for this alternative is illustrated in Fig. 2.34

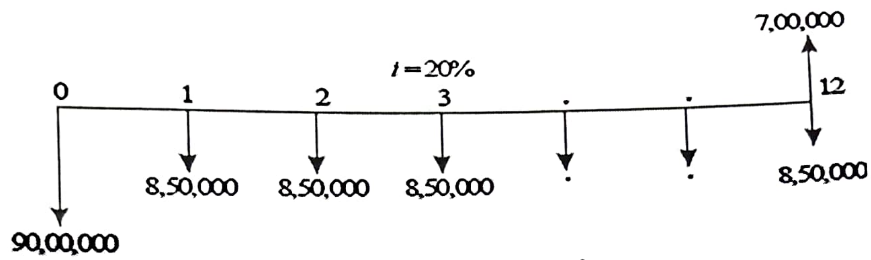


Fig. 2.34 Cash flow diagram for manufacturer 3

The future worth amount of alternative 3 is calculated as

$$FW(i) = P(F/P, i, n) + P(F/A, i, n) - 500{,}000$$

Where $P(F/P, i, n)$ is the single payment future worth sum, $P(F/A, i, n)$ is future sum of annual equal payments and S is salvage value.

$$FW_3(20\%) = 9{,}000{,}000(F/P, 20\%, 12) + 850{,}000(F/A, 20\%, 12) - 700{,}000$$
$$= 9{,}000{,}000(8.916) + 850{,}000(39.581) - 700{,}000 = \#113{,}187{,}850$$

The future worth cost of alternative 2 is less than that of the other two alternatives. Therefore, M/s. Krishna castings should buy the annealing furnace from manufacturer 2.

2.3.3 Annual Equivalent Method

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost based comparison will be selected as the best alternative.

Revenue-Dominated Cash Flow Diagram

A generalized revenue-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is presented in Fig. 2.35

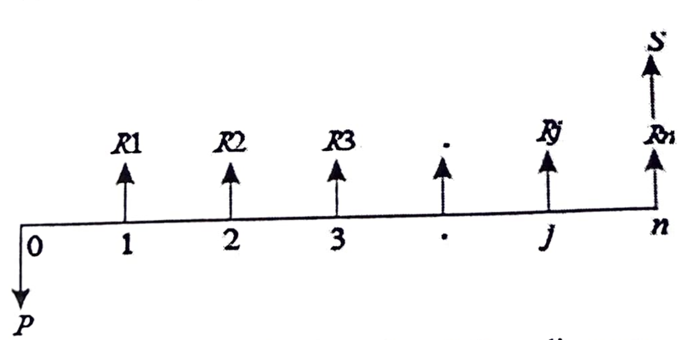


Fig. 2.35: Revenue-dominated cash flow diagram

In Fig. 2.37, *P* represents an initial investment, *Rj* the net revenue at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year.

The first step is to find the net present worth of the cash flow diagram using the following expression for a given interest rate, *i*:

$$PW(i) = -P + \frac{R1}{(1+i)^1} + \frac{R2}{(1+i)^2} + \cdots + \frac{Rj}{(1+i)^j} + \cdots + \frac{Rn}{(1+i)^n} + \frac{S}{(1+i)^n}$$

In the above formula, the expenditure is assigned with a negative sign and the revenues are assigned with a positive sign.

In the second step, the annual equivalent revenue is computed using the following formula:

$$A = PW(i)\frac{i(1+i)^n}{(1+i)^n - 1} = PW(i)(A/P, i, n)$$

where **(*A*/*P*, *i*, *n*)** is called *equal payment series capital recovery factor.*

If we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent revenues are to be computed and compared. Finally, the alternative with the maximum annual equivalent revenue should be selected as the best alternative.

Cost-Dominated Cash Flow Diagram

A generalized cost-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is illustrated in Fig. 2.36

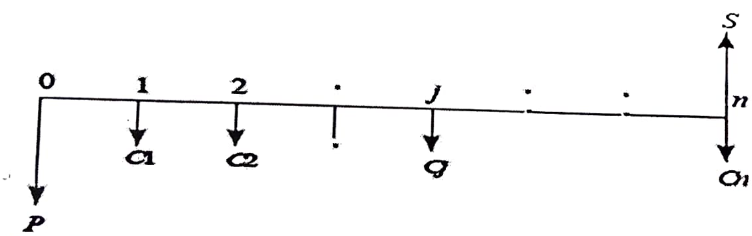


Fig. 2.36: Cost-dominated cash flow diagram

In Fig. 2.36, *P* represents an initial investment, *Cj* the net cost of operation and maintenance at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year.

The first step is to find the net present worth of the cash flow diagram using the following relation for a given interest rate, *i*.

$$PW(i) = P + \frac{C1}{(1+i)^1} + \frac{C2}{(1+i)^2} + \cdots + \frac{Cj}{(1+i)^j} + \cdots + \frac{Cn}{(1+i)^n} + \frac{S}{(1+i)^n}$$

In the above formula, each expenditure is assigned with positive sign and the salvage value with negative sign. Then, in the second step, the annual equivalent cost is computed using the following equation:

$$A = PW(i)\frac{i(1+i)^n}{(1+i)^n - 1} = PW(i)(A/P, i, n)$$

where **(A/P, i, n)** is called as equal-payment series capital recovery factor

As in the previous case, if we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent costs are to be computed and compared. Finally, **the alternative with the minimum annual equivalent cost should be selected as the best alternative.**

If we have some non-standard cash flow diagram, then we will have to follow the general procedure for converting each and every transaction to time zero and then convert the net present worth into an annual equivalent cost/ revenue depending on the type of the cash flow diagram. Such procedure is to be applied to all the alternatives and finally, the best alternative is to be selected.

Example 18: A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

Manufacturer	Down Payment (#)	Yearly equal installment (#)	No of installment
1	500,000	200,000	15
2	400,000	300,000	15
3	600,000	150,000	15

Determine the best alternative based on the annual equivalent method by assuming i = 20%, compounded annually.

Solution Alternative 1

Down payment, P = #500,000
Yearly equal installment, A = #200,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for manufacturer 1 is shown in Fig. 2.37

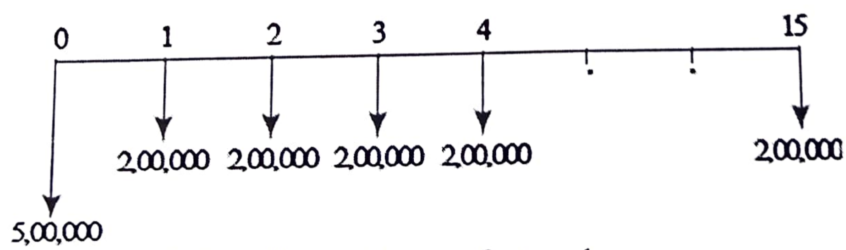


Fig. 2.37: Cash flow diagram for manufacturer 1

The annual equivalent cost expression of the above cash flow diagram is

$$AE(i) = P(A/P, i, n) + A$$

Where $P(A/P, i, n)$ is the annual equivalent of a present worth and A, annual equal installment

$$AE_1(20\%) = 500{,}000(A/P, 20\%, 15) + 200{,}000 = 500{,}000(0.2139) + 200{,}000$$
$$= \#306{,}950$$

Alternative 2

Down payment, P = #400,000
Yearly equal installment, A = #300,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for the manufacturer 2 is shown in Fig. 2.38

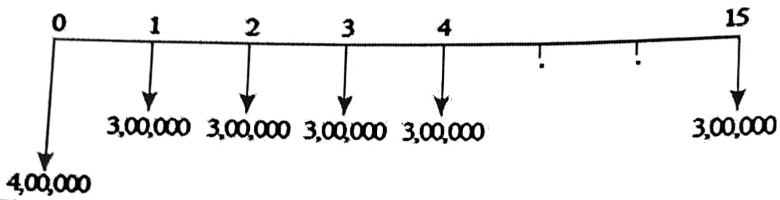


Fig. 2.38: Cash flow diagram for manufacturer 2

The annual equivalent cost expression of the above cash flow diagram is

$$AE(i) = P(A/P, i, n) + A$$

Where $P(A/P, i, n)$ is the annual equivalent of a present worth and A, annual equal installment

$$AE_2(20\%) = 400{,}000(A/P, 20\%, 15) + 300{,}000 = 400{,}000(0.2139) + 300{,}000$$
$$= \#385{,}560$$

Alternative 3

Down payment, P = #600,000
Yearly equal installment, A = #150,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for manufacturer 3 is shown in Fig. 2.39

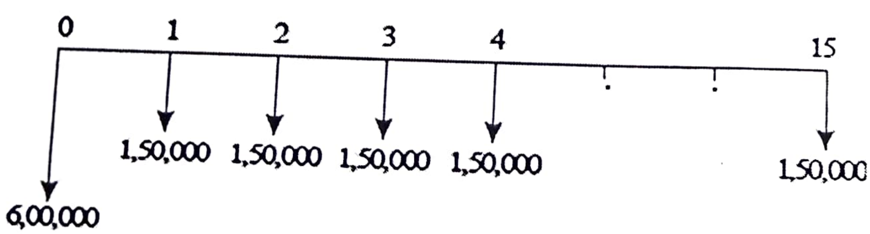


Fig. 2.39: Cash flow diagram for manufacturer 3

The annual equivalent cost expression of the above cash flow diagram is

$$AE(i) = P(A/P, i, n) + A$$

Where $P(A/P, i, n)$ is the annual equivalent of a present worth and A, annual equal installment

$$AE_3(20\%) = 600{,}000(A/P, 20\%, 15) + 150{,}000 = 600{,}000(0.2139) + 150{,}000$$
$$= \#278{,}340$$

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

Example 19: A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternatives	
	A	B
Investments (#)	-150,000	175,000
Annual equal return (#)	+60,000	+70,000
Salvage value (#)	+15,000	+35,000

Determine the best alternative based on the annual revenue method by assuming $i = 25\%$.

Solution Alternative A

Initial investment, $P = \#150{,}000$
Annual equal return, $A = \#60{,}000$
Salvage value at the end of machine life, $S = \#15{,}000$
Life = 5 years
Interest rate, $i = 25\%$, compounded annually

The cash flow diagram for alternative A is shown in Fig. 2.40

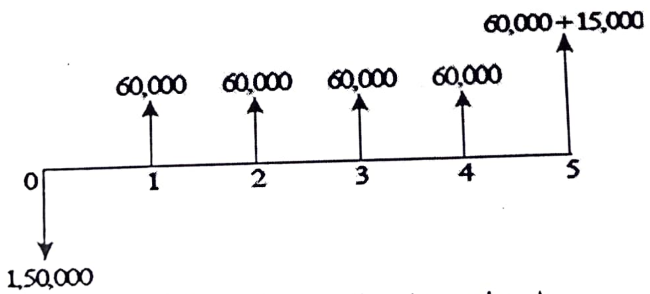


Fig. 2.40: Cash flow diagram for alternative A

The annual equivalent revenue expression of the above cash flow diagram is as follows:

$$AE(i) = P(A/P, i, n) + A + S(A/F, i, n)$$

Where $P(A/P,i,n)$ is the annual equivalent of the present worth, A, is annual equal return and $S(A/F,i,n)$ is the annual equivalent of a future sum

$$AE_A(25\%) = -150{,}000(A/P,25\%,5) + 60{,}000 + 15{,}000(A/F,25\%,5)$$
$$= -150{,}000(0.3718) + 60{,}000 + 15{,}000(0.1218) = \#6{,}057$$

Alternative B

Initial investment, $P = \#175{,}000$
Annual equal return, $A = \#70{,}000$
Salvage value at the end of machine life, $S = \#35{,}000$
Life = 5 years
Interest rate, $i = 25\%$, compounded annually

The cash flow diagram for alternative B is shown in Fig. 2.41

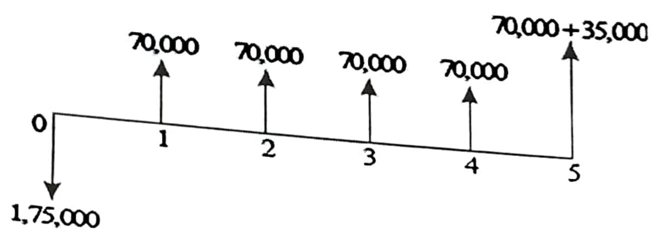


Fig: 2.41: Cash flow diagram for alternative B

The annual equivalent revenue expression of the above cash flow diagram is

$$AE(i) = P(A/P,i,n) + A + S(A/F,i,n)$$

Where $P(A/P,i,n)$ is the annual equivalent of the present worth, A, is annual equal return and $S(A/F,i,n)$ is the annual equivalent of a future sum

$$AE_A(25\%) = -175{,}000(A/P,25\%,5) + 70{,}000 + 35{,}000(A/F,25\%,5)$$
$$= -175{,}000(0.3718) + 70{,}000 + 35{,}000(0.1218) = \#9{,}198$$

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

Example 20: A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time. The minimum attractive rate of return for the firm is 15%. The following data are to be used in the analysis:

	Machine X	Machine Y
First cost	#150,000	#240,000
Estimated life	12 years	12 years
Salvage value	#0	#6,000
Annual maintenance cost	#0	#4,500

Which machine would you choose? Base your answer on annual equivalent cost.

