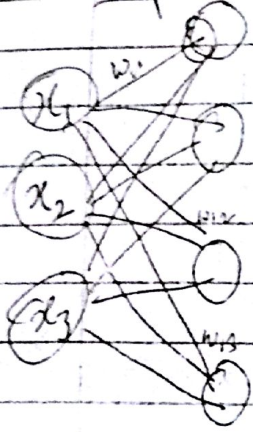
Kohonen

Self Organizing Map (KSOM or SOM)



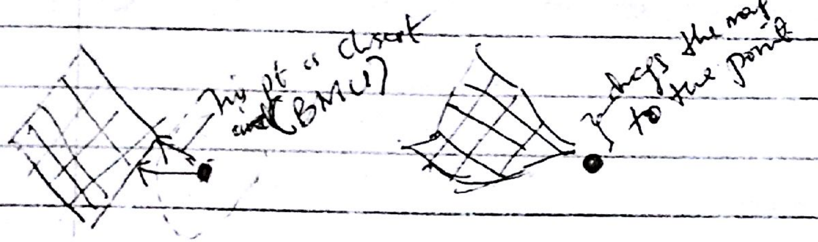
node 1: (W_{11}, W_{12}, W_{13}) Dist. $\sqrt{\sum(x_i - w_{1i})^2} = 1.2$

node 2: $(W_{21}, W_{22}, W_{23}) = 0.4$

node 3 (W_{31}, W_{32}, W_{33}) Dist = 0.4

node 4 (W_{41}, W_{42}, W_{43}) Dist = 1.1

The closest is node 3. Then we will ~~settle~~ call node no 3 the Best Matching Unit (BMU).



Important to know:

1. SOMs retain topology of the input set
2. SOMs reveal correlations that are not easily identified
3. SOMs classify data without supervision.
4. SOMs don't require a target vector → no back propagation
5. No lateral connections b/w output nodes.

   No Activation functions.

Steps of SOM

1. Initialize the weights w_{ij}, Random values may be assumed. Initialize the learning rate α.
2. Calculate square of the Euclidean distance i.e., for each $j = 1$ to m

$$D(j) = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i - w_{ij})^2$$

3. Find Winning unit index J, so that $D(J)$ is minimum
4. For all unit j within a specific neighbourhood of J and for all i, calculate new weights

$$w_{ij}(new) = w_{ij}(old) + \alpha(x_i - w_{ij}(old))$$

5. Update learning rate "α" using the formula

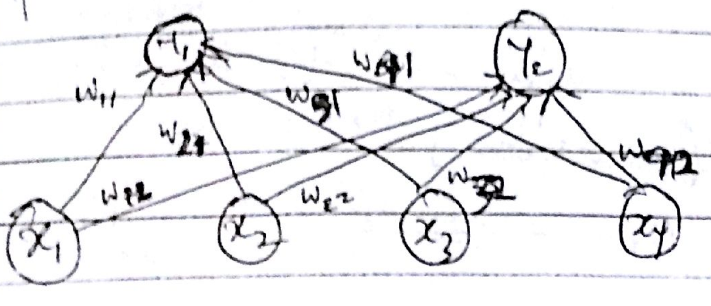
Question

Q. Construct SOM to cluster four given vectors [0 0 1 1], [1 0 0 0], [0 1 1 0] and [0 0 0 1]. Number of cluster to be formed is 2. Assume an initial learning rate of 0.5.

Soln

No of input vector, $n=4$

No of cluster, $M=2$



Initialize weight randomly b/w 0 & 1

$$W_{ij} = \begin{bmatrix} 0.2 & 0.9 \\ 0.4 & 0.7 \\ 0.6 & 0.5 \\ 0.8 & 0.3 \end{bmatrix}$$

(input vector 1) weights connected from x_1 to Y_1 (cluster 1)

input vector 2 weights connected from x_2 to Y_2 (cluster 2)

First input vector

$x_1 = [0, 0\ 1\ 1]$

Calculate the Euclidean distance

$$D_{(j)} = \sum_i^4 (w_{ij} - x_i)^2$$

Euclidean dist. b/w input vector x_1 & cluster Y_1

$$D_{(1)} = (0.2-0)^2 + (0.4-0)^2 + (0.6-1)^2 + (0.8-1)^2$$
$$= 0.04 + 0.16 + 0.16 + 0.04$$
$$= 0.4$$

$$D_{(2)} = (0.9-0)^2 + (0.7-0)^2 + (0.5-1)^2 + (0.3-1)^2$$
$$= 0.81 + 0.49 + 0.25 + 0.49$$
$$= 2.04.$$

(Best matching unit)

$\therefore D(1) < D(2)$, Therefore winning cluster is $j=1$ i.e. Y_1

Update weights on winning cluster using $j=1$

$w_{ij}(new) = w_{ij}(old) + \alpha[x_i - w_{ij}(old)]$ where $j=1$

$w_{i1}(new) = w_{i1}(old) + \alpha[x_i - w_{i1}(old)]$

Let n = new & O = old.