Solution Machine X

First cost, P = #1,50,000
Life, n = 12 years
Estimated salvage value at the end of machine life, S = #0.
Annual maintenance cost, A = #0.
Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine X is illustrated in Fig. 2.42

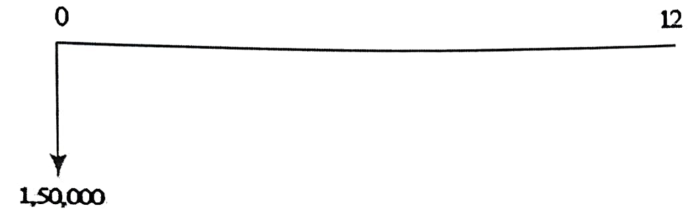


Fig. 2.42 Cash flow diagram for machine X.

The annual equivalent cost expression of the above cash flow diagram is

$$AE = P(A/P, i, n)$$

$$AE_X = 150{,}000(A/P, 15\%, 12) = 150{,}000(0.1845) = \#27{,}675$$

Machine Y

First cost, P = #240,000
Life, n = 12 years
Estimated salvage value at the end of machine life, S = #60,000
Annual maintenance cost, A = #4,500
Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine Y is depicted in Fig. 2.43

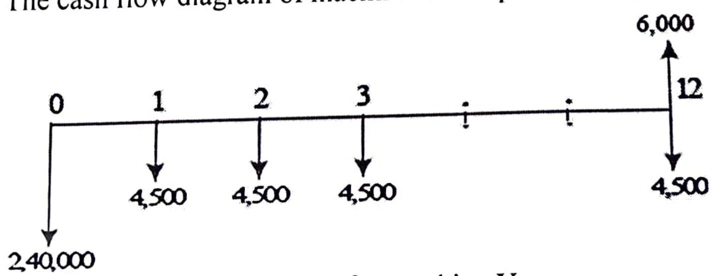


Fig 2.43: Cash flow diagram for machine Y

The annual equivalent cost expression of the above cash flow diagram is

$$AE(i) = P(A/P, i, n) + A + S(A/F, i, n)$$

Where $P(A/P, i, n)$ is the annual equivalent of the present worth, A, is annual equal return and $S(A/F, i, n)$ is the annual equivalent of a future sum

$$AE_Y(15\%) = 240{,}000(A/P, 15\%, 12) + 4{,}500 - 6{,}000(A/F, 15\%, 12)$$
$$= 240{,}000(0.1845) + 4{,}500 - 6{,}000(0.0345) = \#48{,}573$$

The annual equivalent cost of machine X is less than that of machine Y. So, machine X is the more cost effective machine.

Example 21: A company must decide whether to buy machine A or machine B.

	Machine A	Machine B
Initial cost (#)	300,000	600,000
Useful life (years)	4	4
Salvage value at the end of machine life (#)	200,000	300,000
Annual maintenance (#)	30,000	0

At 15% interest rate, which machine should be purchased?

Solution Machine A

Initial cost = #300,000
Useful life (years) = 4
Salvage value at the end of machine life = #200,000
Annual maintenance = #30,000
Interest rate = 15%, compounded annually

The cash flow diagram of machine A is depicted in Fig. 2.44

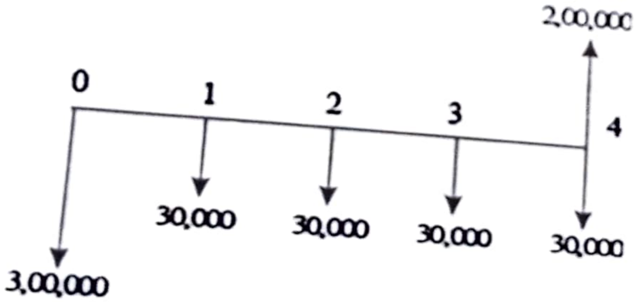


Fig. 2.44: Cash flow diagram for machine A

The annual equivalent cost expression of the above cash flow diagram is

$$AE(i) = P(A/P, i, n) + A + S(A/F, i, n)$$

Where $P(A/P, i, n)$ is the annual equivalent of the present worth, A, is annual equal return and $S(A/F, i, n)$ is the annual equivalent of a future sum

$$AE(15\%) = 300{,}000(A/P, 15\%, 4) + 30{,}000 - 200{,}000(A/F, 15\%, 4)$$
$$= 300{,}000(0.3503) + 30{,}000 - 200{,}000(0.2003) = \#95{,}030$$

Machine B

Initial cost = #600,000

Useful life (years) = 4
Salvage value at the end of machine life = #300,000
Annual maintenance = #0.
Interest rate = 15%, compounded annually

The cash flow diagram of machine B is illustrated in Fig. 2.45

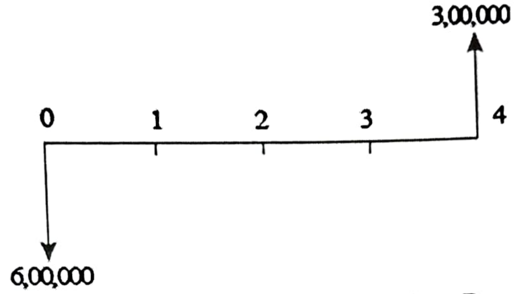


Fig 2.45: Cash flow diagram for machine B

The annual equivalent cost expression of the above cash flow diagram is

$$AE(i) = P(A/P, i, n) + A + S(A/F, i, n)$$

Where $P(A/P, i, n)$ is the annual equivalent of the present worth, A, is annual equal return and $S(A/F, i, n)$ is the annual equivalent of a future sum

$$\begin{aligned} AE(15\%) &= 600{,}000(A/P, 15\%, 4) - 300{,}000(A/F, 15\%. 4) \\ &= 600{,}000(0.3503) - 300{,}000(0.2003) = \#150{,}090 \end{aligned}$$

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

2.2.3 Rate of Return Method

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative. In this type of analysis, **the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign.**

A generalized cash flow diagram to demonstrate the rate of return method of comparison is presented in Fig. 2.46

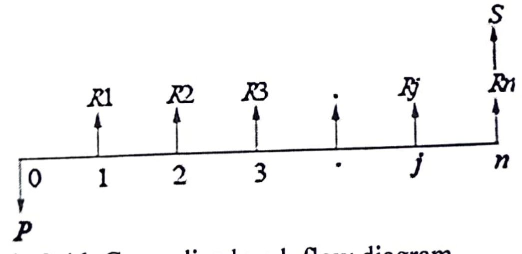


Fig 2.46: Generalized cash flow diagram.

In the above cash flow diagram, P represents an initial investment, Rj the net revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram at a given interest rate, i.

$$PW(i) = -P + \frac{R1}{(1+i)^1} + \frac{R2}{(1+i)^2} + \dots + \frac{Rj}{(1+i)^j} + \dots + \frac{Rn}{(1+i)^n} + \frac{S}{(1+i)^n}$$

Now, the above function is to be evaluated for different values of i until the present worth function reduces to zero, as shown in Fig. 2.47

In the figure, the present worth goes on decreasing when the interest rate is increased. The value of i at which the present worth curve cuts the X-axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of i at which the present worth function reduces to zero.

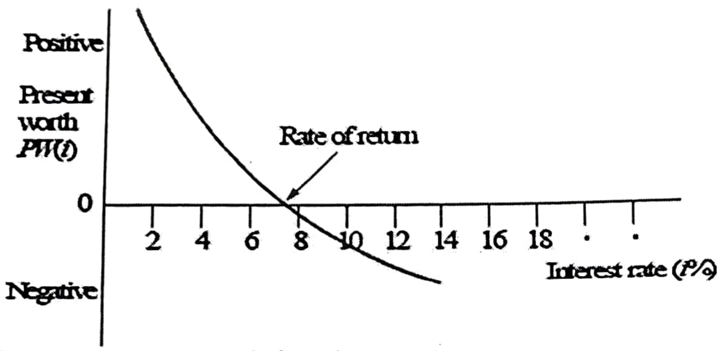


Fig 2.47: Present worth function graph.

So, one has to start with an intuitive value of i and check whether the present worth function is positive. If so, increase the value of i until $PW(i)$ becomes negative. Then, the rate of return is determined by interpolation method in the range of values of i for which the sign of the present worth function changes from positive to negative.

Example 22: A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash flow (#)	-100,000	30,000	30,000	30,000	30,000	30,000

Solution

Initial investment = #100,000
Annual equal revenue = #30,000
Life = 5 years

The cash flow diagram for this situation is illustrated in Fig. 2.48

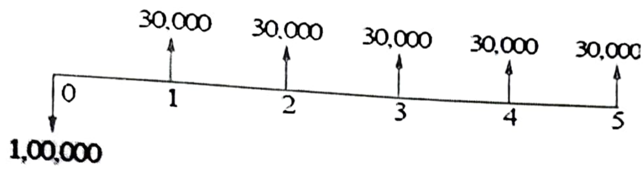


Fig 2.48: Cash flow diagram

The present worth function for the business is

$$PW(i) = P + A(P/A, i, n)$$

Where $A(P/A, i, n)$ is equal payment series present worth factor

When i = 10%

$$PW(10\%) = -100{,}000 + 30{,}000(P/A, 10\%, 5) = -100{,}000 + 30{,}000(3.7908)$$
$$= \#13{,}724$$

When i = 15%

$$PW(15\%) = -100{,}000 + 30{,}000(P/A, 15\%, 5) = -100{,}000 + 30{,}000(3.3522) = \#566$$

When i = 18%

$$PW(18\%) = -100{,}000 + 30{,}000(P/A, 18\%, 5) = -100{,}000 + 30{,}000(3.1272)$$
$$= -\#6{,}184$$

$$i = 15\% + \frac{566 - 0}{566 - (-6184)} \times 3\% = 15\% + 0.252\% = 15.252\%$$

Therefore, the rate of return for the new business is 15.252%.

Example 23: A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is #2,000,000. The annual net profit is #350,000. Find the rate of return for the new business.

Solution

Life of the product line (n) = 10 years
Initial outlay = #2,000,000
Annual net profit = #350,000
Scrap value after 10 years = 0

The cash flow diagram for this situation is shown in Fig. 2.49

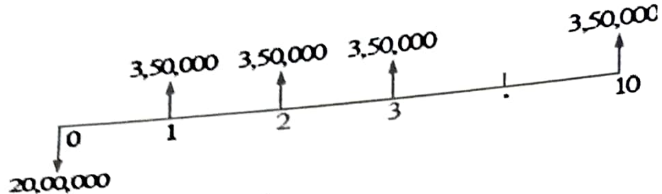


Fig. 2.49: Cash flow diagram

The formula for the net present worth function of the situation is

$$PW(i) = P + A(P/A, i, n)$$

Where $A(P/A, i, n)$ is equal payment series present worth factor

$$PW(i) = -2{,}000{,}000 + 350{,}000(P/A, i\%, 10)$$

When i = 10%

$$PW(10\%) = -2{,}000{,}000 + 350{,}000(P/A, 10\%, 10) = -2{,}000{,}000 + 350{,}000(6.1446) \\ = \#150{,}610$$

When i = 12%

$$PW(12\%) = -2{,}000{,}000 + 350{,}000(P/A, 12\%, 10) = -2{,}000{,}000 + 350{,}000(5.6502) \\ = \# - 22{,}430$$

$$i = 10\% + \frac{150{,}610 - 0}{150{,}610 - (-22{,}430)} \times 2\% = 11.74\%$$

Therefore, the rate of return of the new product line is 11.74%

Example 24: A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	Alternatives		
	A1	A2	A3
Investment	#150,000	#210,000	#255,000
Annual net income	#45,570	#58,260	#69,000

Find the best alternative based on the rate of return method of comparison.

Solution Calculation of rate of return for alternative $A1$

Initial outlay = #150,000
Annual profit = #45,570
Life = 5 years
The cash flow diagram for alternative $A1$ is shown in Fig. 2.50

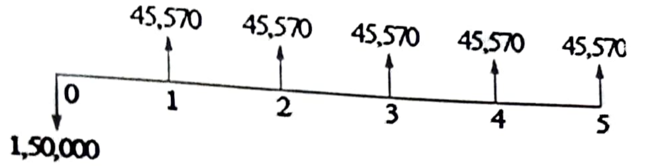


Fig 2.50: Cash flow diagram for alternative $A1$

The formula for the net present worth of alternative $A1$ is given as

$$PW(i) = P + A(P/A, i, n)$$

Where $A(P/A, i, n)$ is equal payment series present worth factor

$$PW(i) = -150{,}000 + 45{,}570(P/A, i\%, 5)$$

When i = 10%

$$PW(10\%) = -150{,}000 + 45{,}570(P/A, 10\%, 5) = -150{,}000 + 45{,}570(3.7908)$$
$$= \#22{,}746.76$$

When i = 12%

$$PW(12\%) = -150{,}000 + 45{,}570(P/A, 12\%, 5) = -150{,}000 + 45{,}570(3.6048)$$
$$= \#14{,}270.74$$

When i = 15%

$$PW(15\%) = -150{,}000 + 45{,}570(P/A, 15\%, 5) = -150{,}000 + 45{,}570(3.3522)$$
$$= \#2{,}759.75$$

When i = 18%

$$PW(18\%) = -150{,}000 + 45{,}570(P/A, 18\%, 5) = -150{,}000 + 45{,}570(3.1272)$$
$$= -\#7{,}493.50$$

Therefore, the rate of return of the alternative $A1$ is

$$i = 15\% + \frac{2{,}759.75 - 0}{2{,}759.75 - (-7{,}493.50)} \times 3\% = 15\% + 0.81\% = 15.81\%$$

Calculation of rate of return for alternative A2

Initial outlay = #210,000
Annual profit = #58,260
Life of alternative $A2$ = 5 years
The cash flow diagram for alternative $A2$ is shown in Fig. 2.51

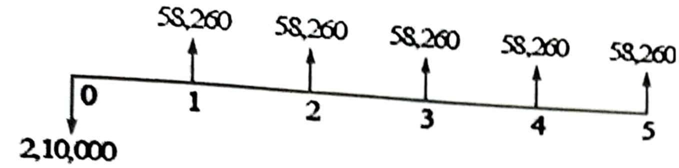


Fig. 2.51: Cash flow diagram for alternative $A2$

The formula for the net present worth of this alternative is

$$PW(i) = P + A(P/A, i, n)$$

Where $A(P/A, i, n)$ is equal payment series present worth factor

$$PW(i) = -210{,}000 + 58{,}260(P/A, i\%, 5)$$

When i = 12%

$$PW(12\%) = -210{,}000 + 58{,}260(P/A, 12\%, 5) = -210{,}000 + 58{,}260(3.6048)$$
$$= \#15.65$$

When i = 13%

$$PW(13\%) = -210{,}000 + 58{,}260(P/A, 13\%, 5) = -210{,}000 + 58{,}260(3.5172)$$
$$= \# - 5{,}087.93$$

Therefore, the rate of return for alternative $A2$ is

$$i = 12\% + \frac{15.65 - 0}{15.65 - (-5{,}087.93)} \times 1\% = 12\% + 0\% = 12\%$$

Calculation of rate of return for alternative A3

Initial outlay = #255,000
Annual profit = #69,000
Life of alternative $A3$ = 5 years

The cash flow diagram for alternative $A3$ is depicted in Fig. 2.52

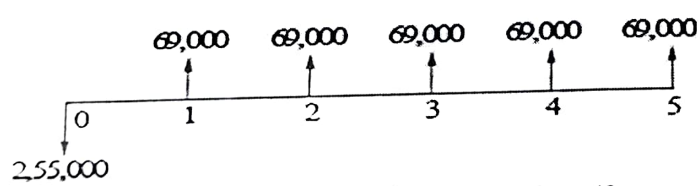


Fig. 2.52: Cash flow diagram for alternative $A3$

The formula for the net present worth of this alternative $A3$ is

$$PW(i) = P + A(P/A, i, n)$$

Where $A(P/A, i, n)$ is equal payment series present worth factor

$$PW(i) = -255{,}000 + 69{,}000(P/A, i\%, 5)$$

When i = 11%

$$PW(11\%) = -255{,}000 + 69{,}000(P/A, 11\%, 5) = -255{,}000 + 69{,}000(3.6959) = \#17.1$$

When i = 12%

$$PW(12\%) = -255{,}000 + 69{,}000(P/A, 12\%, 5) = -255{,}000 + 69{,}000(3.6048)$$
$$= \# - 6{,}268.80$$

Therefore, the rate of return for alternative $A3$ is

$$i = 11\% + \frac{17.1 - 0}{17.1 - (-6{,}268.80)} \times 1\% = 11\%$$

The rate of return for alternative $A3$ is less than the minimum attractive rate of return of 12%. So, it should not be considered for comparison. The remaining two alternatives are qualified for consideration. Among the alternatives $A1$ and $A2$, the rate of return of alternative $A1$ is greater than that of alternative $A2$. Hence, alternative $A1$ should be selected.

Example 24: A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	Initial investment (#)	Yearly revenue (#)
Alternative 1	500,000	170,000
Alternative 2	800,000	270,000

Solution Alternative 1

Initial outlay = #500,000
Annual revenue = #170,000
Life of alternative 1 = 5 years

The cash flow diagram for alternative 1 is illustrated in Fig. 2.53

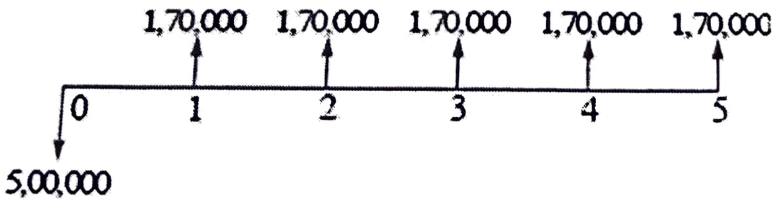


Fig 2.53: Cash flow diagram for alternative 1.

The formulae for the net present worth of alternative 1 are as follows:

$$PW(i) = P + A(P/A, i, n)$$

Where $A(P/A,i,n)$ is equal payment series present worth factor

$$PW_1(i) = -500{,}000 + 170{,}000(P/A, i\%, 5)$$

$$PW_1(15\%) = -500{,}000 + 170{,}000(P/A, 15\%, 5) = -500{,}000 + 170{,}000(3.3522)$$
$$= \#69{,}874$$
$$PW_1(17\%) = -500{,}000 + 170{,}000(P/A, 17\%, 5) = -500{,}000 + 170{,}000(3.1993)$$
$$= \#43{,}881$$
$$PW_1(20\%) = -500{,}000 + 170{,}000(P/A, 20\%, 5) = -500{,}000 + 170{,}000(2.9906)$$
$$= \#8{,}402$$
$$PW_1(22\%) = -500{,}000 + 170{,}000(P/A, 22\%, 5) = -500{,}000 + 170{,}000(2.8636)$$
$$= \# - 13{,}188$$

Therefore, the rate of return for alternative *1* is

$$i = 20\% + \frac{8{,}402 - 0}{8{,}402 - (-13.188)} \times 2\% = 20.78\%$$

Alternative 2

Initial outlay = #800,000
Annual revenue = #270,000
Life = 5 years

The cash flow diagram for alternative 2 is depicted in Fig. 2.54

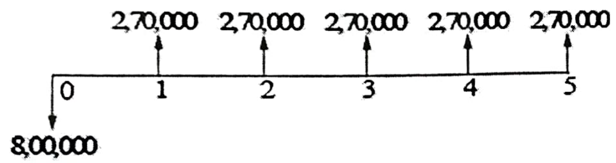


Fig 2.54: Cash flow diagram for alternative 2.

The formula for the net present worth of alternative 2 is:

$$PW(i) = P + A(P/A,i,n)$$

Where $A(P/A,i,n)$ is equal payment series present worth factor

$$PW_2(i) = -800{,}000 + 270{,}000(P/A, i\%, 5)$$
$$PW_2(20\%) = -800{,}000 + 270{,}000(P/A, 20\%, 5) = -800{,}000 + 270{,}000(2.9906)$$
$$= \#7{,}462$$
$$PW_2(22\%) = -800{,}000 + 270{,}000(P/A, 22\%, 5) = -800{,}000 + 270{,}000(2.8636)$$
$$= \# - 26{,}828$$

Therefore, the rate of return for alternative 2 is

$$i = 20\% + \frac{7{,}462 - 0}{7{,}462 - (-26{,}828)} \times 2\% = 20.435\%$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

Production planning identifies four categories of operations:

i. Plan
ii. Source
iii. Make
iv. Deliver

Plan: This refers to all the operations needed to plan and organize the operations in the other three categories. We will investigate three operations in this category in some detail: demand forecasting, product pricing, and inventory management.

Source: Operations in this category include the activities necessary to acquire the inputs to create products or services. We look at two operations here. The first, procurement, is the acquisition of materials and services. The second operation, credit and collections, is not traditionally seen as a sourcing activity but it can be thought of as, literally, the acquisition of cash. Both these operations have a big impact on the efficiency of production.

Make: This category includes the operations required to develop and build the products and services that a supply chain provides. Operations that we discuss in this category are product design, production management, and facility and management.

Deliver: These operations encompass the activities that are part of receiving customer orders and delivering products to customers. The three operations we review are management, product delivery, and return processing. These are the operations that constitute the core connections between companies in a supply chain.

3.1 Demand Forecasting and Planning (Plan)

Production management decisions are based on forecasts that define which products will be required, what amount of these products will be called for, and when they will be needed. The demand forecast becomes the basis for companies to plan their internal operations and to cooperate among each other to meet market demand.

All forecasts deal with four major variables that combine to determine what market conditions will be like. Those variables are:

i. Supply
ii. Demand
iii. Product Characteristics
iv. Competitive Environment

Supply is determined by the **number of producers** of a product and by **the lead times** that are associated with a product. The more producers there are of a product and the shorter the lead times, the more predictable this variable is. When there are only a few suppliers or when lead times are longer, then there is more potential uncertainty in a market. Like variability in demand, uncertainty in supply makes forecasting more difficult. Also, longer lead times associated with a product require a longer time horizon over which forecasts must be made. Demand forecasts must cover a time period that encompasses the combined lead times of all the components that go into the creation of a final product.

Demand refers to the overall market demand for a group of related products or services. Is the market growing or declining? If so, what is the yearly or quarterly rate of growth or decline? Or maybe the market is relatively mature and demand is steady at a level that has been predictable for some period of years. Also, many products have a seasonal demand pattern. For example, snow skis and heating oil are more in demand in the winter, and tennis rackets and sun screen are more in demand in the summer. Perhaps the market is a developing market—the products or services are new and there is not much historical data on demand or the demand varies widely because new customers are just being introduced to the products. Markets where there is little historical data and lots of variability are the most difficult when it comes to demand forecasting.

Product characteristics include the features of a product that influence customer demand for the product. Is the product new and developing quickly like many electronic products or is the product mature and changing slowly or not at all, as is the case with many commodity products? Forecasts for mature products can cover longer timeframes than forecasts for products that are developing quickly. It is also important to know whether a product will steal demand away from another product. Can it be substituted for another product? Or will the use of one product drive the complementary use of a related product? Products that either compete with or complement each other should be forecasted together.

Competitive environment refers to the actions of a company and its competitors. What is the market share of a company? Regardless of whether the total size of a market is growing or shrinking, what is the trend in an individual company's market share? Is it growing or declining? What is the market share trend of competitors? Market share trends can be influenced by product promotions and price wars, so forecasts should take into account such events that are planned for the upcoming period. Forecasts should also account for anticipated promotions and price wars that will be initiated by competitors.

3.1.1 Forecasting Methods

There are four basic methods to use when doing forecasts. Most forecasts are done using various combinations of these four methods. Chopra and Meindl define these methods as:

i. Qualitative
ii. Causal
iii. Time Series
iv. Simulation

Qualitative methods rely upon a person's intuition or subjective opinions about a market. These methods are most appropriate when there is little historical data to work with. When a new line of products is introduced, people can make forecasts based on comparisons with other products or situations that they consider similar. People can forecast using production adoption curves that they feel reflect what will happen in the market.

Causal methods of forecasting assume that demand is strongly related to particular environmental or market factors. For instance, demand for commercial loans is often closely correlated to interest rates. So if interest rate cuts are expected in the next period of time, then loan forecasts can be derived using a causal relationship with interest rates. Another strong causal relationship exists between price and demand. If prices are lowered, demand can be expected to increase and if prices are raised, demand can be expected to fall.

Time series methods are the most common form of forecasting. They are based on the assumption that historical patterns of demand are a good indicator of future demand. These methods are best when there is a reliable body of historical data and the markets being forecast are stable and have demand patterns that do not vary much from one year to the next. Mathematical techniques such as moving averages and exponential smoothing are used to create forecasts based on time series data. These techniques are employed by most forecasting software packages.

Simulation methods use combinations of causal and time series methods to imitate the behavior of consumers under different circumstances. This method can be used to answer questions such as what will happen to revenue if prices on a line of products are lowered or what will happen to market share if a competitor introduces a competing product or opens a store nearby

Few companies use only one of these methods to produce forecasts. Most companies do several forecasts using several methods and then combine the results of these different forecasts into the actual forecast that they use to plan their businesses. Studies have shown that this process of creating forecasts using different methods and then combining the results into a final forecast usually produces better accuracy than the output of any one method alone.

Regardless of the forecasting methods used, when doing forecasts and evaluating their results it is important to keep several things in mind. First of all, short-term forecasts are inherently more accurate than long-term forecasts. The effect of business trends and conditions can be much more accurately calculated over short periods than over longer periods. When Wal-Mart began restocking its stores twice a week instead of twice a month, the store managers were able to significantly increase the accuracy of their forecasts because the time periods involved dropped from two or three weeks to three or four days. Most long range, multiyear forecasts are highly speculative.

Aggregate forecasts are more accurate than forecasts for individual products or for small market segments. For example, annual forecasts for soft drink sales in a given metropolitan area are fairly accurate but when these forecasts are broken down to sales by districts within the metropolitan area, they become less accurate. Aggregate forecasts are made using a broad base of data that provides good forecasting accuracy. As a rule, the more narrowly focused or specific a forecast is, the less data is available and the more variability there is in the data, so the accuracy is diminished.

Finally, forecasts are always wrong to a greater or lesser degree. There are no perfect forecasts and businesses need to assign some expected degree of error to every forecast. An accurate forecast may have a degree of error that is plus or minus 5 percent. A more speculative forecast may have a plus or minus 20 percent degree of error. It is important to know the degree of error because a business must have contingency plans to cover those outcomes. What would a company do if raw material prices were 5 percent higher than expected? What would it do if demand was 20 percent higher than expected?