$$w_{11}(n) = w_{11}(0) + 0.5[x_1 - w_{11}(0)]$$
$$= 0.2 + 0.5(0 - 0.2) = \underline{0.1}$$

$$w_{12}(n) = w_{12}(0) + 0.5[x_1 - w_{12}(0)]$$
$$= 0.4 + 0.5(0 - 0.4) = \underline{0.2}$$

$$w_{13}(n) = w_{13}(0) + 0.5(x_1 - w_{13}(0)]$$
$$= 0.6 + 0.5(1 - 0.6) = \underline{0.8}$$

$w_{14}(n) = W_{14}(o) + 0.5(x_i - w_{14}(o))$

$= 0.8 + 0.5(1 - 0.8) = \underline{\underline{0.9}}$

After processing the 1st input vector, we get updated weight matrix,

$$W_{ij} = \begin{bmatrix} 0.1 & 0.9 \\ 0.2 & 0.7 \\ 0.8 & 0.5 \\ 0.9 & 0.3 \end{bmatrix}$$

we then use the updated input vector for computing the second input vector. ~~weight~~ Euclidean distance.

Second input vector

$x_2 = [1\ 0\ 0\ 0]$

Calculate the Euclidean distance.

$D(1) = (0.1-1)^2 + (0.2-0)^2 + (0.8-0)^2 + (0.9-0)^2$

$= 0.81 + 0.04 + 0.64 + 0.81 = \underline{\underline{2.3}}$

$D(2) = (0.9-1)^2 + (0.7-0)^2 + (0.5-0)^2 + (0.3-0)^2$

$= 0.01 + 0.49 + 0.25 + 0.09 = \underline{\underline{0.84}}$

$D(2) < D(1)$ $J = 2 \rightarrow$ Therefore winning cluster is $j=2$, $Y=2$

Update weight ~~using~~ using winning cluster $j=2$

$W_{ij}(n) = W_{ij}(0) + \alpha(x_i - W_{ij}(0))$

$W_{i2}(n) = W_{i2}(0) + \alpha(x_i - W_{i2}(0))$

$W_{12}(n) = W_{12}(0) + \alpha(x_1 - W_{12}(0))$

$= 0.9 + 0.5(1 - 0.9) = \underline{\underline{0.95}}$

$W_{22}(n) = W_{22}(0) + \alpha(x_i - W_{22}(0))$

$= 0.7 + 0.5(0 - 0.7) = \underline{\underline{0.35}}$

$W_{32} = 0.5 + 0.5(0 - 0.5) = \underline{\underline{0.25}}$

$W_{42} = 0.3 + 0.5(0 - 0.3) = \underline{\underline{0.15}}$

$$W_{ij} = \begin{bmatrix} 0.1 & 0.95 \\ 0.2 & 0.35 \\ 0.8 & 0.25 \\ 0.9 & 0.15 \end{bmatrix}$$

Third input vector $x_3 = [0\ 1\ 1\ 0]$

$D(1) = 1.5$, $D_2 = 1.91$

$D(1) < D_2$. Winning cluster is $J=1$

$W_{11}(n) = 0.05$

$w_{21}(n) = 0.6$

$w_{31}(n) = 0.9$

$w_{41}(n) = 0.45$

$$w_{ij} = \begin{bmatrix} 0.05 & 0.95 \\ 0.60 & 0.35 \\ 0.90 & 0.25 \\ 0.45 & 0.15 \end{bmatrix}$$

<u>Fourth Input Vector</u>

$x_4 = [0\ 0\ 0\ 1]$

$D_1 = 1.475$, $D_2 = 1.81$

$D_1 < D_2$, winning Cluster is $J = 1$

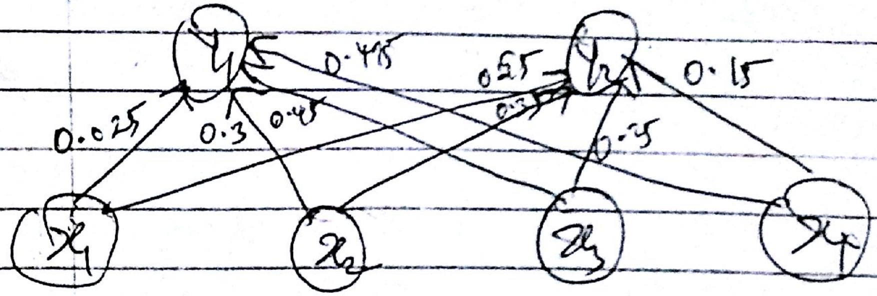
$w_{11}(n) = 0.025$

$w_{12}(n) = 0.3$

$w_{13}(n) = 0.45$

$w_{14}(n) = 0.475$

$$w_{ij} = \begin{bmatrix} 0.025 & 0.95 \\ 0.30 & 0.35 \\ 0.45 & 0.25 \\ 0.475 & 0.15 \end{bmatrix}$$



final Architecture