3.1.2 Time Series Methods

A *time series* is a set of observations of a variable at regular intervals over time. In *decomposition analysis*, the components of a time series are generally classified as trend T, cyclical C, seasonal S, and random or irregular R. *(Note:* Autocorrelation effects are sometimes included as an additional factor).

The forecast value (Ye) is commonly expressed as a multiplicative or additive function of its components; examples here will be based upon the commonly used multiplicative model.

$Y_c = T.\ S.\ C.\ R$ *multiplicative model* (3.1)

$Y_c = T + S + C + R$ *additive model* (3.2)

where T is Trend, S is Seasonal, C is Cyclical, and R is Random components of a series.

Trend is a gradual long-term directional movement in the data (growth or decline).

Seasonal effects are similar variations occurring during corresponding periods, e.g., December retail sales. Seasonal can be quarterly, monthly, weekly, daily, or even hourly indexes.

Cyclical factors are the long-term swings about the trend line. They are often associated with business cycles and may extend out to several years in length.

Random component are sporadic (unpredictable) effects due to chance and unusual occurrences. They are the residual after the trend, cyclical, and seasonal variations are removed.

Moving Average Method

A centered moving average (MA) is obtained by summing and averaging the values from a given number of periods repetitively, each time deleting the oldest value and adding a new value. Moving averages can smooth out fluctuations in any data, while preserving the general pattern of the data (longer averages result in more smoothing). However, they do not yield a forecasting equation, nor do they generate values for the ends of the data series.

$$MA = \frac{\sum x}{Number\ of\ Period}$$

A weighted moving average (MA_{wt}) allows some values to be emphasized by varying the weights assigned to each component of the average. Weights can be either percentages or a real number.

$$MA_{wt} = \frac{\sum(Wt)x}{\sum Wt}$$

Example 1: *Shipments (in tons) of welded tube by an aluminum producer are shown below:*

Year	1	2	3	4	5	6	7	8	9	10	11
Tons	2	3	6	10	8	7	12	14	14	18	19

(a) Compute a 3-year moving average, and use it to forecast shipments in year 12. *(b)* Using a weight of 3 for the most recent data, 2 for the next, and 1 for the oldest, forecast shipments in year 12.

Table 5.1: 3 – year moving average

Year	Shipment (tons)	3-year moving total	3-year moving average
1	2	-	-
2	3	11	3.7
3	6	19	6.3
4	10	24	8.0
5	8	25	8.3
6	7	28	9.3
7	13	34	11.3
8	14	41	13.7
9	14	46	15.3
10	18	51	17
11	19	-	-

(a) See Table 5.1 for computations of the MA. The MA forecast for year 12 would be that of the latest average, 17.0 tons.

$$MA_{wt} = \frac{\sum(Wt)x}{\sum Wt} = \frac{(1)(14) + (2)(18) + (3)(19)}{1 + 2 + 3} = 17.8\ tons$$

Least Squares Method

Least squares are a mathematical technique of fitting a trend to data points. The resulting *line of best fit* has the following properties:

(1) the summation of all vertical deviations about it is zero,
(2) the summation of all vertical deviations squared is a minimum, and
(3) the line goes through the means X and Y. For linear equations, the line of best fit is found by the simultaneous solution for a and b of the following two *normal equations:*

$$\sum Y = na + b\sum x$$

$$\sum XY = a\sum X + b\sum x^2$$

The above equations can be used in the form shown above and are used in that form for regression. However, with time series, the data can also be coded so that $\Sigma X = 0$. Two terms then dropout, and the equations are simplified to:

$$\sum Y = na$$

$$a = \frac{\Sigma Y}{n}$$

$$\sum XY = b\sum x^2$$

$$b = \frac{\Sigma XY}{\Sigma x^2}$$

To code the time series data, designate the center of the time span as X = 0 and let each successive period be ±1 more unit away. (For an even number of periods, use values of ±0.5, 1.5, 2.5, etc.).

Example 2: *Use the least square method to develop a linear trend equation for the data from example 1. State the equation and forecast a trend value for year 16.*

Year	X year coded	Y shipments (tons)	XY	X^2
1	-5	2	-10	25
2	-4	3	-12	16
3	-3	6	-18	9
4	-2	10	-20	4
5	-1	8	-8	1
6	0	7	0	0
7	1	12	12	1
8	2	14	28	4
9	3	14	42	9
10	4	18	72	16
11	5	19	95	25
	0	113	181	110

We have

$$a = \frac{\Sigma Y}{n} = \frac{113}{11} = 10.30$$

$$b = \frac{\Sigma XY}{\Sigma x^2} = \frac{181}{110} = 1.6$$

The forecasting linear equation is of the form $Y = a + bX$

$$Y = 10.3 + 1.6X$$

For 16 years forecast,

$$Y = 10.3 + 1.6(16) = 35.9\ tons$$

Exponential Smoothing Method

Exponential smoothening is a moving-average forecasting technique that weights past data in an Exponential manner so that most recent data carry more weight in the moving average.

With simple Exponential smoothening, the forecast F_t is made up of the last period forecast F_{t-1} plus a portion, α, of the difference between the last periods actual demand A_{t-1} and last period forecast F_{t-1}.

$$F_t = F_{t-1} + \alpha\,(A_{t-1} - F_{t-1})$$

Example 3: *A firm uses simple exponential smoothing with α = 0.1 to forecast demand. The forecast for the week of February 1 was 500 units, whereas actual demand turned out to be 450 units.*

a) Forecast the demand for the week of February 8.
b) Assume that the actual demand during the week of February 8 turned out to be 505 units. Forecast the demand for the week of February 15, Continue forecasting through March 15, assuming that subsequent demands were actually 516, 488, 467, 554 and 510 units.

Solution

(a) $F_t = F_{t-1} + \alpha\,(A_{t-1} - F_{t-1}) = 500 + 0.1(450 - 500) = 495$ unit
(b) Arranging the procedure in tabular form, we have

Week	Actual demand A_{t-1}	Old forecast F_{t-1}	Forecast error $A_{t-1} - F_{t-1}$	Correction $\alpha\,(A_{t-1} - F_{t-1})$	New forecast $F_t = F_{t-1} + \alpha\,(A_{t-1} - F_{t-1})$
Feb 1	450	500	- 50	- 5	495
8	505	495	10	1	496
15	516	496	20	2	498
22	488	498	- 10	- 1	497
Mar 1	467	497	- 30	- 3	494
8	554	494	60	6	500
15	510	500	10	1	501

The smoothing constant, α, is a number between 0 and 1 that enters multiplicatively into each forecast but whose influence declines exponentially as the data become older. Typical values range from 0.01 to 0.40. An Ion α gives more weight to the past average and will effectively dampen high random variation. High α values are more responsive to changes in demand (e.g., from new-product introductions, promotional campaigns). An α of 1 would reflect total adjustment to recent demand, and the forecast would be last period's actual demand. A satisfactory α can generally be determined by trial-and-error modeling (on computer) to see which value minimizes forecast error.

Simple exponential smoothing yields only an average. It does not extrapolate for trend effects. No α value will fully compensate for a trend in the data. An α value that yields an approximately equivalent degree of smoothing as a moving average of *n* periods is:

$$\alpha = \frac{2}{n+1}$$

Regression and Correlation Methods

Regression and correlation techniques quantify the statistical association between two or more variables.

(a) ***Simple regression*** expresses the relationship between a dependent variable *Y* and a independent variable *X* in terms of the slope and intercept of the line of best fit relating the two variables.

(b) ***Simple correlation*** expresses the degree or closeness of the relationship between two variables in terms of a correlation coefficient that provides an indirect measure of the variability of points from the line of best fit. Neither regression nor correlation gives proof of a cause-effect relationship.

Regression

The simple linear regression model takes the form $Y = a + bX$, where *Y* is the dependent variable and X the independent variable. Values for the slope *b* and intercept α are obtained by using the *normal equations* written in the convenient form:

$$b = \frac{\sum XY - n\overline{XY}}{\sum X^2 - nX^2} \qquad (1)$$

$$a = \bar{Y} - b\bar{X} \qquad (2)$$

In Equations (1) and (2), $X = (\Sigma X)/n$ and $Y = (\Sigma Y)/n$ *Y* are the means of the independent and dependent variables respectively, and *n* is the number of pairs of observations made.

Example 4: *The general manager of a building materials production plant feels that the demand for plasterboard shipments may be related to the number of construction permits issued in the county during the previous quarter. The manager has collected the data shown in Table.*

(a) Compute values for the slope b and intercept a.

(b) Determine a point estimate for plasterboard shipments when the number of construction permits is 30.

Construction period (*X*)	Plasterboard shipments (*Y*)
15	6
9	4
40	16

	6
20	13
25	9
25	10
15	16
35	

(a) Solution

X	Y	XY	X^2	Y^2
15	6	90	225	36
9	4	36	81	16
40	16	640	1600	256
20	6	120	400	36
25	13	325	625	169
25	9	225	625	81
15	10	150	225	100
35	16	560	1225	256
184	**80**	**2146**	**5006**	**950**

n = 8 pairs of observations

$$\bar{X} = \frac{\Sigma X}{n} = \frac{184}{8} = 23$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{80}{8} = 10$$

$$b = \frac{\sum XY - n\overline{XY}}{\sum X^2 - nX^2} = \frac{2146 - 8(23)(10)}{5006 - 8(23)(23)} = 0.395$$

$$a = \bar{Y} - b\bar{X} = 10 - 0.395(23) = 0.91$$

The regression equation is

$$Y = 0.91 + 0.395X$$

When $X = 30$

$$Y = 0.91 + 0.395 \times 30 = 12.76 \cong 13\ shipments.$$

Forecast Application

Forecasts should be sufficiently accurate to plan for future activities. Low-accuracy methods may suffice; higher accuracy usually costs more to design and implement. *Long-term forecasts*—used for location, capacity, and new-product decisions-require techniques with long-term horizons. *Short-term forecasts*— such as those for production-and-inventory control, labour levels, and cost controls–can rely more on recent history.

Example 5: *A food processing company uses a moving average to forecast next month's demand. Past actual demand (in units) is as shown in Table*

(a) Compute a simple 5-month moving average to forecast demand for month 52.
(b) Compute a weighted 3-month moving average, where the weights are highest for the latest months and descend in order of 3, 2 and 1.

Month	Actual Demand
43	105
44	106
45	110
46	110
47	114
48	121
49	130
50	128
51	137

Solution

$$MA = \frac{\sum X}{Number\ of\ period} = \frac{114 + 121 + 130 + 128 + 137}{5} = 126\ units$$

$$MA_{wt} = \frac{\sum(wt)X}{\sum wt} = \frac{(3 \times 137) + (2 \times 128) + (1 \times 130)}{6} = \frac{797}{6} = 133\ units$$

Example 9: *For* $N = 7$ *years of (Coded) time series data,* $\Sigma Y = 56$, $\Sigma XY = 70$, *and* $\Sigma X^2 = 28$.

(a) Find the intercept and slope of the linear trend line.
(b) Forecast the Y-value for 6 years distant from the origin.

Solution

$$intercept = \frac{\sum Y}{N} = \frac{56}{7} = 8.0$$

$$slope = \frac{\sum XY}{\sum X^2} = \frac{70}{28} = 2.5$$

$$Y = a + b(X) = 8.0 + 2.5(6) = 23.0$$

Scheduling

Scheduling can be defined as *"prescribing of when and where each operation necessary to manufacture the product is to be performed"*. It is also defined as *"establishing of times at which to begin and complete each event or operation comprising a procedure"*. The principle aim of scheduling is to plan the sequence of work so that production can be systematically arranged towards the end of completion of all products by due date.

Principles of Scheduling

The principle of optimum task size: scheduling tends to achieve maximum efficiency when the task sizes are small, and all tasks of same order of magnitude.

Principle of optimum production plan: the planning should be such that it imposes an equal load on all plants.

Principle of optimum sequence: scheduling tends to achieve the maximum efficiency when the work is planned so that work hours are normally used in the same sequence.

Inputs to Scheduling

i. ***Performance standards***: the information regarding the performance standards (standard times for operations) helps to know the capacity in order to assign required machine hours to facility.
ii. Units: in which loading and scheduling is to be expressed.
iii. ***Effective capacity*** of the work centre.
iv. ***Demand pattern*** and extent of flexibility to be provided for rush orders,
v. ***Overlapping*** of operations
vi. ***Individual jobs*** schedules.

Techniques of Scheduling

Master Scheduling (MS): it shows the dates on which important production items are to be completed. It's a weekly or monthly break-up of the production requirements for each product. Whenever any order is received, it is accommodated first in the MS considering the availability of the machine and labour. It helps production managers for advance planning & to have check over the production rate and efficiency. See table 7.1.

Table 7.1: Aggregate plan and master schedule for electric motors

Aggregate Plan

Month	J	F	M	A	M	J	J	A	S
Number of orders	40	25	55	30	30	50	30	60	40

Master Schedule

Month	J	F	M	A	M	J	J	A	S
AC Motors									
5hp	15	-	30	-	-	30	-	-	10
25hp	20	25	25	15	15	15	20	30	20

DC motors									
20hp	-	-	-	-	-	-	10	10	-
WR motors									
10hp	5	-	-	15	15	5	-	25	10

Shop Manufacturing Schedule: after preparing the MS, shops schedules (SS) are prepared. It assigns a definite period of time to a particular shop for manufacturing products in required quantity. It shows how many products are to be made, and on what day or week.

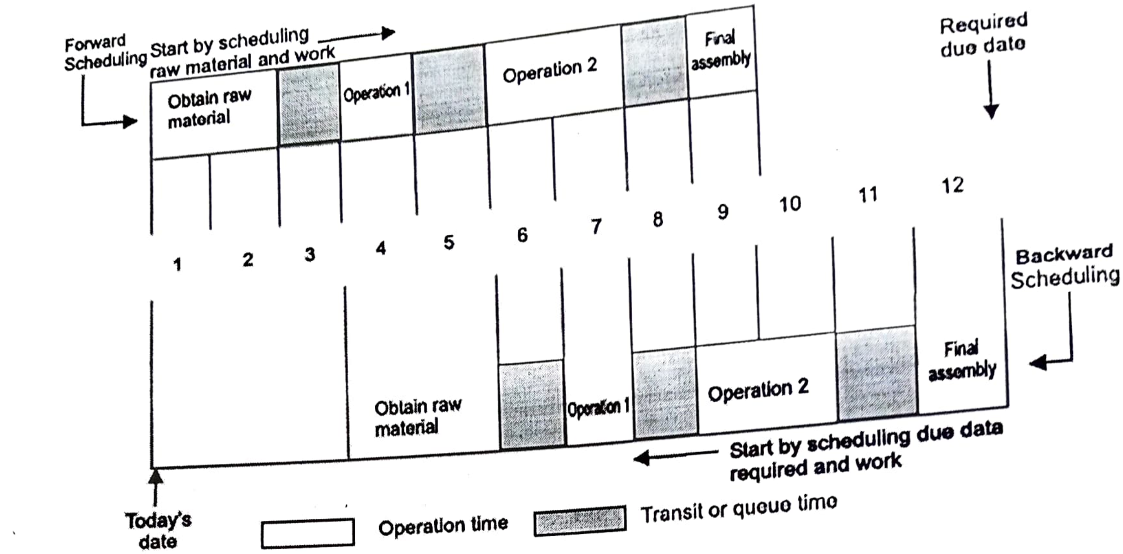
Backward or Reverse Scheduling: External due date considerations will directly influence activity scheduling in certain structures. The approach adopted in scheduling activities in such cases will often involve a form of reverse scheduling with the use of **bar or Gantt charts.** *A major problem with such reverse or 'due date' scheduling is in estimating the total time to be allowed for each operation,* in particular the time to be allowed for waiting or queuing at facilities. Some queuing of jobs (whether items or customers) before facilities is often desirable since, where processing times on facilities are uncertain, high utilization is achieved only by the provision of such queues.

Forward Scheduling: for a manufacturing or supply organization a forward scheduling procedure will in fact be the opposite of that described above. This approach will be particularly relevant where scheduling is *undertaken on an internally oriented basis* and the objective is to determine the date or times for subsequent activities, given the times for an earlier activity, e.g. a starting time. In the case of supply or transport organizations, the objective will be *to schedule forward from a given start date*, where that start date will often be the customer due date, e.g. the date at which the customer arrives into the system. In these circumstances, therefore, *forward scheduling will be an appropriate method for dealing with externally oriented scheduling activities.*

Optimized Production Technique (OPT): It is a program help to recognizes the existence of *bottlenecks* through which the flow gets restricted. It consists of modules that contain data on products, customer orders, work center capacities, etc, as well as algorithms to do the actual scheduling. *A key feature of the program is to simulate the load on the system, identify bottleneck (as well as other) operations,* and develop alternative production schedule.

Example 1: A job is due to be delivered at the end of 12th week. It requires a lead time of 2 weeks for material acquisition, 1 week of run time for operation – 1, 2 weeks for operation – 2, and 1 week for final assembly. Allow 1 week of transit time prior to each operation. Illustrate the completion schedule under (a) forward, and (b) backward scheduling methods.

Solution: the solution is illustrated in fig (7.2).



Scheduling Criteria

Scheduling criteria is also called as scheduling methodology. Key to multiprogramming is scheduling. Different scheduling algorithm have different properties. The criteria used for comparing these algorithms include the following:

Throughput: is the rate at which processes are completed per unit time.

Turnaround time: this is the how long a process takes to execute a process. It is calculated as the time gap between the submission of a process and its completion.

Waiting time: is the sum of the time periods spent in waiting in the ready queue.

Response time: is the time it takes to start responding from submission time. It is calculated as the amount of time it takes from when a request was submitted until the first response is produced.

Scheduling Policies

In general, scheduling policies may be *pre-emptive or non-pre-emptive.*

Non-pre-emptive Scheduling: in non-pre-emptive mode, once if a process enters into running state, it continues to execute until it terminates or blocks itself to wait for Input / Output or by requesting some operating system service.

Pre-emptive Scheduling: in pre-emptive mode, currently running process may be interrupted and moved to the ready State by the operating system. When a new process arrives or when an interrupt occurs, pre-emptive policies may incur greater overhead than non-pre-emptive version but pre-emptive version may provide better service.

Sequencing and Dispatching Phase

Sequencing activities are closely identified with detailed scheduling as they specify **the order** in which jobs are to be processed at the various work centres. ***Dispatching*** is concerned with starting the processes. It gives necessary ***authority*** to start a particular work, which has already been planned under "routing" and "scheduling". For starting the work, essential orders and instruction are given. Therefore, the definition of ***dispatching*** is '***release*** *of orders and instructions for starting of the production for an item in accordance with the 'route sheet' and schedule charts'.*

Dispatching function include:

i. Implementing the schedule in a manner that retains any order priorities assigned at the planning phase.
ii. Moving the required materials from stores to the machines, and from operation to operation.
iii. Authorizing people to take work in hand as per schedule
iv. Distributing machine loading and schedule charts, route sheet, and other instruction and forms.
v. Issuing inspection orders, stating the type of inspection at various stages.
vi. Ordering tool-section to issue tools, jigs and fixtures.

Dispatching or Priority Decision Rules

Job shops generally have many jobs waiting to be processed. ***The principle method job dispatching is by means of priority rules***, which are simplified guidelines (heuristics) to determine the sequence in which jobs will be processed. The use of priority rule dispatching is an attempt to formalize the decision of the experienced human dispatcher.

Some of the rules used job assignment are: first come, first served (FCFS), earliest due date (EDD), longest processing time (LPT), and preffered customer order (PCO). These rules can be classified as: *Static or dynamic.*

Static rules do not incorporate an updating feature. They have priority indices that stay constant as jobs travel through the plant, where as dynamic rules change with time and queue characteristics.

Scheduling Algorithms

Scheduling algorithms or scheduling policies are mainly used for short-term scheduling. The main objective of short-term scheduling is to allocate process time in such a way as to optimize one or more aspects of system behaviour. Scheduling algorithms decide which of the processes in the ready queue is to be allocated based on the type of scheduling policy and whether that policy is either pre-emptive or non-pre-emptive. For scheduling arrival time and service time are also will play a role. List of scheduling algorithms are as follows:

i. First-come, first-served scheduling (FCFS) algorithm
ii. Shortest job first Scheduling (SJF) algorithm
iii. Shortest Remaining Time (SRT) algorithm
iv. Round-Robin Scheduling algorithm
v. Multilevel Queue Scheduling algorithm

vi. Multilevel Feedback Queue Scheduling algorithm

First-come, First-served Scheduling (FCFS)

First-come, First-served Scheduling follow first in first out method. As each process becomes ready, it joins the ready queue. When the current running process ceases to execute, the oldest process in the Ready queue is selected for running. That is first entered process among the available processes in the ready queue. The average waiting time for FCFS is often quite long. It is non-pre-emptive.

Advantages

i. Better for long processes
ii. Simple method (i.e., minimum overhead on processor)
iii. No starvation

Disadvantages

i. Convoy effect occurs. Even very small process should wait for its turn
ii. Throughput is not emphasized.

Shortest Job First Scheduling (SJF)

The process with the shortest expected processing time is selected for execution, among the available processes in the ready queue. Thus, a short process will jump to the head of the queue over long jobs. If the process time of two processes are the same then FCFS scheduling is used to break the tie. SJF scheduling algorithm is probably optimal. SJF can be pre-emptive or non-pre-emptive. A pre-emptive SJF algorithm will pre-empt the currently executing process if the process time of the next newly arrived process may be shorter than what is left to the currently executing process. A non-pre-emptive SJF algorithm will allow the currently running process to finish. Pre-emptive SJF Scheduling is sometimes called Shortest Remaining Time First algorithm.

Advantages

i. It gives superior turnaround time performance to shortest process next because a short job is given immediate preference to a running longer job.
ii. Throughput is high

Disadvantages

i. Elapsed time (i.e., execution-completed-time) must be recorded, it results an additional overhead on the processor
ii. Starvation maybe possible for the longest processes.

Priority Scheduling

Each process is assigned a priority. The ready list contains an entry for each process ordered by its priority. The process at the beginning of the list (highest priority) is picked first.

A variation of this scheme allows pre-emption of the current process when a higher priority process arrives.

Another variation of the policy adds aging scheme, where the priority of a process increases as it remains in the ready queue; hence, will eventually execute to completion.

Round-Robin Scheduling

This type of scheduling algorithm is basically designed for time sharing system. It is similar to FCFS with pre-emption added. Round-Robin Scheduling is also called as time-slicing scheduling and it is a pre-emptive version based on a clock. That is a clock interrupt is generated at periodic intervals. When the interrupt occurs, the currently running process is placed in the ready queue and the next ready job is selected on a First-come, First-serve basis. This process is known as time-slicing, because each process is given a slice of time before being pre-empted.

In round-robin scheduling, the principal design issue is the length of the time quantum or time-slice to be used. If the quantum is very short, then short processes will move quickly.

Advantages

i. Round-robin is effective in a general-purpose, times-sharing system or transaction-processing system
ii. Fair treatment for all processes
iii. Overhead on processor is low
iv. Good response time for short processes

Disadvantages

i. Care must be taken in choosing quantum value
ii. Processing overhead is there in handling clock interrupt
iii. Throughput is low if time quantum is too small

Multilevel Queue

Multi-level queue (MLQ) scheme solves the mix job problem by maintaining separate "ready" queues for each type of job class and apply different scheduling algorithms to each.

Multilevel Feedback Queue

This is a variation of MLQ, where processes (jobs) are not permanently assigned to a queue when they enter the system. In this approach, if a process exhausts its time quantum (i.e., it is CPU-bound), it is moved to another queue with a longer time quantum and a lower priority. The last level uses FCFS algorithm in this scheme.

Johnson's Algorithm

Step 1: find the minimum among various t_{i1} and t_{i2}.

Step 2a: if the minimum processing time requires machine 1, place the associated job in the first available position in sequence. Go to step 3.

Step 2b: if the minimum processing time requires machine 2, place the associated job in the last available position in sequence. Go to step 3.

Step 3: removed the assigned job from consideration and return to step 1 until all positions in sequence are filled.

The above algorithm is illustrated using the following problem:

Example 2: Consider the following two machines and six jobs flow shop-scheduling problem. Using Johnson's algorithm, obtain the optimal sequence, which will minimize the make-span.

Job	Machine 1	Machine 2
1	5	4
2	2	3
3	13	14
4	10	1
5	8	9
6	12	11

Solution

Job	Machine 1	Machine 2
1	5	4
2	2	3
3	13	14
4	~~10~~	~~1~~
5	8	9
6	12	11

Job	Machine 1	Machine 2
1	5	4
~~2~~	~~2~~	~~3~~
3	13	14
4	~~10~~	~~1~~
5	8	9
6	12	11

Job	Machine 1	Machine 2
~~1~~	~~5~~	4
~~2~~	~~2~~	~~3~~
3	13	14
4	~~10~~	~~1~~
5	8	9
6	12	11

Job	Machine 1	Machine 2
~~1~~	~~5~~	4
~~2~~	~~2~~	~~3~~
3	13	14
4	~~10~~	~~1~~
~~5~~	~~8~~	~~9~~
6	12	11

Job	Machine 1	Machine 2
~~1~~	~~5~~	4
~~2~~	~~2~~	~~3~~
3	13	14
4	~~10~~	~~1~~
~~5~~	~~8~~	~~9~~
~~6~~	~~12~~	~~11~~

Job	Machine 1	Machine 2
~~1~~	~~5~~	4
~~2~~	~~2~~	~~3~~
~~3~~	~~13~~	~~14~~
4	~~10~~	~~1~~
~~5~~	~~8~~	~~9~~
~~6~~	~~12~~	~~11~~

The optimal sequence is **2-5-3-6-1-4**

The make-span is determined as shown below.

Time in M/C 2 = max M/C 1 Time-out of the current job, M/C 2 Time-out of the previous job	
	Processing Time

Jobs	Machine 1		Machine 2	
	Time-in	Time-out	Time-in	Time-out
2	0	2	2	5
5	2	10	10	19
3	10	23	23	37
6	23	35	37	48
1	35	40	48	52
4	40	50	52	53

The make-span for this schedule is 53

Example 3.3: consider the following 3 machines and 5 jobs flow shop problem:

	Processing Time		
Job	Machine 1	Machine 2	Machine 3
1	8	5	4
2	10	6	9
3	6	2	8
4	7	3	6
5	11	4	5

Solution: we can extend the Johnson's algorithm to this problem. So the modified problem may be given as follows:

Job	Machine 1	Machine 2
1	13	9
2	16	15
3	8	10
4	10	9
5	15	9

Job	Machine A	Machine B
1	13	9
2	16	15
~~3~~	~~8~~	~~10~~
4	10	9
5	15	9

Job	Machine A	Machine B
1	13	9
2	16	15
~~3~~	~~8~~	~~10~~
4	~~10~~	~~9~~
5	15	9

Job	Machine A	Machine B
~~1~~	~~13~~	~~9~~
2	16	15
~~3~~	~~8~~	~~10~~
4	~~10~~	~~9~~
5	15	9

Job	Machine A	Machine B
~~1~~	~~13~~	~~9~~
2	16	15
~~3~~	~~8~~	~~10~~
4	~~10~~	~~9~~
~~5~~	~~15~~	~~9~~

Job	Machine A	Machine B
~~1~~	~~13~~	~~9~~
~~2~~	~~16~~	~~15~~
~~3~~	~~8~~	~~10~~
4	~~10~~	~~9~~
~~5~~	~~15~~	~~9~~

The optimal sequence for the above problem: **3-2-5-1-4**

The Assignment Problem and the Hungarian Method

The Assignment Problem: suppose we have n resources to which we want to assign to n tasks on a one-to-one basis. Suppose also that we know the cost of assigning a given resource to a given task. We wish to find an optimal assignment one which minimizes total cost.

The mathematical model: let c_{ij} be the cost of assigning the ith resource to the jth task. We define the cost matrix to be n x n matrix

$$C = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & \vdots & & \vdots \\ c_{n,1} & c_{n,2} & \cdots & c_{n,3} \end{bmatrix}$$

An assignment is a set of n entry position in the cost matrix, no two of which lie in the same row or column. The sum of the n entries of an assignment is its cost. An assignment with the smallest possible cost is called an **optimal assignment**.

Theorem: if a number is added to or subtracted from all of the entries of any one row or column of a cost matrix, then an optimal assignment for the resulting cost matrix is also an optimal assignment for the original cost matrix.

The Hungarian Method

The Hungarian Method is an algorithm which finds an optimal assignment for a given cost matrix.

The following algorithm applies the above theorem to a given n x n cost matrix to find an optimal assignment.

Step 1: Subtract the smallest entry in each row from all the entries of its row.

Step 2: Subtract the smallest entry in each column from all the entries of its column.

Step 3: Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the minimum number of such lines is used.

Step 4: Test of Optimality: (i) If the minimum number of covering lines is n, an optimal assignment of zeros is possible and we are finished. (ii) If the minimum number of covering lines is less than n, an optimal assignment of zeros is not yet possible. In that case, proceed to step 5.

Step 5: Determine the smallest entry not covered by any line. Subtract this entry from each uncovered row, and then add it to covered column. Return to step 3.

Example 3.4: You the sales manager for a toy manufacturer, and you currently have three salesreps on the road meeting buyers. Your salesreps are in Lagos, Enugu and Port Harcourt.

You want them to fly to three cities: Aba, Abuja and Kano. The table below shows the cost of airplanes tickets between these cities.

From / To	Aba	Abuja	Kano
Lagos	250	400	350
Enugu	400	600	350
Port Harcourt	200	400	250

Where will you send each of your salesreps in order to minimize cost?

Solution

In matrix form we have

$$\begin{bmatrix} 250 & 400 & 350 \\ 400 & 600 & 350 \\ 200 & 400 & 250 \end{bmatrix}$$

Step 1: Subtract 250 from Row 1, 350 from Row 2, and 200 from Row 3.

$$\begin{bmatrix} 250 & 400 & 350 \\ 400 & 600 & 350 \\ 200 & 400 & 250 \end{bmatrix} \approx \begin{bmatrix} 0 & 150 & 100 \\ 50 & 250 & 0 \\ 0 & 200 & 50 \end{bmatrix}$$

Step 2: Subtract 0 from Column 1, 150 from Column 2, and 0 from Column 3.

$$\begin{bmatrix} 0 & 150 & 100 \\ 50 & 250 & 0 \\ 0 & 200 & 50 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & 100 \\ 50 & 100 & 0 \\ 0 & 50 & 50 \end{bmatrix}$$

Step 3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

$$\begin{bmatrix} 0 & 0 & 100 \\ 50 & 100 & 0 \\ 0 & 50 & 50 \end{bmatrix}$$

Step 4: Since the minimum number of lines is 3, an optimal assignment of zeros is possible and we are finished.

$$\begin{bmatrix} 0 & [0] & 100 \\ 50 & 100 & [0] \\ [0] & 50 & 50 \end{bmatrix}$$

Here is the same assignment made to the original cost matrix.

$$\begin{bmatrix} 250 & [400] & 350 \\ 400 & 600 & [350] \\ [200] & 400 & 250 \end{bmatrix}$$

Example 5: A construction company has four large bulldozers located at four different garages. The bulldozers are to be moved to four different construction sites. The distances in miles between the bulldozers and the construction sites are given below.

Bulldozer/Site	A	B	C	D
1	90	75	75	80
2	35	85	55	65
3	125	95	90	105
4	45	110	95	115

How should the bulldozers be moved to the construction sites in order to minimize the total distance travelled.

Solution

Step 1: Subtract 75 from Row 1, 35 from Row 2, 90 from Row 3, and 45 from Row 4.

$$\begin{bmatrix} 90 & 75 & 75 & 80 \\ 35 & 85 & 55 & 65 \\ 125 & 95 & 90 & 105 \\ 45 & 110 & 95 & 115 \end{bmatrix} \approx \begin{bmatrix} 15 & 0 & 0 & 5 \\ 0 & 50 & 20 & 30 \\ 35 & 5 & 0 & 15 \\ 0 & 65 & 50 & 70 \end{bmatrix}$$

Step 2: Subtract 0 from Column 1, 0 from Column 2, 0 from Column 3, and 5 from Column 4.

$$\begin{bmatrix} 15 & 0 & 0 & 5 \\ 0 & 50 & 20 & 30 \\ 35 & 5 & 0 & 15 \\ 0 & 65 & 50 & 70 \end{bmatrix} \approx \begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 50 & 20 & 25 \\ 35 & 5 & 0 & 10 \\ 0 & 65 & 50 & 65 \end{bmatrix}$$

Step 3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

$$\begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 50 & 20 & 25 \\ 35 & 5 & 0 & 10 \\ 0 & 65 & 50 & 65 \end{bmatrix}$$

Step 4: Since the minimum number of lines is less than 4, we have to proceed to step 5.

Step 5: Note that 5 is the smallest entry not covered by any line. Subtract 5 from each uncovered row.

$$\begin{bmatrix} 15 & 0 & 0 & 0 \\ 0 & 50 & 20 & 25 \\ 35 & 5 & 0 & 10 \\ 0 & 65 & 50 & 65 \end{bmatrix} \approx \begin{bmatrix} 15 & 0 & 0 & 0 \\ -5 & 45 & 15 & 20 \\ 30 & 0 & -5 & 5 \\ -5 & 60 & 45 & 60 \end{bmatrix}$$

Now add 5 to each covered column

$$\begin{bmatrix} 15 & 0 & 0 & 0 \\ -5 & 45 & 15 & 20 \\ 30 & 0 & -5 & 5 \\ -5 & 60 & 45 & 60 \end{bmatrix} \approx \begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 45 & 20 & 20 \\ 35 & 0 & 0 & 5 \\ 0 & 60 & 50 & 60 \end{bmatrix}$$

Now return to step 3

Step 3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

$$\begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 45 & 20 & 20 \\ 35 & 0 & 0 & 5 \\ 0 & 60 & 50 & 60 \end{bmatrix}$$

Step 4: Since the minimum number of lines is less than 4, we have to return to step 5.

Step 5: Note that 20 is the smallest entry not covered by any line. Subtract 20 from each uncovered row. Then add 20 to each covered column.

$$\begin{bmatrix} 20 & 0 & 5 & 0 \\ 0 & 45 & 20 & 20 \\ 35 & 0 & 0 & 5 \\ 0 & 60 & 50 & 60 \end{bmatrix} \approx \begin{bmatrix} 20 & 0 & 5 & 0 \\ -20 & 25 & 0 & 0 \\ 35 & 0 & 0 & 5 \\ -20 & 40 & 30 & 40 \end{bmatrix}$$

$$\begin{bmatrix} 20 & 0 & 5 & 0 \\ -20 & 25 & 0 & 0 \\ 35 & 0 & 0 & 5 \\ -20 & 40 & 30 & 40 \end{bmatrix} \approx \begin{bmatrix} 40 & 0 & 5 & 0 \\ 0 & 25 & 0 & 0 \\ 55 & 0 & 0 & 5 \\ 0 & 40 & 30 & 40 \end{bmatrix}$$

Now return to step 3.

Step 3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

$$\begin{bmatrix} 40 & 0 & 5 & 0 \\ 0 & 25 & 0 & 0 \\ 55 & 0 & 0 & 5 \\ 0 & 40 & 30 & 40 \end{bmatrix}$$

Step 4: Since the minimum number of lines 4, an optimal assignment of zeros is possible and we are finish.

$$\begin{bmatrix} 40 & 0 & 5 & [0] \\ 0 & 25 & [0] & 0 \\ 55 & [0] & 0 & 5 \\ [0] & 40 & 30 & 40 \end{bmatrix}$$

Since the total cost for this assign is 0, it must be an optimal assignment.

Here is the same assignment made to the original cost matrix

$$\begin{bmatrix} 90 & 75 & 75 & [80] \\ 35 & 85 & [55] & 65 \\ 125 & [95] & 90 & 105 \\ [45] & 110 & 95 & 115 \end{bmatrix}$$

BASIC PRINCIPLES OF WORK-STUDY AND MOTION ECONOMIC

WORK STUDY

"**Work study** is a generic term for those techniques, method study and work measurement which are used in the examination of human work in all its contexts. And which lead systematically to the investigation of all the factors which affect the efficiency and economy of the situation being reviewed, in order to effect improvement."

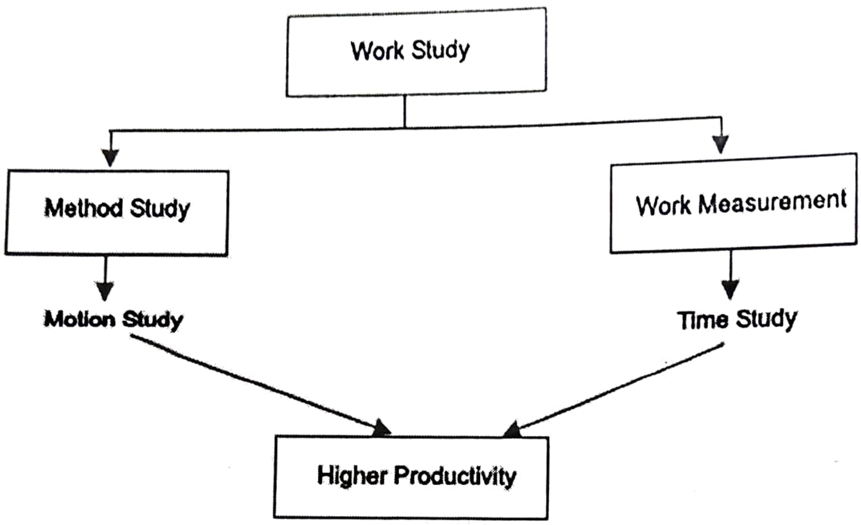


Fig. 2.1: *Framework of work study*

Work study is a means of enhancing the production efficiency (productivity) of the firm by elimination of waste and unnecessary operations. It is a technique to identify non-value adding operations by investigation of all the factors affecting the job. It is the only accurate and systematic procedure oriented technique to establish time standards. It is going to contribute to the profit as the savings will start immediately and continue throughout the life of the product.

Method study and work measurement is part of work study. Part of method study is motion study, work measurement is also called by the name 'Time study'.

Advantages of Work Study

Following are the advantages of work study:

i. It helps to achieve the smooth production flow with minimum interruptions.
ii. It helps to reduce the cost of the product by eliminating waste and unnecessary operations.
iii. Better worker-management relations.
iv. Meets the delivery commitment.
v. Reduction in rejections and scrap and higher utilisation of resources of the organization.
vi. Helps to achieve better working conditions.
vii. Better workplace layout.

viii. Improves upon the existing process or methods and helps in standardisation and simplification.
ix. Helps to establish the standard time for an operation or job which has got application in manpower planning, production planning.

Method Study

Method study enables the production engineer to subject each operation to systematic analysis. The main purpose of method study is to eliminate the unnecessary operations and to achieve the best method of performing the operation.

Method study is also called **methods engineering or work design.** Method engineering is used to describe collection of analysis techniques which focus on improving the effectiveness of men and machines.

According to British Standards Institution (BS 3138): *"Method study is the systematic recording and critical examination or existing and proposed ways or doing work as a means or developing and applying easier and more effective methods and reducing cost."*

Fundamentally method study involves the breakdown of an operation or procedure into its component elements and their systematic analysis.

Objectives of Method Study

Method study is essentially concerned with finding better ways of doing things. It adds value and increases the efficiency by eliminating unnecessary operations, avoidable delays and other forms of waste.

The improvement in efficiency is achieved through:

i. Improved layout and design of workplace.
ii. Improved and efficient work procedures.
iii. Effective utilisation of men, machines and materials.
iv. Improved design or specification of the final product.

The objectives of method study techniques are:

i. Present and analyse true facts concerning the situation.
ii. To examine those facts critically.
iii. To develop the best answer possible under given circumstances based on critical examination of facts.

Scope of Method Study

The scope of method study is not restricted to only manufacturing industries. Method study techniques can be applied effectively in service sector as well. It can be applied in offices, hospitals, banks and other service organizations.

The areas to which method study can be applied successfully in manufacturing are:

i. To improve work methods and procedures.
ii. To determine the best sequence of doing work.
iii. To smoothen material flow with minimum of back tracking and to improve layout.
iv. To improve the working conditions and hence to improve labour efficiency.
v. To reduce monotony in the work.
vi. To improve plant utilisation and material utilisation.
vii. Elimination of waste and unproductive operations.
viii. To reduce the manufacturing costs through reducing cycle time of operations.

Steps or Procedure Involved in Methods Study

The basic approach to method study consists of the following eight steps.

i. **SELECT** the work to be studied and define its boundaries.
ii. **RECORD** the relevant facts about the job by direct observation and collect such additional data as may be needed from appropriate sources.
iii. **EXAMINE** the way the job is being performed and challenge its purpose, place sequence and method of performance.
iv. **DEVELOP** the most practical, economic and effective method, drawing on the contributions of those concerned.
v. **EVALUATE** different alternatives to developing a new improved method comparing the cost-effectiveness of the selected new method with the current method with the current method of performance.
vi. **DEFINE** the new method, as a result, in a clear manner and present it to those concerned, *i.e.*, management, supervisors and workers.
vii. **INSTALL** the new method as a standard practice and train the persons involved in applying it.
viii. **MAINTAIN** the new method and introduce control procedures to prevent a drifting back to the previous method of work.

Symbols Used in Method Study

Graphical method of recording was originated by Gilberth, in order to make the presentation of the facts clearly without any ambiguity and to enable to grasp them quickly and clearly. It is useful to use symbols instead of written description.

O OPERATION

□ INSPECTION

⇨ TRANSPORTATION

D DELAY

∇ STORAGE

Operation O: An operation occurs when an object is intentionally changed in one or more of its characteristics (physical or chemical). This indicates the main steps in a process, method or procedure. An operation always takes the object one stage ahead towards completion. Examples of operation are:

a. Turning, drilling, milling, etc.
b. A chemical reaction.
c. Welding, brazing and riveting.
d. Lifting, loading, unloading.
e. Getting instructions from supervisor.
f. Taking dictation.

Inspection □**:** An inspection occurs when an object is examined and compared with standard for quality and quantity. The inspection examples are:

a. Visual observations for finish.
b. Count of quantity of incoming material.
c. Checking the dimensions.

Transportation →: A transport indicates the movement of workers, materials or equipment from one place to another. ***Example:*** Movement of materials from one workstation to another or Workers travelling to bring tools.

Delay D: Delay (Temporary Storage): A delay occurs when the immediate performance of the next planned thing does not take place. ***Example:*** Work waiting between consecutive operations, Workers waiting at tool cribs or Operators waiting for instructions from supervisor.

Storage ∇: Storage occurs when the object is kept in an authorised custody and is protected against unauthorised removal. For example, materials kept in stores to be distributed to various work.

Example 1. *Develop a Process Chart for making a cheese sandwich.*

SOLUTION. The following chart is one possible solution. The level of detail in process charts depends upon the requirements of the job. Time is often included to aid analysis of value added.

Process Chart

S/N	Distance in meter	Symbol	Process description
1	10	⇨	Move to cabinet
2	-	O	Get loaf of bread
3	-	O	Remove two slice of bread
4	-	O	Lay slice on counter top
5	-	O	Close loaf of bread
6	-	O	Replace loaf of bread on shelf
7	-	O	Open butter
8	-	O	Spread butter on top slice of bread
9	-	□	Inspect sandwich
10	10	⇨	Move to serving area
11	-	O	Serve sandwich

Exmple 2. *Develop a Multiple Activity Chart for doing three loads of laundry, assume you will have access to one washing machine and one dryer.*

SOLUTION: The followingchart is one possible solution. The level of detail in process charts depends upon the requirements of the job. Time is often included to aid analysis of value added.

Multiple Activity Chart

Time	**Operator**	**Machine 1 Washer**	**Machine 2 Dryer**
	Load clothes and detergent in to Machine 1	Being loaded	Idle
Repeat cycle	Idle	Run	Idle
	Remove clothes from Machine 1	Being unloaded	Idle
	Load clothes into Machine 2	Idle	Being loaded
	Load clothes and detergent into Machine 1	Being loaded	Run
	Idle	Run	Run
	Remove clothes from Machine 2	Idle	Being unloaded
	Hang clothes	Idle	Idle

MOTION STUDY

Motion study is part of method study where analysis of the motion of an operator or work will be studied by following the prescribed methods.

Principles of Motion study

There are a number of principles concerning the economy of movements which have been developed as a result of experience and which forms the basis for the development of improved methods at the workplace. These are first used by Frank Gilbreth, the founder of motion study and further rearranged and amplified by Barnes, Maynard and others. The principles are grouped into three headings:

a. Use of the human body.
b. Arrangement of workplace.
c. Design of tools and equipment.

a. **Uses of Human Body:** *When possible:*
   i. The two hands should begin and complete their movements at the same time.
   ii. The two hands should not be idle at the same time except during periods of rest.
   iii. Motions of the arms should be made simultaneously.
   iv. Hand and body motions should be made at the lowest classification at which it is possible to do the work satisfactorily.
   v. Momentum should be employed to help the worker, but should be reduced to a minimum whenever it has to be overcome by muscular effort.
   vi. Continuous curved movements are to be preferred to straight line motions involving sudden and changes in directions.
   vii. 'Ballistic' (*i.e.*, free swinging) movements are faster, easier and more accurate than restricted or controlled movements.

viii. Rhythm is essential to the smooth and automatic performance of a repetitive operation. The work should be arranged to permit easy and natural rhythm wherever possible.

ix. Work should be arranged so that eye movements are confined to a comfortable area, without the need for frequent changes of focus.

b. Arrangement of the Workplace

i. Definite and fixed stations should be provided for all tools and materials to permit habit formation.

ii. Tools and materials should be pre-positioned to reduce searching.

iii. Gravity fed, bins and containers should be used to deliver the materials as close to the point of use as possible.

iv. Tools, materials and controls should be located within a maximum working area and as near to the worker as possible.

v. Materials and tools should be arranged to permit the best sequence of motions.

vi. 'Drop deliveries' or ejectors should be used wherever possible, so that the operative does not have to use his hands to dispose of finished parts.

vii. Provision should be made for adequate lightning, and a chair of type and height to permit good posture should be provided. The height of the workplace and seat should be arranged to allow alternate standing and seating.

c. Design of tools and equipments

i. The colour of the workplace should contrast with that of work and thus reduce eye fatigue.

ii. The hands should be relieved of all work of 'holding' the work piece where this can be done by a jig or fixture or foot operated device.

iii. Two or more tools should be combined where possible.

iv. Where each finger performs some specific movement, as in typewriting, the load should be distributed in accordance with the inherent capacities of the fingers.

v. Handles such as those used on screw drivers and cranks should be designed to permit maximum surface of the hand to come in contact with the handle.

vi. Levers, cross bars and wheel bars should be in such position that operator can manipulate them with least body change and with greatest mechanical advantage.

Principles of Motion Economy

Through the pioneer work of Gilbreth, Ralph M. Barnes and other investigators, certain rules for motion economy and efficiency have been developed. Some of the more important of these principles are the following:

i. The movements of the two hands should be balanced and the two hands should begin and end their motions simultaneously (refer Figure 2.3).

ii. The hands should be doing productive work and should not be idle at the same time except during rest periods.

iii. Motions of the hands should be made in opposite and symmetrical direction and at the same time (refer Figure 2.4).

iv. The work should be arranged to permit it to be performed with an easy and natural rhythm.

v. Momentum and ballistic-type movements should be employed wherever possible in order to reduce muscular effort.
vi. There should be a definite location for all tools and materials, and they should be located in front of and close to the worker.
vii. Bins or other devices should be used to deliver the materials close to the point of use.
viii. The workplace should be designed to ensure adequate illumination, proper workplace height, and provision for alternate standing and sitting by the operator.
ix. Wherever possible, jigs, fixtures, or other mechanical devices should be used to relieve the hands of unnecessary work.
x. Tools should be prepositioned wherever possible in order to facilitate grasping them.
xi. Object should be handled, and information recorded. Only once

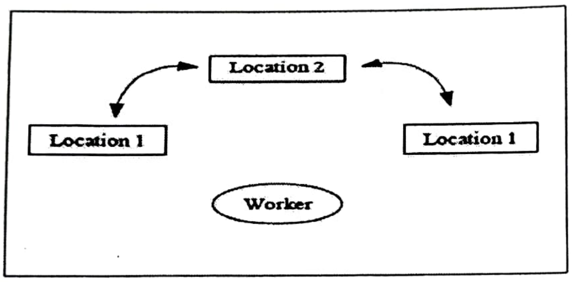


Figure 2.3

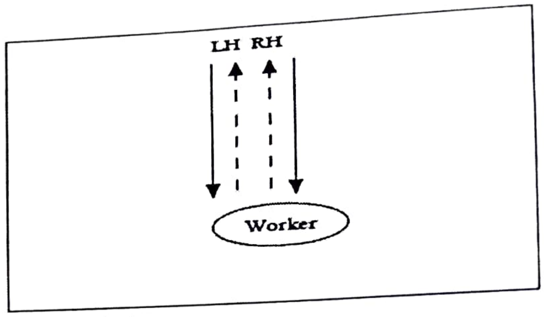
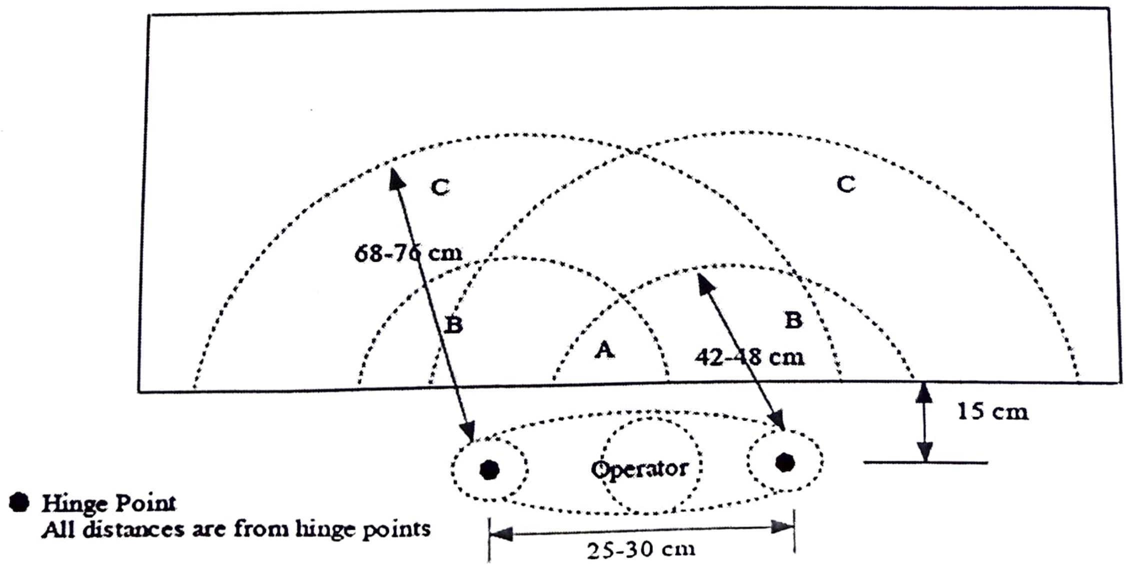


Figure 2.4

Elimination

i. Eliminate possible job, steps, or motion
ii. Eliminate irregularities in a job so as to facilitate automaticity
iii. Eliminate the use of hand as holding device
iv. Eliminate awkward or abnormal motion
v. Eliminate the use of muscle to maintain a fixed position
vi. Eliminate muscular force by using power tools, power feeds, etc.
vii. Eliminate overcoming of momentum
viii. Eliminate danger
ix. Eliminate idle time unless needed for rest



Key: A: Common Working Area; B:Min Working Area; C: Max Working Area

Fig 2.5: Normal and Maximun working areas:

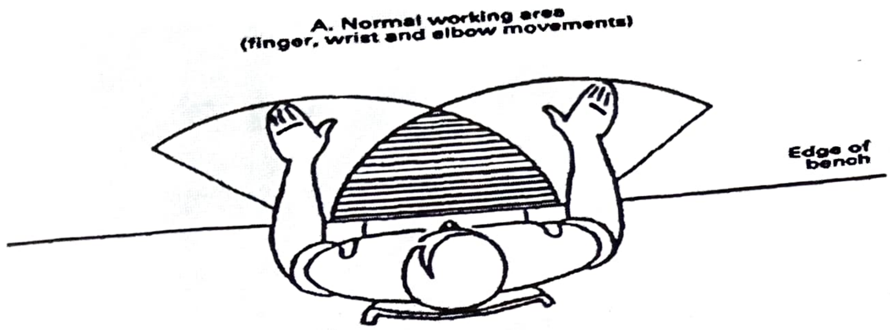


Fig 2.6: minimum working area of an operator

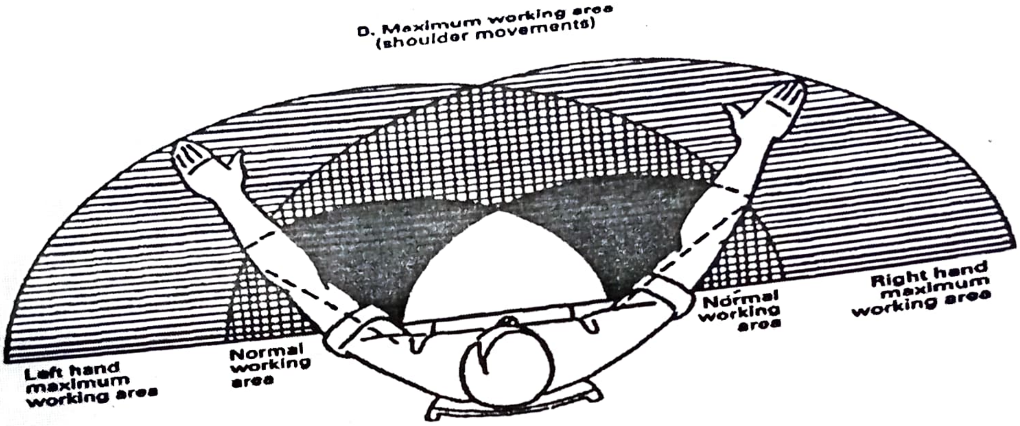


Fig 2.7: maximum working area of an operator

Work Measurement

Work measurement is also called by the name 'time study'. Work measurement is absolutely essential for both the planning and control of operations. Without measurement data, we cannot determine the capacity of facilities or it is not possible to quote delivery dates or costs. We are not in a position to determine the rate of production and also labour utilisation and efficiency. It may not be possible to introduce incentive schemes and standard costs for budget control.

Objectives of Work Measurement

The use of work measurement as a basis for incentives is only a small part of its total application. The objectives of work measurement are to provide a sound basis for:

i. Comparing alternative methods.
ii. Assessing the correct initial manning (manpower requirement planning).
iii. Planning and control.
iv. Realistic costing.
v. Financial incentive schemes.
vi. Delivery date of goods.
vii. Cost reduction and cost control.

viii. Identifying substandard workers.
ix. Training new employees.

Techniques of Work Measurement

For the purpose of work measurement, work can be regarded as:

i. **Repetitive work:** The type of work in which the main operation or group of operations repeat continuously during the time spent at the job. These apply to work cycles of extremely short duration.
ii. **Non-repetitive work:** It includes some type of maintenance and construction work, where the work cycle itself is hardly ever repeated identically.

Various techniques of work measurement are:

i. Time study (stop watch technique),
ii. Synthesis,
iii. Work sampling,
iv. Predetermined motion and time study,
v. Analytical estimating.

Time study and work sampling involve direct observation and the remaining are data based and analytical in nature.

1. **Time study:** A work measurement technique for recording the times and rates of working for the elements of a specified job carried out under specified conditions and for analysing the data so as to determine the time necessary for carrying out the job at the defined level of performance. In other words measuring the time through stop watch is called time study.
2. **Synthetic data:** A work measurement technique for building up the time for a job or pans of the job at a defined level of performance by totalling element times obtained previously from time studies on other jobs containing the elements concerned or from synthetic data.
3. **Work sampling:** A technique in which a large number of observations are made over a period of time of one or group of machines, processes or workers. Each observation records what is happening at that instant and the percentage of observations recorded for a particular activity, or delay, is a measure of the percentage of time during which that activities delay occurs.
4. **Predetermined motion time study (PMTS):** A work measurement technique whereby times established for basic human motions (classified according to the nature of the motion and conditions under which it is made) are used to build up the time for a job at the defined level of performance. The most commonly used PMTS is known as **Methods Time Measurement (MTM).**
5. **Analytical estimating:** A work measurement technique, being a development of estimating, whereby the time required to carry out elements of a job at a defined level of performance is estimated partly from knowledge and practical experience of the elements concerned and partly from synthetic data.

The work measurement techniques and their applications are shown in Table 2.1

TABLE 2.1: Work measurement techniques and their application

S/N	Techniques	Applications	Unit of measurement
1	Time study	Short cycle repetitive jobs. Widely used for direct work.	Centiminute (0.01 min)
2	Synthetic Data	Short cycle repetitive jobs.	Centi minutes
3	Working sampling	Long cycle jobs/heterogeneous operations.	Minutes
4	MTM	Manual operations confined to one work centre.	TMU (1 TMU = 0.006 min)
5	Analytical estimation	Short cycle non-repetitive job.	Minutes

Time Study

Time study is also called work measurement. It is essential for both planning and control of operations.

According to British Standard Institute time study has been defined as *"The application of techniques designed to establish the time for a qualified worker to carry out a specified job at a defined level of performance."*

Steps in Making Time Study

Stop watch time is the basic technique for determining accurate time standards. They are economical for repetitive type of work. Steps in taking the time study are:

i. Select the work to be studied.
ii. Obtain and record all the information available about the job, the operator and the working conditions likely to affect the time study work.
iii. Breakdown the operation into elements. An element is a instinct part of a specified activity composed of one or more fundamental motions selected for convenience of observation and timing.
iv. Measure the time by means of a stop watch taken by the operator to perform each element of the operation. Either continuous method or snap back method of timing could be used.
v. At the same time, assess the operators effective speed of work relative to the observer's concept of 'normal' speed. This is called performance rating.
vi. Adjust the observed time by rating factor to obtain normal time for each element

$$\text{Normal} = \frac{\text{Observed time Rating}}{100}$$

vii. Add the suitable allowances to compensate for fatigue, personal needs, contingencies. etc. to give standard time for each element.
viii. Compute allowed time for the entire job by adding elemental standard times considering frequency of occurrence of each element.
ix. Make a detailed job description describing the method for which the standard time is established.

x. Test and review standards wherever necessary

Computation of Standard Time

Standard time is the time allowed to an operator to carry out the specified task under specified conditions and defined level of performance. The various allowances are added to the normal time as applicable to get the standard time as shown in Fig. 2.8.

Standard time may be defined as the, amount of time required to complete a unit of work: (*a*) under existing working conditions, (*b*) using the specified method and machinery, (*c*) by an operator, able to do the work in a proper manner, and (*d*) at a standard pace.

Thus basic constituents of standard time are:

i. Elemental (observed time).
ii. Performance rating to compensate for difference in pace of working.
iii. Relaxation allowance.
iv. Interference and contingency allowance.
v. Policy allowance.

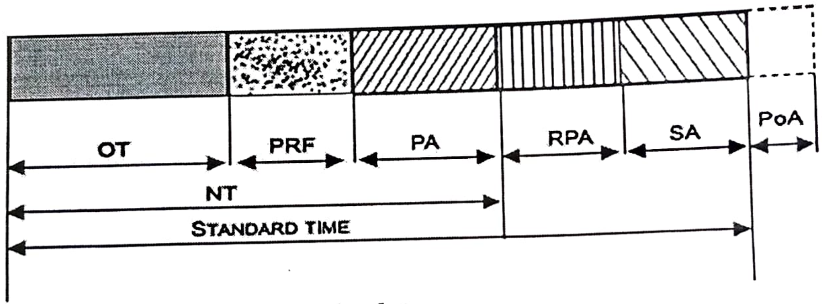


Fig. 2.8: *Components standard time*

OT – Observed Time
PRF – Performance Rating Factor
NT – Normal Time
PA – Process Allowances
RPA – Rest and Personal Allowances
SA – Special Allowances
PoA – Policy Allowances

Allowances

The normal time for an operation does not contain any allowances for the worker. It is impossible to work throughout the day even though the most practicable, effective method has been developed. Even under the best working method situation, the job will still demand the expenditure of human effort and some allowance must therefore be made for recovery from fatigue and for relaxation. Allowances must also be made to enable the worker to attend to his personal needs. The allowances are categorised as: (1) Relaxation allowance, (2) Interference allowance, and (3) Contingency allowance.

i. **Relaxation Allowance:** Relaxation allowances are calculated so as to allow the worker to recover from fatigue. Relaxation allowance is a addition to the basic time intended to provide the worker with the opportunity to recover from the physiological and psychological effects of carrying out specified work under specified conditions and to allow attention to personal needs. The amount of allowance will depend on nature of the job. Relaxation allowances are of two types: fixed allowances and variable allowances. **Fixed allowances** constitute:

(a) **Personal needs allowance:** It is intended to compensate the operator for the time necessary to leave, the workplace to attend to personal needs like drinking water, smoking, washing hands. Women require longer personal allowance than men. A fair personal allowance is 5% for men, and 7% for women.

(b) **Allowances for basic fatigue:** This allowance is given to compensate for energy expended during working. A common figure considered as allowance is 4% of the basic time.

ii. **Variable Allowance:** Variable allowance is allowed to an operator who is working under poor environmental conditions that cannot be improved, added stress and strain in performing the job. The variable fatigue allowance is added to the fixed allowance to an operator who is engaged on medium and heavy work and working under abnormal conditions. The amount of variable fatigue allowance varies from organization to organization.

iii. **Interference Allowance:** It is an allowance of time included into the work content of the job to compensate the operator for the unavoidable loss of production due to simultaneous stoppage of two or more machines being operated by him. This allowance is applicable for machine or process controlled jobs. Interference allowance varies in proportion to number of machines assigned to the operator. The interference of the machine increases the work content.

iv. **Contingency Allowance:** A contingency allowance is a small allowance of time which may be included in a standard time to meet legitimate and expected items of work or delays. The precise measurement of which is uneconomical because of their infrequent or irregular occurrence. This allowance provides for small unavoidable delays as well as for occasional minor extra work: Some of the examples calling for contingency allowance are:

a. Tool breakage involving removal of tool from the holder and all other activities to insert new tool into the tool holder.
b. Power failures of small duration.
c. Obtaining the necessary tools and gauges from central tool store. Contingency allowance should not exceed 5%.

v. **Policy Allowance:** Policy allowances are not the genuine part of the time study and should be used with utmost care and only in clearly defined circumstances. The usual reason for making the policy allowance is to line up standard times with requirements of wage agreement between employers and trade unions. The policy allowance is an increment, other than bonus increment, applied to a standard time (or to some constituent part of it, *e.g.*, work content) to provide a satisfactory level of earnings for a specified level of performance under exceptional circumstances. Policy allowances are sometimes made as imperfect functioning of a division or part of a plant.

Example 3: Assuming that the total observed time for an operation of assembling an electric switch is 1.00 min. If the rating is 120%, find normal time. If an allowance of 10% is allowed for the operation, determine the standard time.

Solution

Observed time or selected time $= 1\text{min}$

Rating = 120%
Allowance = 10%

$$Normal\ time = observed\ time \times \frac{rating}{100} = 1.00 \times \frac{120}{100} = 120min$$

$$Allowance\ @\ 10\% = 1.20 \times \frac{10}{100} = 0.12min$$

$$\therefore Standard\ time = Normal\ time\ + Allowances = 1.20 + 0.12 = 1.32min.$$

Example 4: An operator manufactures 50 jobs in 6 hours and 30 minutes. If this time includes the time for setting his machine. Calculate the operator's efficiency. Standard time allowed for the job was: Setting time = 35 min and Production time per piece = 8 min

Solution:
As standard time = Set up time + Time per piece × No. of pieces produced

∴ Standard time for manufacturing 50 jobs = 35 + 8 × 50 = 435 min = 7 hours and 15 min.

Actual time = 6hours 30mins = 390mins

$$Efficiency\ of\ Operator = \frac{Standard\ time\ \times 100}{Actual\ time} = \frac{435 \times 100}{390} = 11.5\%$$

Example 4 Following datas were obtained by a work study. Man from a study conducted by hours.

i. **Maintenance time**
   a. Get out and put away tools = 12.0 min/day
   b. Cleaning of machine = 5.0 min/day
   c. Oiling of machine = 5.0 min/day
   d. Replenish coolant supply = 3.0 min/day
ii. **Interruption**
   a. Interruption by foreman = 5.0 min/day
   b. Interruption by porter etc. = 4.0 min/day
iii. Delay time due to power failure etc. = 6.0 min/day
iv. Personal time = 20.0 min/day

Calculate total allowances, total available cycle time productive hours, considering a working day of 8 hours.

Solution:
Total allowance (sometimes also known as station time) = Total maintenance time + Interruption time + Delay time + Personal time = (12.0 + 5 + 5 + 3.0) + (5.0 + 4.0) + 6.0 + 20.0 = 25.0 + 9.0 + 6.0 + 20.0 = 60.0 min per day

∴ Total available cycle time = Total work period – Total allowances = 480 – 60 = 420 **min/day**

$$Production\ time = \frac{Time\ available}{Number\ of\ hours} = \frac{420}{8} = 52.5min$$

Example 5: Find out the standard time using the following data:

Average time for machine elements = 6 min

Average time for manual elements = 4 min

Performance rating = 110%

Allowances = 10%

Solution:

Normal time = Machinery time + Manual time × Rating = 6 + 4 × 1.1 = 6 + 4.4 = **10.4 min**

∴ Standard time = Normal time + Allowances = 10.4 + 10.4 × 10/100 = 10.4 (1 + 0.1) = 11.44 min.